Generalized Optimal Strategy of n Currencies

Dieudonne Ndong Ovono

Abstract— A tricky way some courageous investors make money is precisely to buy or sell money itself! The exchange of foreign currency (“forex” market) is the purchase, or trade, of a particular currency from an individual or institution and the simultaneous sale of another currency at the equivalent value or current exchange rate. This exchange rate or price fluctuation is based on demand, political, social and economic events surrounding each country currency, this is a stochastic phenomenon, hence hard to predict, of course there is a strong potential for loss.

Currencies are always traded in pairs. When someone says they are “buying the EUR/USD”, they are buying Euros and simultaneous sale of another currency at the equivalent value. Currencies are always traded in pairs. The average amount of time spent in state j having started in state i before reaching the absorbing state D. To know how long a stock will spend, for instance, on state I and N before arriving in state D, one could assume D is an absorbing state provided the process began in a non-absorbing state i, we reach the absorbing state j. More details can be found in reference [6], pp.230-234.

The expected number of trials before absorption for each non-absorbing state is found by adding the entries in the corresponding row of the fundamental matrix T. Probability of being absorbed: The (i,j)th entry in the matrix product T.S gives the probability that, starting in non-absorbing state i, we reach the absorbing state j. More details can be found in reference [6], pp.230-234.

The Rise and Fall of Stock Prices: Let I (increase), D (decrease) and N (no change) be the three states of a stock price, the transition matrix P is a 3 x 3 matrix, where rows I(I), D(D) and N(N) are initial states I, D and N and columns 1, 2 and 3 are final states I, D and N. In the long run, if t=t₁ t₂ t₃ is the fixed probability vector derived from P, the entries t₁, t₂ and t₃ indicate respectively that a stock will have increased its price 100t₁% of the time, decrease it price 100t₂% of the time and remained the same 100t₃% of the time.

The average amount of time between visits to state i (called mean recurrence average) is given by the reciprocal of the iᵗʰ component of the fixed vector t. The average amount of time between visits to state i (called mean recurrence average) is given by the reciprocal of the iᵗʰ component of the fixed vector t.

Index Terms—Markov Chain, Forex Market, Trading Strategy, Expected Payoff, Two Players Game, Gambler Ruin.

I. INTRODUCTION

This paper presents a generalized approach to determine the optimal strategy when trading several pairs of currency in forex stock market. The model developed herein is based on stochastic mathematics, namely Markov Chains, briefly reviewed in section II herein, (11), Chap 7 through 10 and [6], Chap 4). Then we use advance matrix calculus, ([11], Chap 1 to 3), ([2], Chap 1), ([3], Appendix A), ([4]) and ([5]), and combine the concepts of “gambler’s ruin problem”, “two-person-zero-sum game”, rise and fall of stock market” and “mixed strategy” into a generalized approach. We then use our generalized formulæ to determine the optimal currency to trade on. In the meantime we demonstrate that the trader is a loser by default.

In section III, we present our “generalized optimal strategy” technique that uses an n x n matrix to represent several pairs of currency. The concept was developed by the author and therefore has no direct reference.

The last section illustrates the technique using data from a financial experiment conducted by the author in trading forex.

II. THEORETICAL FOUNDATIONS

A. Markov Chains and Games

In a Markov chain, let pᵢⱼ denotes the probability of remaining in state Eᵢ. If pᵢᵢ=1, then Eᵢ is called an absorbing state. A Markov chain is said to be absorbing Markov chain if and only if it contains at least one absorbing state and it is possible to go from any non-absorbing state to an absorbing state in one or more trials.

• Gambler’s Ruin Problem:

For an absorbing Markov chain that has a transition matrix P of the form

\[
P = \begin{bmatrix}
I & 0 \\
S & Q
\end{bmatrix}
\]

Where S is of dimension s x r and Q is of dimension s x s. Let the matrix \( T = [I - Q]^{-1} \). \( T \) is called the fundamental matrix of the Markov chain. The entries of \( T \) give the expected number of times the process is in each non-absorbing state provided the process began in a non-absorbing state.

