Modeling of Piezoelectric Energy Harvesting for Low Power Generation

Ahmed A. Telba, *IAENG, Member, Senior Member, IEEE,* Wahied Gharieb,

***Abstract*—Energy harvesting has received growing attention over the last decade. The research motivation in this field is due to the reduced power requirement of small electronic components, such as the wireless sensor networks used in structural health monitoring applications. The ultimate goal in this research field is to power such small electronic devices by using the vibration energy available in their environment. If this can be achieved, the requirement of an external power source as well as the maintenance requirement for periodic battery replacement can be minimized in this paper the modeling ,simulation and experimental results of the piezoelectric Energy harvesting in both Matlab ,LabVIEW then analytical had been done This paper describes a dynamic model describe piezoelectric harvester and identifying model parameters with few simple measurements and standard laboratory equipment.**

***Index Terms*— Power Harvesting, Mechanical Vibration, Piezoelectric Materials, modeling and simulation**

I. INTRODUCTION

From literature the idea of vibration-to-electricity conversion first appeared in a journal article by Williams and Yates [1-2] in 1996. They described the basic transduction mechanisms that can be used for this purpose and provided a lumped-parameter base excitation model to simulate the electrical power output for electromagnetic energy harvesting. As stated by Williams and Yates [2], the three basic vibration-to-electric energy conversion mechanisms are the electromagnetic [2-4], electrostatic [5-6] and piezoelectric [7-8] transductions. Over the last decade, several articles have reported to use these transduction mechanisms for low power generation from ambient vibrations. Piezoelectric transduction has received the greatest attention especially in the last few years. Piezoelectricity is a form of coupling between the mechanical and electrical behaviors certain materials. The materials exhibiting the piezoelectric effect are called the piezoelectric materials. The piezoelectric effect is usually divided into two parts as the direct and the converts of piezoelectric effects. In the simplest terms, when a piezoelectric materials is squeezed (mechanically strained) an electric charge collects at the electrodes located on its Surface. This called the direct piezoelectric effect and it was first demonstrated by the Currie brothers in 1880[1] if the same material is subjected to a voltage drop (i.e. an electrical potential difference applied across its electrodes), it deforms mechanically. This is called the converse piezoelectric effect and it was deduced mathematically (after the discovery of the direct piezoelectric effect) from the fundamental principles of thermodynamics by Gabriel Lippmann in 1881 and then confirmed experimentally by the Curie brothers. Piezoelectric sensors are used in a variety of applications to convert mechanical energy to electrical energy such as: pressure-sensing applications, detecting imbalances of rotating machine parts, ultrasonic level measurement, flow rate measurement, sound transmitters (buzzers), sound receivers (microphones), …etc [4]. This paper is organized as follows. Section1 presents the literature work and research motivations in Energy harvesting using Piezoelectric. Section 2 describes the generic model and dynamic model for piezoelectric energy harvester. Simulation and experimental results are presented and discussed in Section 3. Finally, conclusion is drawn in Section 4.

Manuscript received July 14, 2016; revised July 27 2016. This work was supported in part by the Research Center of Collage of Engineering, King Saud University. Ahmed. A Telba is with King Saud University, Electrical Engineering Department, P.O. Box 800, Riyadh 11421, Saudi Arabia, E-mail: atelba@ksu.edu.sa

II. GENERIC MODEL OF PIEZOELECTRIC ENERGY HARVESTER

A. Review Stage

From dynamic point, the piezoelectric can be equivalent as a damped mass-spring mechanic system by a linear time-invariant second order differential equation; as the applied voltage generates force, as shown in figure 1, and the equation can be written as in 1 [5], [9-10]:

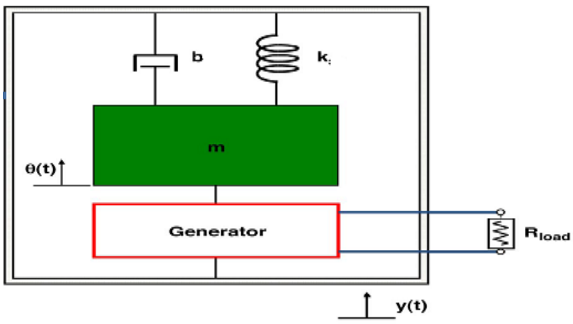

Fig.1 Generic model of piezoelectric energy harvester

Where m: The effective mass, b: The damper coefficient, k: The stiffness, f : The generated force from applied voltage, d: The ratio of the displacement to the applied voltage, v : applied voltage on piezoelectric Energy harvesting for the analysis, it is assumed that the mass of the vibration source is much greater than the mass of seismic mass in the

