Hybrid High-Fidelity Modeling of Radar Scenarios using Atemporal, Discrete Event, and Time-Step Simulation

YuanPin Cheng, Don Brutzman, Phillip E. Pace, and Arnold H. Buss

Abstract—Many simulation scenarios attempt to seek a balance between model fidelity and computational efficiency. Unfortunately, scenario realism and model level of detail are often reduced due to the complexity of experimental design and corresponding limitations of computational power. Such simplifications can produce misleading results. For example if the Radar Cross Section (RCS) effects in response to time-varying target aspect angle are ignored.

A hybrid, high-fidelity sensor model can be achieved by using a Time-Step (TS) approach with precomputed atemporal response factors (such as RCS) each situated on active entities that interact within an overall Discrete Event Simulation (DES) framework. This paper further applies regression analysis to the cumulative results of 100 replications times 255 scenarios to provide additional insight. This new methodology adapts the best aspects of each simulation paradigm to integrate multiple high-fidelity physically based models in a variety of tactical scenarios with tractable computational complexity.

Index Terms—Discrete Even Simulation (DES), regression model, Nearly Orthogonal Latin Hyper Cube (NOLH), Radar Range Equation, Hybrid sensor model.

I. INTRODUCTION

There is always a dilemma for simulation developers and users. On the one hand, modelers want the simulation to contain as many details as possible, so the results could be closer to the real world, and on the other hand, a simulation needs to be executed efficiently and smoothly. In this dilemma, “better is the enemy of good enough.” As the models get more complicated, the computational complexity and cumbersome system are the price paid, especially when dealing with a high complexity physical model, like a radar system, in simulation. Better levels of detail create high computational complexity which negatively impacts efficiency.

A search radar-targets simulation scenario, seeking for radar’s time-to-detect targets, contains numerous factors that contribute to model performance, such as target’s distance, antenna sweeping period, aspect angle of targets and target speed etc. Some parameters are sensitive to the performance of the whole system and the computation is nearly too complex to accomplish in a real time simulation, such as Radar Cross Sections (RCS) of targets from different aspect angles.

Wang et al., in their study of constructing radar system simulations, propose a universal program framework that considers the radar-target scenario using an object template. The template divides the simulation into three subsystems classified by functions. These functions include: 1) Scene target echo generation system, 2) Radar information process system, 3) Radar control, display and estimation system [1].

A reoccurring problem in approaches to dynamic target-sensor scenarios, when a sensor’s detection algorithm is based on the range between sensor and target [2], or sensor detectable perimeters [3] this neglects the factor of aspect angle of targets. The change of aspect angle can lead to drastic variation in RCS response. The maximum detectable range of sensor derived from radar range equation [Eq. (1)] also fluctuates along with aspect angle. RCS can have a huge influence on received signal power at the receiver, and has been recommended as a topic of future work in Radar Modeling and Simulation (M&S) research by Inggs et al. [4].

Wang et al.’s study has incorporated RCS values into modeling by a ground clusters numerical Weibull model in a Radar M&S focusing on Terrain Environment effects [5]. The cluster backscatter RCS is the function of terrain range and aspect angle relative to the sensor in the Weibull model that has been previously measured and built in prior research.

However, when integrating a non-parameterized RCS model generated from an arbitrary target, a simulation application might produce incoherent responses due to considerable computational latency from generating target RCS.

To fill in these gaps in target-sensor scenarios at run time, this research integrates modeling structure from three categories with a multi-stage approach as shown in Figure.1. The three categories are: 1) Atemporal construction, 2) Discrete event simulation construction, 3) Time step construction. There are two major contributions of this structure approach:

1) Integrating a high-fidelity physical radar range equation, which considers dynamic position between sensors and targets through simulation, into a comprehensive detection algorithm. This work constructs a hybrid model with three modeling constructs that mitigate the disparate simulation time mechanisms of each paradigm.

   • Atemporal models, i.e. models that cannot be interactive in real time simulation scenario due to its tremendous computational latency, such as target’s RCS response.

   • Discrete Event Simulation (DES) structure for precise and efficient event execution.

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Y. Cheng is the Army Major of the Republic of China, Taiwan with the Modeling, Virtual Environments, and Simulation Institute (MOVES), Naval Postgraduate School (NPS), Monterey, CA, 93943 USA e-mail: (ycheng@nps.edu).

D. Brutzman and A. H. Buss are with MOVES, Naval Postgraduate School, Monterey, CA, 93943 USA e-mail: (brutzman, abuss@nps.edu)

P. E. Pace is with the Center for Joint Services Electronic Warfare and the Dept. of Electrical and Computer Engineering, Naval Postgraduate School, Monterey, CA, 93943 USA e-mail: (ppecz@nps.edu)
DIFFERENTIAL TIME STEP (TS) STRUCTURE TO REPRESENT INHERENT TIME STEP BEHAVIOR. IN THIS MODEL, IT EMULATES RADAR ANTENNA MECHANICAL SWEEPING BEHAVIOR.

