Computationally Efficient DOA and Range Estimation for Near-Field Source with Linear Antenna Array

Hao Chen, Xinggan Zhang, Yechao Bai, and Jingji Ma

***Abstract*—This letter presents an effective direction of arrival (DOA) and range estimation algorithm for a single near-field source. In the proposed algorithm, a correlation function is firstly constructed based on the array configuration. Then, the phase which contains the information about the DOA and range is extracted. By utilizing the least square operation, the closed-form solutions of DOA and range are obtained. Compared to the existing high-order statistics based estimate algorithm which involves spectral search and eigenvalue decomposition (EVD), the proposed algorithm can provide higher computation efficiency and improved estimate accuracy. Simulations are carried out to verify the effectiveness of the proposed algorithm.**

***Index Terms*—Near-field, linear antenna array, source localization, closed-form solution.**

I. Introduction

DIRECTION of arrival (DOA) and range estimation are important topics of source localization, which have extensive application prospects in microphone array, radar, sonar, and navigation. In the far-field scenario (FFS), the sources are located far from the array, and the range of all sources is infinity. Hence only the DOA should be estimated. On the other hand, for the near-field sources (NFS), the sources are located close to the array, and the received data is a coupling of the DOA and range. As a result, source localization for NFS is more complicated than the FFS [1].

Recently, a large number of algorithms have been developed for localization of the NFS. Two-dimensional (2-D) multiple signal classification (MUSIC) algorithm have been presented in [2]. To reduce the computational complexity, W. Zhi et al. subdivided the uniform linear array (ULA) into two subarrays [3]. By utilizing the rotational invariance property, 2-D spectrum search is transformed to one-dimensional (1-D) search, thereby enhancing computational efficiency. J. Liang et al. proposed a two-stage MUSIC algorithm, in which twice 1-D MUSIC were carried out to achieve source localization [4]. To some extent, this algorithm reduces the computational complexity. However, twice 1-D spectral search and the eigenvalue decomposition (EVD) implementation still consume great computational burden. The simplified high-order estimation (SHOE) algorithm [5] constructs a non-Hermitian fourth order cumulant matrix, by exploiting twice 1-D spectral search, the localization is achieved. Another well-known subspace-based algorithm is the high-order estimation of signal parameters via rotational invariance technique (ESPRIT) algorithm [6],[7], which can avoid the tremendous spectral search. Constructing several fourth-order cumulants, high-order ESPRIT algorithm makes use of the multiple rotational invariance properties to accomplish range and bearing estimation. Although the maximum likelihood estimator (MLE) [8],[9] and weighted linear prediction method [10] are developed, the computational efficiency is depended on an iterative procedure.

Manuscript received June 21, 2017. This work was supported in part by the National Natural Science Foundation of China under Grant 41671352, Basic Research Program of Jiangsu Province under Grant BK20151391, and Natural Science Program of Universities in Anhui Province under Grant KJ2017A329.

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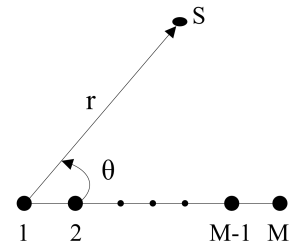

Fig. 1: Near-field ULA configuration.

In this letter, we present a computationally efficient DOA and range estimation algorithm for a single near-field source. Unlike the algorithms in aforementioned literature, the proposed algorithm does not require spectrum search or the EVD operation, which can reduce the computational complexity to a great extent. By constructing a correlation function based on the array configuration, the proposed algorithm provides a closed-form solution of the DOA and range based on the least square (LS) approach. Compared to the SHOE [5] and TLS-ESPRIT [6] algorithm, the proposed algorithm possesses significant efficiency advantage and can achieve better estimation accuracy.

II. Signal model

Consider a uniform linear array (ULA) with M identical sensors, the distance between two adjacent elements is d. As illustrated in Fig. 1, a single narrowband source impinging on the ULA. The signal received by the mth sensor is then given by

$$y_m(l) = s(l)e^{j\varphi_m} + n_m(l) \tag{1}$$

where $l = 1, 2, \cdots, L$, and L is the number of snapshots, $s(l)$ is the source signal, $n_m(l)$ is the additive sensor noise, φ_m

