Abstract— The method of strengthening concrete structures with FRP composites has existed for over a decade in civil engineering rehabilitation projects; the most common way to strengthen structures is in bending, but also wrapping of girders and beams is quite common. A time dependent reliability analysis of RC girders shear strengthened by CFRP wraps and subject to both pitting corrosion and FRP degradation is conducted. Based on environmental harshness classified in relevant literature, three levels of corrosion vulnerability, and correspondingly surface chloride concentration as a random variable, are considered for chloride ion attack to steel reinforcement. The FRP degradation progress due to the environmental effects was modeled as two range of FRP's retention in strength according to existing studies. Monte Carlo simulation (MCs) was used in time dependent reliability model to consider uncertainties in vulnerabilities and calculate the reliability indices (β) during the extended lifecycle of strengthened beam. The model was applied to a worked example designed based on ACI440.2R-08. Delamination and debonding of FRP sheets roughly parallel to the concrete surface are the most predominant mechanisms which results in premature fracture of the strengthened beam. Therefore in this paper the premature fractures limit state function is defined in reference to the provisions of ACI440.2R-08 and its corresponding time dependent probability of failure during the extended lifecycle is calculated using Monte Carlo simulation method. Since the target reliability index according to ACI440.2R-08 is 3.5, the proposed method can precisely predict the time of deviation from code provisions. As the mechanism of shear fracture in beam elements is catastrophic, by estimating the deviation time, this model can denounce the onset of the brittle and premature fracture likelihoods in different environmental aggressiveness. Furthermore via sensitivity analysis of different strengthening configurations and material characteristics, the influence of design variables on probability of different fracture modes is assessed. Also it is shown that the trend and inclination of time-β diagram after the onset of the brittle fracture and probability of premature fracture modes can predict approximately the governed fracture mode and related time.

Index Terms— Shear fracture, Fiber Reinforced Polymer, Time dependent reliability analysis, Deterioration models.

I. INTRODUCTION

Over the last decades, FRPs have emerged as a noteworthy rival to the more conventional civil engineering materials for rehabilitation of as-built infrastructures [1]. This applicability has been due to the well-known advantages of FRP composites including their high strength-to-weight ratio, excellent fatigue resistance in cyclic loading, good corrosion resistance, and versatility in coping with different sectional shapes [2-4]. Therefore, many researchers and related associations such as American Concrete Institute (ACI) provided practical design guidelines for all aspects of application FRPs for strengthening structural elements [5]. In these guidelines, the provisions specified for bending design are more well-understood than for shear because the shear equations have been developed by semi-empirical background [6] and aren't quite comprehensive or are treated very briefly [7]. The reason for this is that the interaction of concrete and reinforcement is complicated and difficult to describe in terms of mechanical properties. When a strengthening material like FRPs is added to the concrete, the shear mechanism will be even more complicated [7].

Shear strengthening of RC members with CFRPs is a research problem with no consensus understanding and agreement amongst subject matter experts [8]. Moreover, the brittle shear fracture of RC beams is catastrophic compared to the flexural mode [9] that takes the shear capacity of an RC beam as its critical strength. The source of the existing disagreements are mainly related to uncertainties have been recognized as the role of materials properties and degradations in long term operation [10], [11], the effect of design parameters such as the size of the beam section, fiber orientation, strip width and spacing between FRP strips [12-14] and governed fracture mechanism among probable failure modes [6], [15]. In spite of all these efforts accomplished so far, there are still limited and some gaps in the knowledge base and hence definite rooms for improvement to reach a holistic and sustainable solution.

There are several modes of deterioration affecting FRP strengthened structures. Chloride induced pitting corrosion of steel stirrup and FRP's degradation are known as the more effective deterioration mechanisms which are considered in this work. Although several reliability analyses have been developed so far to assess the flexural strengthened RC beam [16-18] the application of reliability based evaluation of shear strengthened RC beam with FRPs is scarce [13]. Since environmental uncertainties make the governed...
fracture mechanism as a time variant event; it is necessary to evaluate the trustworthiness of existing provisions by time dependent reliability method. In this study, three levels of deterioration effectiveness were considered for stirrup pitting corrosion and two levels of retention in strength were applied for CFRP degradation. This paper assesses every probable fracture mechanism of shear strengthened RC beam with FRPs. According to ACI440.2R-08 provisions, there are four possible shear fractures which should be validated along the project life. These include delamination, debonding, premature concrete crushing and rapture of FRP wraps. The probabilities of occurrence of these modes of failure have temporal variability due to the different environmental harshness during the extended lifetime. The merit of this paper is that it develops a time dependent reliability model to monitor the safety in respect to all possible fracture modes and to recognize the probability of occurrence of any mechanism of failure in different environmental conditions. Monte-Carlo simulation Model has been employed to evaluate the corrosion initiation time, related rate of reinforcing area reduction and probability of defined limit states in each year after strengthening. Since the target reliability index according to ACI 440.2R-08 assumed to be 3.5, the proposed procedure can indicate the initiation of deficiencies to control the governed mode.

