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A Variable Step-Size Sparsity-Estimating PNLMS Algorithm

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***Abstract*—We present a variable step-size, sparsity-estimating PNLMS (VSS-SE-PNLMS) algorithm. The proposed VSS-SE-PNLMS algorithm has the following good properties. (1) It has good convergence speed from the beginning to the steady-state stage, (2) it reaches the desired steady-state misadjustment as designed, and (3) it can be used in sparse channels and dispersive systems as well. Many simulations show that our VSS-SE-PNLMS performs better than several published algorithms in sparse channels and dispersive systems as well.**

***Index Terms*—Adaptive filters, NLMS, proportionate NLMS , sparse system, variable step-size NLMS**

I. Introduction

Adaptive filtering algorithms have been widely employed in many signal processing applications such as equalization, active noise control, acoustic echo cancellation, and so forth. The normalized least-mean-square (NLMS) adaptive filter is the most popular due to its simplicity. The stability of the basic NLMS is controlled by a fixed step-size. This parameter also governs the rate of convergence, speed of tracking ability and the amount of steady-state excess mean-square error (MSE). Aiming to solve the conflicting objectives of fast convergence and low excess MSE, a number of variable step-size NLMS (VSS-NLMS) have been presented in the past two decades [1]-[3].

In some practical applications, impulse response of unknown systems like the network echo channel, wireless multipath channel, and hands-free mobile telephony can be assumed to be sparse, i.e., having a few coefficients with large magnitude among many insignificant ones [4]-[10]. The proportionate NLMS (PNLMS) algorithm [4] is the first algorithm that exploits the sparseness of channels in network echo cancellation to get significantly faster adaptation than the standard NLMS algorithm. The step-size of PNLMS is not a constant for all filter coefficients; it varies from tap to tap and is roughly proportional to the magnitude of the current tap weight estimate. The PNLMS algorithm has faster initial convergence and tracking than the NLMS filter when the echo path is sparse. In the applications of dispersive and/or not sparse enough channel, the PNLMS converges slower than the NLMS algorithm. Several modifications such as the IPNLMS [5] algorithm, which introduces a controlled mixture of PNLMS and NLMS adaptation, was proposed as a better alternative for whatever the nature of impulse response of the unknown system.

While the PNLMS algorithm has very good convergence speed in the initial period, it begins to slow down dramatically at the point still with large MSE [6]. Deng *et al*. derived the optimal proportionate step-size for PNLMS to achieve the fastest convergence and proposed a μ-law PNLMS (MPNLMS) algorithm, which has fast convergence in the initial stage and maintains the fast rate until it gets the steady-state period [6]. However, this MPNLMS algorithm was found to converge slower than NLMS for dispersive channels. By employing a recursive estimate of the channel sparseness, an improved MPNLMS (IMPNLMS) was developed [8].

By using individual activation factors for each filter coefficient, an individual activation factor PNLMS (IAF-PNLMS) was introduced recently [9]. IAF-PNLMS has a better distribution of the adaptation energy over the filter taps than the conventional PNLMS does, and therefore, it performs better than PNLMS and IPNLMS algorithms for highly sparse channels. To further improve the convergence performance after the initial fast convergence period, the concept of μ-law PNLMS was employed jointly with individual activation factor scheme to produce an IAF-MPNLMS algorithm [10].

The purpose of this paper is to introduce a new PNLMS algorithm that has the following good properties. (1) It has good convergence speed from the beginning to the steady-state stage. (2) It reaches the desired steady-state misadjustment as designed. (3) It can be used in sparse channels and dispersive systems as well. To achieve this goal, we employ a variable step-size scheme [3] in the proportionate NLMS filter equipped with a mechanism estimating channel sparsity recursively, and this results in a variable step-size, sparsity-estimating PNLMS (VSS-SE-PNLMS) algorithm. Simulation results show that the proposed VSS-SE-PNLMS performs better than several published algorithms in sparse channels and dispersive systems as well.