ISBN: 978-988-14047-5-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

is the phase shift between the signal received by the sensor 1 and sensor m of the single source. In near-field scenario, φ_m can be expressed as [11],[12]

$$\varphi_m = \frac{2\pi}{\lambda}\left(\sqrt{r^2 + (md)^2 - 2rmd\sin\theta} - r\right) \tag{2}$$

where λ is wavelength, r is range, θ is DOA. When the source is in the Fresnel region [1], which is satisfying $0.62(D^3/\lambda)^{1/2} < r < 2D^2/\lambda$, with D is the array aperture, by applying the second-order Taylor expansion, we have

$$\varphi_m = \eta m + \gamma m^2 + O\left(\frac{d^2}{r^2}\right) \tag{3}$$

where $O\left(d^2/r^2\right)$ is the remainder term of Taylor formula that is neglected here, and

$$\eta = -\frac{2\pi d}{\lambda}\sin\theta \tag{4}$$

$$\gamma = \frac{\pi d^2}{\lambda r}\cos^2\theta \tag{5}$$

By omitting the Taylor remainder, the received signal in (1) can be represented as

$$y_m(l) = s(l)e^{j\left(-\frac{2\pi d}{\lambda}\sin\theta\right)m + j\left(\frac{\pi d^2}{\lambda r}\cos^2\theta\right)m^2} + n_m(l) \tag{6}$$

III. Proposed Algorithm

Under above signal model, we firstly construct a correction function. The $(p, q)th$ element in the covariance matrix of received signal is given as

$$\begin{aligned} R_{p,q} &= E[y_p(l)y_q^*(l)] \\ &= \sigma_s^2 e^{j\left(-\frac{2\pi d}{\lambda}\sin\theta\right)(p-q) + j\left(\frac{\pi d^2}{\lambda r}\cos^2\theta\right)\left(p^2-q^2\right)} + \sigma_n^2 \end{aligned} \tag{7}$$

where $(\cdot)^*$ represents the complex conjugate, σ_s^2 and σ_n^2 are the power of signal and noise, respectively. Extracting the phase of $R_{p,q}$, we have

$$\begin{aligned} \psi_{p,q} &= \left(-\frac{2\pi d}{\lambda}\sin\theta\right)(p-q) + \left(\frac{\pi d^2}{\lambda r}\cos^2\theta\right)\left(p^2-q^2\right) \\ &= -\frac{2\pi d}{\lambda}\left[(p-q)\sin\theta - \left(p^2-q^2\right)\frac{d}{2r}\cos^2\theta\right] \end{aligned} \tag{8}$$

To guarantee there is no phase ambiguity in $\psi_{p,q}$, it should satisfy the condition that $d/\lambda \le 1/4$. Note that the problem of phase ambiguity can also be solved by [13]. Rewrite (8) as

$$\psi_{p,q} = -\frac{2\pi d}{\lambda}\begin{bmatrix} p-q \\ q^2-p^2 \end{bmatrix}^T \begin{bmatrix} \sin\theta \\ \frac{d}{2r}\cos^2\theta \end{bmatrix} \tag{9}$$

where $(\cdot)^T$ represents the transpose. Assume that $p - q = v$, we can express (9) in matrix form as

$$\boldsymbol{\psi} = \mathbf{HW} \tag{10}$$

where

$$\boldsymbol{\psi} = [\psi_{1,1+v}, \psi_{2,2+v}, \cdots, \psi_{M-v,M}]^T \tag{11}$$

$$\mathbf{H} = -\frac{2\pi d}{\lambda}\begin{bmatrix} v & (1+v)^2 - 1^2 \\ v & (2+v)^2 - 2^2 \\ \vdots & \vdots \\ v & M^2 - (M-v)^2 \end{bmatrix} \tag{12}$$

$$\mathbf{W} = \begin{bmatrix} \sin\theta \\ \frac{d}{2r}\cos^2\theta \end{bmatrix} \tag{13}$$

TABLE I: Comparison of the computation complexity.

Algorithms	Computation complexity
TLS-ESPRIT	$O(4\times 9(\frac{M}{2})^2L + \frac{4}{3}(\frac{3M}{2})^3)$
SHOE	$O(9M^2L + \frac{4}{3}M^3 + TM^2)$
Proposed method	$(M-v)L + (M-v)(L-1)$

Note that v is a constant, which can be chosen as $1, 2, \cdots, M-1$. To exploit the greatest degree of array aperture, we chose $v = 2$ in all of the simulations.

