Generalized Optimal Strategy of n Currencies

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Abstract— **A tricky way some courageous investors make money is precisely to buy or sell money itself! The exchange of foreign currency ("forex" market) is the purchase, or trade, of a particular currency from an individual or institution and the simultaneous sale of another currency at the equivalent value or current exchange rate. This exchange rate or price fluctuation is based on demand, political, social and economic events surrounding each country currency, this is a stochastic phenomenon, hence hard to predict, of course there is a strong potential for loss.**

Currencies are always traded in pairs. When someone says they are "buying the EUR/USD", they are buying Euros and selling Dollars. One challenge is to predict the rise and fall of the value so that one can make the decision to buy or sell a position. Another challenge is to select the best currency pair in term of return on investment. The innovative approach developed in this paper attempts to solve these challenges.

Index Terms—**Markov Chain, Forex Market, Trading Strategy, Expected Payoff, Two Players Game, Gambler Ruin.**

I. INTRODUCTION

THIS paper presents a generalized approach to determine the optimal strategy when trading several pairs of currency in forex stock market. The model developed herein is based on stochastic mathematics, namely Markov Chains, briefly reviewed in section II herein, ([1], Chap 7 through 10 and [6], Chap 4). Then we use advance matrix calculus, ([1], Chap 1 to 3), ([2], Chap 1), ([3], Appendix A), ([4]) and ([5]), and combine the concepts of "gambler's ruin problem", "two-person zero-sum game", rise and fall of stock market" and "mixed strategy" into a generalized approach. We then use our generalized formulae to determine the optimal currency to trade on. In the meantime we demonstrate that the trader is a loser by default.

In section III, we present our "generalized optimal strategy" technique that uses an n x n matrix to represent several pairs of currency. The concept was developed by the author and therefore has no direct reference.

The last section illustrates the technique using data from a financial experiment conducted by the author in trading forex.

II. THEORETICAL FOUNDATIONS

A. Markov Chains and Games

In a Markov chain, let p_{ii} denotes the probability of remaining in state E_i. If $p_{ii}=1$, then E_{ii} is called an **absorbing state**. A Markov chain is said to be **absorbing Markov chain** if and only if it contains at least one absorbing state and it is possible to go from any non-absorbing state to an absorbing state in one or more trials.

- Gambler's Ruin Problem:

For an absorbing Markov chain that has a transition matrix P of the form

$$P=\begin{bmatrix} I_r & | & 0 \\ - & | & - \\ S & | & Q \end{bmatrix}$$

Where S is of dimension s x r and Q is of dimension s x s. Let the matrix $T=[I_s-Q]^{-1}$. T is called the **fundamental matrix** of the Markov chain. The entries of T give the expected number of times the process is in each non-absorbing state provided the process began in a non-absorbing state.

The expected number of trials before absorption for each non-absorbing state is found by adding the entries in the corresponding row of the fundamental matrix T.

Probability of being absorbed: The (i,j)th entry in the matrix product T.S gives the probability that, starting in non-absorbing state i, we reach the absorbing state j. More details can be found in reference [6], pp.230-234.

- The Rise and Fall of Stock Prices:

Let I (increase), D (decrease) and N (no change) be the three states of a stock price, the transition matrix P is a 3 x 3 matrix, where rows 1(I), 2(D) and 3(N) are initial states I, D and N and columns 1, 2 and 3 are final states I, D and N. In the long run, if $t=[t_1\ t_2\ t_3]$ is the fixed probability vector derived from P, the entries t_1, t_2 and t_3 indicate respectively that a stock will have increased its price $100t_1$% of the time, decrease it price $100t_2$% of the time and remained the same $100t_3$% of the time.

To know how long a stock will spend, for instance, on state I and N before arriving in state D, one could assume D is an absorbing state by replacing the current probability p_{22} (or p_{DD}) with 1 and make all other entries in the D row (or row 2) 0. The matrix thereof could be subdivided into identity (I), zero (O), absorbing (S) and non-absorbing (Q) matrices. The (i,j)th entry of $T=[I_s-Q]^{-1}$ gives the average time spent in state j having started in state i before reaching the absorbing state D.

The average amount of time between visits to state i (called **mean recurrence average**) is given by the reciprocal of the i^{th} component of the fixed vector t.

B. Two Person Games

A **two-person game** is any conflict or competition between two people. If whatever is lost (or gained) by player I is gained (or lost) by player II, the game is said to be a **zero-sum** game. Such a game can be represented by an m x n matrix $A=[a_{ij}]$ called a **game matrix** or **payoff matrix**, where each entry is termed **payoff** and in which player I chooses any of the m rows and player II chooses any of the

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n columns. A **strategy** for a given matrix is the decision for player I to select rows and player II to select columns. A game defined by a matrix is said to be **strictly determined** if and only if there is an entry of the matrix that is the smallest element in its row and is also the largest element in its column. This entry is then called the **saddle point** and is the **value** of the game. If the value is positive, the game favors player I, if the value is negative, the game favors player II and if the value is zero, the game is **fair**. The row containing the saddle point is the best strategy of player I and the column containing the saddle point is the best strategy of player II, such strictly determined games are called **game of pure strategy**.