II. THE CORROSION AND DEGRADATION MODELING

Pitting corrosion is a non-homogeneous corrosion occurring along with reinforcing bars; in fact it shows randomness due to the variability of concrete and steel material properties, humidity, concrete cover, surface cracking, etc. [19]. For FRPs, several environmental harshness such as hydrothermal effects (the combination of moisture and heat), ultraviolet radiation, fiber alkalinity, cold and freeze-thaw cycling, etc. degrade the nominal strength in different levels of influence. In this paper three states of chloride surface concentration considered for corrosion initiation and two levels of retention in FRP strength, totaling in six levels of environmental harshness as described in Table 1. In addition, one state of aging without any environmental effects was considered to compare the deteriorations intensity. Based on the probabilistic pitting corrosion parameters reported in Table 2, Monte Carlo simulation is employed to evaluate the corrosion initiation time and current based on the Equation (1) and equation (2) respectively [20]. The pitting depth and area reduction of reinforcing are calculated probabilistically based on the equations presented in [20].

\[
T_{pi} = \frac{e^{cT}}{4D \left( erf \frac{c}{C_0} \right) \left( C_0 \right)}
\]

\[
T_{\text{cor}} = \frac{e^{cT}}{4D \left( erf \frac{c}{C_0} \right) \left( C_0 \right)}
\]

\[
\ln(i_{\text{cor}}) = 0.0312RH - 4736/T + 1.695 wc - 0.391 e + 14.589
\]

Where in equation (1) \( T_{\text{init}} \) is the time to corrosion initiation (years), \( e \) is the depth of cover (mm), \( D_t \) is the diffusion coefficient for chloride in concrete (mm²/year), \( c = cT / \) denotes the inverse complementary standard error function, and \( C_{0i} \) and \( C_s \) are the threshold and surface chloride concentrations, respectively (kg/m³). The statistical descriptions of corrosion parameters are presented in Table 2. FRPs degradation is modeled using Arrhenius rate equations developed by Karbhari and Abanilla [11] which are expressed in Equation (3) and (4) as follows:

\[
\text{Ret.F}_1 = \frac{-3.66Ln(1) + 106.07}{100}
\]

\[
\text{Ret.F}_2 = \frac{-3.66Ln(1) + 106.07}{100} \times 1.05
\]

where \( \text{Ret.F}_1 \) and \( \text{Ret.F}_2 \) are the reduction factors considered for calculation of retention in strength of FRPs subjected to high and moderate degradation condition respectively and \( t \) is the time of exposure expressed in days.

III. FORMULATION OF PROBLEM

The shear fracture of RC beams strengthened with FRP wraps may occur in four possible mechanisms. These fracture mods make a series system that the critical limit state occurs when an actual demand exceed its resistance. In series system in any point in time, i.e., instantaneous probability of failure of system is governed by the weakest link. In general, if it is assumed that for a series system \( m \) demand events \( S_i \) occur within the time interval \( (0, t_m) \) discretized at times \( t_i \) (\( i = 1, 2, ..., m \)), the critical limit state for a beam comprising \( n \) elements is
\[
G_{m}(X) = \text{Min}\{L_i(t_j) - S_j(t_j)\} \quad \text{for } i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n
\]  

(5)

where \(L_i(t_j)\) represents the structural resistance/capacity level whilst \(S_j\) is the load effect/demand in the \(j\)th mechanism at time \(t_j\). Since deteriorating structures tend to have high variability of resistance due to temporal uncertainties of corrosion phenomena, fractures are not clearly independent and the cumulative probability of failure during this lifetime is:

\[
P_f(t_m) = 1 - \text{Pr}\{G_{m}(X) > 0 \cap G_{1}(X) > 0 \cap \ldots \cap G_{m}(X) > 0 \}
\]  

(6)

