

Dynamic Programming applied to Medium Term Hydrothermal Scheduling

Thais G. Siqueira and Marcelo G. Villalva

Abstract—The goal of this work is to present a better understanding of the behavior of dynamic programming operating policies applied to the resolution of medium-term operational planning problem. Hydro plants located in different regions in Brazil will be considered for the analysis. The randomness of inflows will be treated and considered in the resolution of the problem using Dynamic Programming approaches. Then these results will be simulated from series of inflows.

Index Terms—hydro plants, hydrothermal scheduling, inflow uncertainty, dynamic programming, simulation

I. INTRODUCTION

The medium term hydrothermal scheduling (MTHS) problem is quite complex due to some of its characteristics, specially the randomness of inflows. The MTHS aims to determine, for each stage (month) of the planning period (years), the amount of generation at each hydro and thermal plant which attends the load demand and minimizes the expected operation cost along the planning period.

Stochastic dynamic programming (SDP) has been the most suggested technique to solve the MTHS problem since it can adequately cope with the uncertainty of inflows and the nonlinear relations among variables. Although efficient in the treatment of river inflows as random variables described by probability distributions, the SDP technique is limited by the so-called "curse of dimensionality" since its computational burden increases exponentially with the number of hydro plants. In order to overcome this difficulty one common solution adopted is to represent the hydro system by an aggregate model, as it is the case in the Brazilian power system. Alternatives to stochastic models for MTHS can be developed through operational policies based on deterministic models. The advantage of such approaches is their ability to handle multiple reservoir systems without the need of any modeling manipulation. Although some work has been done in the comparison between deterministic and stochastic approaches for MTHS, the discussion about the best approach to the problem is far from ending. The purpose of this paper is to present a discussion about different policies based on Dynamic Programming to solve MTHS. Hydro plants located in different regions of Brazil will be considered as case studies. The uncertainty of inflows will be modelled and the Box-Cox transformation will be used. One determinist model and three other stochastic models will be considered for solving the problem and finally these results will be simulated using inflows series. This paper is organized as follows: Section 1 presents the formulation of

the MTHS problem, section 2 shows four different Dynamic Programming approaches that will be analysed in this paper. Section 3 reports the numerical results for the case studies and discusses the features and sensitivities of the different models. Section 4 summarizes the conclusions.

II. MTHS FORMULATION

The MTHS problem, in systems composed of a single hydro plant, considering the uncertainty of inflows, can be formulated as a nonlinear stochastic programming problem seeking the minimization of the expected operational cost and is given by:

$$\min E_y \left\{ \lambda_t \sum_{t=1}^{T-1} \psi_t(d_t - p_t) \right\} + \alpha_T(x_T) \quad (1)$$

subject to:

$$x_t^{med} = \frac{x_t + x_{t+1}}{2} \quad (2)$$

$$p_t = k [\phi(x_t^{med}) - \theta(u_t) - \delta(q_t)] q_t, \quad \forall t \quad (3)$$

$$x_t = x_{t-1} + (y_t - q_t)\beta, \quad \forall t \quad (4)$$

$$u_t = q_t + s_t, \quad \forall t \quad (5)$$

$$x_t \in X_t \quad (6)$$

$$v_t \in V_t \quad (7)$$

$$s_t \geq 0 \quad (8)$$

$$x_0 \text{ given} \quad (9)$$

In the above equations E_y : represents the expected value of the inflows ; T is the planning period; t is the index of the planning stages; λ_t : is a discount rate to convert cost for present; $\psi_t(\cdot)$ is the thermal cost function at stage t ; g_t is the thermal generation at stage t ; d_t is the energy demand at stage t ; p_t is the hydro generation at stage t , which is the product of a constant k , the water head given by the difference of forebay elevation $\phi(x_t)$ and tailrace elevation $\theta(u_t)$, and the water discharged at stage t ; y_t represents the inflow at stage t ; x_t is the reservoir storage at the end of stage t ; v_t is the water released at stage t ; s_t is the water spilled at stage t ; β is a constant factor that converts flow into volume; X_t and V_t are feasible sets representing bounds for the variables x_t and v_t .

