Proceedings of the World Congress on Engineering and Computer Science 2015 Vol II
WCECS 2015, October 21-23, 2015, San Francisco, USA

# Data Computational Modelling of Multivariable Non-Stationary Noisy Linear Systems by MOESP_AOKI_VAR Algorithm

Johanna B. Tobar and Celso P. Bottura

*Abstract*—**The main objective of this work is to develop a recursive algorithm for identification in the state-space of linear stochastic discrete multivariable non-stationary system; a computational process called MOESP_AOKI_VAR is proposed and implemented to achieve this. The proposed algorithm is based on the subspace methods: Multivariable Output-Error State Space (MOESP), used for computational modelling of systems and on an AOKI algorithm developed by Masanao Aoki, for computational modelling of time series that we call the Aoki algorithm.**

*Index Terms*—**MOESP, Markov parameters, non-stationary system, time series, identification, Aoki.**

## I. Introduction

An initial study of different kind of systems for identification, based on the state-space, is performed. Additionally, a structure to be used in the problem resolution of computational modelling for non-stationary noisy linear systems is proposed. Through this study, non-stationary systems are treated as a group of invariant models with respect to the time. It is also considered that the matrix system $A_K, B_K, C_K, D_K$, presents small changes with respect to the time. This is translated into continuous and slow changes within the matrices and it allows for the generation of a recursive algorithm, which is the main objective of this study. A linear system is considered as the superposition of its deterministic and stochastic part. A MOESP_VAR algorithm is used for modelling the deterministic part, whilst the stochastic part is modelled by the use of AOKI_VAR algorithm. Finally, the algorithm is tested by using a benchmark.

## II. FOUNDATION

### A. Stationary Deterministic Linear System

The representation of the deterministic linear system in state spaces has the following form:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k + Du_k \end{cases} \tag{1}$$

where, $x_k \in R^n$ is the state vector, $u_k \in R^m$ is the input vector and $y_k \in R^l$ is the output vector. The $A, B, C$ and $D$ matrices are considered constant for every $k$ instant.

Johanna Tobar is with the Department of Energy Sciences and Mechanics, University of the Armed Forces, Quito, Ecuador, e-mail: jbtobar@espe.edu.ec.
Celso Bottura is with the Department of Machines, components and intelligent systems, State University of Campinas, Campinas, Brasil, e-mail: bottura@dmcsi.fee.unicamp.br .

### B. Stationary Noisy Linear System

In order to quantify the uncertainty and external perturbations of the system, $v_k$ *and* $w_k$ are added in the state-space equations. These terms are considered as inputs where non-control exists:

$$\begin{cases} x_{k+1} = Ax_k + v_k \\ y_k = Cx_k + w_k \end{cases} \tag{2}$$

The pertubation vectors $v_k \in R^n$ and $w_k \in R^l$ are random variables with zero means. The sequences $(v_k,\ k = 0, \pm1, \pm2, ...)$ *and* $(w_k,\ k = 0, \pm1, \pm2, ...)$ are considered stochastic processes of Gaussian white noise.
Additionally, the stochastic process can be represented by defining the error vector as $e_k = \begin{bmatrix} w_k \\ v_k \end{bmatrix}$, with $E[e_k] = 0 \quad \forall k$, and its innovation representation is the following:

$$\begin{cases} x_{k+1} = Ax_k + Ke_k \\ y_k = Cx_k + e_k \end{cases} \tag{3}$$

where, $e_k$ is a white noise sequence and its covariance matrix is given by $\Delta = E\left(e_k e_k^T\right)$.

When referring to the covariance domain, the Markov parameters of the system can be represented as:

$$\Lambda_i = \begin{cases} C\Pi_0 C^T + R & i = 0 \\ G^T \left(A^T\right)^{-i-1} C^T & i < 0 \\ CA^{i-1}G & i \geq 1 \end{cases} \tag{4}$$

where, $G$ is also presented as $G = A\Pi_0 C^T + S$, yielding as result the following:

$$\begin{cases} R = \Lambda_0 - CPC^T \\ Q = P - APA^T \\ S = M - APC^T \end{cases} \tag{5}$$

The stochastic realization problem consists of finding one or more models in the state-space through process statistical data such us covariance. For further information read Caceres, Angel Fernando Torrico (2005), Tamariz, Annabell (2005) and Barreto, G. (2002).

