

# Indoor Location Estimation based on the Statistical Spatial Modeling and Radial Distributions

Kosuke Okusa and Toshinari Kamakura

**Abstract**—We study the problem of analyzing indoor location estimation by statistical radial distribution model. In this study, we suppose the observed distance data between transmitter and receiver as a statistical radial distribution. The proposed method is based on the marginal likelihoods of radial distribution generated by positive distribution among the several transmitter radio sites placed in the room. To demonstrate the effectiveness of our method, we conducted two sets of experiments, assessing the accuracy of location estimation of static case and dynamic case. In static experiment, subject is stationary state in some places in the chamber. This experiment is able to measure the precise performance of proposed method. In dynamic experiment, subject is move around in the chamber. This experiment is able to measure the suitability for practical use of proposed method. As a result, our method shows high accuracy for the indoor spatial location estimation compared to other previous methods..

**Index Terms**—Indoor location estimation, Maximum likelihood estimation, Radial distribution, Statistical spatial modeling, ToA dataset

## I. INTRODUCTION

WE study the problem of analyzing indoor spatial location estimation by statistical radial distribution model. Recently we daily use the global positioning systems (GPS) for obtaining the location for car navigation. These systems are very convenient, but we sometimes need to the location estimation in the indoor environments for obtaining the nursing care information like in hospitals. Indoor location estimation based on the GPS is very difficult, because the GPS signals are difficult to receive in the indoor situation.

A study on the indoor spatial location estimation is very important in the fields of the marketing science, design for the public space. For instance, indoor spatial location estimation is one of the important methods to make the space planning based on the evacuation model, and shop layout planning [6], [7], [11]–[13].

Recently, indoor spatial location estimation is mainly based on Received Signal Strength (RSS) method [8], [19], [24], Angle of Arrival (AoA) method [15], [21] and Time of Arrival (ToA) method [5], [20], [23].

RSS is cost-effective approach, this method is able to using general radio signals (e.g. Wi-Fi networks). However, the signal strength is affected by signal reflections and attenuation and hence it is not robust. Therefore, RSS method's location estimation accuracy is very low. AoA is most high accuracy approach, this method is able to use signal arrival directions and estimated distances. However, this system is very high

cost, because this system requires array signal receivers. On the other hand, ToA method is only using the distance between transmitter and receiver. This method's accuracy is higher than RSS method, and cost is lower than AoA method. For this reason, previous research has suggested that ToA method is most suitable for the practical indoor location estimation system [17].

In this research, we use the ToA data based measurement system. Previous research's location estimation algorithm is mainly based on the least squares method. However least squares method is difficult to process the outlier value, and such data is frequently observed in the ToA data.

For this problem, the proposed method is based on the marginal likelihoods of radial distribution generated by positive distribution among the several transmitter radio sites placed in the room. We shall compare the proposed statistical method and other previous methods and conclude that our iterative method is promising for practical use.

To demonstrate the effectiveness of our method, we conducted two sets of experiments, assessing the accuracy of location estimation of static case and dynamic case. In static experiment, subject is stationary state in some places in the chamber. This experiment is able to measure the precise performance of proposed method. In dynamic experiment, subject is move around in the chamber. This experiment is able to measure the suitability for practical use of proposed method. As a result, our method shows high accuracy for the indoor spatial location estimation.

The organization of the rest of the paper is as follows. In Section II, we discuss the feature and problem of ToA data. In Section III, we present our models for indoor location estimation based on the statistical distribution and optimization algorithm. In Section IV, we present some performance results showing the effectiveness of our model based on the two experiments. We conclude with a summary in Section V.

## II. TIME OF ARRIVAL (TOA) DATA

ToA is one of the methods of estimate the distance between transmitter and receiver. This method is computed from radio signal's travel time between transmitter and receiver. When the transmitter and receiver's time has been completely synchronized, distance  $d$  between transmitter and receiver is calculated as follows:

$$d = C(r_t - r_r) \quad (1)$$

where  $r_t, r_r$  are transmitted and received time, respectively.  $C$  is the speed of light. In the ideal circumstance,  $d$  provides accurate distance between transmitter and receiver, which called Line-of-Sight. In this case, subject location is easy to estimate by trilateration (Figure 1).

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K. Okusa is with the Department of Communication Design Science, Faculty of Design, Kyushu University, Fukuoka, 8158540 Japan e-mail: okusa@design.kyushu-u.ac.jp.