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III. DYNAMIC PROGRAMMING MODELS FOR MTHS

For solving the problem (1)-(8) by SDP, the optimization problem is divided into stages and at each stage the optimal control variable is chosen in order to minimize the expected cost for each state of the system. The optimization process is based on a previous knowledge of the future possibilities and its consequences, satisfying the Bellman optimality principle, [2]. Thus the total operation cost from stage until the end of the planning period is obtained with the sum of the present cost at stage t with the optimal future cost of the following stages, which were previously determined. Since the problem is a stochastic one, the optimal control at each stage is obtained based on the probability distribution of the inflow at that stage, [3], [4], [7]. The dynamic programming recursive equation is given by:

$$F_t(x_t) = \min_{q_t, s_t} \{ \psi_t(d_t - p_t) + F_{t+1}(x_{t+1}) \} \quad (10)$$

The recursive equation (10) is solved for each stage t subject to equations (2) -(8).

The four policies based on dynamic programming models considered in this work will be detailed bellow.

A. Deterministic Dynamic Programming

In the Deterministic Dynamic Programming (DDP) the inflow for each month m is known previously and calculated based on the historical values of each hydro plant. In this approach the long term average, \bar{y}_m , provides the inflow arithmetical mean for each month for all N years of the historical.

$$\bar{y}_m = \frac{1}{N} \sum_{r=1}^N y_{r,m} \quad (11)$$

DDP can be considered as a particular case of SDP where the probability is assumed one if the inflow long term average for a certain month occurs.

The recursive equation for this particular case, where Q_t is the decision search space, can be written as:

$$\alpha_{t-1}(x_{t-1}) = \min_{q_t \in Q_t} \{ \psi_t(d_t - p_t) + \alpha_t(x_t) \} \quad (12)$$

where:

$$x_t = x_{t-1} + (\bar{y}_t - q_t)\beta \quad (13)$$

B. Independent Stochastic Dynamic Programming

If one solves dynamic programming considering the inflows monthly independent, the recursive equation will be similar to DDP and the only difference is the future cost that will be weighted by their probabilities p_i considering the inflow discretization divided in Ny parts.

Thus, the recursive equation of ISDP is given by:

$$\alpha_{t-1}(x_{t-1}) = \min_{q_t \in Q_t} \sum_{i=1}^{Ny} \{ \psi_t(d_t - p_t^i) + \alpha_t(x_t^i) \} . p_i \quad (14)$$

where:

$$x_t^i = x_{t-1} + (y_t^i - q_t)\beta \quad (15)$$

and

$$p_t^i = k \left[\phi\left(\frac{x_t^i + x_{t+1}}{2}\right) - \theta(q_t) - \delta(q_t) \right] q_t \quad (16)$$

C. Stochastic Dual Dynamic Programming

In the Stochastic Dual Dynamic Programming (SDDP) one supposes that in the beginning of each month the inflow that will occur is known. Each final month state is represented by a pair (stored volume at the end of the month; inflow of this month) [7]. The inflow distribution is represented by a inflow set and its probabilities. Each inflow is analysed separately, resulting in different optimal individual decisions. For each combination of storage level and inflow, according to its discretization, an optimal decision is found. For a given storage level, each optimal decision takes to a total cost of operation. Thus, an expected cost is calculated with these different costs.

For this approach one considers the following recursive equation:

$$\alpha_{t-1}(x_{t-1}) = \sum_{j=1}^{Ny} p_j \left\{ \min_{q_t \in Q_t} [\psi_t(d_t - p_t^j) + \alpha_{t-1}(x_t^j)] \right\} \quad (17)$$

$$x_t^j = x_{t-1} + (y_t^j - q_t^j)\beta \quad (18)$$

$$p_t^j = k \left[\phi\left(\frac{x_t^j + x_{t-1}}{2}\right) - \theta(q_t) - \delta(q_t) \right] q_t \quad (19)$$

D. Dependent Stochastic Dynamic Programming

When the inflows uncertainty is considered through a Markov chain, [6], leading to the dependent SDP (DSDP), the state variable changes to include the inflow of the previous stage, the probabilities are now calculated from the conditional probability density function and the recursive equation is modified to:

$$F_t(x_t) = \min_{q_t, s_t} \{ \psi(g_t) + E_{q_t | q_{t-1}} \{ F_{t+1}(x_{t+1}) \} \} \quad (20)$$

Again, one solves the recursive equation (20) for each stage t according to equations (2)-(8).