ISBN: 978-988-14047-1-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

generator and the vibration source is unaffected by the movement of the generator. Then the differential equation of the movement of the mass with respect to the generator housing from the dynamic forces on the mass can be derived as follows[11,12]

$$m.\frac{d^2z(t)}{dt^2}+b.\frac{dz(t)}{d(t)}+k.z(t)=-m.\frac{d^2y(t)}{dt^2} \tag{1}$$

Rewritten (1) in the form of Laplace transform as

$$m.s^2.z(s)+b.s.z(s)+k.z(s)=-m.a(s) \tag{2}$$

$$a(t)=\frac{d^2y(t)}{dt^2} \tag{3}$$

Where a (t) is the acceleration,

The transfer function of vibration micro generation is

$$\frac{z(s)}{a(s)}=\frac{1}{s^2+\frac{b}{m}s+\frac{k}{m}}=\frac{1}{s^2+\frac{\omega_r}{Q}s+\omega_r{}^2} \tag{4}$$

Where quality factor $Q=\frac{\sqrt{km}}{b}$

If we compare this form to resonant frequency ω_r

$$\omega_r=\sqrt{\frac{k}{m}}$$

If we go other side for the Equivalent Circuit of the ordinary differential equation:

If we compare between the equation of an equivalent electrical circuit and kinetic energy harvester can be found

$$-m.a(s)=s.z(s)\left(ms+b+\frac{k}{s}\right) \tag{5}$$

Equivalent electrical circuit equation can be rewrite as

$$-I(s)=E(s)\left(sc+\frac{1}{R}+\frac{1}{sL}\right) \tag{6}$$

Where

$$I(s)=m.a(s),E(s)=s.Z(s),C=m,R=\frac{1}{b},L=\frac{1}{k}$$

From on equation .6 the equivalent circuit is a parallel resonance circuit as shown in Fig. 2.

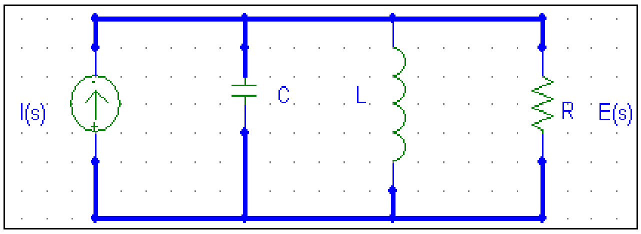

Fig.2 Equivalent circuit of kinetic energy harvesters

From the Beam Equation if the equation of motion for a cantilever with length, L, width, w, and height h like the one depicted in figure .3 is given by the Euler-Bernoulli beam equation [20]

$$\frac{\partial^2U(z,t)}{\partial t^2}\rho\Gamma+\frac{\partial^4U(z,t)}{\partial z^4}EI=0 \tag{7}$$

where U(z, t) is the displacement in the y-direction, ρ is the density, Γ = w*h is the cross-sectional area, E is the Young's modulus, and I is the area moment of inertia of the beam.
The solution to this differential equation is a harmonic,
$U(z,t)=U(z)e^{-j\omega_n t}$,
where ω_n is the resonant frequency. By insertion into equation .7 the spatial solution can be found

$$\frac{d^4U(z,t)}{dz^4}=k_n^4U(z,t),k_n^4=\frac{\omega_n\rho\Gamma}{EI} \tag{8}$$

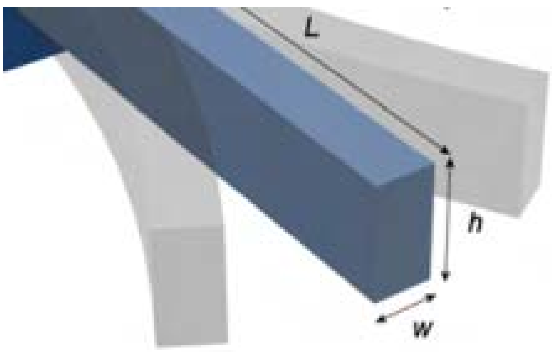

Fig.3: Schematic of a vibrating cantilever

In figure.3 the cantilever has the length L, width w, and height h. The density of the cantilever is and Young's modulus is E then the relation given by equation [13]

$$\frac{d^4U(z,t)}{dz^4}=k_n^4U(z,t),k^4=\frac{\omega^2\ \rho\Gamma}{EI} \tag{9}$$