T. Kamakura is with the Department of Science and Engineering, Chuo University, Tokyo, 1128551 Japan e-mail: kamakura@indsys.chuo-u.ac.jp.

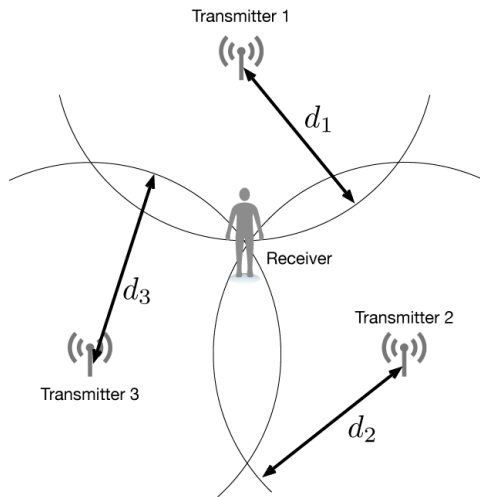


Fig. 1. Location estimation by trilateration (ideal case)

However, in many cases, distance  $d$  includes error components, which called Non Line-of-Sight (NLoS) [3], [22](Figure 2).

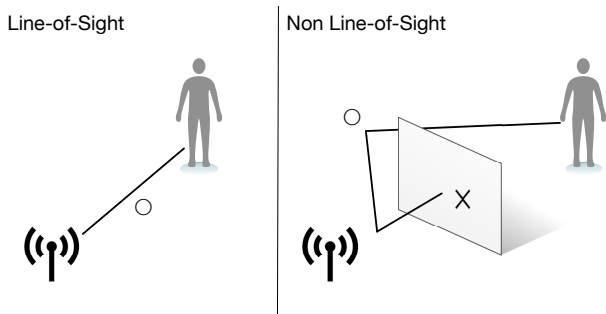


Fig. 2. LoS, NLoS circumstance

NLoS conditions is mainly caused by obstacles between transmitter and receiver, signal reflections. In this case, observed distance  $d$  will be larger than true distance. Fujita *et. al.* [4] reported that observed distance of LoS and NLoS are defined as follows:

$$d_{k,LoS} = \sqrt{(x - c_{k1})^2 + (y - c_{k2})^2} + e_k \quad (2)$$

$$d_{k,NLoS} = \sqrt{(x - c_{k1})^2 + (y - c_{k2})^2} + e_k + b_k \quad (3)$$

LoS case, observed value is distributed from true distance with error term  $e_k \sim N(0, \sigma_k^2)$ , where  $N(\cdot)$  is normal distribution. On the other hand, NLoS case, observed value is contains the additional bias term  $b_k \sim U(0, B_{\max})$ , where  $U(\cdot)$  is uniform distribution.  $B_{\max}$  means possible maximum bias value of observed value.

Figure 3 shows example of error density between true distance and observed distance. Here solid line represents LoS case density, and dashed line represents NLoS case. Figure 3 indicates that NLoS case's estimated distance will not distributed in true distance.

To cope with this, some researchers focus on the model based maximum likelihood estimation approach [10], [14], [16], but these methods were modeled by 1-D distribution. ToA data is only provided distance data, and not provided angle data. In this paper, we can assume that the observed data is a 2-D distribution and propose the statistical radial distribution.

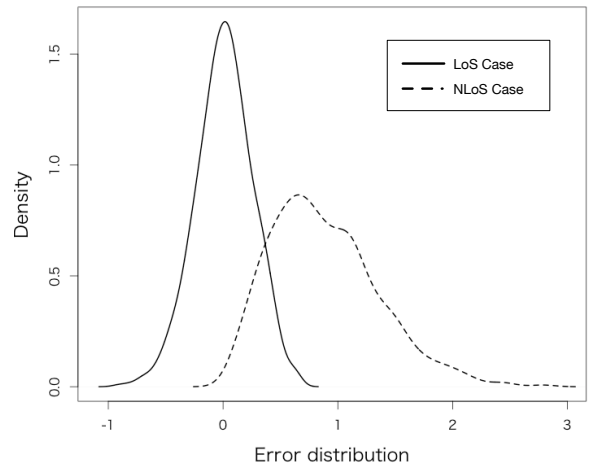


Fig. 3. Error density between true distance and observed distance

### III. INDOOR LOCATION ESTIMATION ALGORITHM

In this section, we describe the indoor location estimation algorithm. The proposed method is based on the marginal likelihoods of radial distribution generated by positive distribution. Firstly, we explain the proposed radial distribution. Secondly, we discuss the location estimation algorithm based on the radial distributions.