### C. Non-stationary Deterministic Linear System.

A non-stationary deterministic linear system is represented by the following state-space equations:

$$\begin{cases} x_{j,k+1} = A_{j,k}x_{j,k} + B_{j,k}u_{j,k} \\ y_{j,k} = C_{j,k}x_{j,k} + D_{j,k}u_{j,k} \end{cases} \tag{6}$$

ISBN: 978-988-14047-2-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

being $j \in [j_0, j_0 + n - 1]$ and $k \in [k_0, k_0 + T - 1]$ , where $j_0$ is the first interval of experiment, $k_0$ is the first instant of the experiment, $n$ is the total number of simple experiments and $T \geq n$. Equation (6) can also be expressed as:

$$y_H = O_k X_H + T_k U_H \tag{7}$$

To get detailed information about the process to obtain the equation (7), consult the matrices $y_H$,$O_k$,$X_H$,$T_k$ y $U_H$ on [2].

### D. Non-stationary Noisy Linear System

A non-stationary noisy linear system is expressed as follows:

$$\begin{cases} x_{j,k+1} = A_{j,k}x_{j,k} + B_{j,k}u_{j,k} + v_{j,k} \\ y_{j,k} = C_{j,k}x_{j,k} + D_{j,k}u_{j,k} + w_{j,k} \end{cases} \tag{8}$$

being $j \in [j_0, j_0 + n - 1]$ and $k \in [k_0, k_0 + T - 1]$ , where $j_0$ is the first interval of the experiment, $k_0$ is the first instant of time of the experiment, $n$ is the total number of simple experiments, $T \geq n$ and $v_{j,k} \in R^n$ *and* $w_{j,k} \in R^l$ are random variables of null arithmetic mean, and the sequences $(v_k, k = 0, \pm1, \pm2, ...)$ *and* $(w_k, k = 0, \pm1, \pm2, ...)$ are non-stationary stochastic processes that are generated by the non-stationary stochastic system represented by the state-space equation:

$$\begin{cases} x_{j,k+1} = A_{j,k}x_{j,k} + K_{j,k}e_{j,k} \\ y_{j,k} = C_{j,k}x_{j,k} + e_{j,k} \end{cases} \tag{9}$$

where, $e_k$ is the white noise stochastic process.

### E. Time Variant Identification

The identification algorithm works on the assumption that time-varying systems can be treated as a set of time-invariant models for a given time interval. Thus, the identification of time-varying systems consists of a set of $n$ time-invariant models which describes the system for the defined experiment.

The following expression relates the input variables to the state vector in a time variant linear system in the$k^{th}$ moment:

$$x_k = A_{(k-1)}x_0 + \sum_{l=0}^{k-1} A^{(k-l-1)}B_l u_l \tag{10}$$

where, $A_{(n)}$ and $A^{(n)}$ represent the transition matrixes that satisfy:

$$\begin{cases} A_{(0)} = A_0 \\ A^{(0)} = I \\ A_{(n)} = A_n A_{(n-1)} = A_n \dots A_2 A_1 A_0 \\ A^{(n)} = A_n A^{(n-1)} = A_n \dots A_2 A_1 \end{cases} \tag{11}$$

A set of indices are stated to interpret the identification of problem parameters; additionally, the set of indices $j, k$ on $u_{j,k}$ indicates the sample input for instant $k^{th}$ and for system experimentation range $j^{th}$ (8). Furthermore, $j \in [j_{0,} j_0 + n - 1]$ and $k \in [k_{0,} k_0 + T - 1]$, where $j_0$ is the first range of experimentation, $k_0$ is the first instant of time, $n$ is the total number of experiments or tests and $T$ is the time required for a single experiment.