In this work one represents the inflow uncertainty of the hydro plants by a Normal probability density function with Box-Cox transformation.

IV. CASE STUDIES

Two hydro plants were considered for the case studies: Serra da Mesa in the Tocantins River in northern region of Brazil, Furnas in the São Francisco River, located in the southeastern region of Brazil. In order to get equilibrated hydrothermal systems, the thermal plant capacity, in MW, was considered equal to the installed capacity of the hydro plants and the load demand was assumed constant.

The cost function associated with non-hydraulic complementary generation was given by the following quadratic function:

TABLE I
STATISTICAL OF NATURAL INFLOWS OF HYDRO PLANTS.

Hydro plant	Serra da Mesa	Sobradinho	Furnas
Minimal inflow	97.0	570.0	204.0
Maximal inflow	6163.0	15676.0	3757.0
Average	775.5	2668.4	918.5
Deviation	700.5	1984.4	615.7

TABLE II
HYDRO PLANTS FEATURES

Hydro plant	Installed Capacity (MW-Month)	Reservoir (hm ³)	Discharge (m ³ /s)
Serra da Mesa	1200	43250	98/1165.9
Furnas	1312	22950	196/1692

$$\psi_t = 0.02(d_t - p_t)^2 \quad (21)$$

The control policies were implemented and simulated in a monthly basis throughout the inflow historical sequence, which in this case begins in 1931.

Table 1 shows some relevant statistical data of the hydro plants inflows. This information was used for modeling the inflow with the probability density function.

In the optimization process the planning period T considered for the recursive equation solution was equal to 120 with terminal cost null, $\alpha_T(x_T) = 0, \forall x_T$.

For all DP policies, the discretization adopted for state variable was 100, and for the stochastic ones, the inflow variable was discretized into 10 possible values. A normal probability density function with Box-Cox transformation was adopted for modeling the inflows and the optimal parameters were found by the likelihood test. The optimal values for Serra da Mesa and Furnas reservoir were $\lambda^* = -0.318$ and $\lambda^* = -0.539$, respectively.

Cubic splines were used in the the interpolation of future costs.

Table 2 shows some operative data of the hydro plants selected for the case studies.

The forebay and tailrace elevations, $\phi(x_t)$ and $\theta(q_t)$, were calculated by 4th degree polynomial functions. The coefficient values of these polynomial function are shown in Table 3. These data will be used in the calculus of PD policies.

The DDP, ISDP, and DSDP were simulated through historical records since 1931 and the NFN-SDP was simulated using an inflow forecast given by neural fuzzy network

TABLE III
FOREBAY AND TAILRACE ELEVATIONS POLYNOMIAL COEFFICIENTS OF HYDRO PLANTS

Coefficients		Serra da Mesa	Furnas
$\phi(x_t)$	a_0	3.877328E+02	7.361261E+02
	a_1	3.487404E-03	3.193892E-03
	a_2	-8.567909E-08	-1.608703E-07
	a_3	1.233703E-12	5.076109E-12
	a_4	-7.135002E-18	-6.504317E-17
$\theta(q_t)$	b_0	3.327979E+02	6.716328E+02
	b_1	1.342970E-03	1.017380E-03
	b_2	8.819558E-08	-1.799719E-07
	b_3	-1.627669E-11	2.513280E-11
	b_4	0.000000E+00	0.000000E+00

TABLE IV
SERRA DA MESA SIMULATION RESULTS

Policy	Generation (MW)		Cost (\$)
	Average	Deviation	Average
AO	840.3	132.7	4135
DPP	815.7	177.9	4839.4
ISDP	818.1	165.3	4712.4
NFN-SDP	817.5	165.0	4724.3
DSDP	816.0	161.2	4726.5

TABLE V
FURNAS SIMULATION RESULTS

Policy	Generation (MW)		Cost (\$)
	Average	Deviation	Average
AO	727.8	148.3	7271.1
DDP	709.2	198.1	8029.0
ISDP	704.6	183.8	8037.7
NFN-SDP	707.9	209.2	8158.2
DSDP	708.1	192.9	8016.4

model. In this case, the forecast were based on neural networks and fuzzy logical [1]. An absolute optimal solution was considered as an upper bound for the operative policies analysed. This approach considers the total knowledge of the inflows.