Monte-Carlo simulation is used herein to simulate equation 6 for 100 years after strengthening to evaluate the probability of failure of strengthened beam (PF) (e.g. \(t_f = 1, 2,\ldots, 100\)). The reliability index (\(\beta_{PF}\)) was then evaluated at \(\beta_{PF} = \Phi^{-1}(P_f(t_f))\), where \(\Phi^{-1}(.)\) is the inverse of the standard normal distribution function. Based on the ACI 440 to prevent the failure of strengthened beam against brittle and premature fractures the guideline considered several constraints to be satisfied in order to calculate shear capacity. According to the ACI 440 reference study [6], the FRPs’ delamination is the most probable shear fracture mode which controls the attainment to the intended design shear capacity. Based on the ACI 440 to prevent the failure of strengthened beam against brittle and premature fractures the guideline considered several constraints to be satisfied in order to calculate shear capacity. According to the ACI 440 reference study [6], the FRPs’ delamination is the most probable shear fracture mode which controls the attainment to the intended design shear capacity.

IV. POSSIBLE SHEAR FRACtURES

In the FRP systems that do not enclose the entire section (two- and three-sided wraps) different states of premature fractures restrict the shear capacity of the beam. In ACI440.2r-08 the shear fracture of strengthened beams and its related capacity have been controlled by effective design strain of FRP wraps. In U-shaped FRP wraps the bond stresses between concrete and FRPs have been analyzed to determine the corporation of design strain that can be achieved to control the delamination of FRPs from the concrete before the loss of aggregate interlock of the section [27]. Based on the ACI440 provisions which have defined the bond reduction factor (\(k_b\)) to calculate the effective design strain (\(\varepsilon_{fr}\)), the probability of failure and corresponding limit state functions to control the delamination fracture of FRP wraps can be expressed as:

\[
P_r \left( G_{D(1)} \leq 0 \right) = P_r \left( 0.004 - \varepsilon_{fr} \leq 0 \right)
\]

(7)

\[
P_r \left( G_{D(2)} \leq 0 \right) = P_r \left( 0.75 - k_b \leq 0 \right)
\]

(8)

 Debonding is the second type of premature fracture which occurs in reinforced concrete members having comparatively long shear spans. Debonding may start at flexural cracks or both shear and flexural cracks near the region of maximum moment and is intensified in sections having a high shear-moment ratio. This failure progresses through a thin mortar-rich layer comprising the surface of the concrete beam. According to ACI 440 provisions which have retrieved from [2], the debonding probability of failure and its limit state function can be formulated as per equation (9).

\[
P_r \left( G_{D(h)} \leq 0 \right) = P_r \left( 0.9 \varepsilon_{fr} - 0.41 \sqrt{\frac{f_y}{nE_f t_f}} \leq 0 \right)
\]

(9)

where \(\varepsilon_{fr}\) is the design rupture strain of FRP reinforcement (mm/mm), \(n\) is the number of FRP laminates, \(E_f\) is the tensile modulus of elasticity of FRP (MPa) and \(t_f\) is nominal thickness of one ply of FRP reinforcement (mm). In accordance with ACI 318-05, to prevent brittle shear failure of strengthened beam, the provided shear capacity of added materials must be restricted by a specified value. Based on ACI440, the value for contribution of steel and FRP reinforcement must be restricted by 4 times of provided concrete shear capacity (\(V_C\)). Therefore, for reliability analysis of shear strengthened reinforced concrete beam with FRP the third limit state function, corresponding to the brittle concrete fracture is formulated as per equation (10) and its probability of failure can be calculated as:

\[
P_r \left( G_{hr} \leq 0 \right) = P_r \left( 4V_C - V_s - V_{FRP} \leq 0 \right)
\]

(10)

