On the Artificial Neural Networks used for the Forward Problem for the Electrical Impedance Equation

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***Abstract*—To achieve a recognition of a solution for the forward value problem of the electrical impedance equation, two different artificial neural networks are used and compared: the multilayer perceptron and the backpropagation neural networks.**

***Index Terms*—Artificial Neural Network, Backpropagation, Electrical Impedance Equation, Multilayer Perceptron.**

I. Introduction

THE artificial neural networks (ANN) [14] are commonly used to recognise or classify the information contained in a data base. The information depends upon the problem that is trying to be solved, but in general, to obtain an approximation is very difficult or the computational resources that needs is very high. For this reason ANNs help to obtain a faster recognition or classification for this complicated problem.

In this case, the main problem is the forward problem for the electrical impedance equation posed by A. P. Calderon in 1980 [4], that is an easy task in comparison with the inverse problem for the same equation. The equation that is trying to be solved is the follows:

$$\mathrm{div}\,(\sigma \mathrm{grad} u) = 0, \tag{1}$$

where the σ is the conductivity and the u denotes the electric potential for a domain Ω with boundary Γ. This equation is also known as the electrical impedance equation and it can be solved through the Pseudoanalytic function theory [1] and [6], using the Taylor series in formal powers method, exposed in [8] and [9].

Several works utilize ANNs to recognize or classify the data they possess [2], [14] and [7]. In [10], the usage of the ANNs, in combination of genetic algorithms and multilayer perceptron [5] is employ to recognise an earthquake by its wave. Acoording to [12], in which a Bayesian multilayer perception neural network (BMLP-ANN)[11] is used to analyse the information, in order to relate it with the boundary measurements, and employed before with the finite-element method (FEM). In [13] and [3], a radial basis function that is a variation of ANN employed together with the FFM, this method works with the information of the forward problem of electrical impedance equation for training and testing. Its information successfully recognise the solution related with resistivity in the electrodes and inside the domain.

In current pages, ANNs intend to recognize the information contained in a data set, which is obtained by the Taylor series in formal powers. This information represents the forward problem solution of the electrical impedance equation. The analysis is performed to prove that ANN could be used to find a faster solution, once the ANNs are correctly trained.

This work is distributed as follows: section II is dedicated to study the multilayer perceptron and backpropagation. In section III we presented the procedure to use both ANNs for recognizing the solution. The section IV performs the analysis of the information constructed for the forward problem. Finally, section V closed this work.

II. Preliminaries

An artificial neural network (ANN) is a computational model inspired in biological neural networks (BNN). It consists of an interconnection group of pseudo neurons [2]. Akin to the BNN, ANN has the same structure where the axon is represented by the weights; dendrite is expressed by input of the system; the body that is defined by the activation function and the synapses that is described by the connection between neurons.

The ANN simulates the functionality and structure of BNN, such as the human brain does. Then, ANN can have multiple inputs and outputs, and it can have several amounts of neurons per hidden layer that exist in the system, and its purpose is to interconnect the inputs with the outputs.

The ANN mathematically represents the dynamics of the information flow; this function is called network function.

$$f(x) = K\left(\sum_i w_i \cdot g_i(x)\right), \tag{2}$$

where w_1 denotes the weights, K refers to an activation function and $g_i(x)$ is the collection of functions $g_i(x) = (g_1, g_2, ..., g_3)$ that represents the function of neurons in ANN. The activation function is represented by any desire function, the most commonly used is the step function represented by:

$$g(x) = \begin{cases} 1 & \text{if } x \geq 0; \\ 0 & \text{if } x < 0. \end{cases} \tag{3}$$

In the figure 1, a simple neuron is shown.

It is important to study the ANN learning paradigms, such as the supervised learning in which the solution is looked after by an expert. Another learning paradigm is the

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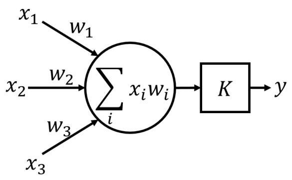

Fig. 1. Simple neuron of artificail neural network.

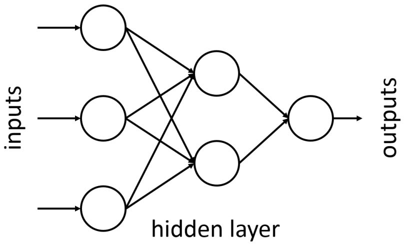

Fig. 2. Basic multilayer perceptron.

reinforcement learning that consists in punish or reward the action depending upon the knowledge of an expert. Other learning paradigm is the unsupervised learning in which the solution is reached without any supervision.