C. *Mixed Strategies*

The **expected payoff** E of player I in a two-person zero-sum game, defined by the matrix A, in which the row vector P and column vector Q define the respective strategy probabilities of player I and player II is

$$E = PAQ$$

D. *Optimal Strategy in "Two-Person, Zero-Sum Games" with 2X2 Matrices*

Using the model of "Rise and Fall of Stock market prices" presented earlier, we obtain:

$$E(1)=P.A.Q= [p_1\ p_2]\begin{bmatrix} a_{11} & a_{1n} \\ a_{n1} & a_{nn}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$$

$$E(2)=P.A.Q= [p_1\ p_2]\begin{bmatrix} a_{11} & a_{1n} \\ a_{n1} & a_{nn}\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$$

$$E(1)= E(2) \text{ and } p_1 + p_2 = 1$$

leading to the following theorem:

The optimal strategy for player I is P=[p_1 p_2] where

$$p_1 = \frac{a_{22}-a_{21}}{a_{11}+a_{22}-a_{12}-a_{21}},\ p_2 = \frac{a_{11}-a_{12}}{a_{11}+a_{22}-a_{12}-a_{21}},$$
$$a_{11} + a_{22} - a_{12} - a_{21} \neq 0$$

The optimal strategy for player II is $Q = \begin{bmatrix}q_1\\q_2\end{bmatrix}$ where

$$q_1 = \frac{a_{22}-a_{12}}{a_{11}+a_{22}-a_{12}-a_{21}},\ p_2 = \frac{a_{11}-a_{21}}{a_{11}+a_{22}-a_{12}-a_{21}},$$
$$a_{11} + a_{22} - a_{12} - a_{21} \neq 0$$

The expected payoff E corresponding to the optimal strategy is

$$E = PAQ = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

III. GENERALIZED "OPTIMAL STRATEGY IN TWO PERSON ZERO-SUM GAME" TO AN N X N MATRIX

A. *Definitions and Notations*

The "Game or Payoff" Matrix, *A:*
Let *A* be a n x n matrix representing the Markov Chain "game matrix" or "payoff matrix" in the "two-person zero-Sum games". *A* is represented as followed:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn}\end{bmatrix}$$

Trader, Player-I Probability Vector, P:
P: the best currency pair investment or the "optimal strategy" for the trader, 1 x n probability distribution vector to trade n pairs of currencies. The trader plays rows and is represented by "player-I" in the "two-person zero-sum game" model.

$$P = [p_1\ p_2\ \dots\ p_{n-1}\ p_n]^T$$

Forex Market, Player-II Probability Vector, Q:
Q: n x 1 probability distribution of the Forex market conditions (up or down) for each pair of currency. The Forex market plays columns and is represented by "player-II" in the "two-person zero-sum game" model.

$$Q=\begin{bmatrix} q_1 \\ \vdots \\ q_{n-1} \\ q_n\end{bmatrix},$$

Expected Payoff:
The Expected Payoff is defined by:

$$E=P.A.Q$$

B. *Trader Best Strategy, Optimal Currency*

Matrix Calculus:
Let *U*, *V* be column vectors and *A* be an *n x n* matrix. Let q_y be the *y*^*th*^ entry of *V*, it can then be shown that (refer to [4] for details on matrix calculus):

$$\frac{\partial U^T AV}{\partial V} = U^T A$$

Hence, for

$$V = [q_1 \dots q_y \dots q_n]^T$$

We obtain

$$\frac{\partial U^T AV}{\partial q_y} = U^T A \frac{\partial V}{\partial q_1} = U^T A \begin{bmatrix}0\\ \vdots \\ 1 \\ \vdots \\ 0\end{bmatrix}$$

Optimal *P*, Function of *A* entries:
Assuming player-I chooses rows 1, 2, …, n-1 with probabilities $p_1, p_2, \dots, p_{n-1}$, then player-I chooses row n with probability $p_n = 1 - \sum_1^{n-1} p_i$, so that $\sum_{i=1}^{n} p_i = 1$ with $p_i \geq 0$, leading to

$$P = [p_1\ p_2\ \dots\ p_{n-1}\ (1 - \sum_1^{n-1} p_i)]^T$$

It follows that the optimal payoff E_y with regard to the y^{th} row, when player-II choses that row again player-I is

$$E_y = [p_2 \dots p_{n-1}\left(1 - \sum_1^{n-1} p_i\right)]\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn}\end{bmatrix}\begin{bmatrix}0\\ \vdots \\ \text{y}^{\text{th}}\text{ entry} = 1 \\ \vdots \\ 0\end{bmatrix}$$

$$E_y = [p_1\ p_2\ \dots\ p_{n-1}\ (1 - \textstyle\sum_1^{n-1} p_i)]\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn}\end{bmatrix}\begin{bmatrix}0\\ \vdots \\ 1 \\ \vdots \\ 0\end{bmatrix}$$

$$E_y = p_2 \dots p_{n-1}\ (1 - \textstyle\sum_1^{n-1} p_i)]\begin{bmatrix}a_{1y}\\ \vdots \\ a_{ny}\end{bmatrix}$$

$$=p_1 a_{1y} + p_2 a_{2y} \dots p_{n-1} a_{(n-1)y} + (1 - \textstyle\sum_1^{n-1} p_i)\, a_{ny}$$
$$=\textstyle\sum_{i=1}^{n-1} p_i\ a_{iy} + (1 - \sum_1^{n-1} p_i)\, a_{ny}$$

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$$E_y = \sum_{i=1}^{n-1} p_i\ a_{iy} + (1 - \sum_1^{n-1} p_i)\ a_{ny}$$

It follows that, for $y+1 \leq n$

$$E_{y+1} = \sum_{i=1}^{n-1} p_i\ a_{i(y+1)} + (1 - \sum_1^{n-1} p_i)\ a_{n(y+1)}$$

Using calculus of variation, (refer to ([2] Chap.2) for details on extremals), the optimal payoff is obtained for

$$E_1 = E_2 = \dots = E_y = E_{y+1} = \dots = E_n$$

Where $E_i = \frac{\partial E}{\partial q_i}$ is the derivative with regard to the q_i situated at the $i^{th} = y^{th}$ entry of Q.

