A Mathematical Simulation for Bank Valuation

Enahoro A. Owoloko *Member, IAENG*, Godwin O. S. Ekhaguere and Nicholas A. Omoregbe

Abstract - In this work, we concluded the process of formulating a mathematical model for bank valuation established in an earlier work. We formulated the mathematical model for the valuation of bank and developed a simulation interface for the valuation. When the appropriate parameters are keyed into the simulation interface, the value of the bank is obtained. The algorithm and the simulation interface are as displayed in appendices A and B.

Keywords: Mathematical model, valuation, algorithm, simulation.

I. INTRODUCTION

The banking sectors are usually the first victims at any recession, globally or locally. In their proclamation, [1] asserted that little has been written on bank valuation through the contingent claim approach.

Most of the work done on or related to financial intermediation are usually found in textbooks and practitioners guide which include ([14], [7], [13], [11], [12], [8], [2], [3], [4], [6], and [9]). The research work on banks intermediations are so rare that [5] as stated in [14] asserts that bank valuation is one of the unresolved issues in financial research.

In an earlier work, [14], stated that the characteristics of the banking business motivating a distinct valuation approach can be because of the following: first due to their significant role for the economy; banking is typically a heavily regulated industry, covering a wide range of provisions, such as market entry, deposit insurance, reserve e requirements, or capital structure. Secondly, banks operate on both sides of their balance sheets, actively seeking profit not only in lending but also in rising capital.

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E. A. Owoloko and Omoregbe work with the Department of Mathematics and Computer Science, Covenant University as lecturers. While, G.O.S. Ekhaguere is of the Department of Mathematics, University of Ibadan.
(corresponding author phone: +2348023397922;
e-mail: alfred.owoloko@covenantuniversity.edu.ng

In their work [1] proposed a valuation model for banks derived from [10]. The Black-Scholes pricing model and the concept of matched maturity marginal value of funds (MMMVF) was adopted. However, the model proposed by [1] had some short falls; which was addressed and improved on by [14].

II. MATHEMATICAL FORMULATION

In our model, the value of a bank is a function of the following variables loans, expected growth in loans, deposit, expected growth in deposits, variable cost, cash balance, loss-carry forward, accumulated property, plant and equipment and time. This is written as

$$V \equiv \left(L, \mu_L, D, \mu_D, \gamma, X, Y, P, t\right) \qquad (1)$$

The objective of the model is to determine the value V of the bank at the current time $t = 0$.

In their work ([15], [16]), assert that this value is given as

$$V\left(0\right) = e^{-rT}\left[E_Q\left[\left(X\left(T\right)\right) + M\left(\Pi\left(T\right) - C\left(T\right)\right)\right]\right] \qquad (2)$$

Where,

$V\left(0\right) =$ value of the bank at present time

$X\left(T\right) =$ Outstanding cash balance at time T

$M =$ Multiplier

$C\left(T\right) =$ Cost at time T

$r =$ Risk free rate

$\Pi\left(T\right) =$ Net income at time T

$e^{-rT} =$ Continuously compounded discount factor

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$E_Q =$ Equivalent martingale measure

The draw-back of (2), is that some of the variables are path-dependent, that is, depends on historical cash flows. To take care of this draw-back, a Monte Carlo simulation is then used to determine the value of the bank putting the parameters in (2) into consideration. Therefore, the value $V(t)$ of a bank at an arbitrary time $t \in [0,T]$ is given by

$$V(t) = e^{rt}e^{-rT}\left\{E_Q\left[X(T-t) + M\left(\Pi(T-t) - C(T-t)\right)\right]\right\}$$
$$= e^{-r(T-t)}\left\{E_Q\left[X(T-t) + M\left(\Pi(T-t) - C(T-t)\right)\right]\right\} \quad (3)$$

For $t \in [0,\mathrm{T}]$

(2) follows from (3) by putting $t = 0$.

