Application of Common Fixed Point Theorem on Fuzzy Metric Space

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Abstract — This paper introduces the notion of common fixed point theorem for three mappings in fuzzy metric space for various applications on 2 and 3-metric spaces with examples. Here, the result is generalized and improved too.

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I. INTRODUCTION

FIXED point theory is an important area of functional analysis. The concept of fuzzy sets was introduced by Zadeh in 1965 [31]. Since then many authors have extensively developed the theory of fuzzy sets and their applications. Especially, Deng [8], Erceg [10], Kaleva and Seikkala [23], Kramosil and Michalek [25] have introduced the concept of fuzzy metric spaces in several ways. Authors who have studied the fixed point theory in these fuzzy metric spaces are Badard [1] and Bose and Sahani [2] for fuzzy mappings. There are many view points of the notion of a metric space in fuzzy topology. Coincidence point and minimization theorems in fuzzy metrics, fuzzy sets and systems Chang and Cho[6]. We primarily split them into two groups.

Now it is natural to expect 3-metric space which is suggested by the quantity function. The method of introducing this is naturally different from 2-metric space theory. Here we have to use simplex theory from algebraic topology. The first group is formed by those results in which a fuzzy metric on a set X is treated as a map d : X x X → R+ where $X \subset I^{x}[10]$ or X = the totality of all fuzzy points of a set [2], and Hu [18] satisfying some collection of axioms or that are analogous to the ordinary metric axioms. Thus, in such an approach numerical distances are set up between fuzzy objects. We keep in the second group results in which the distance between objects is fuzzy, the objects themselves may be fuzzy or not. The most interesting references in this direction are [9], [23], [24]. Gahler in a series of papers [13], [14], [15] investigated 2-metric spaces.

It is pertinent to recall here that Sharma, Sharma and Iseki [29] studied contraction type mappings in 2-metric space for the first time. Later on Wenzhi [30] and many others initiated the study of probabilistic 2-metric spaces (or 2-PM spaces).

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It is known that 2-metic space is a real valued function of a point triples on a set X, whose abstract properties are suggested by the area function in Euclidean space.

II. PRELIMINARIES

Definition 2.1 [28]

A binary operation * “ [0, 1] x [0, 1] → [0, 1] is called a continuous t-norm if ([0, 1], *) is an abelian topological monoid with unit 1 such that a*b ≤ c*d whenever a ≤ c and b ≤ d for all a, b, c, d ∈[0, 1]. Examples of t-norm are a*b = ab and a*b = min{a,b}.

Definition 2.2 [25]

The 3-tuple (X, m, *) is called a fuzzy metric space (shortly, FM-space), if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in X^2 x [0, ∞] satisfying the following conditions : for all x, y, z ∈ X and s, t, > 0

(FM-1) M(x, y, 0) = 0

(FM-2) M(x, y, t) = 1, for all t > 0 if and only if x = y

(FM-3) M(x, y, t) = M(y, x, t)

(FM-4) M(x, y, t)* M(y, z, s) ≤ M(x, z, t + s)

(FM-5) M(x, y, .) : [0, 1] → [0, 1] is left continuous.

It follows here that, (X, M, *) denotes a fuzzy metric space. Note that M(x, y, t) can be thought of as the degree of nearness between x and y with respect to t. We identify x = y with M(x, y, t)=1 for all t >0 and M(x, y, t) = 0 with ∞. In the following example, we show that every metric induces a fuzzy metric.

Example 2.3[16]

Let (X, d) be a metric space. Define a*b = ab (or a*b = min{a, b}) and for all x, y ∈ X and t > 0,

$$M(x, y, t) = \frac{1}{t + d(x, y)}$$

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Then (X, M, *) is a fuzzy metric space. This is called a fuzzy metric M induced by the metric d the standard fuzzy metric. On the other hand, note that no metric exists in X satisfying (1.a).

Definition 2.4 [17]

Let (X, M, *) be a fuzzy metric space :

(1) A Sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$, (denoted by $\lim_{n\to\infty} = x_n = x$) if

$$\lim_{n\to\infty} M(x_n, x, t) = 1$$

For all t > 0.

(2) A Sequence $\{x_n\}$ in X is called a Cauchy sequence if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$$

For all t > 0 and p > 0.

(3) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Remark 2.5

Since * is continuous, it follows from (FM-4) that the limit of the sequence in FM-space is uniquely determined.

Let (X, M, *) be a fuzzy metric space with the following condition :

(FM-6) $\lim_{n\to\infty} M(x, y, t) = 1$ for all x, y $\in$ X

Lemma 2.6 [17]

For all x, y $\in$ X, M(x, y, .) is nondecreasing.