The problem to be solved is represented by the state-space model:

$$\begin{bmatrix} x_{j,k+1} \\ y_{j,k} \end{bmatrix} = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \begin{bmatrix} x_{j,k} \\ u_{j,k} \end{bmatrix} \tag{12}$$

based on the following output data sequence:

$$Y_{j,k} = \begin{bmatrix} y_{j_0,k_0} & y_{j_0,k_0+1} & \cdots & y_{j_0,k_0+T-1} \\ y_{j_0+1,k_0} & y_{j_0+1,k_0+1} & \cdots & y_{j_0+1,k_0+T-1} \\ \vdots & \vdots & \cdots & \vdots \\ y_{j_0+n-1,k_0} & y_{j_0+n-1,k_0+1} & \cdots & y_{j_0+n-1,k_0+T-1} \end{bmatrix} \tag{13}$$

The problem is also based on the input sequence $U_{j,k}$ for the same series of experiments and same time interval. The matrix $Y_{j,k}$ represents the set of $(n-1)$ intervals of experimentation. It allows to develop general expressions that govern the system and it also relates the inputs and outputs at a start time $k_0$ and establishes a correct experimentation interval $j$, for a discrete time variant system, which is represented in state space by:

$$\begin{cases} x_{j,k+1} = A_k x_{j,k} + B_k u_{j,k} \\ y_{j,k} = C_k x_{j,k} + D_k u_{j,k} \end{cases} \tag{14}$$

for the next instant, the expression is:

$$\begin{cases} x_{j,k+2} = A_{k+1}x_{j,k+1} + B_{k+1}u_{j,k+1} \\ y_{j,k+1} = C_{k+1}x_{j,k+1} + D_{k+1}u_{j,k+1} \end{cases} \tag{15}$$

plugging in equation (15) into equation (14) yields:

$$\begin{cases} x_{j,k+2} = A_{k+1}A_k x_{j,k} + A_{k+1}B_k u_{j,k} + B_{k+1}u_{j,k+1} \\ yy_{j,k+1} = C_{k+1}A_k x_{j,k} + C_{k+1}B_k u_{j,k} + D_{k+1}u_{j,k+1} \end{cases} \tag{16}$$

and this process is repeated successively. Solving equation (14) for any moment of time $k_0 \geq 0$, the solution can be written as:

$$y_{j,l} = \begin{cases} C_{l,j} + D_l u_{j,l} & + \quad l = 0 \\ C_l A_{(l-1)} x_{j,l} + & \\ + \sum_{i=0}^{l-1} C_l A^{(l-i-1)} B_i u_{j,i} + D_l u_{j,l} & l > k_0 \end{cases} \tag{17}$$

Equation (14) can be rewritten in a shorter form by the extended model:

$$Y_H = O_k X_H + T_k U_H \tag{18}$$

*1) Determination of the extended model:* For an experiment $j$, the matrix $U_H$ for the stationary case is equal to the matrix $U_H$ from the time-varying case. Data from a single experiment is assumed. To solve the non-stationary discrete case is necessary to assume that data is available from a single experiment.

*2) Recursive Algorithm for the deterministic part:* A recursive algorithm is applied for time-varying systems that assumes small variations in a predefined range of operations in the system matrixes. A recursive scheme is implemented to systems that vary slowly with time.

ISBN: 978-988-14047-2-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

*QR Factorization Update:*

a.- Let's assume that the following measurements were already processed by the algorithm:
$[u_{j0,k0} \quad u_{j0,k0+1} \quad \dots \quad u_{j0,k0+T-1}]^T$
and
$[y_{j0,k0} \quad y_{j0,k0+1} \quad \dots \quad y_{j0,k0+T-1}]^T$

b.- Being the QR factorization:

$$\begin{bmatrix} R_{j0,11} & 0 \\ R_{j0,21} & R_{j0,22} \end{bmatrix} \begin{bmatrix} Q_{j0,1} \\ Q_{j0,2} \end{bmatrix} \tag{19}$$

where, $j_0$ represents the set of input-output values processed in the most recent experiment.

c.- Finally, suppose that during the time interval $[j_0, j_0 + n - 1]$ the model in state space is invariant and equal to:

$$\begin{cases} x_{k+1} = A_{j0}x_k + B_{j0}u_k \\ y_k = C_{j0}x_k + D_{j0}u_k \end{cases} \tag{20}$$