The expected number of trials before absorption for each non-absorbing state is found by adding the entries in the corresponding row of the fundamental matrix T.

Probability of being absorbed: The \((i,j)\)th entry in the matrix product \( T.S \) gives the probability that, starting in non-absorbing state \( i \), we reach the absorbing state \( j \). More details can be found in reference [6], pp.230-234.

• The Rise and Fall of Stock Prices:

Let I (increase), D (decrease) and N (no change) be the three states of a stock price, the transition matrix P is a 3 x 3 matrix, where rows I(I), D(D) and N(N) are initial states I, D and N and columns 1, 2 and 3 are final states I, D and N. In the long run, if \( t=[t₁ t₂ t₃] \) is the fixed probability vector derived from P, the entries \( t₁, t₂ \) and \( t₃ \) indicate respectively that a stock will have increased its price 100\( t₁ \)% of the time, decrease it price 100\( t₂ \)% of the time and remained the same 100\( t₃ \)% of the time.

To know how long a stock will spend, for instance, on state I and N before arriving in state D, one could assume D is an absorbing state by replacing the current probability \( p_{22} \) (or \( p_{33} \)) with 1 and make all other entries in the D row (or row 2) 0. The matrix thereof could be subdivided into identity (I), zero (O), absorbing (S) and non-absorbing (Q) matrices. The \((i,j)\)th entry of \( T = [I - Q]^{-1} \) gives the average time spent in state j having started in state i before reaching the absorbing state D.

The average amount of time between visits to state i (called mean recurrence average) is given by the reciprocal of the \( i^{th} \) component of the fixed vector \( t \).

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n columns. A strategy for a given matrix is the decision for player I to select rows and player II to select columns. A game defined by a matrix is said to be strictly determined if and only if there is an entry of the matrix that is the smallest element in its row and is also the largest element in its column. This entry is then called the saddle point and is the value of the game. If the value is positive, the game favors player I, if the value is negative, the game favors player II and if the value is zero, the game is fair. The row containing the saddle point is the best strategy of player I and the column containing the saddle point is the best strategy of player II. Thus, strictly determined games are called game of pure strategy.

C. Mixed Strategies
The expected payoff $E$ of player I in a two-person zero-sum game, defined by the matrix $A$, in which the row vector $P$ and column vector $Q$ define the respective strategy probabilities of player I and player II is

$$E = PAQ$$

D. Optimal Strategy in “Two-Person, Zero-Sum Games” with 2X2 Matrices

Using the model of “Rise and Fall of Stock market prices” presented earlier, we obtain:

$$E(1) = PAQ = [p_1 p_2] \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = a_{11}q_1 + a_{12}q_2$$

$$E(2) = PAQ = [p_1 p_2] \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 0 \\ q_1 \end{bmatrix} = a_{12}q_1$$

leading to the following theorem:

The optimal strategy for player I is $P = [p_1 p_2]$ where

$$p_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

$$p_2 = \frac{a_{11} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

The optimal strategy for player II is $Q = [q_1 q_2]$ where

$$q_1 = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

$$q_2 = \frac{a_{11} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

The expected payoff $E$ corresponding to the optimal strategy is

$$E = PAQ = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

III. GENERALIZED “OPTIMAL STRATEGY IN TWO PERSON ZERO-SUM GAME” TO AN N X N MATRIX

A. Definitions and Notations

The “Game or Payoff” Matrix, $A$:

Let $A$ be a $n \times n$ matrix representing the Markov Chain “game matrix” or “payoff matrix” in the “two-person zero-Sum games”. $A$ is represented as follows:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$$

Trader. Player-I Probability Vector, $P$:

$P$: the best currency pair investment or the “optimal strategy” for the trader, a $1 \times n$ probability distribution vector to trade $n$ pairs of currencies. The trader plays rows and is represented by “player-I” in the “two-person zero-sum game” model.

$$P = [p_1, p_2, \ldots, p_{n-1}, p_n]^T$$

Forex Market, Player-II Probability Vector, $Q$:

$Q$: a $n \times 1$ probability distribution of the Forex market conditions (up or down) for each pair of currency. The Forex market plays columns and is represented by “player-II” in the “two-person zero-sum game” model.

$$Q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_{n-1} \\ q_n \end{bmatrix}$$

Expected Payoff:

The Expected Payoff is defined by:

$$E = PAQ$$

B. Trader Best Strategy, Optimal Currency

Matrix Calculus:

Let $U$, $V$ be column vectors and $A$ be an $n \times n$ matrix. Let $q_y$ be the $y^{th}$ entry of $V$, it can then be shown that (refer to [4] for details on matrix calculus):

$$\frac{\partial U^T AV}{\partial y} = U^T A$$

Hence, for

$$V = [q_1 \ldots q_y \ldots q_n]^T$$

We obtain

$$\frac{\partial U^T AV}{\partial q_y} = U^T A$$

Optimal $P$, Function of $A$ entries:

Assuming player-I chooses rows $1, 2, \ldots, n-1$ with probabilities $p_1, p_2, \ldots, p_{n-1}$, then player-I chooses row $n$ with probability $p_n = 1 - \sum_{i=1}^{n-1} p_i$, so that $\sum_{i=1}^{n} p_i = 1$ with $p_i \geq 0$, leading to

$$P = [p_1, p_2, \ldots, p_{n-1}, (1 - \sum_{i=1}^{n-1} p_i)]^T$$

It follows that the optimal payoff $E_y$ with regard to the $y^{th}$ row, when player-II chooses rows again player-I is

$$E_y = [p_1, p_2, \ldots, p_{n-1}, (1 - \sum_{i=1}^{n-1} p_i)] \begin{bmatrix} a_{11} & \cdots & a_{1y} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{ny} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$E_y = p_1 a_{1y} + p_2 a_{2y} + \cdots + p_{n-1} a_{(n-1)y} + (1 - \sum_{i=1}^{n-1} p_i) a_{ny}$$
The value of the system of n-1 equations with n-1 unknowns \( \sum_{i=1}^{n-1} p_i a_{iy} + (1 - \sum_{i=1}^{n-1} p_i) a_{ny} \).

It follows that, for \( y + 1 \leq n \)
\[
E_{y+1} = \sum_{i=1}^{n-1} p_i a_{i(y+1)} + (1 - \sum_{i=1}^{n-1} p_i) a_{n(y+1)}
\]

Using calculus of variation, (refer to [2] Chap.2) for details on extremals), the optimal payoff is obtained for
\[
E_x = \frac{\partial E}{\partial q_i} = \sum_{i=1}^{n-1} p_i a_{iy} + (1 - \sum_{i=1}^{n-1} p_i) a_{ny}
\]

Where \( E_x = \frac{\partial E}{\partial q_i} \) is the derivative with regard to the \( q_i \)
situated at the \( i^{th} \) entry of \( Q \).

Leading to a system of n-1 equations and equations of the form:
\[
E_y = \sum_{i=1}^{n-1} p_i a_{iy} + (1 - \sum_{i=1}^{n-1} p_i) a_{ny} = \sum_{i=1}^{n-1} p_i (a_{iy} - a_{ny}) = a_{n(y-1)} - a_{ny}
\]

This leads to a system of n-1 equations with n-1 unknowns \( p_1, p_2, \ldots, p_n \). Such a system can be represented by
\[
M.P_{i=1...n-1} = X
\]

Where \( M \) is n-1 x n-1 matrix of the system of equations, \( X \) is n-1 x 1 column vector and \( P_{i=1...n-1} \) is the 1 x n-1 column vector containing the first n-1 probabilities of the player-I.