2) FORMING A META-MODEL THAT IS RELATIVELY SIMPLE COMPARED WITH THE HYBRID MODEL, WHICH IS GIVEN DESIGN OF EXPERIMENTS (DOE) [6] AND GENERATING RESULTS. THE META-MODEL ENCAPSULATES PARAMETERS THAT HAVE BEEN CONSIDERED IN DOE'S WITHOUT CARRYING BULKY COMPUTATIONAL LOADING FOR SIMULATION APPLICATIONS.

This paper is organized as follows:

Section II, to ensure that maximum detail is incorporated into model parameters, a hybrid model which integrates atemporal, TS structure into DES is used. There will be an introduction of Discrete Event Simulation (DES) sensor structure, and a hybrid sensor structure which incorporates TS and atemporal factors into a DES structure.

Section III analyzes how aspect angle affects RCS, and the extent of variation in the sensor maximum detectable range (derived from radar range equations) due to RCS.

Section IV will first introduce designs of experience (DOE) which are set up efficiently using Nearly Orthogonal Latin Hyper Cube (NOLHC) from Cioppa [6]. Then applying regression analysis from the hybrid model DOE results, a new numerical meta-models is generated that incorporates the parameters considered in the hybrid simulation framework.

II. MODEL CONSTRUCTION

A. Event Driven Sensors and Targets

DES sensor models can be executed efficiently and accurately in the simulation scenario because of its event driven structure. There are no redundant computational procedures between scheduled events. Two DES sensors explain this concept [3]:

1) Cookie cutter sensor: The simplest sensor model. The detection algorithm is based on a detection perimeter of the sensor. In Figure 2 scenario, when simulation starts at $t_{-1}$, an aircraft flies from orange diamond along the blue dotted line with speed $s$. At simulation time $t_0$, the target hits the detectable perimeter (green star) of sensor and then the sensor declares that the target was detected at $t_0$. The target was undetected when it reached the red square at $t_1$. The event schedule is shown in Table I.

2) Constant rate sensor: An exponential random variable with a mean time-to-detect (TDD) ($\mu$) represents the time-to-detect a target after the target enters the detection perimeter (the green star in Figure 2). Time-to-detect $t_d \sim \text{Exponential}(\mu)$ represents a sequence of Bernoulli trials with identical probability until the first successful detection. The $t_d$ brings stochastic time delay into the detection algorithm, which makes the constant rate sensor more realistic than the cookie cutter model. Event schedule is shown in Table II.

There are two points of ambiguity with this design:

- There is no way to map a clear relationship between the mean time-to-detect of $t_d$ and the desired parameters within radar equations.
- There is only one mean time-to-detect $\mu$ for the whole detectable area of a sensor. This assumption does not account for the RCS variation of a target at different angles.

A hybrid approach allows the DES framework to handle these difficulties.

B. Hybrid sensor

DES models are sometimes less intuitive than Time Step (TS) models to build. DES models also present challenges when attempting to integrate parameters that have complex outcomes, such as complex RCS responses of targets, since all possible interactive events need to be considered in model structures. To address such problems, this research integrates atemporal and Time Step (TS) simulation methodologies into a DES framework to build a flexible hybrid sensor model. This hybrid structure can integrate the highly detailed factors of atemporal and TS sensor structure with the reasonable computational efficiency of DES models.

- Atemporal RCS simulation response that is put in a table and carried by sensor class (the event graph shown in Figure 3) for represent RCS response of specific target which we applied in this scenario. Due to its huge computational latency, this atemporal factor is isolated in order to execute it in real-time or differential simulation time scenario, such as DES.
- TS structure is applied to emulate the sweeping mechanism of search radar antenna for each scan. A ping event is constantly scheduled to represent the antenna sweeping intervals in the system event graph Figure 6. The sensor checks target position and angle, deriving maximum detectable range at each sweeping event.

The sweeping schedule starts when the target reached the sensor maximum unambiguous Ru (green star on perimeter in Figure 3). This DES structure saves irrelevant computational power outside possible detectable range of the sensor. Once the target is inside the detection circle in Figure 2, the sensor starts pinging the target in period of $\Delta t = t_1 - t_0 = t_2 - t_1 \cdots = t_n - t_{n-1}$ while the target moves in distance $\Delta d = s \cdot \Delta t$ at each time step. During sweeping time step $t_0 \cdots t_n$, the aspect angle of target changes from $\theta_{0} \cdots \theta_{n}$ respectively. Precise RCS is available corresponding to angle changes for maximum detectable range ($R_{max}$) which are computed from equation (1). In this scenario, the target does...
not reach the Rmax even though it is within Ru at t0, t1 (green star and first yellow circle). When the target arrives at t2 (second yellow circle), the target’s position is within Rmax (the blue line figure in the center). That is the moment the sensor detecting the target. The full discrete event simulation schedule is shown in TABLE III.