It can also be represented in a shorter form by relating the different data matrices as:

$$Y_H = \mathcal{O}_k X_H + \mathcal{T}_{j0,k} U_H \tag{21}$$

*3) Recursive algorithm for the stochastic part: See [2] :*

*F. State-Space Deterministic-Stochastic Modelling of the Non-stationary System*

One way of representing non-stationary discrete multivariate noisy linear systems, with exogenous time variable inputs in the state space is:

$$\begin{cases} x_{j,k+1} = A_{j,k}x_{j,k} + B_{j,k}u_{j,k} + v_{j,k} \\ y_{j,k} = C_{j,k}x_{j,k} + D_{j,k}u_{j,k} + w_{j,k} \end{cases} \tag{22}$$

with

$$E\left[\begin{pmatrix} v_{j,k} \\ w_{j,k} \end{pmatrix} \begin{pmatrix} v_{j,s}^T & w_{j,s}^T \end{pmatrix}\right] = \begin{array}{ll} \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} & k = s \\ \quad 0 & k \neq s \end{array} \tag{23}$$

A theorem for the decomposition by superposition of noisy time variant linear systems is proposed as follows:

**Theorem 1.** *By superposition, a variant time noisy linear model* S, *in an innovative form given by:*

$$S : \begin{cases} x_{j,k+1} = A_{j,k}x_{j,k} + B_{j,k}u_{j,k} + K_{j,k}e_{j,k} \\ y_{j,k} = C_{j,k}x_{j,k} + D_{j,k}u_{j,k} + e_{j,k} \end{cases} \tag{24}$$

can be decomposed into the following two subsystems:

$$S_d : \begin{cases} x^d_{j,k+1} = A_{j,k}x^d_{j,k} + B_{j,k}u_{j,k} \\ y^d_{j,k} = C_{j,k}x^d_{j,k} + D_{j,k}u_{j,k} \end{cases} \tag{25}$$

and

$$S_e : \begin{cases} x^e_{j,k+1} = A_{j,k}x^e_{j,k} + K_{j,k}e_{j,k} \\ y^e_{j,k} = C_{j,k}x^e_{j,k} + e_{j,k} \end{cases} \tag{26}$$

where, the superscripts *d* and *e* refer to the determistic and the stochastic subsystems $S_d$ and $S_e$ respectively and $y_{j,k} = y^d_{j,k} + y^e_{j,k}$. The noisy signal state is: $x_{j,k} = \begin{bmatrix} x^d_{j,k} \\ x^e_{j,k} \end{bmatrix}$ where,

$$A = \begin{bmatrix} A^d & 0 \\ 0 & A^e \end{bmatrix}$$

$$B = \begin{bmatrix} B^d \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} C^d & C^e \end{bmatrix}$$

$$D = D^d, \quad K = K^e$$

Proof: See [1]

## III. MOESP_AOKI_VAR Algorithm

The proposed algorithm MOESP_AOKI_VAR collects the MOESP_VAR proposed in [2], [3] and the MOESP_AOKI proposed in [1], [2] and the AOKI_VAR in [13], [14] respectively. Therefore, the algorithm MOESP_AOKI_VAR is as follows:

1) Obtain the Hankel matrices $Y_H$ and $U_H$
2) Perform the QR factorization:

$$\begin{bmatrix} U_H \\ Y_H \end{bmatrix} = \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \tag{27}$$

where, $R_{11}$ and $R_{22}$ are invertible square matrices.

3) Compute the SVD of $R_{22}$ as:

$$R_{22} = \begin{bmatrix} U_H & U_H^{\perp} \end{bmatrix} \begin{bmatrix} \sum_n & 0 \\ 0 & \sum_2 \end{bmatrix} \begin{bmatrix} V_n^T \\ (V_n)^T \end{bmatrix} \tag{28}$$

4) Solve the equation system:

$$U_H^{(1)} A_T = U_H^{(2)} \tag{29}$$

5) Update: The recursive algorithm shown in section II.E.2 is applied
Thus, obtaining the matrices $A^d_{j,k}, B^d_{j,k}, C^d_{j,k}, D^d_{j,k}$, of $y^d_{j,k}$ for each $k$ time instant and $j$ intervals with respect to the time.
6) Determine the signal generated by the matrices $H^A, H^M, H^C, H, Y_-, Y_+$

$$Y_- = \begin{bmatrix} \bar{y_1} & \bar{y_2} & \bar{y_3} & \cdots & \bar{y_{N-1}} \\ 0 & \bar{y_1} & \bar{y_2} & \cdots & \bar{y_{N-2}} \\ 0 & 0 & \bar{y_1} & \cdots & \bar{y_{N-3}} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & \bar{y_{N-k-1}} & \bar{y_{N-k}} \end{bmatrix}$$