**Summary Results for P:**

\[
\begin{align*}
M &= [m_{ik}], \ m_{ik} = (a_{ik} - a_{i(k+1)}) - a_{nk} + a_{n(k+1)}; \\
X &= [x_i], \ x_i = a_{n(i+1)} - a_{ni}, \ i = 1, \ldots, n-1; \\
P_{i=1...n-1} &= \{p_1, p_2, \ldots, p_{n-1}\}, \\
&\Rightarrow P_{i=1...n-1} = M^1X, \text{ and } p_{n-1} = 1 - \sum_{k=1}^{n-1} p_k, \text{ for } i = 1, 2, \ldots, n-1.
\end{align*}
\]

**C. Forex Market Optimal Up and Down**

A reasoning similar to the previous section leads to the following:

**Optimal Q, Function of A entries:**
\[
\sum_{j=1}^{n} q_j = 1, p_j \leq 1 \Rightarrow q_n = 1 - \sum_{j=1}^{n-1} q_j \Rightarrow
\]
\[
Q = \begin{bmatrix}
q_1 \\
\vdots \\
q_{n-1} \\
(1 - \sum_{j=1}^{n-1} q_j)
\end{bmatrix}
\]

If player-I chooses a given row \( x \) of \( A \), all entries of \( P \) are equal to 0 except the \( x^{th} \) entry which is 1. The expected payoff is:
\[
E_x = P.A.Q = \begin{bmatrix}
a_{11} & \ldots & a_{1n} \\
\vdots & \ddots & \vdots \\
a_{n1} & \ldots & a_{nn}
\end{bmatrix}
\begin{bmatrix}
q_1 \\
\vdots \\
q_{n-1} \\
(1 - \sum_{j=1}^{n-1} q_j)
\end{bmatrix}
\]
\[
E_x = \{q_1 a_{x1} + q_2 a_{x2} + \ldots + q_{n-1} a_{x(n-1)} + (1 - \sum_{j=1}^{n-1} q_j) a_{xn}\}
\]

The Optimal Expected Payoff, \( E \):

**Formula, Assembling E:**

Following the previous sections, \( E \) is entirely defined by the entries of \( A \).
\[
E = P.A.Q = \{p_1, p_2, \ldots, p_{n-1}, p_n\}
\]

1- Assembling \( P \):
\[
M = [m_{ik}], \ m_{ik} = (a_{ik} - a_{i(k+1)}) - a_{nk} + a_{n(k+1)}, i = 1, 2, \ldots, n-1; \\
X = [x_k], \ x_k = a_{n(k+1)} - a_{nk}, \ k = 1, 2, \ldots, n-1; \\
P_{i=1...n-1} = \{p_1, p_2, \ldots, p_{n-1}\}, \\
&\Rightarrow P_{i=1...n-1} = M^1X, \text{ for the first } (n-1)^{th} \text{ entries and the } n^{th} \text{ entry is given by } p_n = 1 - \sum_{i=1}^{n-1} p_i \\
&\Rightarrow P = \{p_1, p_2, \ldots, p_{n-1}, p_n\} = [M^1X, p_n]^T
\]

2- Assembling \( Q \):
\[
N = [n_{kj}], \ n_{kj} = (a_{kj} - a_{k(k+1)}) - a_{kn} + a_{k(1+n)}, j = 1, 2, \ldots, n-1; \\
Y = [y_k], \ y_k = a_{k(1+n)} - a_{kn}, \ k = 1, 2, \ldots, n-1; \\
Q_{i=1...n-1} = \{q_1, q_2, \ldots, q_{n-1}\}, \text{ for the first } (n-1)^{th} \text{ entries and the } n^{th} \text{ entry is given by } q_n = 1 - \sum_{j=1}^{n-1} q_j \\
&\Rightarrow Q = \{q_1, q_2, \ldots, q_{n-1}, q_n\} = [N^1X, q_n]^T
\]

For \( k = 1, 2, \ldots, n-1 \)

3- Calculating \( E \):

To evaluate \( E \), first one has to calculate \( M^{-1}X \) and \( X \), then evaluate \( (M^{-1}X) \) which gives the \( (n-1) \) first \( p_i \) and then, deduce \( p_n \) by \( p_n = 1 - \sum_{i=1}^{n-1} p_i \). Similarly, evaluate \( N^{-1}Y \), and the \( (n-1) \) first \( q_i \), then deduce \( q_n = 1 - \sum_{j=1}^{n-1} q_j \).
\[
E = P.A.Q = [(M^{-1}X) \quad p_n]^T.A.([N^{-1}Y] \quad q_n)
\]
Or
\[
E = \left[(M^{-1}X) \left(1 - \sum_{i=1}^{n-1} p_i \right) \right]^T A \left[\left(N^{-1}Y \left(1 - \sum_{j=1}^{n-1} q_j \right) \right] \right.
\]