#### A. Radial Distribution

Considering that the obtained distance data are all positive, we propose the following circular distribution based on the 3-parameter Weibull distribution:

$$f(r, \theta) = \frac{1}{2\pi} \left(\frac{m}{\eta}\right) \left(\frac{r-g}{\eta}\right)^{m-1} \exp\left\{-\left(\frac{r-g}{\eta}\right)^m\right\} \quad (4)$$

$(r, g, \eta, m > 0, 0 \leq \theta < 2\pi)$

Here  $g$  is location parameter,  $\eta, m$  are Weibull distribution's shape, scale parameter.

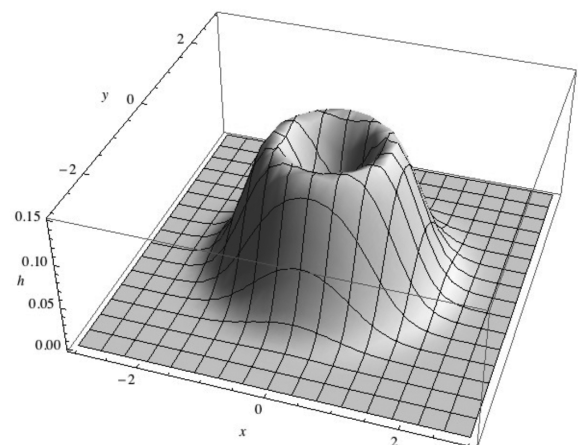


Fig. 4. Radial Weibull Distribution ( $m = 3, \lambda = 0.5, g = 0$ )

From Eq.4, converts from Polar to Cartesian coordinates:

$$g(x, y) = \frac{\lambda m}{2\pi} (\sqrt{x^2 + y^2} - g)^{m-2} \exp\left\{-\lambda(\sqrt{x^2 + y^2} - g)^m\right\} \quad (5)$$

$(x, y, g, \lambda, m > 0)$

Here  $\lambda = 1/\eta^m$  (Figure 4).

Assuming that each transmitter station observers independent measurements, the likelihood based on the data set is calculated as follows:

$$L(\lambda_1, m_1, g_1, \dots, \lambda_K, m_K, g_K) = \prod_{i=1}^K \prod_{j=1}^{n_i} \frac{\lambda_i m_i}{2\pi} (\sqrt{(x_{ij} - c_{i1})^2 + (y_{ij} - c_{i2})^2} - g_K)^{m-2} \exp \left[ -\lambda_i \left\{ \sqrt{(x_{ij} - c_{i1})^2 + (y_{ij} - c_{i2})^2} - g_K \right\}^m \right]. \quad (6)$$

Here  $K$  is the number of stations and for each station  $i$  the sample size is  $n_i$ . The observed data set for station  $i$  is  $(x_{ij}, y_{ij})$ . The coordinates  $(c_{i1}, c_{i2})$  are given transmitter station positions (Figure 1).

We can assume that most high likelihood value point  $(\hat{x}, \hat{y})$  as a subject estimated location (Figure 5). However, maximum likelihood point  $(\hat{x}, \hat{y})$  derivation is difficult solve analytically. Therefore, we propose location estimation algorithm based on the radial distribution and sequential centroid computation.

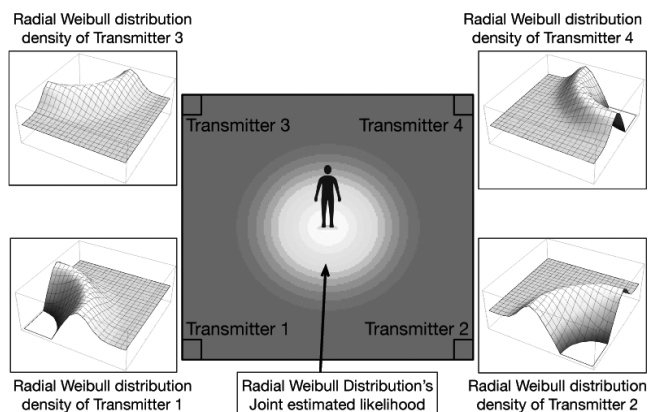


Fig. 5. Location estimation based on the radial Weibull distribution

### B. Location Estimation Algorithm

Previous studies are focused on the least squares method for the location estimation. Least squares method is powerful algorithm, but it is difficult to consider the outlier value, and evaluate the estimate result.