II. VSS-SE-PNLMS Algorithm

Let $d(n)$ be the desired response of the adaptive filter

$$d(n) = \mathbf{x}^T(n)\mathbf{h}(n) + \upsilon(n) = y(n) + \upsilon(n) \,, \tag{1}$$

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where $y(n) = \mathbf{h}^T(n)\mathbf{x}(n)$, $\mathbf{x}(n)$ is the input vector and $\mathbf{h}(n)$ denotes the coefficient vector of the unknown system with length N and $\upsilon(n)$ is the system noise that is independent of $x(n)$. Denoting its coefficient vector at iteration n as $\mathbf{w}(n)$, the estimation error is evaluated as

$$e(n) = d(n) - \mathbf{w}^T(n)\mathbf{x}(n). \quad (2)$$

The sparseness of the channel is estimated recursively as,

$$\xi_{\mathbf{w}}(n) = \lambda\xi_{\mathbf{w}}(n-1) + (1-\lambda)\left[\frac{N}{N-\sqrt{N}}\left(1 - \frac{\|\mathbf{w}(n)\|_1}{\sqrt{N}\|\mathbf{w}(n)\|_2}\right)\right]. \quad (3)$$

The proposed VSS-SE-PNLMS algorithm updates the filter coefficient vector $\mathbf{w}(n)$:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu(n)\mathbf{Z}(n)\mathbf{x}(n)e(n)}{\mathbf{x}(n)^T\mathbf{Z}(n)\mathbf{x}(n) + \delta} \quad (4)$$

where $\mu(n)$ is the variable step-size [3]

$$\mu(n) = \gamma\mu(n-1) + (1-\gamma)\frac{\hat{\sigma}_e^2(n)}{\rho\hat{\sigma}_\upsilon^2}, \quad (5)$$

where $\hat{\sigma}_e^2(n)$ is the estimated MSE, $\hat{\sigma}_\upsilon^2(n)$ is the estimated noise power, γ is the forgetting factor, and ρ is a positive parameter that controls the steady-state excess MSE. A complete set of recursions of the VSS-SE-PNLMS are presented in Table I.

III. SIMULATION RESULTS

We present simulation results that demonstrate the good properties of the proposed VSS-SE-PNLMS algorithm. The adaptive filter is used to identify an unknown system with impulse response $\mathbf{h}(n)$ of length N. We evaluate conventional NLMS, three proportionate NLMS algorithms (IPNLMS [5], IMPNLMS [8], and IAF-MPNLMS [10]), and our VSS-SE-NLMS algorithm in the system identification scenario. The parameter settings are tabulated in Table II. The unknown system of length 128 changes signs at iteration 5000 so that we can investigate the tracking performance of the filters. The input signal is a white Gaussian process with σ_x^2 adjusted to make the output of the channel has a unit power. The additive noise is a white Gaussian with $\sigma_\upsilon^2 = 0.001$. We have conducted simulations for sparse channel and dispersive channel as well. The mean-square error curves (in dB) are presented in Figs. 1 and 2 for sparse system and dispersive system, respectively. Results are ensemble averages over 200 independent runs. It is easy to see that our algorithm performs very well for both convergence rate and final steady-state error.

IV. CONCLUSIONS

The VSS-SE-PNLMS algorithm introduced in this paper has been shown to perform with good convergence speed from the beginning to the steady-state stage in sparse channels and dispersive systems as well. It can reach the desired steady-state misadjustment as designed. Simulations show that our VSS-SE-PNLMS performs better than several published algorithms in sparse channels and dispersive systems as well.

Table I, VSS-SE-PNLMS Algorithm

Initialization:
$\mu(0)=1$, $\xi_{\mathbf{w}}(0)=0$
$w(0)=[0,0,...,0]^T$, $\hat{\sigma}_e^2(0)=e^2(1)$, $\phi_i(0)=0$
Update: $n=1,\ 2,\ 3,\ldots$
$\hat{y}(n)=\mathbf{w}^T(n)\mathbf{x}(n)$
$e(n)=d(n)-\hat{y}(n)$
$\xi_{\mathbf{w}}(n)=\lambda\xi_{\mathbf{w}}(n-1)+(1-\lambda)\left[\frac{N}{N-\sqrt{N}}\left(1-\frac{\Vert\mathbf{w}(n)\Vert_1}{\sqrt{N}\Vert\mathbf{w}(n)\Vert_2}\right)\right]$
$\alpha(n)=2\xi_{\mathbf{w}}(n)-1$
$\phi_i(n)=\beta\phi_i(n-1)+(1-\beta)\lvert w_i(n)\rvert$
$z_i(n)=\frac{1-\alpha(n)}{2N}+(1+\alpha(n))\frac{\phi_i(n)}{2\sum_{i=1}^{N}\phi_i(n)+\varepsilon}, i=1,2,3,...N$
$\mathbf{Z}(n)=diag[z_1(n),z_2(n),...,z_N(n)]$
$\hat{\sigma}_e^2(n)=\gamma\hat{\sigma}_e^2(n-1)+(1-\gamma)e^2(n)$
$\mu(n)=\gamma\mu(n-1)+(1-\gamma)\frac{\hat{\sigma}_e^2(n)}{\rho\hat{\sigma}_v^2}$
$\mathbf{w}(n+1)=\mathbf{w}(n)+\frac{\mu(n)\mathbf{Z}(n)\mathbf{x}(n)e(n)}{\mathbf{x}(n)^T\mathbf{Z}(n)\mathbf{x}(n)+\delta}$