Alternative Assembling Method for E:

1. Assembling M and N, X and Y:
The representation can be rearranged as followed:
\[
M = [m_{ik}], \quad m_{ik} = a_{n(k+1)} - a_{nk} - (a_{i(k+1)} - a_{ik}), \quad i=1, 2, \ldots , n-1, \quad k=1, 2, \ldots , n-1
\]
\[
X = [x_k], \quad x_k = a_{n(k+1)} - a_{nk}, \quad k=1, 2, \ldots , n-1
\]
\[
N = [n_{kj}], \quad n_{kj} = a_{(k+1)n} - a_{kn} - (a_{i(k+1)} - a_{ik}), \quad j=1, 2, \ldots , n-1
\]
\[
Y = [y_k], \quad y_k = a_{(k+1)n} - a_{kn}, \quad k=1, 2, \ldots , n-1
\]

2. Assembling P:
Clearly, the entries of the matrix M are obtained from the sub-matrix \(A_{(n-1)x(n-1)}\) by replacing the \(i\)th row, \(i \leq n-1\), by
\[
(m_{i[1...(n-1)]}) = (a_{n[i...(n-1)]}) - (a_{[i...(n-1)]})
\]
Or by replacing the \(j\)th column, \(j \leq n-1\), by
\[
(m_{[1...(n-1)]j}) = (a_{[1...(n-1)]j}) - (a_{[1...(n-1)]j})
\]
Where:
\[
{m_{i[1...(n-1)]}} = \text{the } i\text{th row of } M, \text{ with entries from column } 1 \text{ to } n-1
\]
\[
{m_{[1...(n-1)]j}} = \text{the } j\text{th column of } M, \text{ with entries from row } 1 \text{ to } n-1
\]
\[
{a_{i[1...(n-1)]j}} = \text{the } j\text{th column of } A, \text{ with entries from row } 1 \text{ to } n-1
\]
\[
{a_{[1...(n-1)]j}} = \text{the } (j+1)\text{th column of } A, \text{ with entries from row } 1 \text{ to } n-1
\]
\[
{a_{[1...(n-1)]j}} = \text{the } n\text{th row of } A, \text{ with entries from column } 2 \text{ to } n
\]
\[
{a_{n[1...(n-1)]j}} = \text{the } n\text{th row of } A, \text{ with entries from column } 1 \text{ to } n-1
\]
Etc...

3. Assembling Q:
Conversely, the entries of the matrix N are obtained from the sub-matrix \(A_{(n-1)x(n-1)}\) by replacing the \(j\)th column, \(j \leq n-1\), by
\[
(n_{[1...(n-1)]j}) = (a_{[2...(n-1)]n}) - (a_{[1...(n-1)]n})
\]
Or by replacing the \(i\)th row \(A, \leq n-1\), by
\[
(n_{[1...(n-1)]}) = (a_{[1...(n-1)]}) - (a_{[1...(n-1)]})
\]
Where:
\[
{n_{[1...(n-1)]j}} = \text{the } j\text{th column of } N, \text{ with entries from row } 1 \text{ to } n-1
\]
\[
{n_{[1...(n-1)]}} = \text{the } j\text{th row of } N, \text{ with entries from column } 1 \text{ to } n-1
\]