Watabe and Kamakura [23] reported that the sequential centroid computation based approach is better performance to estimate the subject's location in outlier situation. However, this method is iteration calculation based approach, and it does not have a evaluation function. Additionally, this method requires preliminary experiment to correct the biases.

In this research we modify the Watabe and Kamakura [23] algorithm (we called "modified SCC"). We can assume that joint probability of radial Weibull distributions as a evaluation function, and we evaluate the iteration step based on the likelihood value.

Modified SCC contains three calculation steps. 1st step is parameter initialization and calculate the initial location, 2nd step is update the estimated location based on the centroid computation, and 3rd step is evaluate the estimated location based on the radial Weibull distribution. More details of each calculation steps at following sections.

1) 1st step: parameter initialization and calculate the initial location: We defined  $d_{t,k}$  as the observed distance at time  $t$ , transmitter station  $k$ .  $\tilde{d}_k$  is a minimum value of past  $M$  data of observed value at each station as follows:

$$\tilde{d}_k = \min(d_{(t-M),k}, \dots, d_{(t),k}) \quad (7)$$

$M$  is a arbitrary value at experiment situation, then we set at a value as  $M = 10$  by experiences. Then calibrate  $d_k$  from estimated bias value  $b_k$  as follows:

$$\hat{d}_k = \tilde{d}_k - b_k. \quad (8)$$

where  $b_k$  is NLoS bias value.

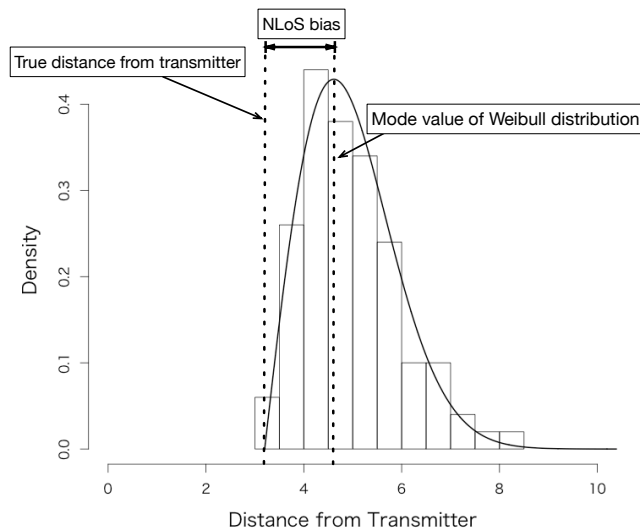


Fig. 6. ToA data NLoS bias

Figure 6 is example of NLoS bias. Observed values are not distributed around the true distance from transmitter. This bias lead to the low accuracy of location estimation. To cope with this, it is assumed that the bias value can approximate to the distance between minimal value and mode value of estimated distributions.

NLoS bias value is calculated by Weibull distribution parameter  $\hat{\eta}_k, \hat{m}_k, \hat{g}_k$  estimation from datasets  $(d_{(t-M),k}, \dots, d_{(t),k})$  at each transmitter station. We set the initial value location parameter  $g_k$  as  $\tilde{d}_k/2$ . From this estimation, we can assume that the NLoS bias as mode value of Weibull distribution as follows:

$$b_k = \hat{\eta} \left( 1 - \frac{1}{\hat{m}} \right)^{\frac{1}{\hat{m}}}. \quad (9)$$

Watabe and Kamakura [23] method is difficult to estimate the each transmitter station's bias value. On the other hand, modified SCC method can be sequentially estimate the bias value.

Then we set the step count  $s$  as 0, and initial location  $(\hat{x}_s, \hat{y}_s)$  was defined as follows:

$$\hat{x}_s = \frac{1}{K} \sum_{k=1}^K x_k, \quad \hat{y}_s = \frac{1}{K} \sum_{k=1}^K y_k. \quad (10)$$

Here  $(x_k, y_k)$  are X-coordinate, Y-coordinate of each transmitter station respectively.