Using the boundary condition of free beam is

ISBN: 978-988-14047-1-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$U(t,0)=0 \quad , \quad \frac{\partial U(z,t)}{\partial z}\Big|_{z=0}=0 \ ,$$

$$\frac{\partial^2 U(z,t)}{\partial z^2}\Big|_{z=L}=0 \ , \quad \frac{\partial^3 U(z,t)}{\partial z^3}\Big|_{z=L}=0$$

Using the boundary condition to solve equation (8) [21]

$$U_n(z)=A_n(\cos k_n z-\cosh k_n z)+B_n(\sin k_n z-\sinh k_n z), \quad (10)$$

$$A_n/B_n=-1.362,-0982,-1.001,-1.000,...., \quad (11)$$

where n denotes the modal number. The modal constants are determined

$$\cos(k_n L)\cosh(k_n L)=-1 \quad (12)$$

Having a value of
$k_n L = C_n$ = 1.875 ,4.694, 7.855, 10.996 [14]
The first four Eigen modes are shown in figure 2, the Eigen functions can be normalized to the desired value, but a convenient measure is to normalize so that A_n = 1 where by[15]

$$\int_0^L U_m(z)U_n(z)dz=L\delta_{mn} \quad (13)$$

The Eigen frequencies of the cantilever can be found from equation 8 but still the area moment of inertia needs to be calculated. The area moment of inertia when bending a structure around the x-axis is given by

$$I_x=\int_A y^2 dA \quad (14)$$

where y is the distance from the z-axis. In case of a rectangular, uniform cantilever the area moment of inertia is[16] If y

$$I_{cant}=\int_{-h/2}^{h/2}\int_{-w/2}^{-w/2} y^2 dydx=\frac{w^3 h}{12} \quad (15)$$

assuming small bending, For a coated cantilever the calculations are slightly more complicated then frequencies are from equation .8[17]

$$\omega_n=\frac{c_n^{\ 2}}{L^2}\sqrt{\frac{EI}{\rho\Gamma}}=\frac{c^2{}_n}{2\sqrt{3}}\frac{w}{L^2}\sqrt{\frac{E}{\rho}} \quad (16)$$

It is commonly used to simplify the beam-dynamics with that of a harmonic oscillator whereby the cantilever is assigned a spring constant and an effective mass.

$$\omega_n\equiv\sqrt{\frac{k}{m_{eff}}}$$

$$m_{eff}=\frac{3m_0}{C_n^4}, m_0=\rho\Gamma L, k=\frac{3EI}{L^3} \quad (17)$$

The effective mass depends on the mode of vibration as can also be seen from figure 2 since the mass participating in the vibration changes with the mode.
Solving of non homogeneous Ordinary Differential Equation is the 2nd order ODE is non homogeneous [18]. To solve it, let us use the corresponding homogeneous Ordinary Differently Equation first, that is,

$$J''(t)+m\,J'(t)+nJ(t)=0 \quad (18)$$

Using Leibnitz's rule this leads to

$$J=e^{\lambda t}, J'=\lambda e^{\lambda t}, J''=\lambda^2 e^{\lambda t}$$

Then the general solution of the homogeneous ODE is

$$J=Ae^{-t}+Be^{2t} \quad (19)$$

where A and B are arbitrary constants.
Let us now find a particular solution of the original non-homogeneous ODE. The right hand side of the ODE suggests that we try where α and β are constants to be determined

$$I=\alpha\sin(t)+\beta\cos(t)$$

$$I'=\alpha\cos(t)-\beta\sin(t)$$

$$I''=-\alpha\sin(t)-\beta\cos(t)$$

Differentiating, we obtain Substituting into the non homogeneous ODE [19], we obtain

$$-\alpha\sin(t)-\beta\cos(t)+3\alpha\cos(t)-3\beta\sin(t)$$

$$+2\alpha\sin(t)+2\beta\cos(t)\equiv\cos(t)$$

$$[\alpha-3\beta]\sin(t)+[3\alpha+\beta]\cos(t)\equiv\cos(t)$$

The non homogeneous ODE is satisfied if we choose α and β satisfying
α − 3β =0 and 3α + β =1, that is, α = 3/10 and β = 1/10

III. Experimental results

In this section, the proposed system used for real-time measurements is shown in Fig. 4. The main core of the setup includes National instruments simulation and real time measurements tool LabVIEW which is used to simulate and measure the real data from the piezoelectric Energy harvesting system through the data acquisition card (DAQ NI 6361 24 Chanel Data Transfer System). In Fig. 5 LabVIEW real time measurements consists of measurements of output Signal voltage and the center frequency of oscillation for measurements at different output voltage, the output of the measurements file transferred to excel sheet to draw the output. Figure 6 shows the real time measurements of Piezoelectric Energy harvesting. Experiments used to measure the relation between the acceleration of the mini shaker and both the center frequency and the maximum output voltage of the Harvester.