But,

$$X(t) = \Pi(1+r) + Y(t) + Dep(t) - Capx(t)$$

$$\Pi(t) = \overline{w} + L(t)(1+r) - D(t)(s-1)$$

$$C(t) = \gamma(t)L(t) + F \quad (4)$$

$$Y(t) = \left[\Pi(t) - C(t) - Dep(t)\right](1-\tau_c)$$

According to [14], substituting (4) into (3) gives (5).

$$p_1 = (1+r)(2-r-\tau_c)$$

$$p_2 = M(1+r)$$

$$p_3 = (s-1)(2+r-\tau_c)$$

$$p_4 = M(s-1)$$

$$p_5 = (1-\tau_c - M)$$

Equation (5) cannot be evaluated directly [14]. This is resolved by using Monte Carlo simulation. To achieve this, we partition the interval $[0,T]$ as follows;

$$0 = t_0 < t_1 < \ldots < t_{N-1} < t_N = T.$$

Then, from (5), we have (6).

Equation (6) is used in the simulation. To apply the simulation, a summary of parameter used is given in table 1. A photo shot of the simulation software is given in Figure 1

$$V(t) = e^{-r(T-t)}\left\{E_Q\left[(p_1 - p_2)L(T-t) + (p_4 - p_3)D(T-t) - p_5\gamma(T-t)L(T-t) + \Lambda\right]\right\} \quad (5)$$

Where;

$$\Lambda = \left[\overline{w}(2+r-\tau_c) - F(1-\tau_c - M) - \tau_c Dep(T-t) - Capx(T-t) - M\overline{w}\right]$$

$$V(t) = e^{-r(T-t)}\left\{\frac{1}{N}\left[(p_1 - p_2)\sum_{t=0}^{N} L(t_i) + (p_4 - p_3)\sum_{t=0}^{N} D(t_i) - p_5\sum \gamma(t_i)L(t_i) + \Lambda\right]\right\} \quad (6)$$

Where;

$N =$ number of partitions

$$D(t_i) = D(t_{i-1})\exp\left\{\left[\left(\mu_D - \lambda\sigma_D\right) - \frac{\sigma_D^2}{2}\right]\right\}\Delta t_i + \sigma_D\varepsilon_i\sqrt{\Delta t_i}$$

$$L(t_i) = L(t_{i-1})\exp\left\{\left[\left(\mu_L - \lambda\sigma_L\right) - \frac{\sigma_L^2}{2}\right]\right\}\Delta t_i + \sigma_L\varepsilon_i\sqrt{\Delta t_i}$$

$$\gamma(t_i) = \overline{\mu} - \left(\overline{\mu} - \gamma(t_{i-1})e^{-k\Delta t_i} + \xi\varepsilon_i\sqrt{\frac{1-e^{-2k\Delta t_i}}{2k}}\right)$$

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Table 1: Summary of parameters

Parameters	Notations
Loan Parameters	
Initial Loan	L_0
Initial expected growth rate in loan	μ_0^L
Initial volatility of loans	σ_0^L
Initial volatility of expected growth rate in loans	φ_0^L
Long-term growth rates in loans	$\overline{\mu_L}$
Long-term volatility of growth rate in loans	$\overline{\sigma_L}$
Deposit Parameters	
Initial Deposit	D_0
Initial expected growth rate in deposit	μ_0^D
Initial volatility of deposit	σ_0^D
Initial volatility of expected growth rate in deposit	φ_0^D
Long-term volatility of growth rate in deposit	$\overline{\sigma_D}$
Long-term growth rate in deposit	$\overline{\mu_D}$
Cost Parameters	
Initial variable cost	γ_0
Long-term variable cost	$\overline{\gamma}$
Volatility of initial variable cost	ξ_0
Volatility of long-term variable cost	$\overline{\xi}$
Fixed cost	F
Other Parameters	
Initial cash balance	X_0
Time increment of the period	M
Horizon for the estimation	T
Tax rate	τ_c
Risk-free rate	r_f
Mean-reversion rate	k
Multiplier	M
Depreciation rate	$Dep.$
Accumulated property & equipment	Accum. Prop
Capital expenditure (investment property)	CX
Long-term investment (investment in securities)	CR
Market price	λ