A. *Multilayer perceptron*

The multilayer perceptron (MLP-ANN) [5] is a feed-forward artificial neural network model that maps sets of input data onto a set of appropriate output. MLP-ANN belongs to the supervised learning paradigm and possesses multiple layers that consist of the input and output layer, and one or more hidden layers connecting the input with the output. Each layer consists of several nodes in a direct graph. To understand better the MLP-ANN, the figure 2 illustrates the multilayer perceptron.

The algorithm 1 shows the multilayer perceptron neural network method.

Algorithm 1 Multilayer perceptron.

Initialize the weights and threshold.
Do
  Calculate the actual output.
  Update the weights.
While not fulfil the stop criterion **Or** the number of iterations are fulfil.

B. *Backpropagation*

The backpropagation (BANN) [11] is a commonly method used to train ANN, and it is usually used together with an optimization method. Moreover, akin to MLP-ANN; the BANN possesses an input and output layer and hidden layers. MLP-ANN differs from BANN because in this method, the output is returned to its previous layer to update the weights, this process is done until all the weights in the network are updated, this process helps to obtain a better solution. The stop criterion consists in recognise or classify correctly a pattern; alternatively other stop criteria could be used such as satisfying a threshold. The figure 3 shown the backpropagation network.

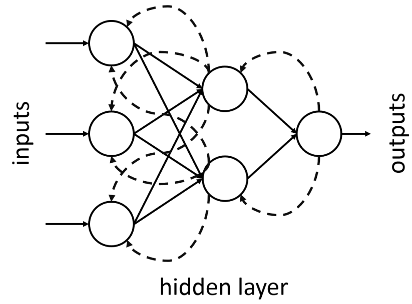

Fig. 3. Backpropagation.

The algorithm 2 illustrates the backpropagation neural network method.

Algorithm 2 Backpropagation algorithm

Initialize the weights.
Do
  Forward-Pass *(is like the ANN doing before)*.
  give the desire output.
  Calculate the error (desire - output)
  Compute the weights from the hidden layer.
  Compute the weights from the input layer.
  Update the weights in the network.
While not fulfil the stop criterion **Or** fulfil recognition rates

III. Methodology

This section is fully dedicated to understand the way the artificial neural networks (ANN) are used. The main idea is to use ANN, such like multilayer perceptron (MLP-ANN) and backpropagation (BANN), to recognise a solution from a data set constructed by Taylor series in formal powers method; this method comes from the Pseudoanalytic Function Theory, and it proves to be a good alternative to find a solution for the electrical impedance equation.

Applying both MLP-ANN and BANN requires a data set that it is used as the input of the system; this data set is constructed using the orthonormal system obtained by Taylor series in formal power method, and it represents several solutions for the forward value problem for the electrical impedance equation. This data set contains good and bad approximations, and the task is to recognise the valid approximations from all the data set.

Once the data set is obtained, MLP-ANN and BANN are used to recognise the solution, however, the construction of the MLP-ANN and BANN is not a trivial task. The architecture is very difficult to design and depending upon the hidden layers and the neurons in each layer; the computing time could increase considerably. Hence, the architecture for both ANNs in the present work is the same, in which the number of inputs consist in 16 inputs and a single output.

The number of hidden layer are set in a mirror scheme; it means that the same number in the initial hidden layer is in the third, and the second hidden layer consists of

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TABLE I
CONFIGURATION FOR THE SIMULATIONS.

ANN Architecture	$16, \{8, 4, 8\}, 1$
Activation Function	$g(x) = 1 \leftarrow x \geq 0$ $g(x) = 0 \leftarrow x < 0$
Net. Function	$f(x) = K \sum_i w_i g_i(x)$

four neurons, meanwhile the first and third layers are represented by eight neurons. The full architecture is expressed as $16, \{8, 4, 8\}, 1$. Consequently, the activation function (3) previously shown, is employed to determine if is considered or not as a solution by satisfying the threshold.

The following table I defines the configuration for both ANNs.

To summarize, the methodology consists in three phases:

- The construction of the data set.
- The development of the artificial neural network.
- Analysing the recognition obtained.