Tables 4 and 5 show the simulation results for all policies investigated: the hydrothermal generation average and its deviation and the operation cost for Serra da Mesa and Furnas, respectively.

The discharged water trajectories for the DP policies: DDP (red), ISDP (green), SDP (black) and DSDP (blue) can be visualized in Figure 1 and Figure 3 for Serra da Mesa and Furnas hydro plants, respectively.

The behavior of stored volumes for the DP policies including the optimal absolute solution (full line in black): DDP (red), ISDP (blue), NFN-SDP (green) and DSDP (pink) can be visualized in Figure 2 and Figure 4 for Serra da Mesa and Furnas hydro plants, respectively.

According to the obtained simulation results for Serra da Mesa hydro plant the lower mean cost (\$ 4712.4) was associated to ISDP policy. The higher mean generation, that was equal to 818.1 MW, was also provided by ISDP. DDP was the policy with worse performance, presenting the higher mean cost equal to \$ 4839.4.

The difference between stochastic approaches, ISDP and DSDP, was equal to 0.29%. Analysing ISDP versus NFN-SDP there was a cost gain of 0.25% over ISDP. Comparing DDP versus DSDP the difference was 2.3%. The performance difference between the best and worse policy was at about 2.7%.

Thus, one concludes that due to the fact that DSDP explores the mensal correlation between the months of the year, there is no guarantee of this policy presents the best result.

The comparison of the obtained results for Serra da Mesa hydro plant are shown in Figure 1, that shows the discharged water, and Figure 2, that shows the stored water behavior for all policies investigated.

Figures 3 and 4 show the discharged water trajectories and the stored volume behavior for all DP policies investigated.