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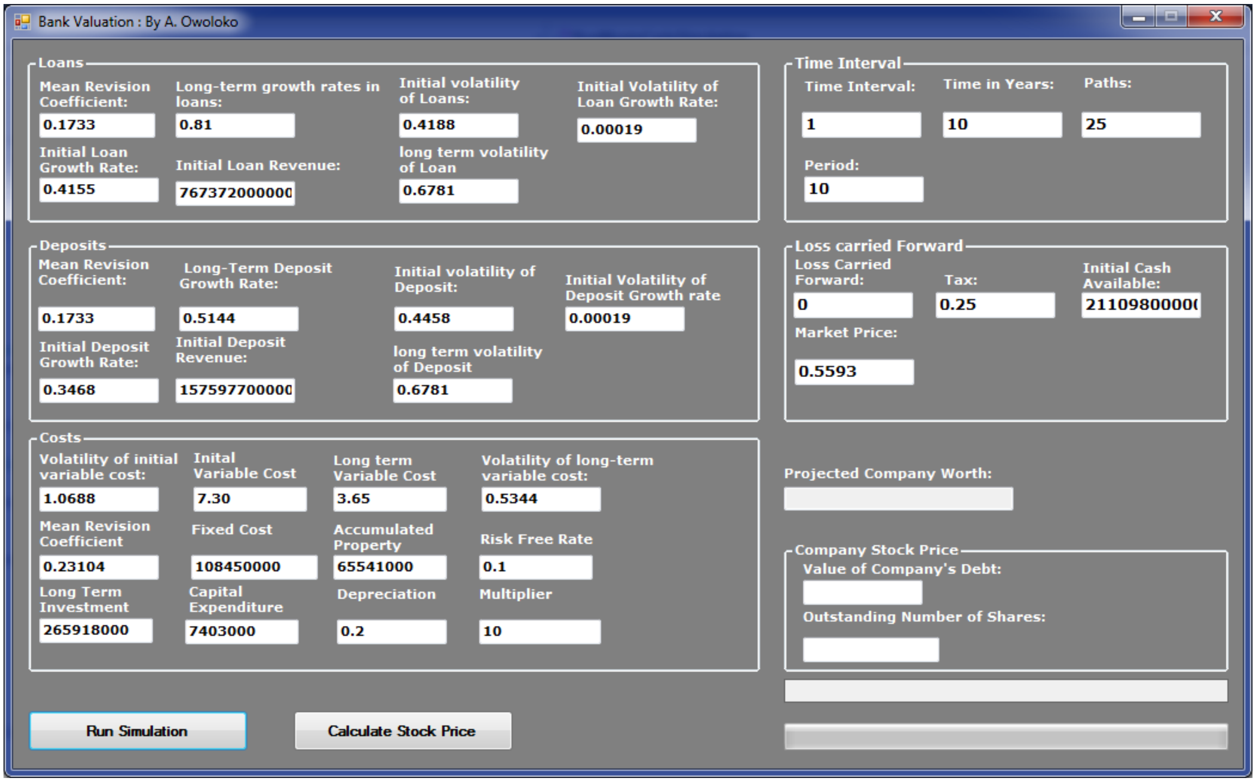

Fig. 1. Photoshot of the custom graphical user interface (GUI) driven application in running the Monte Carlo simulation, the programming language used was VB.NET along with the .NET Framework

III. CONCLUSION

In this work, we have been able to deploy a new mathematical model for bank valuation with its algorithm and simulation interface. This is an improvement over existing models of [1], [3] and [12]. This work is a continuation of an earlier work as in [14].

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