The sensor might never detect a target if the target never reaches Rmax during all time steps. The parameters contributed to the sensor model are:
1) Target RCS table, 2) Target Speed, 3) Target aspect angle, 4) Radar sweeping period, 5) Radar pulse repetition interval (prr), 6) Antenna 3dB azimuth beam-width.

III. RADAR RANGE EQUATION

The maximum detectable range \( R_{\text{max}} \) is derived by radar range equation as follows:

\[
R_{\text{max}} = \sqrt{\frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2 \cdot \sigma_T \cdot N_{pr}}{4\pi^2 \cdot K \cdot T_0 \cdot B \cdot F_R \cdot (S/N)_1 \cdot X_{\text{detection}}}}
\]  

and the maximum unambiguous range \( R_u \) of the sensor due to the radar pulse repetitive frequency constrain is derived as
The detectable range of targets from the sensor has to satisfy both equations (1) and (2) i.e. the threshold range $R_{\text{detect}}$ in which the sensor will detect targets needs to meet these two criteria: $R_{\text{detect}} < R_u$ and $R_{\text{detect}} < R_{\text{max}}$.

The parameters, applied in radar range equation, have been classified into two categories in this study: 1) the parameters that can only be configured by the radar system. The RCS of a flat plate at broadside, $A$ is the area of plate

$$\sigma_{\text{Plate}} = \frac{4\pi A^2}{\lambda} \bigg|_{\lambda=0.1m,f=3GHz,A=1m^2} = 1000(m^2)$$

The pulse integration improvement factor is determined by the number of pulses returned from a point target [7] during a scan of radar with a pulse repetition rate $prf(\text{Hz})$, an antenna 3dB beamwidth $\theta_{\text{3dB}}$ (we only consider azimuth angle effect in this study), and antenna revolutions per minute ($\text{rpm}$).

First, the effective radar signal illumination time on target ($ToT$) per revolution is calculated as follow:

$$ToT = \frac{60s}{\text{rpm} \times 360^\circ \times \theta_{\text{3dB}} AZ} \text{ (second)}$$

then the number of effective pulses provided to the integration improvement factor is

$$N_{\text{1int}} = ToT \times prf$$

A mono-static radar is assumed in this scenario, which means the transmitting and receiving signal are from the same source at the same station. A coherent integration improvement factor can be applied in this system.
C. Target Fluctuation: Swerling III model $\chi^2_{df=4}$

Target detection algorithm utilizing the square law detector assumes a constant RCS. This work was extended by Peter Swerling to four distinct cases of target RCS fluctuation [9]. We apply Swerling III, a chi-square probability density function with four degree of freedom, in this study. This factor models the fluctuation loss from targets between each antenna scan. The RCS is assumed to be constant within a scan and independent from scan to scan [10]; these conditions are the same as those in this simulation scenario. The sensor model generates stochastic time-to-detect results due to this fluctuation loss model which is represented by a Chi-Squared random variable with degree of freedom = 4.

IV. DESIGN OF EXPERIMENTS AND DATA ANALYSIS

Regression analysis is a statistical method that investigates the relationship between two or more variables related in a non-deterministic manner [11]. The relationship is expressed in the form of an equation or more explanatory predictor variables. To learn the system behavior or predict the outcome, associated with the input parameters of the system, requires sufficient experiment data that cover as many detail levels in parameters as possible. Tallavajhula [12] applies data-driven methods to learn the relationship between input parameters and system state and construct a high fidelity model for a planar range sensor.

There are two crucial considerations for design of experiment (DoE):

- The number of available experiments must be statistically significant for designer and analyzer. Nevertheless, because of realistic constraints such as budget, time or available exercise space to conduct experiments, experimenters are usually unable to get unlimited experimental results for analysis. Efficiently designing experiments is important.
- How well the space is filled reflecting the system parameters from the DoE is also important. The more detailed levels are included in the DoE, the more the relationship between system and parameters will be captured by the analysis. This need for detail must be balanced with the need for efficiency. A well-covered and succinct DoE is necessary.

A. Design of Experiment (DoE) by Nearly Orthogonal Latin Hypercube (NOLH)

To analyze the interactive relationship between all input parameters in the model, a DoE that efficiently fills space in the range of parameters is crucial to reveal the relationship between parameters and system state. In full factorial design, if applying five factors with 250 levels of each factor, the number of design points $= 250^5 \approx 9.7 \times 10^{11}$. For each design point executing 100 repetitions, the total simulation execution amount will be up to $\approx 9.7 \times 10^{13}$. Even though in this computer based simulation, the number of experiments that can be conducted is not restricted, this execution amount is still unbearable.