Lemma 2.7 [7]

Let $\{y_n\}$ be a sequence in a fuzzy metric space (X, M, *) with the condition (FM-6). If there exists a number $q \in (0, 1)$ such that

$$M(y_{n+2}, y_{n+1}, qt) \geq M(y_{n+1}, y_n, t) \qquad (1.b)$$

For all t > 0 and n = 1, 2, … then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2.8 [27]

If, for all $x, y \in X$, t > 0 and for a number $q \in (0,1)$, then x = y.

Lemmas 1, 2, 3 and remark 1 hold for fuzzy 2-metric spaces and fuzzy 3-metric spaces also.

Definition 2.9

A function M is continuous in fuzzy metric space iff whenever $x_n \to x$, $y_n \to y$, then

$$\lim_{n\to\infty} M(x_n, y_n, t) = M(x, y, t)$$

For each t > 0.

Definition 2.10

Two mappings A and S on fuzzy metric space X are weakly commuting iff,

$$M(Asu, SAu, t) \geq M(Au, Su, t)$$

For all $u \in X$ and t > 0

Definition 2.11

A binary operation *: [0, 1] x [0, 1] x [0, 1] → [0, 1] is called a continuous t-norm if ([0, 1], *) is an abelian topological monoid with unit 1 such that $a_1*b_1*c_1 \leq a_2*b_2*c_2$ whenever $a_1 \leq a_2$, $b_1 \leq b_2$, $c_1 \leq c_2$ for all a_1, a_2, b_1, b_2 and c_1, c_2 are in [0, 1].

Definition 2.12

The 3-tuple (X, M, *) is called a fuzzy 2-metric space if, X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in X^3 x [0, ∞] satisfying the following conditions: for all x, y, z, u $\in$ X and t_1, t_2, t_3 > 0.

(FM′-1) M(x, y, z, 0) = 0

(FM′-2) M(x, y, z, t) = 1, t > 0 and when at least two of the three points are equal (symmetry about three variables)

(FM′-3) M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t) (symmetry about three variables)

(FM′-4) M(x, y, z, $t_1 + t_2 + t_3$) ≥ M(x, y, u, t_1) * M(x, u, z, t_2) *M(u, y, z, t_3) (This corresponds to tetrahedron inequality in 2-metric space). The function t value M(x, y, z, t) may be interpreted as the probability that the area of triangle is less than t.

(FM′-5) M(x, y, z, .) : [0, 1) → [0, 1] is left continuous.

Definition 2.13

Let (X, M, *) be a fuzzy 2-metric space :

(1) A sequence $\{x_n\}$ in fuzzy 2-metric space X is said to be convergent to a point $x \in X$, if

$$\lim_{n\to\infty} M(x_n, x, a, t) = 1$$

For all $a \in X$ and t > 0

(2) A sequence $\{x_n\}$ in fuzzy 2-metric space X is called a Cauchy sequence, if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, a, t) = 1$$

For all $a \in X$ and t > 0, p > 0.

(3) A fuzzy 2-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.14

A function M is continuous in fuzzy 2-metric space iff, whenever $x_n \to x$, $y_n \to y$, then

$$\lim_{n\to\infty} M(x_n, y_n, a, t) = M(x, y, a, t)$$

For all $a \in X$ and t > 0

Definition 2.15

Two mappings A and S on fuzzy metric space X are weakly commuting iff

$$M(Asu, SAu, a, t) \ge M(Au, Su, a, t)$$

For all $u, a \in X$ and t > 0

Definition 2.16

A binary operation * : $[0, 1]^4 \to [0, 1]$ is called a continuous t-norm if ([0,1],*) is an abelian topological monoid with unit 1 such that $a_1*b_1*c_1*d_1 \le a_2*b_2*c_2*d_2$ whenever $a_1 \le a_2$, $b_1 \le b_2$, $c_1 \le c_2$, $d_1 \le d_2$ for all a_1, a_2, b_1, b_2, c_1, c_2 and d_1, d_2 are in [0, 1].

Definition 2.17

The 3-tuple (X, M, *) is called a fuzzy 2-metric space if, X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in $X^4 \times [0, \infty]$ satisfying the following conditions: for all x, y, z, w, u $\in$ X and $t_1, t_2, t_3, t_4 > 0$.