$$Y_+ = \begin{bmatrix} \bar{y_2} & \bar{y_3} & \bar{y_4} & \cdots & \bar{y_N} \\ \bar{y_3} & \bar{y_4} & \bar{y_5} & \cdots & 0 \\ \bar{y_4} & \bar{y_5} & \bar{y_6} & \cdots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \bar{y_{j+1}} & \bar{y_{j+2}} & \bar{y_{j+3}} & \cdots & 0 \end{bmatrix}$$

$$H = \frac{Y_+ Y_-^T}{N} = \begin{bmatrix} \Lambda_1 & \Lambda_2 & \cdots & \Lambda_k \\ \Lambda_2 & \Lambda_3 & \cdots & \Lambda_{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \Lambda_j & \Lambda_{j+1} & \cdots & \Lambda_{j+k} \end{bmatrix}$$

$$H^A = \begin{bmatrix} \Lambda_2 & \Lambda_3 & \cdots & \Lambda_{k+1} \\ \Lambda_3 & \Lambda_4 & \cdots & \Lambda_{k+2} \\ \vdots & \vdots & \ddots & \vdots \\ \Lambda_{j+1} & \Lambda_{j+2} & \cdots & \Lambda_{j+k+1} \end{bmatrix}$$

$$H^M = \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \vdots \\ \Lambda_j \end{bmatrix}$$

$$H^C = \begin{bmatrix} \Lambda_1 & \Lambda_2 & \cdots & \Lambda_k \end{bmatrix}$$

7) Obtain the SVD for the covariance Hankel matrix

$$H = U\sum^{1/2}\sum^{1/2}V^T$$

8) Calculate the matrices $A^e_{j,k}, C^e_{j,k}, K^e_{j,k}$
9) Validate.

## IV. Experimentation and results

The proposed algorithm is initially defined for T intervals of experimentation thus getting the system matrices identification. The presented MOESP_AOKI_VAR is assessed T times in order to determine T sets of matrices corresponding to each experiment.

If $\nabla$ represents small increments, $L_j$ is an integer number for each $I_j$ experimentation interval stated by:

$$I_j = [k_j - L_j\nabla, \quad k_j + L_j\nabla] \tag{30}$$

to validate the proposed algorithm a benchmark is implemented [4]. The identification at time instant $k_j$ (which is the middle point of each interval $I_j$) is determined as $k_{j+1} = k_j + v\nabla$, where $v$ is an integer number, $\nabla$ represents an increment with respect to the simulation time and it is given by $\nabla = M\nabla j$, where $\nabla j$ is the $j-th$ sampling period and $M$ is an integer number. $L_j = 500\forall j$ is defined for this study. The benchmark system is:

The deterministic part is given by the following matrices

$$A_k = \begin{bmatrix} -0.3 & a_k \\ 1 & -1 \end{bmatrix} \tag{31}$$

where,

$$a_k = -\frac{1}{3} - \frac{1}{10}sen(\frac{2\pi k}{400})$$

and the remaining matrices are consired constants:

$$B_k = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}; \qquad C_k = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \tag{32}$$

$$D_k = 0$$

The system input is randomically changing for each iteration of the algorithm.

The proposed algorithm presents the following results for $k = 1$:

$$A_{j,k} = \begin{bmatrix} -0.3000 & 0 \\ 0 & -0.4040 \end{bmatrix}$$

$$B_{j,k} = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$C_{j,k} = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$$

$$C_{j,k}B_{j,k} = \begin{bmatrix} 1 & 4 \\ 0 & 3 \end{bmatrix}$$

$$C_{j,k}A_{j,k}B_{j,k} = \begin{bmatrix} -0.6121 & -1.5121 \\ -0.2081 & -1.1081 \end{bmatrix}$$

The deterministic-stochastic identification of the system under noise is realized. The obtained results after running the first part of the algorithm MOESP_AOKI_VAR are shown:

$$A^d_{j,k} = \begin{bmatrix} -0.3000 & 0 \\ -0.0000 & -0.4077 \end{bmatrix}$$

$$B^d_{j,k} = \begin{bmatrix} 4.1559 & 3.7636 \\ -3.1924 & 1.5997 \end{bmatrix}$$

$$C^d_{j,k} = \begin{bmatrix} 0.7705 & 0.6867 \\ 0.5137 & 0.6677 \end{bmatrix}$$

$$C^d_{j,k}B^d_{j,k} = \begin{bmatrix} 1.0098 & 3.9984 \\ 0.0031 & 3.0014 \end{bmatrix}$$

$$C^d_{j,k}A^d_{j,k}B^d_{j,k} = \begin{bmatrix} -0.6478 & -1.5118 \\ -0.2308 & -1.1086 \end{bmatrix}$$