Leading to n-1 equalities and equations of the form:

$$E_y = E_{y+1} = \dots = E_n \text{ for } y=1, \dots, n-1$$

$$E_y = E_{y+1} \Leftrightarrow \sum_{i=1}^{n-1} p_i\ a_{iy} + (1 - \sum_1^{n-1} p_i)\ a_{ny} = \sum_{i=1}^{n-1} p_i\ a_{i(y+1)} + (1 - \sum_1^{n-1} p_i)\ a_{n(y+1)}$$

$$\Leftrightarrow \sum_{i=1}^{n-1} p_i\ (a_{iy} - a_{i(y+1)} - a_{ny} + a_{n(y+1)}) = a_{n(y+1)} - a_{ny}$$

This leads to a system of n-1 equations with n-1 unknown p_1 $p_2, \dots, p_{n-1}$. Such a system can be represented by

$$M.P_{1\dots(n-1)} = X$$

Where M is n-1 x n-1 matrix of the system of equations, X is n-1 x 1 column vector and $P_{1\dots(n-1)}$ is the 1 x n-1 column vector containing the first n-1 probabilities of the player-I:

Summary Results for *P*:

$$M = [m_{ik}],\ m_{ik} = (a_{ik} - a_{i(k+1)} - a_{nk} + a_{n(k+1)})$$

$$X = \{x_i\},\ x_i = a_{n(i+1)} - a_{ni}$$

$$P_{1\dots(n-1)} = \{p_i\} = [p_1\ p_2\ \dots\ p_{n-1}]^T$$

$$\boldsymbol{\Rightarrow P_{1\dots(n-1)} = M^{-1}.X \text{ and } p_n = 1 - \sum_{k=1}^{k=n-1} p_k,}$$

$$\textbf{for i=1, 2, \dots, n-1}$$

C. *Forex Market Optimal Up and Down*

A reasoning similar to the previous section leads to the following:

Optimal *Q*, Function of *A* entries:

$$\sum_{j=1}^{n} q_j = 1, p_j \leq 1 \Rightarrow q_n = 1 - \sum_1^{n-1} q_j \Rightarrow$$

$$Q = \begin{bmatrix} q_1 \\ \vdots \\ q_{n-1} \\ (1 - \sum_1^{n-1} q_j) \end{bmatrix}$$

If player-I chooses a given row *x* of *A*, all entries of *P* are equal to 0 except the x^{th} entry which is 1. The expected payoff is:

$$E_x = P.A.Q =$$

$$[\ 0 \dots\ 1 \dots\ 0] \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_{n-1} \\ (1 - \sum_1^{n-1} q_j) \end{bmatrix}$$

$$E_x = [q_1\ a_{x1} + q_2 a_{x2} \dots q_{n-1}\ a_{x(n-1)} + (1 - \sum_1^{n-1} q_j)\ a_{xn}]$$

$$= \sum_{j=1}^{n-1} q_j\ a_{xj} + (1 - \sum_1^{n-1} p_j)\ a_{xn}$$

$$E_x = E_{x+1}, \text{ for } x=1, \dots, n-1$$

$$E_x = E_{x+1} \Leftrightarrow \sum_{j=1}^{n-1} q_j\ a_{xj} + (1 - \sum_1^{n-1} q_j)\ a_{xn} = \sum_{j=1}^{n-1} q_j\ a_{(x+1)j} + (1 - \sum_1^{n-1} q_j)\ a_{(x+1)n}$$

$$\Leftrightarrow \sum_{j=1}^{n-1} q_j\ (a_{xj} - a_{(x+1)j} - a_{xn} + a_{(x+1)n}) = a_{(x+1)n} - a_{xn}$$

We obtain a system of n-1 equations of n-1 unknowns $q_1, q_2, \dots\ q_{n-1}$, which can be represented by

$$N.Q_{1\dots(n-1)} = Y$$

Where N is n-1 x n-1 matrix of the system of equations, Y is n-1 x 1 column vector and $Q_{1\dots(n-1)}$ is the 1 x n-1 vector containing the first n-1 probabilities of the player-II:

Summary Results for *Q*:

$$N = [n_{kj}],\ n_{kj} = (a_{kj} - a_{(k+1)j} - a_{kn} + a_{(k+1)n})$$

$$Y = [y_j],\ y_j = a_{(j+1)n} - a_{jn}$$

$$Q_{1\dots(n-1)} = \{q_j\} = [q_1\ q_2\ \dots\ q_{n-1}]^T$$

$$\boldsymbol{\Rightarrow Q_{1\dots(n-1)} = N^{-1}.Y \text{ and } q_n = 1 - \sum_{j=1}^{j=n-1} q_l,}$$

$$\textbf{for k=1, 2, \dots, n-1}$$

The Optimal Expected Payoff, *E:*

Formula, Assembling *E*:
Following the previous sections, E is entirely defined by the entries of *A*.

$$E = P.A.Q = [p_1\ p_2\ \dots\ p_{n-1}\ p_n] \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_{n-1} \\ q_n \end{bmatrix}$$

1- Assembling *P*:

$M = [m_{ik}],\ m_{ik} = (a_{ik} - a_{i(k+1)} - a_{nk} + a_{n(k+1)})$, i=1, 2, …, n-1

$X = [x_k],\ x_k = a_{n(k+1)} - a_{nk}$

$\Rightarrow [p_1\ p_2\ \dots\ p_{n-1}]^T = M^{-1}.X$, for the first *(n-1)*th *entries* and the n^{th} entry is given by $p_n = 1 - \sum_{i=1}^{i=n-1} p_i$