(FM″-1) M(x, y, z, w, 0) = 0

(FM″-2) M(x, y, z, w, t) = 1, t > 0
(only when the three simplex $\langle x, y, z, w\rangle$ degenerate)

(FM″-3) M(x, y, z, w, t) = M(x, w, z, y, t)
= M(y, z, w, x, t) = M(z, w, x, y, t) = ……

(FM″-4) $M(x, y, z, w, t_1 + t_2 + t_3 + t_4) \ge M(x, y, z, u, t_1) * M(x, y, u, w, t_2) * M(x, u, z, w, t_3) * M(u, y, z, w, t_4)$

(FM″-5) M(x, y, z, w, .) : [0, 1) → [0, 1] is left continuous.

Definition 2.18

Let (X, M, *) be a fuzzy 3-metric space :

(1) A sequence $\{x_n\}$ in fuzzy 3-metric space X is said to be convergent to a point $x \in X$, if

$$\lim_{n\to\infty} M(x_n, x, a, b, t) = 1$$

For all $a, b \in X$ and t > 0

(2) A sequence $\{x_n\}$ in fuzzy 3-metric space X is called a Cauchy sequence, if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, a, b, t) = 1$$

For all $a, b \in X$ and t > 0, p > 0.

(3) A fuzzy 3-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.19

A function M is continuous in fuzzy 3-metric space iff, whenever $x_n \to x$, $y_n \to y$, then

$$\lim_{n\to\infty} M(x_n, y_n, a, b, t) = M(x, y, a, b, t)$$

For all $a, b \in X$ and t > 0

Definition 2.20

Two mappings A and S on fuzzy metric space X are weakly commuting iff,

$$M(Asu, SAu, a, b, t) \ge M(Au, Su, a, b, t)$$

For all $u, a, b \in X$ and t > 0

Fisher [12] proved the following theorem for three mappings in complete metric space:

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Lemma 2.21

Let S and T be continuous mappings of a complete metric space (X, d) into itself. Then S and T have a common fixed point in X iff, there exists a continuous mapping A of X into $S(X) \cap T(X)$ which commute with S and T and satisfy :

$$d(Ax, Ay) \leq \alpha\, d(Sx, Ty)$$

for all $x, y \in X$ and $0 < \alpha < 1$. Indeed S, T and A have a unique common fixed point.

III. MAIN RESULTS

Theorem 3.1

Let (X, M, *) be a complete fuzzy metric space with the condition (FM-6) and let S and T be continuous mappings of X in X. Then S and T have a common fixed point in X, if there exists continuous mapping A and B of X into $S(X) \cap T(X)$ which commute with S and T and

$$M(Ax, By, kt) \geq \min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, t), M(Ax, By, t), M(Sx, By, t)\}$$

For all $x, y \in X$, $t > 0$ and $0 < q < 1$. Then A, B, S and T have a unique common fixed point in X.

Proof

Let x_0 be any arbitrary point in X. Construct a sequence $\{y_n\}$ in X such that $y_{2n-1} = x_{2n-1} = Ax_{2n-2}$ and $y_{2n} = Sx_{2n} = Bx_{2n+1}$, $n = 1, 2, 3...$ This can be done by (i). By using contractive condition, we obtain

$$M(y_{2n+1}, y_{2n+2}, kt) = M(Ax_{2n}, Bx_{2n+1}, kt)$$
$$\geq \min\{M(Sx_{2n}, Tx_{2n+1}, t), M(Ax_{2n}, Sx_{2n}, t), M(Bx_{2n+1}, Tx_{2n+1}, t), M(Ax_{2n}, Tx_{2n+1}, t), M(Ax_{2n}, Bx_{2n+1}, t), M(Sx_{2n}, Bx_{2n+1}, t)\}$$

$$= \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n}, t)\}$$

$$= \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), 1, M(y_{2n+1}, y_{2n}, t), 1\}$$

$$= M(y_{2n}, y_{2n+1}, t).$$

That is, $M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)$

Similarly, we have $M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n-1}, y_{2n}, t)$,

So, we get $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$ (3.1.1)

But (X, M, N, *, ◊) is complete. Hence, there exists a point z in X such that $\{y_n\} \to z$.

Also, we have $\{Ax_{2n-2}\}, \{Tx_{2n-1}\}, \{Sx_{2n}\}, \{Bx_{2n+1}\} \to z$.