As it is mention, the construction of the data set falls directly over the Taylor series in formal powers method that is not included in this work, and its main purpose is to approximate the solution to the forward problem of the electrical impedance equation. Then, the development of the artificial neural networks proposed in this work, and finally, the analysis of the results obtained, paying attention to the possibility to perform a faster recognition.

The training and the testing are performed by using a certain amount of information chosen randomly from the data set, For both ANNs used in this work; the training is realised by employing the 50%, 60% and 70% of the information, and the testing is performed by the remaining information that is not used in the training.

To determine the error in the training phase all the selected data in this phase is used for testing, and the error showed in the tables are indeed, the recognition of the training data. Meanwhile, for the testing phase, the remaining data is used to perform the analysis by unknown data.

IV. RESULTS

In present work, the Artificial Neural Network (ANN) is used to compute a fast solution to the forward problem of the electrical impedance equation. The main idea is to employ the data set obtained by the Taylor series in formal power method (NPSM). In this method a solution matrix is constructed by an orthonormalization process for approximating the solution of the forward value problem. Then, in the data set every row in the matrix represent a possible solution to this problem. However, the matrix possesses good and bad approximations.

The task is to compute and analyse the data in order to determine which row is or not a solution. This process will be done automatically for each data set constructed based on the conductivity distribution within the domain. These distributions inside could be geometrical or analytical.

In this study, it is used four different conductivity distributions; the first is the polynomial conductivity; the second is the sinusoidal; the third is a circle at center and the last five disk structure. All the experiments were performed using the methodology exposed before, in which the ANNs commonly employed are: the Multi-Layer Perceptron (MLP-ANN) and the Backpropagation (BANN) neural networks.

Using the parameters that are shown in table (I), the ANN represents with 1 if the solution and 0 if none. The results are separated and using only the data set that represents the solution to the forward value problem. In the further subsections, the analysis is performed using the conductivity distributions mention before.

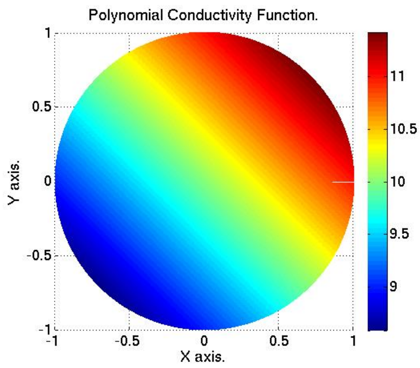

Fig. 4. Polynomial conductivity distribution. $\sigma = x + y + 10$.

TABLE II
MULTILAYER PERCEPTRON FOR POLYNOMIAL CONDUCTIVITY DISTRIBUTION.

Exp.	50 - 50		60 - 40		70 - 30	
	Train	Test	Train	Test	Train	Test
1	95%	80%	97%	85%	99%	90%
2	90%	83%	98%	91%	100%	85%
3	93%	81%	95%	88%	95%	95%
4	99%	80%	96%	89%	87%	91%
5	92%	90%	93%	95%	99%	96%
6	91%	91%	85%	80%	90%	95%
7	96%	90%	94%	98%	98%	99%
8	100%	99%	99%	93%	91%	88%
9	94%	84%	96%	90%	93%	93%
10	90%	85%	95%	93%	97%	91%
Avg.	94%	86.3%	94.8%	90.2%	94.9%	92.3%

A. Polynomial conductivity

For this sample, both MLP-ANN and BANN are used for their data set. To perform the analysis, both ANNs are trained and tested 10 times and reporting their performance on a table and the average of all the experiments done.

The data set is constructed using the Taylor series in formal powers method used to approximate the forward problem of the electrical impedance equation. In this study the imposed electric potential u and the conductivity σ distribution are shown below:

$$\begin{aligned} \sigma &= x + y + 10, \\ u &= \ln(x + y + 10). \end{aligned} \tag{4}$$

The figure 4, illustrates the conductivity σ distribution within the unit disk domain.

The data set that is used for training the MPL-ANN and BANN contains 1500 possible solutions for the forward problem of (1), this data set contains 750 good approximations and 750 bad approximations. The information selected is chosen randomly, and the parameters for both ANNs are presented in table I. The MLP-ANN is performed and showed its training and testing phase result in the table II.

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TABLE III
BACKPROPAGATION FOR POLYNOMIAL CONDUCTIVITY DISTRIBUTION.