$\Rightarrow P = [p_1\ p_2\ \dots\ p_{n-1}\ p_n]^T = [M^{-1}X \quad p_n]^T$

2- Assembling *Q*:

$N = [n_{kj}],\ n_{kj} = (a_{kj} - a_{(k+1)j} - a_{kn} + a_{(k+1)n})$, j=1, 2, …, n-1

$Y = [y_k],\ y_k = a_{(k+1)n} - a_{kn}$

$\Rightarrow [q_1\ q_2\ \dots\ q_{n-1}]^T = N^{-1}.Y$, for the first *(n-1)*th entries and the n^{th} entry is given by $q_n = 1 - \sum_{j=1}^{j=n-1} q_j$

$\Rightarrow Q = [q_1\ q_2\ \dots\ q_{n-1}\ q_n]^T = [N^{-1}X \quad q_n]^T$

For k=1, 2, … , n-1

3- Calculating *E*:

To evaluate E, first one has to calculate M^{-1} and X, then evaluate $(M^{-1}X)$ which gives the *(n-1)* first $\boldsymbol{p_i}$ and then, deduce p_n by $\boldsymbol{p_n = 1 - \sum_{i=1}^{i=n-1} p_i}$. Similarly, evaluate $N^{-1}Y$, and the *(n-1)* first $\boldsymbol{q_i}$, then deduce $\boldsymbol{q_n = 1 - \sum_{j=1}^{j=n-1} q_j}$.

$$\boldsymbol{E = P.A.Q = [(M^{-1}X) \quad p_n]^T.A.[(N^{-1}Y) \quad q_n]}$$

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Or

$$E = \left[(M^{-1}X)\ \left(1 - \sum_{i=1}^{i=n-1} p_i \right) \right]^T . A . \left[(N^{-1}Y)\ \left(1 - \sum_{j=1}^{j=n-1} q_j \right) \right]$$

Alternative Assembling Method for *E*:

1- Assembling *M and N, X and Y*:

The representation can be rearranged as followed:

M=$\{m_{ik}\}$, $m_{ik} = \mathbf{a}_{n(k+1)} - \mathbf{a}_{nk} - (\mathbf{a}_{i(k+1)} - \mathbf{a}_{ik})$, i=1, 2, … , n-1, k=1, 2, … , n-1

X=$\{x_k\}$, $x_k = \mathbf{a}_{n(k+1)}$ - $\mathbf{a}_{nk}$, k=1, 2, … , n-1

N=$\{n_{kj}\}$, $n_{kj} = \mathbf{a}_{(k+1)n} - \mathbf{a}_{kn} - (\mathbf{a}_{(k+1)j} - \mathbf{a}_{kj})$, j=1, 2, … , n-1, k=1, 2, … , n-1

Y=$\{y_k\}$, $y_k = \mathbf{a}_{(k+1)n}$ - $\mathbf{a}_{kn}$, k=1, 2, … , n-1

2- Assembling *P*:

Clearly, the entries of the matrix M are obtained from the sub-matrix $\mathbf{A}_{(n-1)x\,(n-1)}$ by replacing the $\boldsymbol{i^{th}}$ row, *i*≤n-1, by

$$\{\boldsymbol{m}_{i,[1\ldots(n-1)]}\} = \left(\{\mathbf{a}_{n,[2\ldots n]}\} - \{\mathbf{a}_{n,[1\ldots(n-1)]}\}\right) - \left(\{\mathbf{a}_{i,[2\ldots n]}\} - \{\mathbf{a}_{i,[1\ldots(n-1)]}\}\right)$$

Or by replacing the $\boldsymbol{j^{th}}$ column, *j*≤n-1, by

$$\{\boldsymbol{m}_{[1\ldots(n-1)],j}\} = (\mathbf{a}_{n,(j+1)} - \mathbf{a}_{n,j})\{\mathbf{1}\}_{n-1} - \left(\{\mathbf{a}_{[1\ldots(n-1)],j+1}\} - \{\mathbf{a}_{[1\ldots(n-1)],j}\}\right)$$

Meanwhile, the entries of X are:

$$\mathbf{X} = \{\mathbf{a}_{n,[2\ldots n]}\} - \{\mathbf{a}_{n,[1\ldots(n-1)]}\}$$

Where:

$\{\boldsymbol{m}_{i,[1\ldots(n-1)]}\}$ is the *i*th row of **M**, with entries from column 1 to n-1

$\{\boldsymbol{m}_{[1\ldots(n-1)],j}\}$ is the *j*th column of **M**, with entries from row 1 to n-1

$\{\mathbf{a}_{[1\ldots(n-1)],j}\}$ is the *j*th column of **A**, with entries from row 1 to n-1

$\{\mathbf{a}_{[1\ldots(n-1)],j+1}\}$ is the *(j+1)*th column of **A**, with entries from row 1 to n-1

$\{\mathbf{a}_{n,[2\ldots n]}\}$ is the *n*th row of **A**, with entries from column 2 to n

$\{\mathbf{a}_{n,[1\ldots(n-1)]}\}$ is the *n*th row of **A**, with entries from column 1 to n-1

Etc…

3- Assembling *Q*:

Conversely, the entries of the matrix N are obtained from the sub-matrix $\mathbf{A}_{(n-1)x\,(n-1)}$ by replacing the $\boldsymbol{j^{th}}$ column, *j*≤n-1, by

$$\{\boldsymbol{n}_{[1\ldots(n-1)],j}\} = \left(\{\mathbf{a}_{[2\ldots n],n}\} - \{\mathbf{a}_{[1\ldots(n-1)],n}\}\right) - \left(\{\mathbf{a}_{[2\ldots n],j}\} - \{\mathbf{a}_{[1\ldots(n-1)],j}\}\right)$$