According to the results presented in Table V, comparing

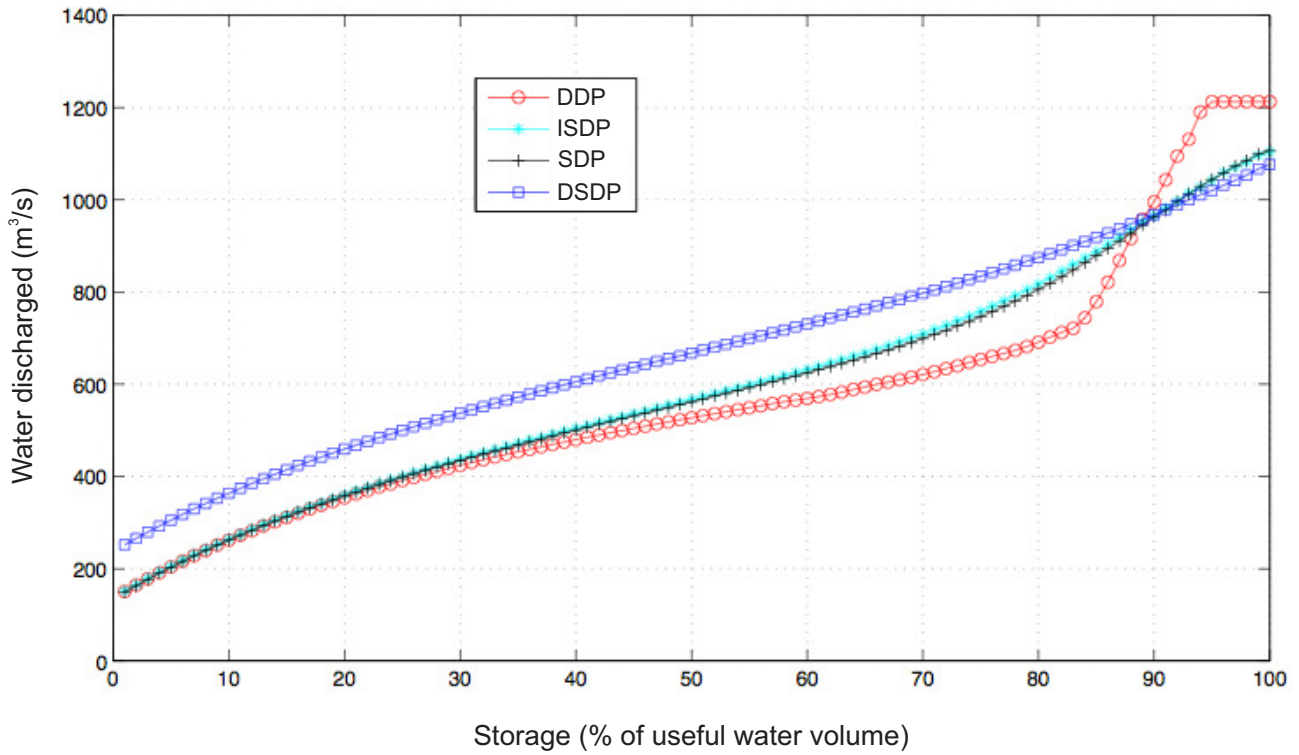


Fig. 1. Discharged water (m^3/s) of Serra da Mesa hydro plant for january.