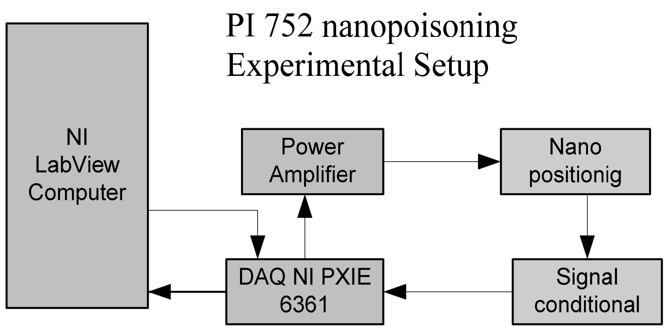

Fig.4 experimental setup

ISBN: 978-988-14047-1-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

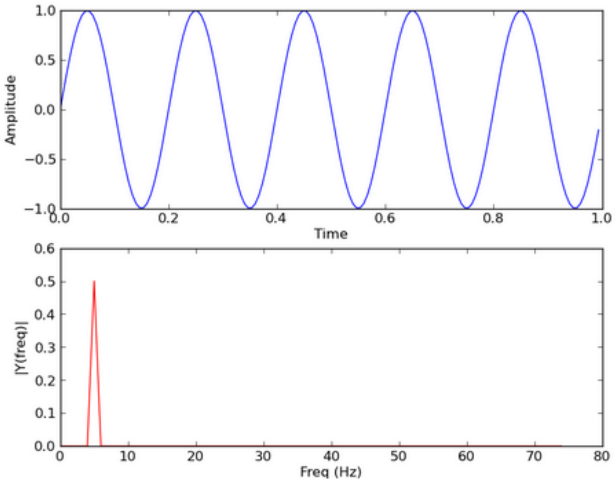

Fig.5 frequency spectrum of real time measurements

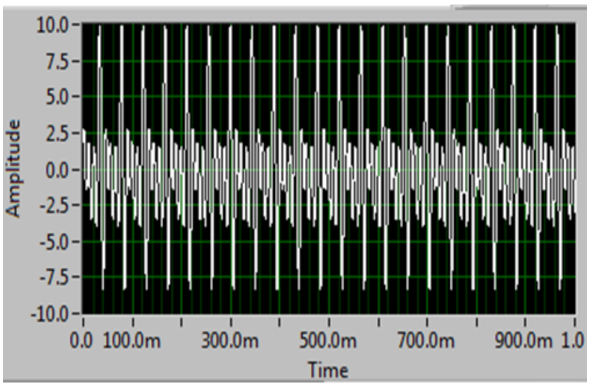

Fig.6 output voltage of energy harvesting

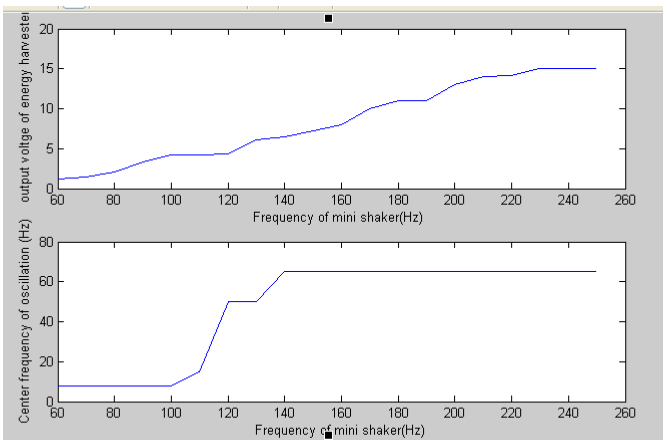

Fig .7 the output voltage and center frequency of Harvester Vs acceleration

IV. Conclusion

This paper describes a reliable dynamic model of piezoelectric energy harvester, measurement the relation between the acceleration and both output voltage and the resonant frequency of the energy harvester a procedure for identifying model parameters with few simple measurements and standard laboratory equipment.