Or by replacing the $\boldsymbol{i^{th}}$ row **A**, *i*≤n-1, by

$$\{\boldsymbol{n}_{i,[1\ldots(n-1)]}\} = (\{\mathbf{a}_{(i+1),n}\} - \{\mathbf{a}_{i,n}\})\{\mathbf{1}\}_{n-1} - \left(\{\mathbf{a}_{(i+1),[1\ldots(n-1)]}\} - \{\mathbf{a}_{i,[1\ldots(n-1)]}\}\right)$$

Meanwhile, the entries of Y are:

$$\mathbf{Y} = \left(\{\mathbf{a}_{[2\ldots n],n}\} - \{\mathbf{a}_{[1\ldots(n-1)],n}\}\right)$$

Where:

$\{\boldsymbol{n}_{[1\ldots(n-1)],j}\}$ is the *j*th column of *N*, with entries from row 1 to n-1

$\{\boldsymbol{n}_{i,[1\ldots(n-1)]}\}$ is the *i*th row of *N*, with entries from column 1 to n-1

$\{\mathbf{a}_{[1\ldots(n-1)],j}\}$ is the *j*th column of **A**, with entries from row 1 to n-1

$\{\mathbf{a}_{[2\ldots n],j}\}$ is the *j*th column of **A**, with entries from row 2 to n.

$\{\mathbf{a}_{i+1,[1\ldots(n-1)]}\}$ is the *(i+1)*th row of **A**, with entries from column 1 to n-1

$\{\mathbf{a}_{[2\ldots n],n}\}$ is the *n*th column of **A**, with entries from row 2 to n

$\{\mathbf{a}_{[1\ldots(n-1)],n}\}$ is the *n*th column of **A**, with entries from row 1 to n-1

Etc…

IV. APPLICATION EXAMPLE WITH ACTUAL DATA

Problem Formulation

TABLE 1 - TRADING DATA

1.3979	1.3976	1.3972	1.3991	1.3891	1.3969	1.3971	1.3991
1.3882	1.3975	1.3974	1.3988	1.3890	1.3976	1.3965	1.3988
1.3872	1.3977	1.3969	1.3984	1.3897	1.3974	1.3968	1.3985
1.3910	1.3980	1.3969	1.3996	1.3901	1.3980	1.3961	1.3981
1.3903	1.3987	1.3970	1.3994	1.3904	1.3986	1.3957	1.3974
1.3885	1.3997	1.3976	1.4000	1.3908	1.3984	1.3965	1.3972
1.3901	1.3993	1.3970	1.3990	1.3912	1.3986	1.3970	1.3970
1.3900	1.3988	1.3974	1.3991	1.3913	1.3987	1.3966	1.3965
1.3893	1.3984	1.3983	1.3988	1.3919	1.3988	1.3965	1.3968
1.3893	1.3977	1.3979	1.3985	1.3917	1.3983	1.3961	1.3968
1.3904	1.3979	1.3980	1.3989	1.3916	1.3984	1.3953	1.3971
1.3903	1.3983	1.3987	1.3990	1.3918	1.3990	1.3973	1.3975
1.3908	1.3981	1.3989	1.3987	1.3952	1.3984	1.3974	1.3965
1.3910	1.3980	1.3982	1.3989	1.3966	1.3981	1.3977	1.3960
1.3906	1.3987	1.3985	1.3988	1.3963	1.3978		1.3952

1. Statistical Approach: Using actual data (Table 1), what are average Euro/USD rate, standard deviation, median and mode? Draw the histogram and frequency polygon.

2. Probability: Using actual trading data over a two months period in 2009, a total of $265 was gained when $1080 was lost, leading to the odds for winning of $265/$1080. What is the probability of winning, losing?

3. Trader's Ruin: Assuming the trader starts with a balance of $500 and will lose or win $2 par trade, using the above win/lose probability figures, what is the probability the initial $500 balance decreases to $494 or increases to $506? If the balance reaches $6, what is the probability of getting broke (Hint: use the Gambler's Ruin Problem model)?

4. Optimal Currency Pair: The trader can exchange several pairs of currencies; meanwhile the forex market will go up and down for each pair. Assuming the trader will win or lose at least the "spread" of each pair as summarized in the below 6x6 matrix for three currency pairs, what is the optimal pair of currencies to trade, amount EUR/USD, GBP/USD and JPY/USD ? (1- Propose a generalization of the 2 x 2 matrix to the "optimal strategy for a 2-person zero-sum game with an n x n matrix". 2- Apply to the case of a 6 x 6 matrix below, each pair having two possibilities of "sell" or "buy" versus its "up and down".

TABLE 2 – SPREAD OF TRADE OR NUMBER OF PIP (PERCENTAGE IN POINT BETWEEN SELL AND BUY)

		Foreign Exchange Market (forex)					
		Up	Down	Up	Down	Up	Down
	Euro Sell	-4	2	0	0	0	0
	Euro Buy	2	-4	0	0	0	0
Trader	GBP Sell	0	0	-6	3	0	0
	GBP Buy	0	0	3	-6	0	0
	JPY Sell	0	0	0	0	-10	5
	JPY Buy	0	0	0	0	5	-10

5. Predicting the Rise and Fall: To decide rather or not we "buy" (the exchange rate will rise) or "sell" (the exchange rate will fall) and at what time we would likely close the position. Using actual data below (Table 2), calculate the average amount of time elapsed between visits to each state (increase, decrease, neutral). (Hint: use the "mean recurrence time" and the Rise and Fall of Stock model).Note: data points are shown in reverse order, they start at {1.4026,1.4014, -12.00} up to {1.4425, 1.4419, 6.00}, read from bottom up, for each set of three columns.