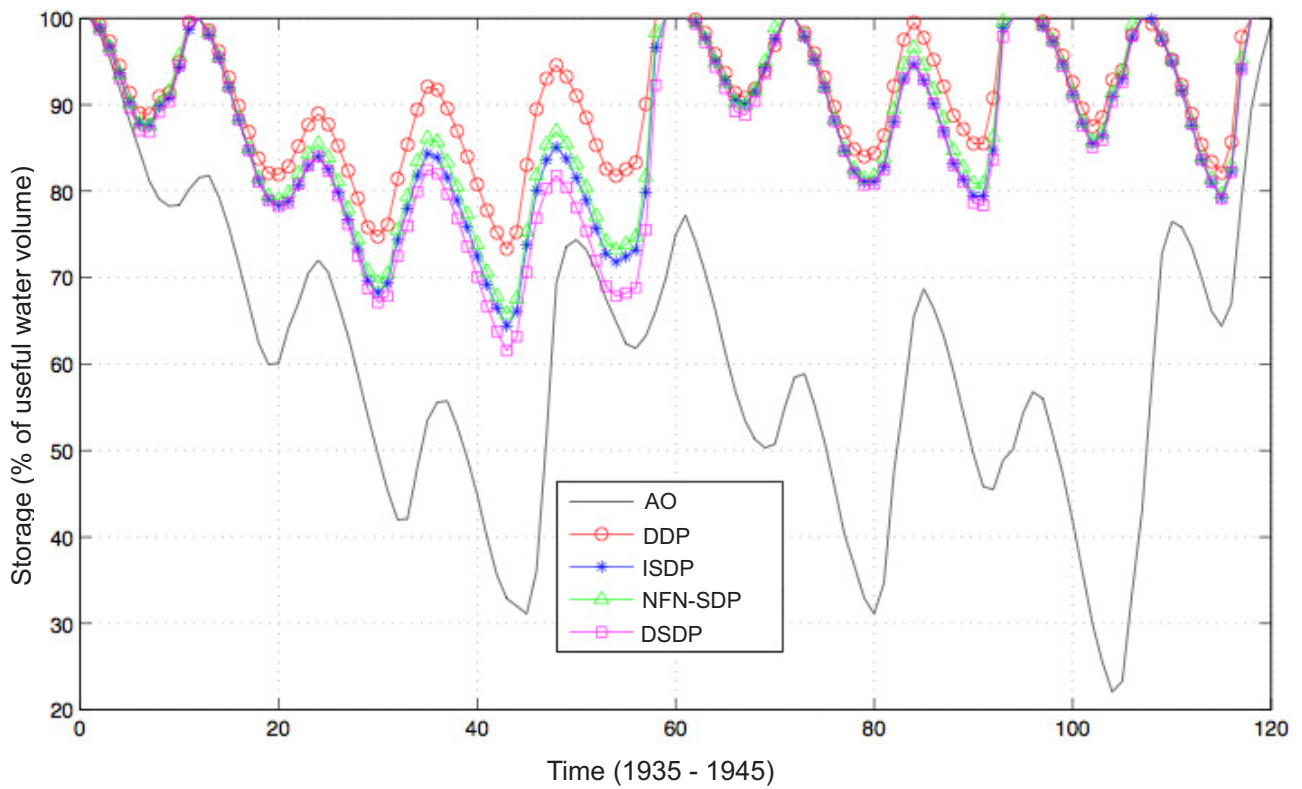


Fig. 2. Trajectory of stored volumes (%v.u.) of Serra da Mesa.

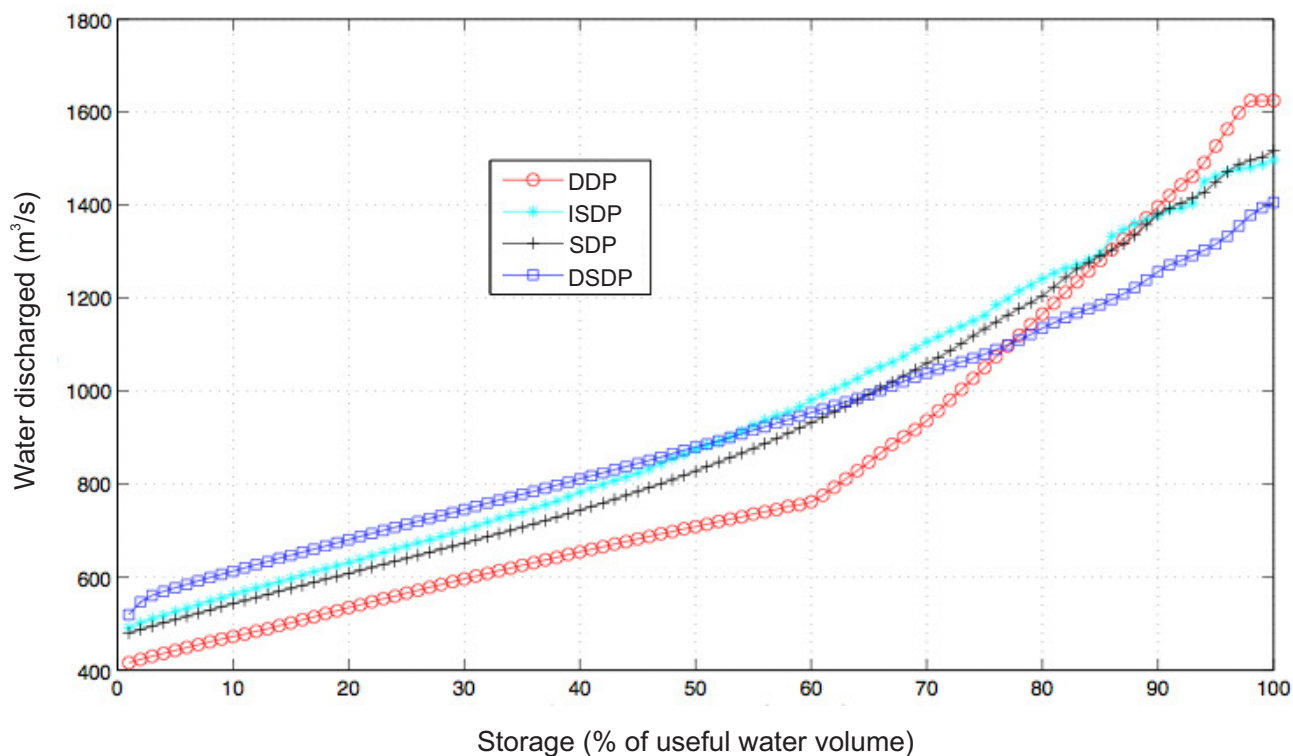


Fig. 3. Discharged water (m^3/s) of Furnas hydro plant for January.