Since, (A, S) is compatible of type (K) and one of the mappings is continuous, using proposition (2.11),

we get $Az = Sz$. (3.1.2)

Since $A(X) \subseteq T(X)$, there exists a point u in X such that $Az = Tu$. Now, by contractive condition we get,

$$M(Az, Bu, kt) \geq \min\{M(Sz, Tu, t), M(Az, Sz, t), M(Bu, Tu, t), M(Az, Tu, t), M(Az, Bu, t), M(Sz, Bu, t)\}$$
$$= \min\{M(Az, Az, t), M(Az, Az, t), M(Bu, Az, t), M(Az, Az, t), M(Az,Au,t), M(Az, Bu, t)\}$$

$M(Az, Bu, kt)\} \geq M(Az, Bu, t)$. (3.1.3)

Thus, we get $Az = Sz = Bu = Tu$. (3.1.4)

To prove $Pz = z$, we have

$$M(Az, Bx_{2n+1}, kt) \geq \min\{M(Sz, Tx_{2n+1}, t), M(Az, Sz, t), M(Bx_{2n+1}, Tx_{2n+1}, t), M(Az, Tx_{2n+1}, t), M(Az, Bx_{2n+1}, t), M(Sz, Bx_{2n+1}, t)\}$$

Taking limit as $n \to \infty$, we get

$$M(Az, z, kt) \geq \min\{M(Sz, z, t), M(Az, Sz, t), M(z, z, t), M(Az, z, t), M(Az, z, t), M(Sz, z, t)\}$$
$$= \min\{M(Az, z, t), 1, 1, M(Az, z, t), M(Az, z, t), M(Az, z, t)\}$$

$M(Az, z, kt) \geq M(Az, z, t)$ (3.1.5)

Hence, we have $Az = Sz = z$

So, z is a common fixed point of A and S.

Also, we get $Bu = Tu = z$ (3.1.6)

Since B and T are weakly compatible, we have $TBu = BTu$.

So, from (6), we get Tz = Bz. (3.1.7)

Again, we get

$M(Ax_{2n-2}, Bz, kt) \geq$
$\min\{M(Sx_{2n-2}, Tz, t), M(Ax_{2n-2}, Sx_{2n-2}, t), M(Bz, Tz, t),$
$M(Ax_{2n-2}, Tz, t), M(Ax_{2n-2}, Bz, t), M(Sx_{2n-2}, Bz, t)\}.$

$M(z, Bz, kt) \geq \min M(z, Tz, t), M(z, z, t), M(Bz, Tz, t),$
$M(z, Tz, t)M(z, Bz, t), M(z, Bz, t)\}$
$= \min\{M(z, Bz, t), 1, 1, M(z, Bz, t),$
$M(z, Bz, t), M(z, Bz, t)\}$

$M(z, Bz, kt) \geq M(z, Bz, t).$ (3.1.8)

Therefore, we have Tz = Bz = z. (3.1.9)

Hence, we get that z is a common fixed point of B and T. From (3.1.5), (3.1.8) and (3.1.9),

we get Az = Sz = Bz =Tz = z. So z is a common fixed point of A, B, S, and T.

For uniqueness, let w be the another common fixed point then Aw= Bw = Sw = Pw = w

$M(Az, Bw, kt) \geq \min\{M(Sz, Tw, t), M(Az, Sz, t),$
$M(Bw, Tw, t), M(Az, Tw, t),$
$M(Az, Bw, t), M(Sz, Bw, t)\}$
$= \min.\{M(Az, Bw, t), 1, 1, M(Az, Aw, t),$
$M(Az, Bw, t), M(Az, Bw, t)\}$

$M(Az, Bw, kt) \geq M(Az, Bw, t).$ (3.1.10)

From (3.1.10), and Lemma (2.1.1), we get Az = Bw, this implies Az = Aw Hence z is a unique fixed point.

COROLLARY 3.2

Let (X, M, *) be a complete fuzzy 2- metric space with the condition (FM-6) and let S and T be continuous mappings of X in X, then S and T have a common fixed point in X, if there exists continuous mapping A and B of X into S(X) ∩ T(X) which commute with S and T and

$M(Ax, By, kt) \geq \min\{M(Sx, Ty, t), M(Ax, Sx, t),$
$M(By, Ty, t), M(Ax, Ty, t),$
$M(Ax, By, t), M(Sx, By, t)\}$

For all x, y ∈ X, t > 0 and 0 < q < 1. Then A, B, S and T have a unique common fixed point in X.

COROLLARY 3.3

Let (X, M, *) be a complete fuzzy 3-metric space with the condition (FM-6) and let S and T be continuous mappings of X in X. Then S and T have a common fixed point in X, if there exists a continuous mapping A of X into S(X)∩T(X) which commute with S and T, and

$M(Ax, Ay, kt) \geq \min\{M(Sx, Ty, t), M(Ax, Sx, t),$
$M(Ay, Ty, t), M(Ax, Ty, t),$
$M(Ax, Ay, t), M(Sx, Ay, t)\}$

For all x, y ∈ X, t > 0 and 0 < q < 1, then A, S and T have a unique common fixed point in X.

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