IV. APPLICATION EXAMPLE WITH ACTUAL DATA

Problem Formulation

1. Statistical Approach: Using actual data (Table 1), what are average Euro/USD rate, standard deviation, median and mode? Draw the histogram and frequency polygon.
2. Probability: Using actual trading data over a two months period in 2009, a total of $265 was gained when $1080 was lost, leading to the odds for winning of $265/$1080. What is the probability of winning, losing?
3. Trader’s Ruin: Assuming the trader starts with a balance of $500 and will lose or win $2 par trade, using the above win/lose probability figures, what is the probability the initial $500 balance decreases to $494 or increases to $506? If the balance reaches $6, what is the probability of getting broke? (Hint: use the Gambler’s Ruin Problem model?)
4. Optimal Currency Pair: The trader can exchange several pairs of currencies; meanwhile the forex market will go up and down for each pair. Assuming the trader will win or lose at least the “spread” of each pair as summarized in the below 6x6 matrix for three currency pairs, what is the optimal pair of currencies to trade, amount EUR/USD, GBP/USD and JPY/USD? (1- Propose a generalization of the 2 x 2 matrix to the “optimal strategy for a 2-person zero-sum game with an n x n matrix”, 2- Apply to the case of a 6 x 6 matrix below, each pair having two possibilities of “sell” or “buy” versus its “up and down”.

<table>
<thead>
<tr>
<th>Foreign Exchange Market (forex)</th>
<th>Up</th>
<th>Down</th>
<th>Up</th>
<th>Down</th>
<th>Up</th>
<th>Down</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro Sell</td>
<td>-4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Euro Buy</td>
<td>-4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Trader GBP Sell</td>
<td>0</td>
<td>-6</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GBP Buy</td>
<td>0</td>
<td>-6</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>JPY Sell</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-10</td>
<td>5</td>
<td>-10</td>
</tr>
<tr>
<td>JPY Buy</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>-10</td>
<td>-10</td>
</tr>
</tbody>
</table>

1. 2 x 2 matrix to the “op
5. Predicting the Rise and Fall: To decide rather or not we "buy" (the exchange rate will rise) or "sell" (the exchange rate will fall) and at what time we would likely close the position. Using actual data below (Table 2), calculate the average amount of time elapsed between visits to each state (increase, decrease, neutral). (Hint: use the "mean recurrence time" and the Rise and Fall of Stock model). Note: data points are shown in reverse order; they start at [1.4026, 1.4014, -12.00] up to [1.4425, 1.4419, 6.00], read from bottom up, for each set of three columns.

Using Excel, the fundamental matrix of the Markov chain is given by $T = [I - Q]^{-1}$:

Using Excel, the fundamental matrix of the Markov chain is rearranged as showed on the following tables.

and the probability of being absorbed (broken) is given by the probability matrix T.S:

### Table 3 - Reordered Data

<table>
<thead>
<tr>
<th>Value</th>
<th>Class</th>
<th>P(a)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Class</td>
<td>P(a)</td>
</tr>
<tr>
<td>0</td>
<td>Class</td>
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<td>Class</td>
<td>P(a)</td>
</tr>
<tr>
<td>0</td>
<td>Class</td>
<td>P(a)</td>
</tr>
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</table>

### Table 4 - Reordered Data (Followed)

<table>
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<tr>
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<th>P(a)</th>
</tr>
</thead>
<tbody>
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<td>Class</td>
<td>P(a)</td>
</tr>
<tr>
<td>0</td>
<td>Class</td>
<td>P(a)</td>
</tr>
<tr>
<td>0</td>
<td>Class</td>
<td>P(a)</td>
</tr>
<tr>
<td>0</td>
<td>Class</td>
<td>P(a)</td>
</tr>
</tbody>
</table>

### Table 5 - Transition Matrix

<table>
<thead>
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<th>Forex Market</th>
<th>$494</th>
<th>$496</th>
<th>$498</th>
<th>$500</th>
<th>$502</th>
<th>$504</th>
<th>$506</th>
</tr>
</thead>
<tbody>
<tr>
<td>$494</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$496</td>
<td>0.803</td>
<td>0.197</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$498</td>
<td>0</td>
<td>0.803</td>
<td>0.197</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$500</td>
<td>0</td>
<td>0</td>
<td>0.803</td>
<td>0.197</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$502</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.803</td>
<td>0.197</td>
<td>0</td>
<td>0</td>
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<tr>
<td>$504</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.803</td>
<td>0</td>
<td>0.197</td>
</tr>
<tr>
<td>$506</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.803</td>
<td>0.197</td>
</tr>
</tbody>
</table>