Figure 1 shows the output of the deterministic model

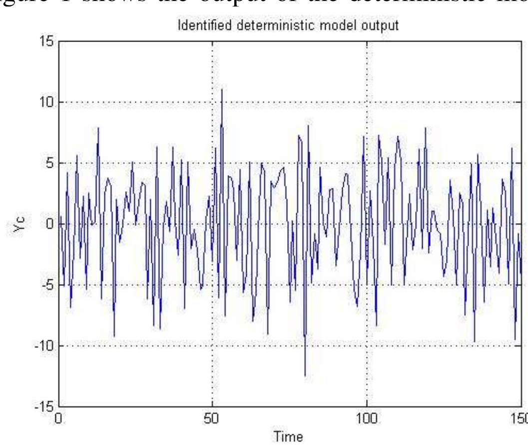


Figure 1. Output of the deterministic subsystem

The results after running the second part of the algorithm MOESP_AOKI_VAR are:

$$\Delta = [1.6092]$$

$$A^e_{j,k} = \begin{bmatrix} -0.2092 & -0.7319 \\ 0.7319 & 0.1218 \end{bmatrix}$$

$$K_{j,k} = \begin{bmatrix} -0.2582 \\ 1.1257 \end{bmatrix}$$

$$C^e_{j,k} = \begin{bmatrix} -2.7437 & 1.0369 \end{bmatrix}$$

ISBN: 978-988-14047-2-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

where, $\Delta$ is the covariance matrix of the noise.

Figure 2 presents the stochastic modeled signal by using the second part of the algorithm MOESP_AOKI_VAR

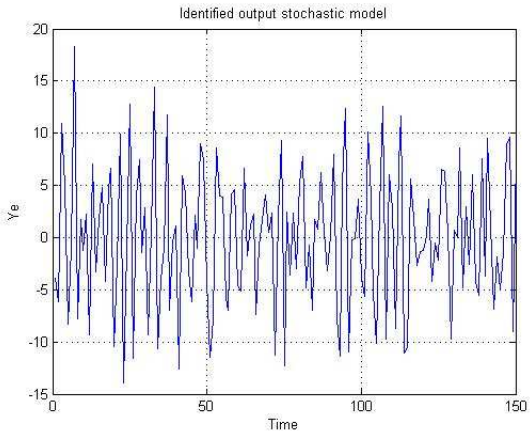


Figure 2. Output of the stochastic subsystem

Figure 3 displays the overlapped signals of the deterministic and stochastic output signals $y_{j,k}^d$ and $y_{j,k}^e$.

Finally, a verification and validation process of the proposed combined algorithm **MOESP_AOKI_VAR** is performed and Figure 4 shows the results.

The algorithm **MOESP_AOKI_VAR** describes satisfactorily the noisy signals.