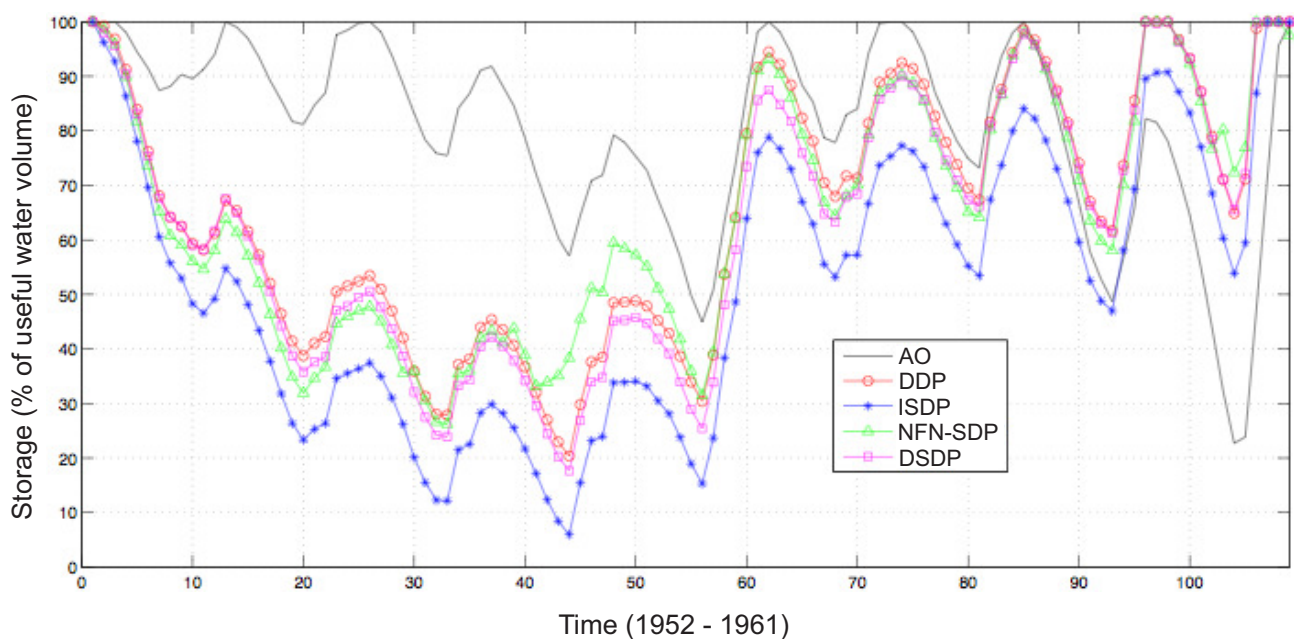


Fig. 4. Trajectory of stored volumes (%v.u.) of Furnas.

the average costs of DDP and DSDP, DDP generates 1.1 MW (0.15%) more, however, its average cost is 0.16% worse.

DDP presented higher average generation equal to 709.2 MW and this can be explained due to the fact that DDP has presented the higher spilled water and took advantage of this water released that could be stored or spilled. The worse performance was associated to NFN-SDP policy with average cost equal to \$ 8158.2.

V. CONCLUSION AND REMARKS

This work has compared deterministic and stochastic models for medium term hydrothermal scheduling for two Brazilian hydro plants located in different regions. Optimal policies for the medium term hydrothermal scheduling were obtained considering four dynamic programming policies according to inflow modelling. The normal probability density function with Box-Cox transformation was considered for inflow modelling. Finally the optimal policies results were simulated through historical inflow records.

Based on the simulation results it is possible to affirm that the policies performance were similar. The differences of average costs between the best and worse policy was nearly, 2.7% and 1.7% for Serra da Mesa and Furnas hydro plants, respectively.

Another conclusion is that the better performance not always was associated to the most sophisticated inflow model. For instance, the better result for Serra da Mesa was given by ISDP that does not consider the inflow time correlation.

Another important conclusion of the numerical results was that DDP presented surprisingly a good performance, only 2.7%, and 0.15% worse than the better SDP policy for Serra da Mesa and Furnas, respectively.

The simulation results have shown that both deterministic approaches have provided quite similar performance, specially regarding the operational cost.

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