Exp.	50 - 50		60 - 40		70 - 30	
	Train	Test	Train	Test	Train	Test
1	93%	75%	94%	82%	98%	87%
2	88%	78%	95%	88%	99%	82%
3	91%	76%	92%	85%	94%	92%
4	97%	75%	93%	87%	86%	89%
5	90%	85%	90%	93%	98%	93%
6	89%	86%	82%	78%	89%	92%
7	94%	85%	91%	96%	97%	96%
8	98%	94%	96%	91%	90%	85%
9	92%	70%	93%	88%	92%	90%
10	88%	80%	92%	91%	96%	88%
Avg.	92%	80.4%	91.8%	87.9%	93.9%	89.4%

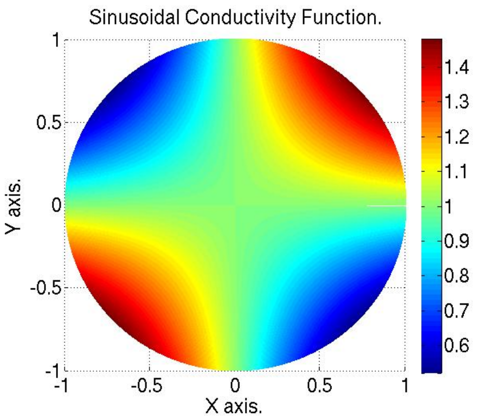

Fig. 5. Sinusoidal conductivity distribution. $\sigma = 2 + \sin(x + y)$

Meanwhile, the results of the BANN are shown in the table III.

Both tables II and III show the recognition rate of the solution. However, the results are very significant because they proof that it is possible to find a faster solution. Using the MLP-ANN is slightly better than the BANN. Furthermore, both ANNs demonstrate that it is best to employ the 70% of the data set for training and 30% for testing.

B. Sinusoidal conductivity

Continuing the study, a sinusoidal conductivity distribution inside a unit disk domain has selected. For this case in which its parameters to construct the data set are the follow:

$$\sigma = 2 + \sin(x + y),$$
$$u = \frac{2}{\sqrt{3}} arctan\left(\frac{2\tan(\frac{x+y}{2})+1}{\sqrt{3}}\right) \tag{5}$$

The figure 5 show the distribution of the conductivity within.

In this case, the data set contains 2000 approximations for the forward problem of (1), and this information contains 1000 good approximations and 1000 bad approximations, all the information contained in the data set is selected randomly, and the parameters for both ANNs are shown in table I.

The table IV shows the behaviour of the MLP-ANN. Performing its results with the data set constructed for this conductivity distribution.

Furthermore, the table V illustrates the behaviour of the BANN for this case.

TABLE IV
MULTILAYER PERCEPTRON FOR SINUSOIDAL CONDUCTIVITY.

Exp.	50 - 50		60 - 40		70 - 30	
	Train	Test	Train	Test	Train	Test
1	75%	70%	95%	84%	93%	87%
2	86%	98%	97%	92%	98%	100%
3	82%	82%	90%	92%	97%	87%
4	88%	85%	93%	90%	91%	90%
5	74%	82%	86%	89%	98%	100%
6	73%	73%	91%	98%	87%	92%
7	95%	70%	94%	87%	90%	89%
8	99%	90%	88%	97%	97%	93%
9	100%	78%	98%	90%	92%	97%
10	87%	74%	96%	87%	88%	94%
Avg.	85.9%	80.2%	92.8%	90.6%	93.1%	92.9%

TABLE V
BACKPROPAGATION.

Exp.	50 - 50		60 - 40		70 - 30	
	Train	Test	Train	Test	Train	Test
1	82%	89%	87%	90%	90%	91%
2	79%	86%	96%	95%	98%	99%
3	94%	88%	83%	87%	95%	96%
4	97%	90%	98%	100%	85%	91%
5	98%	86%	92%	89%	86%	96%
6	85%	80%	85%	86%	92%	90%
7	87%	97%	90%	91%	88%	86%
8	91%	77%	94%	80%	95%	99%
9	85%	84%	100%	83%	93%	85%
10	76%	86%	98%	82%	91%	95%
Avg.	87.4%	86.3%	92.3%	88.3%	91.3%	92.8%

The results shown in both tables V and IV demonstrate that the best way to recognize the solution from the data set is the MLP-ANN. The MLP-ANN is slightly better than the BANN due to the information contained in the data set constructed by its equation (5). Like in the case previously done, the 70% of the information contained in the data set is required for training and the 30% for testing.