TABLE 3 - RE-ORDERED DATA

Open Price	Close Price	Profit	Open Price	Close Price	Profit	Open Price	Close Price	Profit	Open Price	Close Price	Profit
1.4425	1.4419	6.00	1.3906	1.3905	-6.00	1.3963	1.3965	2.00	1.3983	1.3983	2.00
1.3926	1.3926	2.00	1.3901	1.3901	0.00	1.3965	1.3961	-4.00	1.3983	1.3982	4.00
1.3979	1.3981	2.00	1.3891	1.3894	-6.00	1.3972	1.3964	-8.00	1.3971	1.3984	-52.00
1.3882	1.3892	-10.00	1.3890	1.3893	-10.00	1.3974	1.3972	-2.00	1.3968	1.3981	-52.00
1.3872	1.3882	-10.00	1.3897	1.3894	6.00	1.3969	1.3974	-5.00	1.3966	1.3965	6.00
1.3910	1.3911	1.00	1.3901	1.3899	4.00	1.3969	1.3972	-12.00	1.3961	1.3962	2.00
1.3903	1.3898	-5.00	1.3904	1.3900	8.00	1.3970	1.3968	-8.00	1.3957	1.3960	6.00
1.3885	1.3895	8.00	1.3906	1.3907	2.00	1.3976	1.3970	-24.00	1.3968	1.3955	-20.00
1.3901	1.3882	-19.00	1.3912	1.3910	4.00	1.3970	1.3975	-20.00	1.3970	1.3969	2.00
1.3900	1.3905	-5.00	1.3913	1.3911	-4.00	1.3974	1.3973	4.00	1.3966	1.3972	-12.00
1.3893	1.3904	-22.00	1.3919	1.3911	-16.00	1.3963	1.3974	-56.00	1.3968	1.3964	2.00
1.3893	1.3898	4.00	1.3917	1.3919	-4.00	1.3979	1.3981	8.00	1.3961	1.3971	-20.00
1.3904	1.3905	2.00	1.3916	1.3919	-6.00	1.3980	1.3979	4.00	1.3955	1.3970	-60.00
1.3905	1.3901	4.00	1.3918	1.3916	-4.00	1.3987	1.3980	-28.00	1.3975	1.3928	-180.00
1.3906	1.3907	2.00	1.3952	1.3957	-5.00	1.3969	1.3967	8.00	1.3974	1.3945	-116.00
1.3910	1.3915	6.00	1.3966	1.3957	9.00	1.3962	1.3961	-4.00	1.3977	1.3976	4.00

TABLE 4 - RE-ORDERED DATA (FOLLOWED)

Open Price	Close Price	Profit	Open Price	Close Price	Profit	Open Price	Close Price	Profit	Open Price	Close Price	Profit
1.3975	1.3976	2.00	1.3970	1.3975	5.00	1.3978	1.3977	1.00	1.3987	1.3985	2.00
1.3976	1.3979	3.00	1.3969	1.3971	2.00	1.3991	1.3987	4.00	1.3991	1.3989	2.00
1.3975	1.3977	2.00	1.3976	1.3978	2.00	1.3988	1.3988	2.00	1.3985	1.3985	5.00
1.3977	1.3978	2.00	1.3974	1.3975	2.00	1.3984	1.3982	2.00	1.3985	1.3981	4.00
1.3980	1.3983	6.00	1.3980	1.3980	0.00	1.3996	1.3994	2.00	1.3981	1.3981	0.00
1.3987	1.3980	-214.00	1.3986	1.3985	-2.00	1.3994	1.3993	1.00	1.3974	1.3978	-4.00
1.3997	1.3994	6.00	1.3984	1.3985	2.00	1.4000	1.3999	1.00	1.3972	1.3981	-9.00
1.3993	1.3991	4.00	1.3986	1.3978	-32.00	1.3990	1.3988	2.00	1.3970	1.3990	-20.00
1.3988	1.3984	4.00	1.3987	1.3988	4.00	1.3991	1.3988	5.00	1.3968	1.3970	5.00
1.3984	1.3986	-2.00	1.3985	1.3988	8.00	1.3988	1.3987	1.00	1.3968	1.3971	5.00
1.3977	1.3980	3.00	1.3985	1.3985	2.00	1.3985	1.3985	0.00	1.3968	1.3972	4.00
1.3979	1.3980	2.00	1.3984	1.3989	5.00	1.3989	1.3987	2.00	1.3971	1.3975	2.00
1.3983	1.3984	2.00	1.3990	1.3989	-1.00	1.3990	1.3988	2.00	1.3975	1.3975	2.00
1.3981	1.3983	4.00	1.3984	1.3988	4.00	1.3987	1.3984	5.00	1.3968	1.3968	5.00
1.3980	1.3981	1.00	1.3981	1.3982	1.00	1.3989	1.3987	2.00	1.3960	1.3965	5.00
1.3987	1.3985	-2.00	1.3978	1.3979	1.00	1.3988	1.3987	1.00	1.3952	1.3951	-1.00
									1.4026	1.4014	-12.00

Solution

Statistical Analysis

Using Excel and the functions average(), median(), mode() and graphic capabilities, we obtain:

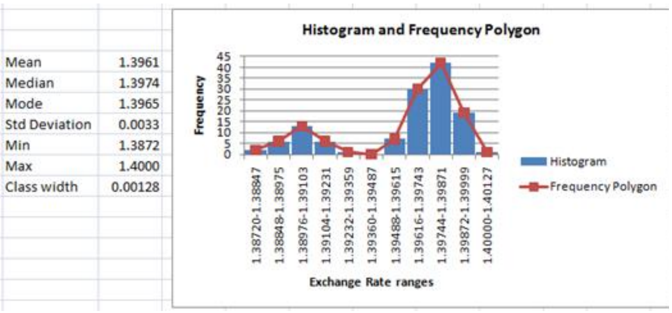

Fig. 1 - Graph and Results from Statistical Analysis

Probability from Odds

If the odds for winning are a/b, the probability for winning is p=a/(a+b)=$265/($265+$1080)=0.197 and the probability of losing is 0.803.