The Nearly Orthogonal Latin Hypercube presented by Cioppa at. el [6] provides an efficient space filling DoE that allows view of 255 levels of up to 29 factors in 255 design points. With 100 repetitions, the total number of simulation to process is 25,500. This makes three billion-to-one difference on simulation executing. In Figure.7 shows that the design points of each factors fill in the DoE space quite evenly with only minor overlapping correlation to each other.

Even though the parameters listed above are set up in conventional units of radar applications, it is important to have all parameter units consistently applied in the simulation to avoid misrepresenting quantity of parameters. In the simulation, the unit of the antenna “Sweeping Integer” parameter is converted from seconds to hours in order to match the length unit within the “Target Speed” parameter in TABLE IV.

B. Linear Regression Estimation of time-to-detect (TTD)

Executing the DoEs through the hybrid sensor model simulation, yields time-to-detect-target data, which is associated with 255 design points. Next, a meta-model is formed by linear regression analysis for predicting the response variable “time-to-detect” the target by the sensor. For a good meta-model, the goal is to make the difference between meta-model and baseline hybrid simulation model small. An simple linear regression model that describes the relationship between response variable $Y$ and predictor variables $X_1, X_2, \cdots, X_p$, is defined as follow [13]:

$$ Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon $$

where $\beta_0, \beta_1, \cdots, \beta_p$, called the regression coefficients and $\epsilon$ is assumed to be a normal random error to represent the
happening is also known: \( P(Y = 0) = 1 − P(Y = 1) \). Logistic regression analysis models the ratio between \( P(Y = 1) \) and \( P(Y = 0) \) with a natural logarithm of the ratio. This expression is called logit of \( Y \):

\[
\ln \left( \frac{P(Y = 1)}{1 - P(Y = 1)} \right) = \text{logit}(Y)
\]

then we can convert the \( \text{logit}(Y) \) back to the probability that \( (Y=1) \) with predictor variables in such equation:

\[
P(Y = 1) = \frac{1}{1 + e^{-(\alpha + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p)}}
\]

A logistic regression analysis is run of the probability of detection (Pd) by a sensor of targets, using predictor parameters of up to a degree of two polynomial and factorial terms. This logistic regression analysis has an R-Square = 0.843. The logistic meta-model determines if the target is detected by the sensor in this DoE with a predicted probability. If the target detection is determined true by comparing the Pd with a uniform random variable [0,1], then time-to-detect the target can be estimated by the linear regression model described in Section IV-B.

D. Meta-Model Validation

The validation of the meta-model was verified by examining the mean (first moment) and variance (second moment) of predicted results. Testing the first moment and second moment between predicted results and baseline data can indicate how well the meta-model represents the hybrid model.

- Two-sample t-test.
  Processing a two sample t-test on TTD from both meta and hybrid models with 100 repetitions in one design point [Figure 9]. Because the t-Ratio = -0.7687 it retains the null hypothesis of equal mean assumption at a 95% confidence interval. In other words, the mean of predicted TTDs are statistically the same as the hybrid sensor model.

- O’Brien unequal variance test.
  The p-value of this unequal variance test was less then 0.0001. It rejects the null hypothesis of equal variance assumption. The test indicates the variance of predicted TTDs is different from the hybrid model in Figure 10.
V. CONCLUSIONS

This work demonstrates a hybrid structure sensor model that successfully integrates high fidelity radar range equations with dynamic RCS response from targets. Based on computationally efficient DES structure with isolated atemporal RCS model, the hybrid model not only introduced angle varying RCS factor into the dynamic sensor-target scenario, but also keep the computational complexity of whole scenario down to reasonable level. A typical desktop PC is capable of executing 250,500 DoE scenario repetitions in a matter of minutes.

Even with these benefits of a DES-based structure for the hybrid model, the embedded TS mechanism for emulating antenna sweeping period is still a key latency for a comprehensive model. Additionally, the model also needs to include maximum detectable range table for the target. These factors still make the model bulky in some sense. This research also utilizes efficient NOLH DoEs which provides a well-filled space of parameters with significantly fewer design points compared with full factorial design.

Finally, a meta-model is fit from the experimental data executed by the DoEs with Least-Squares estimation. The meta-model does present the hybrid model characteristics in certain sense with much more concise structure, even if the fit model showed some heteroscedasticity in prediction results. This difference in variance might result from high order factorial and polynomial terms that had been included in the meta-model. Further data analysis and regression methodologies can also be applied in future work to form a better meta-models of interest.

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REFERENCES

[8] [Online]. Available: https://www.cst.com/Products/CSTMWS