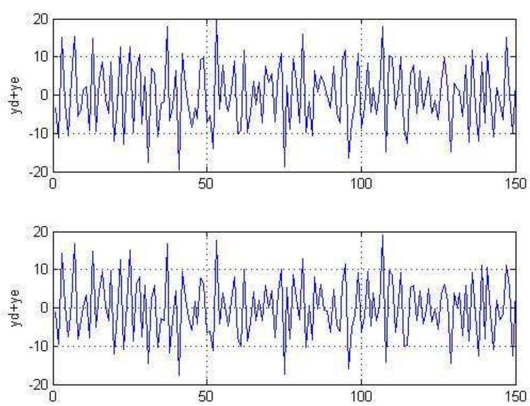


Figure 3. Overlapping signals

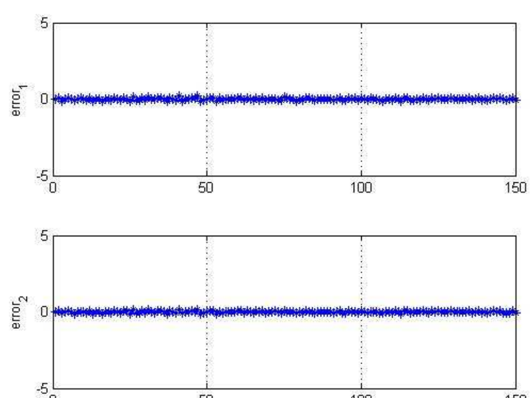


Figure 4. Error

## V. Conclusions

A computational procedure is formulated for the identification of discrete stochastic multivariate linear systems time variant. The considered hypothesis is that the variation in time of the dynamic system is sufficiently slow to guarantee the efficiency of the subspace identification MOESP for a non-stationary noisy linear system. The algorithm MOESP_AOKI_VAR has its foundation in AOKI and MOESP algorithms and it is given by the superposition of them. The obtained results describe that the proposed method guarantee an apropriate behaviour when modeling this kind of systems.

## References

[1] Aoki, Masanao, “State Space Modeling of Time Series”, Springer Verlag, 1987.

[2] Cáceres, Angel Fernando Torrico, "Identificação e Controle Estocásticos Decentralizados de Sistemas Interconectados Multivariáveis no Espaço de Estado", PhD Thesis, FEEC-UNICAMP, 2005.

[3] Tamariz, A.D.R., "Modelagem Computacional de Dados e Controle Inteligente no Espaço de Estados". PhD Thesis, LCSI/FEEC/UNICAMP, Campinas SP, Brasil, 2005.

[4] Tamariz A.D.R., Bottura C.P., Barreto G., "Iterative MOESP Type Algorithm for Discrete Time Variant System Identification", Proceedings of the 13th Mediterranean Conference on Control and Automation, (MED’2005), Limassol, Cyprus, June 27-29, 2005.

[5] Ohsumi, Akira and Kawano T., "Subspace Identification for a Class of Time-Varying Continuous-Time Stochastic Systems Via Distribution Based Approach”, Proceedings 15th IFAC World Congress, Barcelona, July 2002.

[6] Barreto, G., "Modelagem Computacional Distribuída e Paralela de Sistemas e de Séries Temporais Multivariáveis no Espaço de Estado”, PhD Thesis, FEEC-UNICAMP, 2002.

[7] Dewilde, Patrick and Van der Veen, Alle-Jan, "Time-Varying Systems and Computations", Kluwer Academic Publishers, 1998.

[8] Shokoohi, S. and Silverman, L.M., "Identification and Model Reduction of Time-Varying Discrete-Time Systems", Automatica 23, pp. 509-522, 1987.

[9] Tamariz A.D.R., Bottura C.P. and Barreto G., “Algoritmo Iterativo do tipo MOESP para Identificação de sistemas discretos variantes no tempo - Parte I: Formulação”, DINCON, 2003.

[10] Tamariz A.D.R., Bottura C.P. and Barreto G., “Algoritmo Iterativo do tipo MOESP para Identificação de sistemas discretos variantes no tempo - Parte II: Implementação e Experimentação”, DINCON, 2003.

[11] Tamariz A.D.R. and Bottura C.P,“Proposta para Identifacação de sistemas ruidosos multivariaveis no espaço de estado”, DINCON, 2007.

[12] Verhaegen, Michel and Deprettere, E., “A fast recursive MIMO State Space model Identification Algorithm”, Proceedings of the 30th Conference on Decision and Control, Brighton, England, December 1991.

[13] Verhaegen, Michel and Dewilde, Patrick., “Subspace Model Identification - Part 1 : The outputerror state-space model identification class of algorithms”, International Journal of Control, Volume 56, Number 5, 1187-1210, November, 1992.

[14] Verhaegen, Michel and Yu Xiaode, "A Class of Subspace Model Identification Algorithms to Identify Periodically and Arbitrarily Time-varying Systems", Automatica, Vol. 31, N0. 2, pp. 201-216, 1995.

[15] Tobar Johanna, Bottura Celso and Giesbrechtr Mateus, "Computational Modeling of Multivariable Non-Stationary Time Series in the State Space by the AOKI_VAR Algorithm", IAENG International Journal of Computer Science, 2010.

[16] Tobar Johanna, “Propostas para modelagem computacional de series temporais e de sistemas multivariáveis variantes no tempo no espaço de estado” Master Thesis, FEEC-UNICAMP, 2013.