2) *2nd step: update the estimated location based on the centroid computation:* Update the step count  $s \leftarrow s + 1$ , and the new subject location  $(\hat{x}_s, \hat{y}_s)$  from following equation:

$$\begin{aligned} a_{k[s]} &= x_k + \frac{\hat{d}_k(\hat{x}_{s-1} - x_k)}{\sqrt{(\hat{x}_{s-1} - x_k)^2 + (\hat{y}_{s-1} - y_k)^2}} \\ b_{k[s]} &= y_k + \frac{\hat{d}_k(\hat{y}_{s-1} - y_k)}{\sqrt{(\hat{x}_{s-1} - x_k)^2 + (\hat{y}_{s-1} - y_k)^2}} \\ \hat{x}_s &= \sum_{k=1}^K w_k a_{k[s]}, \quad \hat{y}_s = \sum_{k=1}^K w_k b_{k[s]}. \end{aligned} \quad (11)$$

Here  $w_k$  is weighted value from truncated normal distribution  $N_{trunc}(\hat{d}_k; 0, \sigma^2, 0, \infty)$ , variance value  $\sigma^2$  is calculated from  $(d_{(t-M),k}, \dots, d_{(t),k})$ .

3) *3rd step: evaluate the estimated location based on the radial Weibull distribution:* From estimated subject location  $(\hat{x}_s, \hat{y}_s)$  and each estimated Weibull distribution's parameter  $\hat{\eta}_k, \hat{m}_k, \hat{g}_k$ , likelihood value  $l_s = L(\hat{x}_s, \hat{y}_s; \lambda_1, m_1, (g_1 - b_1), \dots, \lambda_K, m_K, (g_K - b_K))$  can compute by Eq.6.

To evaluate the accuracy of estimated location, we maximize the proposed accuracy index as follows:

$$AI_s = l_s + \frac{1}{\delta_s} \rightarrow \max. \quad (12)$$

Here, variation value of estimated location  $\delta_s = \sqrt{(\hat{x}_s - \hat{x}_{(s-1)})^2 + (\hat{y}_s - \hat{y}_{(s-1)})^2}$ . Accuracy index play a role of evaluation function as with AIC [1]. Likelihood value  $l_s$  is monotonic increase value, and variation value  $\delta_s$  is monotonic decrease value, and each values are positive.

In early steps in iteration calculation, variation value becomes larger value, but likelihood value becomes smaller value. On the other hand, in terminal steps in iteration calculation, variation value becomes smaller value, and likelihood value is larger value. In proposed  $AI$  value, we use the inverse of variation value, and it becomes monotonic increase value.

Iterate the Step 2 and Step 3 until maximize the  $AI$  value, and we estimates the subject location.

#### IV. EXPERIMENTAL DETAILS AND RESULTS

To demonstrate the effectiveness of our method, we conducted two sets of experiments. We have two types of data set; one is the data set when the subject with the receiver keeps stopping within a minute on the fixed point (static case experiment). The other is the data set when the subject with the receiver moves on the prescribed trajectory (dynamic case experiment). The latter is, of course, extended to much difficult situation, in case that the subject is randomly walking in the space.

We compare the estimation accuracy with least squares method, Watabe and Kamakura [23] and proposed method. Least squares method is most popular and simple approach for the location estimation, and it used in the GPS. On the other hand, Watabe and Kamakura [23] is indoor location estimation algorithm under the NLoS circumstance, and this algorithm is good performance for the location estimation with compared to other works [2], [9], [22].

#### A. Static Case Experiment

In static experiment, subject is stationary state in some places in the chamber. This experiment is able to measure the precise performance of proposed method.

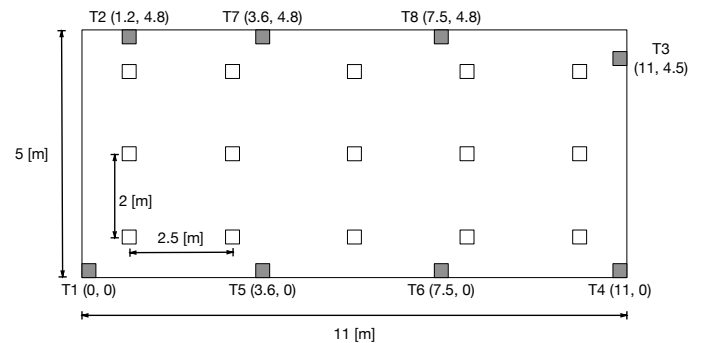


Fig. 7. Static Experiment Circumstance

Figure 7 shows static experiment circumstance. Eight transmitters sets in the 11[m] × 5 [m] space. Gray square in the Figure 7 means transmitter.  $T_x$  and  $(x, y)$  means transmitter number and coordinates. White square in the Figure 4 means receiver, and it locate at 2.5 [m] × 2 [m] interval. We observe subject distance for three minutes at each points.