References

[1] Timoshenko, S.P., 1953, History of Strength of Materials (with a brief account of the history of theory of elasticity and theory of structures), McGraw-Hill Book Company, New York.

[2] Williams, C.B. and Yates, R.B., 1996, “Analysis of a Micro-electric Generator for Microsystems,” Sensors and Actuators A, 52, pp. 8–11.

[3] Glynne-Jones P., Tudor, M.J., Beeby, S.P. and White, N.M., 2004, “An Electromagnetic, Vibration-powered Generator for Intelligent Sensor Systems,” Sensors and Actuators A, 110, pp. 344-349.

[4] Arnold, D., 2007, “Review Of Microscale Magnetic Power Generation,” IEEE Transactions on Magnetics, 43, pp. 3940–3951.

[5] Mitcheson, P., Miao, P., Start, B., Yeatman, E., Holmes, A. and Green, T., 2004, “MEMS Electrostatic Micro-Power Generator for Low Frequency Operation,” Sensors and Actuators A, 115, pp. 523-529.

[6] Roundy, S., Wright, P. and Rabaey, J., 2003, Energy Scavenging for Wireless Sensor Networks, Kluwer Academic Publishers, Boston.

[7] Roundy, S., Wright, P.K. and Rabaey, J.M., 2003, “A Study of Low Level Vibrations as a Power Source for Wireless Sensor Nodes,” Computer Communications, 26, pp.1131-1144.

[8] Jeon, Y. B., Sood, R., Jeong, J. H. and Kim, S., 2005, “MEMS Power Generator with Transverse Mode Thin Film PZT,” Sensors & Actuators A, 122, pp. 16-22.

[9] Beeby, S.P., Tudor, M.J. and White, N.M., 2006, “Energy Harvesting Vibration Sources for Microsystems Applications”, Measurement Science and Technology, 17, pp. R175-R195.

[10] Cook-Chennault, K.A., Thambi, N., Sastry, A.M., 2008, “Powering MEMS Portable Devices – a Review of Non-Regenerative and Regenerative Power Supply Systems with Emphasis on Piezoelectric Energy Harvesting Systems,” Smart Materials and Structures, 17, 043001 (33pp).

[11] Sodano, H., Park G. and Inman, D.J., 2004, “A Review of Power Harvesting from Vibration Using Piezoelectric Materials”, Shock and Vibration Digest, 36, pp. 197-205.

[12] Anton, S.R. and Sodano, H.A., 2007, “A Review of Power Harvesting Using Piezoelectric Materials (2003-2006),” Smart Materials and Structures, 16, pp. R1-R21.

[13] Priya, S., 2007, “Advances in Energy Harvesting Using Low Profile Piezoelectric

[14] Transducers,” Journal of Electroceramics, 19, pp. 167–184. Choi, W.J., Jeon, Y., Jeong, J.H., Sood, R. and Kim, S.G., 2006, “Energy Harvesting MEMS Device Based on Thin Film Piezoelectric Cantilevers,” Journal of Electroceramics, 17, pp. 543–8.

[15] Roundy, S. and Wright, P.K., 2004, “A Piezoelectric Vibration Based Generator for Wireless Electronics,” Smart Materials and Structures, 13, pp.1131-1144.

[16] duToit, N.E., Wardle, B.L. and Kim, S., 2005, “Design Considerations for MEMS-Scale Piezoelectric Mechanical Vibration Energy Harvesters,” Journal of Integrated Ferroelectrics, 71, pp. 121-160.

[17] Sodano, H.A., Park, G. and Inman, D.J., 2004, “Estimation of Electric Charge Output for Piezoelectric Energy Harvesting,” Strain, 40, pp. 49-58.

[18] Lu, F., Lee, H.P. and Lim, S.P., 2004, “Modeling and Analysis of Micro Piezoelectric Power Generators for Micro-Electromechanical-Systems Applications,” Smart Materials and Structures, 13, pp. 57-63.

[19] Chen, S.-N., Wang, G.-J. and Chien, M.-C., 2006, “Analytical Modeling of Piezoelectric Vibration-Induced Micro Power Generator,” Mechatronics, 16, pp. 397-387.

[20] C. B. Williams and R. B. Yates, “Analysis of a micro-electric generator for microsystems,” Sens. Actuators, A, vol. 52, nos. 1-3, pp. 8–11, 1996.

[21] Bendat, J.S. and Piersol, A.G., 1986, Random Data Analysis and Measurement Procedures, John Wiley and Sons, New York.

ISBN: 978-988-14047-1-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)