### Table 6 - Transition Matrix, Rearranged

<table>
<thead>
<tr>
<th>Forex Market</th>
<th>$494</th>
<th>$496</th>
<th>$498</th>
<th>$500</th>
<th>$502</th>
<th>$504</th>
<th>$506</th>
</tr>
</thead>
<tbody>
<tr>
<td>$494</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td>$496</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$498</td>
<td>0.803</td>
<td>0</td>
<td>0.197</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$500</td>
<td>0</td>
<td>0</td>
<td>0.803</td>
<td>0.197</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$502</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.803</td>
<td>0.197</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$504</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.803</td>
<td>0</td>
<td>0.197</td>
</tr>
<tr>
<td>$506</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.803</td>
<td>0.197</td>
</tr>
</tbody>
</table>

### Table 7 - Fundamental Matrix and Number of Trade before Absorption

<table>
<thead>
<tr>
<th>Value</th>
<th>$2</th>
<th>$4</th>
<th>$6</th>
<th>$8</th>
<th>$10</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>1.245</td>
<td>0.305</td>
<td>0.074</td>
<td>0.017</td>
<td>0.003</td>
</tr>
<tr>
<td>Expected number of trade before absorption</td>
<td>1.644</td>
<td>3.267</td>
<td>4.807</td>
<td>6.007</td>
<td>5.824</td>
</tr>
</tbody>
</table>

### Table 8 - Probability of Being Absorbed

<table>
<thead>
<tr>
<th>Value</th>
<th>$494</th>
<th>$506</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.S.</td>
<td>Probability of being Absorbed</td>
<td>T.S.</td>
</tr>
<tr>
<td>$494</td>
<td>0.999</td>
<td>0.001</td>
</tr>
<tr>
<td>$496</td>
<td>0.997</td>
<td>0.003</td>
</tr>
<tr>
<td>$498</td>
<td>0.985</td>
<td>0.015</td>
</tr>
<tr>
<td>$500</td>
<td>0.940</td>
<td>0.060</td>
</tr>
<tr>
<td>$502</td>
<td>0.755</td>
<td>0.245</td>
</tr>
</tbody>
</table>

Starting at $500, the probability of moving to $494 is 0.985; the probability of moving to $506 is 0.015. Starting at $6, the probability of getting broken is 0.985. In conclusion, the probability of losing is very high, meaning trading forex is a very risky business that requires a lot of experience.

### Optimal Currency Trade

The transition matrix of spread is given in Table 2: $m_2$ and $n_2$ are calculated using the Excel INDEX() function. $[x_2]$ and $[x_1]$ are calculated combining Excel MMULT() and INVERSE() functions. The results are given by:

![Fig. 1 - Graph and Results from Statistical Analysis](image1)

**Fig. 1** - Graph and Results from Statistical Analysis

**Probability from Odds**

If the odds for winning are $a/b$, the probability for winning is $p = a/(a+b) = $265/($265+$1080)=0.197 and the probability of losing is 0.803.

**Trader's Ruin**

The transition matrix is rearranged as showed on the following tables.

![Fig. 2 – Calculations of Entries of Matrices M and N](image2)

**Fig. 2** – Calculations of Entries of Matrices M and N

### Solution

**Statistical Analysis**

Using Excel and the functions average(), median(), mode() and graphic capabilities, we obtain:

![Histogram and Frequency Polygon](image3)

**Fig. 3** - Histogram and Frequency Polygon

**Exchange Rate ranges**
The expected payoff is

\[
E = \begin{bmatrix}
0.241935 & 0.241935 & 0.161290 & 0.161290 & 0.096774 & 0.096774
\end{bmatrix}^T
\]

\[
E = -0.4838, \text{ unfavorable to the trader.}
\]

The optimal strategy for the trader is the rows 1 and 2 with highest probabilities of 0.24 as shown in the probability distribution vector \(P\). However, the expected value is negative indicating that even the trader plays rows 1 and 2 (Euro/USD sell and buy), he/she will still lose.