Table II, Simulation Parameters of Our VSS-SE-PNLMS And Other Comparison Algorithms

VSS-SE-PNLMS Parameters:
$\delta_{PROPOSED}=\frac{\delta_{NLMS}}{N}, \varepsilon=0.2, \gamma=0.998, \rho=30$
$\lambda=1-1/4N$, $\mu_{\min}=10^{-5}, \mu_{\max}=1$
NLMS Parameters: $\delta_{NLMS}=0.5, \mu=0.3$
IPNLMS[5] Parameters:
$\delta_{IPNLMS}=\frac{\delta_{NLMS}}{N}, \varepsilon=0.2, \mu=0.3, \alpha=0$
IMPNLMS[8] Parameters:
$\delta_{IMPNLMS}=\frac{\delta_{NLMS}}{N}, \varepsilon=0.2, \mu=0.3, \lambda=0.1$
IAF-MPNLMS[10] Parameters:
$\delta=\frac{\delta_{NLMS}}{N}, \mu=0.3, \mu_{IAF-MPNLMS}=1000$
δ is the regularization parameter

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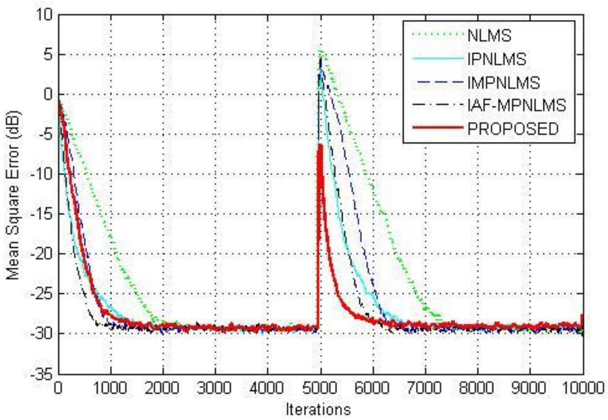

Fig. 1, Mean-square error curves of four comparing algorithms and proposed VSS-SE-PNLMS algorithm for sparse system. The impulse response changes sign at iteration 5000. The input signal is a white Gaussian process, system noise power $\sigma_\upsilon^2 = 0.001$, SNR=30dB, $N = 128$

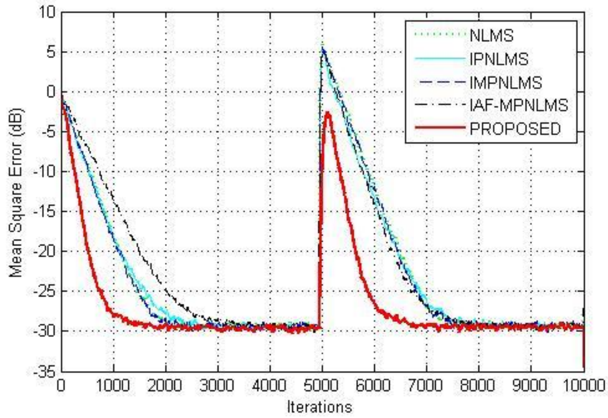

Fig. 2, Mean-square error curves of four comparing algorithms and proposed VSS-SE-PNLMS algorithm for dispersive system. The impulse response changes sign at iteration 5000. The input signal is a white Gaussian process, system noise power $\sigma_\upsilon^2 = 0.001$, SNR=30dB, $N = 128$.