Trader's Ruin

The transition matrix is rearranged as showed on the following tables.

TABLE 5 - TRANSITION MATRIX

Euro/USD		Forex Market $494	$496	$498	$500	$502	$504	$506
	$494	1	0	0	0	0	0	0
	$496	0.803	0	0.197	0	0	0	0
	$498	0	0.803	0	0.197	0	0	0
Trader	**$500**	0	0	0.803	0	0.197	0	0
	$502	0	0		0.803	0	0.197	0
	$504	0	0	0	0	0.803	0	0.197
	$506	0	0	0	0	0	0	1

TABLE 6 - TRANSITION MATRIX, REARRANGED

P=	$494	$506	$496	$498	$500	$502	$504
$494	1	0	0	0	0	0	0
$506	0	1	0	0	0	0	0
$496	0.803	0	0	0.197	0	0	0
$498	0	0	0.803	0	0.197	0	0
$500	0	0	0	0.803	0	0.197	0
$502	0	0	0	0	0.803	0	0.197
$504	0	0.197	0	0	0	0.803	0

Using Excel, the fundamental matrix of the Markov chain is given by $T=[I-Q]^{-1}$:

TABLE 7 - FUNDAMENTAL MATRIX AND NUMBER OF TRADE BEFORE ABSORPTION

$T=[I-Q]^{-1}$, fundamental matrix	$2	$4	$6	$8	$10	Expected number of trade before absorption
$2	1.245	0.305	0.074	0.017	0.003	1.644
$4	1.241	1.546	0.375	0.088	0.017	3.267
$6	1.227	1.528	1.602	0.375	0.074	4.807
$8	1.171	1.458	1.528	1.546	0.305	6.007
$10	0.94	1.171	1.227	1.241	1.245	5.824

and the probability of being absorbed (broken) is given by the probability matrix T.S:

TABLE 8 - PROBABILITY OF BEING ABSORBED

T.S: Probability of being Absorbed	$494	$506	T.S: Probability of being Absorbed	$0	$12
$496	0.999	0.001	$2	0.999	0.001
$498	0.997	0.003	$4	0.997	0.003
$500	**0.985**	**0.015**	$6	**0.985**	**0.015**
$502	0.940	0.060	$8	0.940	0.060
$504	0.755	0.245	$10	0.755	0.245

Starting at $500, the probability of moving to $494 is 0.985; the probability of moving to $506 is 0.015.

Starting at $6, the probability of getting broken is 0.985. in conclusion, the probability of losing is very high, meaning that trading forex is a very risky business that requires a lot of experience.

Optimal Currency Trade

The transition matrix of spread is given in Table 2:

m_{kl} and n_{kl} are calculated using the Excel INDEX() function.

$[y_k]$ and $[x_k]$ are calculated combining Excel MMULT() and INVERSE() functions. The results are given by:

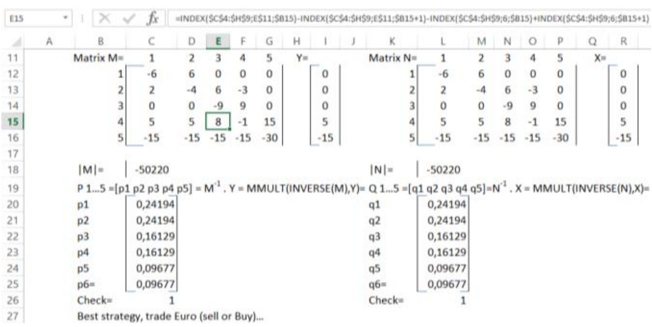

Fig. 2 – Calculations of Entries of Matrices M and N

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The expected payoff is $\boldsymbol{E} =$

$$[0.241935\ 0.241935\ 0.16129\ 0.16129\ 0.096774\ 0.096774]^* \begin{bmatrix} -4 & 2 & 0 & 0 & 0 & 0 \\ 2 & -4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6 & 3 & 0 & 0 \\ 0 & 0 & 3 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 & 5 \\ 0 & 0 & 0 & 0 & 5 & -10 \end{bmatrix} \begin{bmatrix} 0.241935 \\ 0.241935 \\ 0.16129 \\ 0.16129 \\ 0.096774 \\ 0.096774 \end{bmatrix}$$

$$\boldsymbol{E = -0.4838}, \textbf{ unfavorable to the trader.}$$

The optimal strategy for the trader is the rows 1 and 2 with highest probabilities of 0.24 as shown in the probability distribution vector P. However, the expected value is negative indicating that even the trader plays rows 1 and 2 (Euro/USD sell and buy), he/she will still lose.

Predicting the Rise and Fall

Data reporting the past history of currency trading in 2009 were rearranged in three columns. Additional columns "I to I", "I to D", etc were added to return 1 or 0. For instance, in the column H "I to I", row 128 (cell H128), the exchange rate that was increasing from 1.3952 to 1.3960, closed higher at 1.3965, "I to I"=1, all the other "X" to "Y" are zero. We used the IF() and AND() functions as shown to complete all the "X" to "Y" columns and calculate the probability of "X to Y" and statistically fits the transition matrix to the model:

Note: For clarity purposes, only partial data are displayed to show an example (H128) of the formulae used.