### Predicting the Rise and Fall

Data reporting the past history of currency trading in 2009 were rearranged in three columns. Additional columns “I to I”, “I to D”, etc were added to return 1 or 0. For instance, in the column H “I to I”, row 128 (cell H128), the exchange rate that was increasing from 1.3952 to 1.3960, closed higher at 1.3965, “I to I”=1, all the other “X” to “Y” are zero. We used the IF() and AND() functions as shown to complete all the “X” to “Y” columns and calculate the probability of “X to Y” and statistically fits the transition matrix to the model:

**Note:** For clarity purposes, only partial data are displayed to show an example (H128) of the formulae used.

Fig. 3 – Counting Rise and Fall with Excel. The probability X to Y (I to I, I to D, etc) is the probability that a rate that was in state X moves to state Y. For instance 0.397 is the probability that a rate that was increasing continues to increase (I to I).

By considering \(N\) as the absorb state, the results are:

\[
\begin{array}{c|cc|c|c|c|c|c}
\text{N absorbing state} & \text{Identity matrix} & \text{Fundamental matrix} & \text{days to steady} \\
\hline
\text{N} & \text{I} & \text{D} & \text{N} & \text{Check} & \text{Euro/USD increases} & \text{44.81\% of the time} \\
14,32 days & start in I, average time spent in I, before unchanged & T.S & \text{14.32} & \text{16.54} & \text{39.95} \\
16,64 days & start in I, average time spent in D, before unchanged & T.S & \text{15.36} & \text{17.56} & \text{30.90} \\
13,36 days & start in D, average time spent in I, before unchanged & 1.00 & \text{1.00} & \text{1.00} & \text{1.00} \\
17,56 days & start in D, average time spent in D, before unchanged & 1.00 & \text{1.00} & \text{1.00} & \text{1.00} \\
\text{Probability} & \text{Fixed Probability Vector} & & & & \\
\text{T.S} & 0.508 & 0.508 & 0.508 & 0.508 & 0.508 & 0.508 & 0.508 \\
1.00 & 0.508 & 0.508 & 0.508 & 0.508 & 0.508 & 0.508 & 0.508 \\
\text{Fixed Probability Vector} & \text{I} & \text{D} & \text{N} & \text{Check} & \text{Euro/USD increases} & \text{44.81\% of the time} \\
\hline
\text{T.} & 0.448 & 0.521 & 0.031 & 1.000 & \text{Euro/USD increases} & \text{44.81\% of the time} \\
1/1 & 2.352 & 1.921 & 0.031 & \text{Euro/USD decreases} & \text{52.06\% of the time} \\
1/0 & 31.954 & 31.954 & \text{31.954} & \text{31.954} & \text{31.954} & \text{31.954} \\
\text{Mean Recurrence Time} & \text{Conclusion: mostly decreases, so sell!} & & & & \\
\text{t} & \text{Average number of time elapsed between visits to state ith.} & & & & \\
\end{array}
\]

Fig. 4 – Results

### V. CONCLUSION

The generalized approach outlined herein could serve as “a posteriori” (after the fact) estimate that is used to analyze the trade, up to a given point of time, and then allows the trader to reconsider or adjust the current strategy or selection of optimal pair of currency, in term of expected payoff. The tool can then be used as “a priori” (before the fact) indicator for the next currency pairs to trade. This model is a technical analysis tool for selecting the optimal pair when trading multiple currency pairs. It predicts the best pair to focus on and the expected return in term of payoff. Finally, the formulas found in section III, used for a given set of currency, confirms an intuitive result which is that trading forex is intrinsically “unfavorable” to the trader.

### ACKNOWLEDGEMENT

This paper is submitted as a continuation of my works started during my PhD in Applied Mathematics related to finite element implementation. Therefore, I would like to take this opportunity to express my gratitude to those that have been very supportive and of great help, many of my great colleagues, friends, former teachers, parents and close family, my mother Clementine Avome Ndong and my uncle, Paul Ovono Ndong for their moral encouragements throughout my life and studies.

### REFERENCES