C. Circle at center conductivity

Another class of conductivity distribution is needed to characterize the ANNs that it is used throughout this work. The idea is to employ a geometrical conductivity distribution within the unit disk domain, in which the small disk at the center is denoted with σ_1 and the rest of the domain with σ_2. The data set to be used, is constructed by the following expressions:

$$\sigma_1 = 100, \quad \sigma_2 = 10$$
$$u = \frac{x^3+y^3}{3} + 0.5\,(x + y) \tag{6}$$

The figure 6 shown the conductivity distribution within the unit disk domain that is obtained by the expression (6), shown above.

The data set used to perform the analysis by ANNs, contains 750 approximations for the forward problem of (1), the randomly selected data is chosen from the 350 good approximations and 350 bad approximations, the parameters to be used in both ANNs are expose in table I. The results that are shown in table VI, displayed the performance of the MLP-ANN with the data set constructed by (6) shown above.

Simultaneously, the information displayed in table VII, shows the achievement of the BANN.

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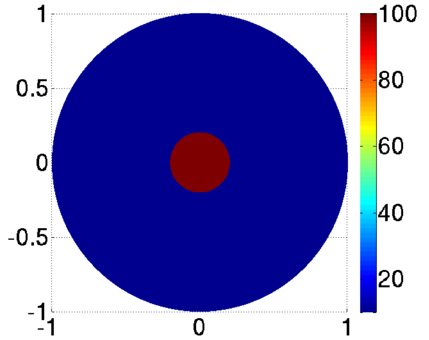

Fig. 6. Circle at center conductivity distribution.

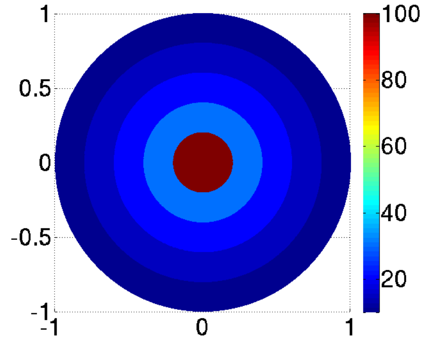

Fig. 7. Circle at center conductivity distribution.

TABLE VI
MULTILAYER PERCEPTRON FOR CIRCLE AT CENTRE CONDUCTIVITY.

Exp.	50 - 50		60 - 40		70 - 30	
	Train	Test	Train	Test	Train	Test
1	73%	86%	88%	87%	89%	95%
2	90%	94%	100%	94%	90%	88%
3	76%	89%	98%	81%	88%	96%
4	79%	83%	89%	84%	94%	97%
5	88%	80%	91%	89%	90%	94%
6	89%	72%	99%	91%	85%	89%
7	97%	75%	97%	97%	100%	89%
8	95%	94%	86%	86%	97%	88%
9	96%	78%	99%	99%	90%	100%
10	99%	70%	91%	91%	98%	93%
Avg.	88%	81.7%	93.8%	87.4%	92.1%	93%

In the light of the last tables VI and VII, they both demonstrated that it is possible to achieve a recognition of the solution by the data set constructed with the information expressed in (6). However, the MLP-ANN is still better than the BANN. For this sample, the information needed to perform the calculation is the 60% of the data set for training and the 30% for testing. It is needed to analyse the data set to understand better the behaviour of the ANNs, because in the case of the BANN, it needs the 70% of the data set for training and 30% for testing.

D. *Five disk structure conductivity*

Continuing with the geometrical conductivity distributions within the unit disk domain, in this study, the proposal conductivity is a five disks structure at the center of the domain. For this case the smallest disk inside is denoted with σ_1, the next disk is σ_2 and consecutively until $sigma_5$.

TABLE VII
BACKPROPAGATION FOR CIRCLE AT CENTER CONDUCTIVITY.

Exp.	50 - 50		60 - 40		70 - 30	
	Train	Test	Train	Test	Train	Test
1	89%	91%	83%	99%	91%	86%
2	79%	81%	99%	80%	92%	97%
3	83%	83%	90%	85%	92%	100%
4	72%	76%	83%	86%	89%	97%
5	74%	71%	85%	96%	100%	93%
6	85%	72%	90%	99%	86%	98%
7	99%	73%	94%	95%	98%	87%
8	82%	95%	99%	98%	89%	86%
9	92%	84%	89%	86%	87%	97%
10	82%	75%	98%	98%	97%	94%
Avg.	83.7%	80.1%	91%	92.2%	92.1%	93.5%

TABLE VIII
MULTILAYER PERCEPTRON FOR FIVE DISKS STRUCTURE AT CENTER CONDUCTIVITY.