H128 =IF(AND($C128>$C129;$D128>$C128);1;0)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1		Time Position Opened	Open Price	Close Price	Profit	Win	Lose	I to I	I to D	I to N	D to I	D to D	D to N	N to I	N to D	N to N
120		2009.07.03 01:43	1,3972	1,3981	-$9	$0	$9	1	0	0	0	0	0	0	0	0
121		2009.07.03 01:37	1,3970	1,3990	-$20	$0	$20	1	0	0	0	0	0	0	0	0
122		2009.07.03 01:18	1,3965	1,3970	$5	$5	$0	0	0	0	1	0	0	0	0	0
123		2009.07.03 01:14	1,3968	1,3971	$3	$3	$0	0	0	0	0	0	0	1	0	0
124		2009.07.03 01:08	1,3968	1,3972	$4	$4	$0	0	0	0	1	0	0	0	0	0
125		2009.07.03 00:55	1,3971	1,3973	$2	$2	$0	0	0	0	1	0	0	0	0	0
126		2009.07.03 00:54	1,3975	1,3973	$2	$2	$0	0	1	0	0	0	0	0	0	0
127		2009.07.03 00:36	1,3965	1,3968	$3	$3	$0	1	0	0	0	0	0	0	0	0
128		2009.07.03 00:16	**1,3960**	**1,3965**	$5	$5	$0	**1**	0	0	0	0	0	0	0	0
129		2009.07.02 21:42	**1,3952**	1,3951	-$1	$0	$1	0	0	0	0	1	0	0	0	0
130		2009.07.02 17:59	1,4026	1,4014	-$12	$0	$12									
131																
132				TOTAL:	-$815	$265	$1 080	25,00	36,00	2,00	28,00	31,00	2,00	4	0	0
133								Subtotal:		63,00			61,00			4
134		Odds for Winning:		265,00	to	$1 080		**0,39683**	**0,571**	**0,0317**	**0,459**	**0,508**	**0,03279**	**1**	**0**	**0**
135		Probability of Winning a/(a+b):		0,19703				**Probability X to Y**								
136		Probability of Losing b/(a+b):		0,80297												
137		Check:		1												

Fig. 3 – Counting Rise and Fall with Excel. The probability X to Y (I to I, I to D, etc) is the probability that a rate that was in state X moves to state Y. For instance 0.397 is the probability that a rate that was increasing continues to increase (I to I).

By considering N as the absorb state, the results are:

N absorbing state	N	I	D	Identity matrix I		Fundamental matrix $T=[I-Q]^{-1}$	I	D	days to steady
N	1	0	0						
I	0.032	0.397	0.571	1	0	I	14.32	16.64	30.95
D	0.033	0.459	0.508	0	1	D	13.36	17.56	30.92

Results Interpretation: | Probability
14,32 days start in I, average time spent in I, before unchanged | T.S
16,64 days start in I, average time spent in D, before unchanged
13,36 days start in D, average time spent in I, before unchanged | 1,00
17,56 days start in D, average time spent in D, before unchanged | 1,00

Probability T.S | Fixed Probability Vector

T.S	P			P.P			P.P.P			P.P.P.P		
	0,397	0,571	0,032	0,452	0,517	0,031	**0,448**	**0,521**	**0,031**	0,448	0,521	0,031
1,00	0,459	0,508	0,033	0,448	0,521	0,031	0,448	0,521	0,031	0,448	0,521	0,031
1,00	1,000	-	-	0,397	0,571	0,032	0,448	0,521	0,031	0,448	0,521	0,031

Fixed Probability Vector

	I	D	N	Check
t=	0,448	0,521	0,031	1,000
1/ti=	**2,232**	**1,921**	**31,954**	

Euro/USD increases **44,81%** of the time
Euro/USD decreases **52,06%** of the time
Euro/USD neutrals **3,13%** of the time
Conclusion: mostly decreases, so sell!

1/ti: Mean Recurrence Time
t: Average number of time elapsed between visits to state ith.

Fig. 4 - Results

V. CONCLUSION

The generalized approach outlined herein could serve as "a posteriori" (after the fact) estimate that is used to analyze the trade, up to a given point of time, and then allows the trader to reconsider or adjust the current strategy or selection of optimal pair of currency, in term of expected payoff. The tool can then be used as "a priori" (before the fact) indicator for the next currency pairs to trade. This model is a technical analysis tool for selecting the optimal pair when trading multiple currency pairs. It predicts the best pair to focus on and the expected return in term of payoff. Finally, the formulas found in section III, used for a given set of currency, confirms an intuitive result which is that trading forex is intrinsically "unfavorable" to the trader.

TABLE 9 - SYMBOLS

Symbol	Description
T	Markov Chain Fundamental Matrix
P	Markov Chain Transition Matrix
I_r	Identity Matrix of Dimension r x r
I_s	Identity Matrix of Dimension s x s
$\boldsymbol{E}$	Expected Pay-off
P	Player I Probability Vector
Q	Player II Probability Vector
M^{-1}	Inverse of the Matrix M
$\mathrm{M}^{-1}\mathrm{X}$	Product of the Matrix M by the Vector X
U^T	Transpose of U
$\{\boldsymbol{m}_{\boldsymbol{i},[\mathbf{1}\ldots(\boldsymbol{n}-\mathbf{1})]}\}$	The i^{th} row of M, with entries from column 1 to n-1
$\{\boldsymbol{n}_{[\mathbf{1}\ldots(\boldsymbol{n}-\mathbf{1})],\boldsymbol{j}}\}$	The j^{th} column of N, with entries from row 1 to n-1

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