In practical, assume $\hat{R}_{p,q}$ is the estimate of $R_{p,q}$, and $\hat{\psi}_{p,q}$ is associated phase, by utilizing the LS method, the estimate of $\hat{\mathbf{W}}$ can be obtained as

$$\hat{\mathbf{W}} = [\hat{w}_1, \hat{w}_2]^T = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\hat{\boldsymbol{\psi}} \tag{14}$$

where $\hat{\boldsymbol{\psi}} = \left[\hat{\psi}_{1,1+v}, \hat{\psi}_{2,2+v}, \cdots, \hat{\psi}_{M-v,M}\right]^T$. According to (13), the estimate of θ and r can be represented as

$$\hat{\theta} = \arcsin(\hat{w}_1) \tag{15}$$

$$\hat{r} = \frac{d\cos^2\hat{\theta}}{2\hat{w}_2} \tag{16}$$

Here we compare the computation complexity of the proposed algorithm with the TLS-ESPRIT and the SHOE, as shown in Table 1. For a single source, the main computation load of the TLS-ESPRIT lies in constructing cumulant matrices and performing the EVD. Summing these two operations, the total computation load of the TLS-ESPRIT is $O(4 \times 9(\frac{M}{2})^2L + \frac{4}{3}(\frac{3M}{2})^3)$. While for the SHOE, twice 1-D spectrum search is needed. For T points spectrum search, the total computation load of the SHOE is roughly $O(9M^2L + \frac{4}{3}M^3 + TM^2)$. In comparison, the proposed algorithm needs neither constructing cumulant matrices nor performing the EVD, it only requires to formulate multiply operation in (7), and LS calculation in (14), and the total computation complexity is $(M-v)L + (M-v)(L-1)$.

IV. Simulation Results

In this section, several simulations are carried out to verify the performance of the proposed algorithm, which are also compared with the SHOE [5], the TLS-ESPRIT [6], and the CRLB [1]. For the simulations, a ULA with $M = 9$ and

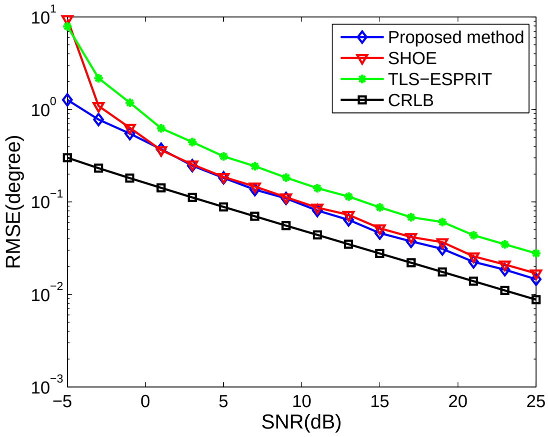

Fig. 2: RMSE versus the SNR of DOA estimation.

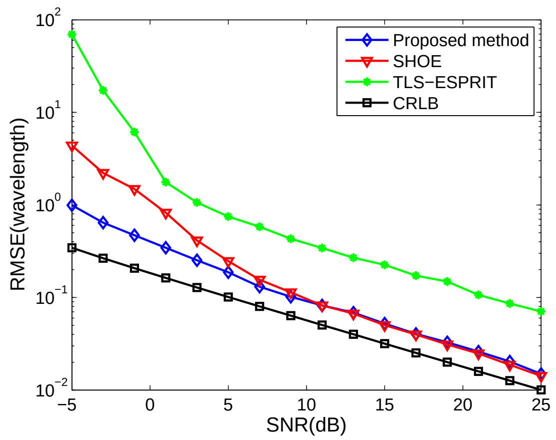

Fig. 3: RMSE versus the SNR of range estimation.

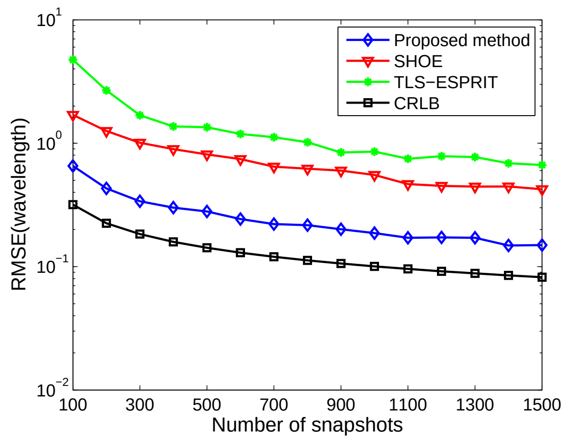

Fig. 5: RMSE versus the number of snapshots of range estimation.