where \(V_s\) and \(V_{FRP}\) are the provided shear capacity of steel

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Mean</th>
<th>COV</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative humidity (RH)</td>
<td>Normal</td>
<td>Depends on</td>
<td>0.10</td>
<td>[20]</td>
</tr>
<tr>
<td>Water cement ratio (w/c)</td>
<td>Lognormal</td>
<td>0.45</td>
<td>0.05</td>
<td>[17]</td>
</tr>
<tr>
<td>Concrete cover (c) (mm)</td>
<td>Lognormal</td>
<td>Depends on</td>
<td>0.15</td>
<td>Assumed</td>
</tr>
<tr>
<td>Reference value of diffusion coefficient (DC def) (mm/year)</td>
<td>Lognormal</td>
<td>1</td>
<td>0.2</td>
<td>[20]</td>
</tr>
<tr>
<td>Temperature (T) (Kelvin)</td>
<td>Normal</td>
<td>Depends on</td>
<td>0.10</td>
<td>[20]</td>
</tr>
<tr>
<td>Surface chloride concentrations (C_s) (kg/m²)</td>
<td>Lognormal</td>
<td>Depends on</td>
<td>0.5</td>
<td>[20]</td>
</tr>
<tr>
<td>Threshold chloride concentrations (C_th) (kg/m²)</td>
<td>Uniform</td>
<td>0.9</td>
<td>0.19</td>
<td>[20]</td>
</tr>
<tr>
<td>Ratio in pitting depth (P)</td>
<td>Extreme Type I</td>
<td>5.65</td>
<td>0.22</td>
<td>[20]</td>
</tr>
<tr>
<td>Age reduction factor (m)</td>
<td>Beta (9.3; 52.7; 0; 1)</td>
<td>0.15</td>
<td>0.30</td>
<td>[22]</td>
</tr>
</tbody>
</table>
stirrups and FRP wraps respectively. Finally it should be noted that according to ACI 440.2R-08, in reliability analysis of the shear capacity of a strengthened RC beam the total capacity is calculated as the sum of resistance provided by each material:

\[ R = V_c + V_s + V_{FRP} \]  \hspace{1cm} (11)

For reliability analysis the considered load demands are amplified based on the load combination presented in equation (12) [28].

\[ S = 1.2DL + 1.6LL \]  \hspace{1cm} (12)

where DL and LL are the static dead and live load effects (KN), respectively. Therefore, the limit state function for shear capacity of shear strengthened RC beam with FRP is formulated as:

\[ P_r = \left\{ \begin{array}{ll}
G_{Cp} & \leq 0 \\
R - S & \leq 0
\end{array} \right. \]  \hspace{1cm} (13)

It is noted that by applying the deterioration mechanisms the terms of ε_s, k_s, V_s and V_{FRP} are time dependent random variables and hence make the governing fracture mechanisms be changed over the time. As suggested in [26], a normal distribution was used to model dead loads and Extreme Type I for live loads. For the dead load the Coefficient of Variation (C.O.V) and the bias factor are considered to be 0.1 and 1.05 respectively. In the present study, a constant value of C.O.V = 0.25 and a bias factor of 1 was used for the maximum live load plus impact load [13], [21]. The bias factor and C.O.V of dead and live loads are shown in Table 3.

V. COMPUTATION OF FAILURE PROBABILITY FOR WORKED EXAMPLE

Uncertainty in the shear capacities is characterized by uncertainty in the design variables contributing to the checking parameters and total shear resistance. The resistance of a component is a function of material strength, section geometry, and dimensions. Statistical descriptions for most design variables of two cases are shown in Table 3. The cases have a same cross section (550×650 mm²), steel stirrups area (157.08 mm²), FRP mechanical and geometrical properties and load effects. However in these worked examples to monitor the influence of corrosion on reliability of fractures during the lifetime, whilst all other basic variables are considered to have identical distributions for both cases, the spacing between steel stirrups and concrete compressive strength have different parameters for these cases as reported in Table 3. Since Case 1 has lower amount of steel reinforcing ratio (A_s/s_c = 0.393), its shear deficiency is compensated by higher FRP thickness (t_F = 0.5 mm). In comparison to Case 2 as the spacing between stirrups was halved the designed FRP thickness for strengthening is calculated as 0.36 cm. 5,000,000 realizations were used to create statistical distribution of random variables in MCs.

VI. RESULTS AND DISCUSSIONS

A. Case 1

The time dependent reliability analysis of Case 1 in EH environmental condition is shown in Figure 1. The results indicate that the governed fracture mode in early times after strengthening is delamination and after initiation of the corrosion process (about 25 years) reinforcing area reduction make an additional difference between delamination and total shear capacity reliabilities. Since the FRP degradation rate especially in EH condition is very intensive in early years (see Figure 2), the reliability index is reached to 3.5 (ACI440 calibrating reliability index) in 7 years (T_s = 7). This result indicates that in sheared strengthened RC beams when the majority of required shear capacity has been provided by FRP wraps the ACI440 environmental factor (CE, Knock down factor) cannot adequately model a safe design for delamination fracture mode [10].