Exp.	50 - 50		60 - 40		70 - 30	
	Train	Test	Train	Test	Train	Test
1	88%	75%	91%	90%	88%	93%
2	93%	76%	96%	93%	96%	89%
3	84%	81%	92%	90%	95%	93%
4	90%	92%	93%	92%	97%	86%
5	100%	98%	80%	81%	89%	95%
6	84%	72%	87%	82%	93%	93%
7	91%	88%	81%	84%	90%	88%
8	85%	81%	81%	95%	87%	91%
9	98%	95%	84%	91%	86%	94%
10	98%	100%	98%	81%	88%	100%
Avg.	91.1%	85.8%	88.3%	87.9%	90.9%	92.2%

The next equations are used to compose the data set to be used further in the ANNs.

$$\begin{gathered} \sigma_1 = 100, \quad \sigma_2 = 30, \quad \sigma_3 = 20, \\ \sigma_4 = 15, \quad \sigma_5 = 10 \\ u = \frac{x^3+y^3}{3} + 0.5\,(x+y) \end{gathered} \tag{7}$$

The figure 7 displayed the conductivity distribution within the unit disk domain.

In the present sample, the data set contains 900 approximation for the forward problem of (1), in which the data is divided in 450 good approximations and 450 bad approximations. The information is selected randomly and to perform the analysis by both ANNs, the parameters to be used are shown in table I. The results presented in table VIII, displayed the performance of the MLP-ANN to realize a recognition of a solution using the data set constructed by (7).

Concurrently, the achievement of BANN is shown in table IX.

Both tables VIII and IX express the performance with different schemes, for this case, the BANN is better than the MLP-ANN. The information used as the data set to perform the recognition is: 60% for training and 40% for testing. These results need to be studied in order to understand this behaviour, because for the MLP-ANN, it is enough used the 50% for training and 50% for testing. Rather than the expected results, in this case the BANN demonstrates be the best way to recognise a solution.

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TABLE IX
BACKPROPAGATION FOR FIVE DISKS STRUCTURE AT CENTER CONDUCTIVITY.

Exp.	50 - 50		60 - 40		70 - 30	
	Train	Test	Train	Test	Train	Test
1	100%	82%	99%	81%	91%	98%
2	72%	92%	97%	80%	88%	97%
3	91%	84%	100%	100%	86%	100%
4	95%	86%	96%	82%	95%	86%
5	99%	78%	88%	87%	100%	91%
6	82%	76%	89%	84%	98%	86%
7	73%	99%	89%	89%	87%	93%
8	91%	93%	93%	82%	92%	94%
9	90%	72%	87%	89%	95%	97%
10	71%	83%	95%	83%	97%	100%
Avg.	86.4%	84.5%	93.3%	85.7%	92.9%	94.2%

V. CONCLUSION

The artificial neural network (ANN) is commonly used to recognize or classify patterns. Furthermore, it helps to optimize an iterative method to save computer resources and obtain a faster solution. The present study is not the exception; the ANNs are used to recognize a solution of a data set constructed for the forward problem of the electrical impedance equation by the Taylor series in formal powers. The advantage of utilizing ANN relies on the possibility to use the information on this problem to find a solution.

However, the ANNs such like the Multi-Layer Perceptron (MLP-ANN) and the Backpropagation Neural Network (BANN) has its limitation, because it always requires a data set to perform its calculus, and the architecture of the ANNs varies a lot depending upon the problem to solve. Once the architecture and the data set are chosen the hard task is to select the correct number of hidden layers that acquire an output, which is the recognition or classification attained.

Considering the obtained results, The MLP-ANN is the best option to recognize a solution, and the BANN is another procedure that achieves a good identification. The data set is constructed depending upon the problem to be solve, but it allows to develop an application which recognises a solution. Is a faster way to compute and recognise a solution. The conductivity distribution inside a unit disk domain proves that different data sets can be used, but the main problem is to consider all the possible conductivity distributions within.

Finally, the data sets computed possess a considerable size, so the time processing increases considerably depend upon the amount of forward problem solutions calculated.

In further works, the ANNs could be employed together with the Finite-Element Method to perform an approximation for the inverse problem of the electrical impedance equation. Moreover, it could study the possibility to use a genetic algorithm to achieve a better solution with the ANNs.

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