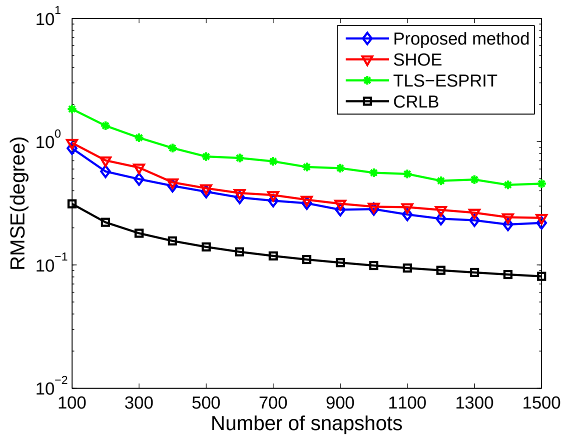

Fig. 4: RMSE versus the number of snapshots of DOA estimation.

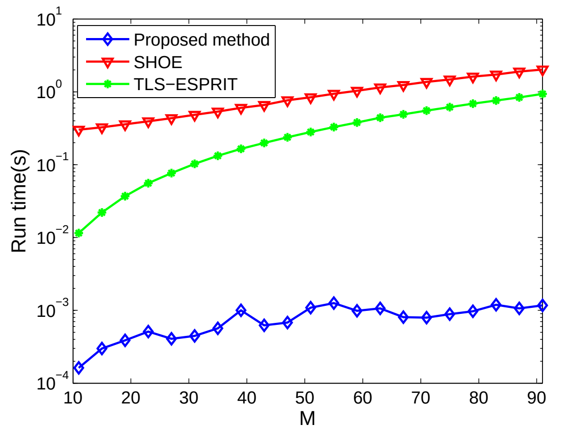

Fig. 6: Run time versus the number of sensors.

$d = \lambda/4$ is utilized, and the noise is additive white Gaussian noise. A single source is located at $(\theta, r) = (30°, 2.3\lambda)$. We define the root mean square error (RMSE) of the estimate DOA and range as

$$RMSE = \sqrt{\frac{1}{K}\sum_{k=1}^{K}(\hat{x}_k - x)^2} \tag{17}$$

where K=500 is the number of Monte Carlo trials, $\hat{x}_i$ is the estimated DOA or range of the kth trail, and x is the corresponding real value.

Figs. 2 and 3 show the RMSE of the DOA and range against the signal-to-noise ratio (SNR), respectively. The number of snapshots is set as $L = 300$, and the SNR is varying from -5 to 25 dB. From these figures, we can see that the proposed algorithm has a lower RMSE than the SHOE and TLS-ESPRIT algorithm for both DOA and range estimation. Note that the estimate performance is distinctly enhanced at low SNR, which is ascribed to the proposed algorithm directly extract phase operation to restrain the influence of noise.

As shown in Figs. 4 and 5, the RMSE of the DOA and range versus the number of snapshots are depicted. The SNR is fixed as 0 dB, and the number of snapshots is varying from 100 to 1500. For the DOA estimation, although the proposed algorithm exhibits similar RMSE to the SHOE, it is superior to that of the TLS-ESPRIT. For the range estimation, the RMSE of the proposed method is lower than that of the other two algorithms.

With regards to the computational efficiency, the simulation time of the proposed algorithm is compared with that of the SHOE and the TLS-ESPRIT. The simulations are carried out at MATLAB platform with a PC of Inter(R) Core(TM) i5-4440 CPU and 8G RAM. The results are averaged over 500 runs. For the search interval is $0.01°$, the computation consume of the SHOE, the TLS-ESPRIT, and the proposed algorithm are 0.3029, 0.0113, and 1.3879×10^{-4} s, respectively. The runtime versus sensor number is depicted in Fig. 6, in which we can see that the proposed algorithm has much lower time consuming than the SHOE and the TLS-ESPRIT.

V. Conclusion

In this letter, a computationally efficient DOA and range estimation algorithm is proposed for near-field source. The key idea of the proposed algorithm is to construct a correlation function matrix whose phase contains the information of the DOA and range. By solving the correlation function matrix, the close-form solutions of the DOA and range are obtained. The proposed algorithm does not need to construct high-order cumulant matrices or spectral search or perform the EVD operation. As a result, it is computationally efficient. Simulation results demonstrate the effectiveness of our algorithm.

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ISBN: 978-988-14047-5-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)