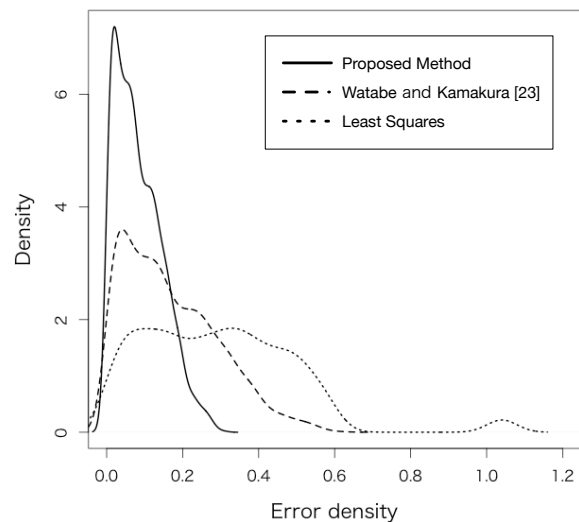


Fig. 8. Error distribution between true and estimated location

TABLE I  
AVERAGE ERROR DISTANCE OF STATIC CASE EXPERIMENT

	Average Error Dist. [m]
Proposed	0.12
Least squares	0.39
Watabe and Kamakura [23]	0.16

Figure 8 shows error distribution between true and estimated location in the static case experiment. X-axis means distance between true and estimated location, and Y-axis mean density of error distance. Solid line means proposed method's error distribution, dotted line means least squares

method's error distribution, and dashed line means Watabe and Kamakura [23]'s error distribution.

Table I shows average error distance of each methods. In Table I, proposed method shows better performance than other methods. Proposed method's average error distance is only little better than Watabe and Kamakura [23]'s result. However, in Figure 8, proposed method's error variance is very smaller than Watabe and Kamakura [23]'s method, thus, this results indicates that proposed method keeps precise estimation in every places.

From Figure 8 and Table I, proposed method shows high accuracy for the indoor spatial location estimation than other methods in static experiment case.

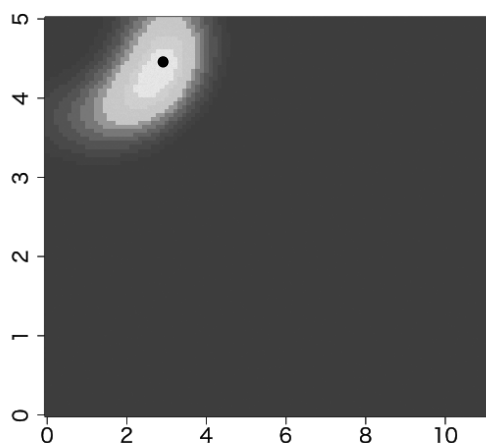


Fig. 9. Joint estimated likelihood of radial Weibull distribution

Figure 9 is circular Weibull distribution's marginal likelihood of static experiment at receiver location (3, 4.5). Figure color means marginal likelihood value, and light tone means high probability. Black circle in the light tone area is receiver location. From this figure, proposed method is able to estimates subject location precisely. In next section, we discuss about dynamic case experiment.

### B. Dynamic Case Experiment

In dynamic experiment, subject is move around in the chamber. This experiment is able to measure the suitability for practical use of proposed method. This situation is much difficult situation than static case, in case that the subject is walking in the space.