B. Case 2

Figure 3 shows the reliability analysis of Case 2 in EH environmental condition for 100 years after strengthening. The governed fracture mode just in a few years after strengthening (less than 10 years) is purely related to delamination fracture. By increasing the debonding probability and initiation of corrosion process the deviation between reliabilities of delamination and shear capacity is growing up. Since the contribution of steel reinforcing has been increased in Case 2 (and FRPs provided capacity has been reduced) by propagation of corrosion process, the reliability of strengthened beam after corrosion initiation is reduced dramatically. Thus, the shape of reliability curve is trended as an inverse S-curve. As shown in Figure 3, the

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Bias factor</th>
<th>Mean Case 1</th>
<th>Mean Case 2</th>
<th>C.O.V</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete compressive strength (f_c) (MPa)</td>
<td>Normal</td>
<td>1.10</td>
<td>30</td>
<td>20</td>
<td>0.18</td>
<td>[21]</td>
</tr>
<tr>
<td>Steel yield strength (f_y) (MPa)</td>
<td>Normal</td>
<td>1.10</td>
<td>400</td>
<td>400</td>
<td>0.125</td>
<td>[23]</td>
</tr>
<tr>
<td>Modulus of CFRP wraps (E_F) (GPa)</td>
<td>Lognormal</td>
<td>1.00</td>
<td>233</td>
<td>233</td>
<td>0.15</td>
<td>[21]</td>
</tr>
<tr>
<td>Fiber Orientation (β_f)</td>
<td>Normal</td>
<td>1.00</td>
<td>90</td>
<td>90</td>
<td>0.05</td>
<td>[13]</td>
</tr>
<tr>
<td>Steel stirrups spacing (S_s) (mm)</td>
<td>Normal</td>
<td>1.00</td>
<td>400</td>
<td>201.4</td>
<td>0.125</td>
<td>Assumed</td>
</tr>
<tr>
<td>Width of CFRP wraps (W_F) (mm)</td>
<td>Normal</td>
<td>1.00</td>
<td>120</td>
<td>120</td>
<td>0.10</td>
<td>Assumed</td>
</tr>
<tr>
<td>Space between CFRP wraps (S_F) (mm)</td>
<td>Normal</td>
<td>1.01</td>
<td>220</td>
<td>220</td>
<td>0.1</td>
<td>[13]</td>
</tr>
<tr>
<td>CFRP strain at failure (ε_F*) (mm/mm)</td>
<td>Weibull</td>
<td>1.10</td>
<td>1.45</td>
<td>1.45</td>
<td>0.022</td>
<td>[23]</td>
</tr>
<tr>
<td>Dead load (DL) (KN)</td>
<td>Normal</td>
<td>1.05</td>
<td>145</td>
<td>130</td>
<td>0.1</td>
<td>[26]</td>
</tr>
<tr>
<td>Live load (LL) (KN)</td>
<td>Extreme Type I</td>
<td>1</td>
<td>145</td>
<td>130</td>
<td>0.25</td>
<td>[13, 24]</td>
</tr>
</tbody>
</table>
reliability of debonding fracture in age 43 is reached to less than the value specified for delamination. Furthermore it is interesting to note that for high steel reinforcing sections like Case 2 in moderate corrosive condition (MH and MM; which mainly FRPs degradation deteriorate the structure), the governed fracture mechanism is mainly controlled by FRP debonding in long term operation (Figure 4). It is indicate that especially for U-shaped RC beams shear strengthening by FRPs in the moderate corrosive harshness, debonding failure mode has a higher priority in strengthening. It is found that the defined brittle shear fracture of concrete, i.e., equation 10, always has a very low probability, so it doesn't have a significant effect on the overall reliability of fractures in different environmental properties.

VII. CONCLUSIONS
The paper has described a time dependent reliability analysis used for shear fracture of reinforced concrete beams strengthened with FRPs in different aggressive environments. The possible shear fracture modes were formulated to recognize governed fracture mode during the extended lifecycle. Two sample cases were used to illustrate the effect of contributing factors in governing fracture mechanisms, i.e., delamination and debonding, during the project lifecycle. It was found that different materials contribution and environmental harshness, cause change the governed shear fracture. The study highlighted the role of deterioration in general and corrosion in particular in occurrence of different fracture mode. Therefore, it suggests that for generalization of this finding more samples with different design requirements and environmental harshness should be evaluated. These samples can be used to calibrate the environmental calibration factor by a time dependent reliability analysis. It lets the codes customize designs based on the environmental classifications and required lifecycle conditions for strengthening.
REFERENCES