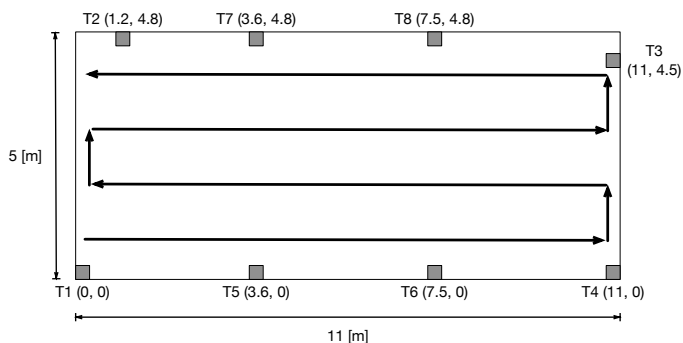


Fig. 10. Dynamic Experiment Circumstance

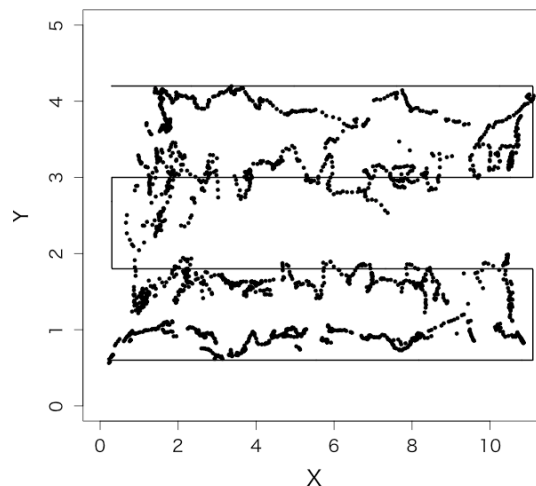


Fig. 11. Estimated trajectories (least squares method)

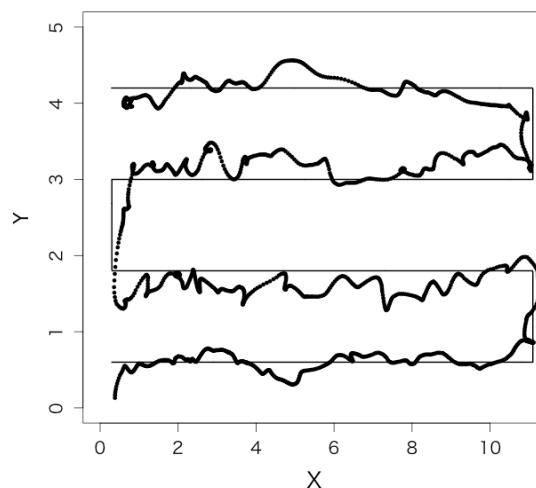


Fig. 12. Estimated trajectories (Watabe and Kamakura [23])

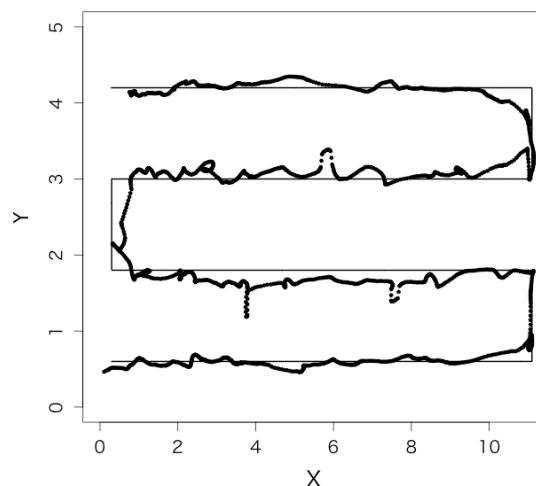


Fig. 13. Estimated trajectories (proposed method)

Figure 10 shows dynamic experiment circumstance. Number of transmitters, and installation locations are same as

static experiment. Arrowed line means subject's movement trajectory. Subject is move along this trajectories by 0.3 [m/sec].

Figure 11, 12, 13 are location estimation results of least squares method (Figure 11), Watabe and Kamakura [23] method (Figure 12), and proposed method (Figure 13). From these figures, proposed method shows high accuracy for the indoor spatial location estimation than other methods.

Table II shows average error distance of each methods. From Table II, proposed method keeps high location estimation accuracy. On the other hand, other method's location estimation accuracy is decreasing. This results indicates, proposed method has robustness performance in case of the moving subject's location estimation case.

From Figure 13 and Table II, proposed method shows better performance than other method same as the static case experiment.

TABLE II  
AVERAGE ERROR DISTANCE OF DYNAMIC CASE EXPERIMENT

	Average Error Dist. [m]
Proposed	0.57
Least squares	0.95
Watabe and Kamakura [23]	0.84

From the the static case experiment and dynamic case experiment, proposed method shows high accuracy to estimate the subject location. This results indicates that the radial Weibull distribution based approach has a high potential for indoor location estimation.

### V. CONCLUSIONS

In this article, we proposed the indoor location estimation model based on the radial Weibull distribution. From experiment results, our model is able to estimate precisely subject's locations compared to other previous methods.

In next phase, we need to implement the indoor location estimation system based on the proposed method and demonstrate it.

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