# The Forward Inverse Problem in Robot Control Signal Selection: EEG Artefact Selection Using Discrimination Kernels with Kaczmarz and Algebraic Reconstruction Technique

Chiemela Onunka *Member IAENG*, Glen Bright, Riaan Stopforth

***Abstract*— Various classification and supervised learning techniques can be applied to nonlinear practical problems. Electroencephalographic (EEG) frequency classification using discriminant kernels was used as an elegant classification technique in analysing the forward inverse problem in EEG artefact selection. Discriminant kernels masked in Kaczmarz and Algebraic Reconstruction Technique used EEG frequency feature to learn and transform the EEG signal during EEG artefact selection. The effective management of EEG data through linearization of the artefact selection model provided an efficient local optimal solution in brain-computer interface development and robot control signal application. The models were used as a microcontroller resource management tool in EEG data management and robot control signal selection.**

**This paper presents the selection of EEG artefact using linear discrimination kernels in analysing the EEG inverse problem. The paper also discussed the implementation of Kaczmarz's and algebraic reconstruction technique (ART) in EEG data management. The selected EEG artefacts were used as robot control signals.**

***Index Terms*— EEG Artefact Selection, Kaczmarz Algorithm, Algebraic Reconstruction Technique**

## I. Introduction

Electroencephalographic (EEG) data extraction, estimation, classification and inverse problems are elements of EEG data management process [1]. The elements of EEG signal analysis are processes which can be analysed through EEG signal forward problem and its inverse solution. EEG data property superposition, scaling and characterization provided inferences which are applicable to controllable linear systems. These inferences form the bases in developing linear and nonlinear mathematical models used in the EEG analysis system. The aim in using discrimination kernels was to find an adequate discriminant within and between EEG data classes [2].

In classifying EEG data, the discrimination kernels used the discriminative features of EEG frequency boundaries in EEG data transformation. The discriminative EEG data features were transformed as a matrix of eigenvectors. The components of the eigenvector matrix were within-class and between-class covariance matrices [3]. Multiclass machine learning process was used in discriminating between EEG data points within EEG sample frequency. The EEG sample frequency represented EEG frequency groups. The discrimination process discriminated EEG data points in the sample as EEG frequency class. Considering EEG artefact selection as an ill-posed or ill-conditioned problem provided the necessary condition required in analysing EEG data as a forward inverse problem. Classical techniques such as the singular value decomposition was not an efficient solution to EEG forward inverse problem when used alone. Singular value decomposition provides the necessary condition in identifying the quantifying metric in solving ill-conditioned EEG inverse problem [4] [5].

The paper presents an efficient model on EEG forward inverse problem analysis using linear discriminants masked with Kaczmarz and Algebraic Reconstruction Technique (ART) in EEG artefact selection process. The Kaczmarz and ART models were used to provide an added advantage by serving as an efficient microcontroller resource management tool during EEG artefact selection. The selected EEG artefacts were used as control signals in controlling a mobile robot [6].

## II. WELL-CONDITIONED EEG DATA

In considering EEG data as well-conditioned data, Hadamard condition of well-posed data was applied to the recorded EEG data. The EEG data had the following characteristics which satisfied the Hadamard condition for well-posed data [7]. There existed a selection solution for all detectable EEG signal frequencies. The selection solution was unique for each of the detectable EEG signal. The selection solution depended on continuous streams of EEG signal. In recording the EEG data, activities and task which fell short of the listed Hadamand conditions rendered the recorded EEG data as ill-conditioned.

### A. *The Multi-Frequency Case*

Given that there were six notable EEG frequency bands, the linear discrimination algorithm was extended to the six EEG frequency groups. The EEG frequency bands are indicated in table 1 [8] [9]. The linear discriminant was used as a multiple discriminant by virtue of its dimension

Manuscript received April 20, 2018.

C. Onunka is with Mangosuthu University of Technology Durban, South Africa (corresponding author to provide phone: +27319077134; e-mail: onunka@mut.ac.za).

G. Bright is with the Mechanical Engineering Department, University of KwaZulu-Natal Durban, South Africa (e-mail: brightg@ukzn.ac.za).

R. Stopforth is with the Mechanical Engineering Department, University of KwaZulu-Natal Durban, South Africa (e-mail: stopforth@ukzn.ac.za)

projection, transformation characteristics and properties.

### B. The EEG Forward Problem in Robot Control

The formulation of an EEG forward problem in robot control strategy is finding, in the shortest time possible an EEG artefact suitable to be used as a control signal.

## III. DISCRIMINATING EEG DATA WITH LINEAR KERNELS

Linear discriminant analysis (LDA) provided high-dimensional data analysis for supervised machine learning. Data classification and dimensional reduction were performed using linear discriminant analysis. EEG data were considered to be randomly generated data and its unequal frequencies were grouped into classes. LDA has a functional capacity to examine the different classes of EEG signal frequencies. The LDA technique maximized the ratio between-EEG class variance and within-EEG class variance. The LDA technique ensured optimum separation of the EEG artefacts [10]. LDA used optimal low-dimensional space to project different EEG data class during data classification process. This process facilitated EEG feature extraction before the EEG data were classified [11]. Multivariate EEG frequencies were transformed to univariate data using discrimination methods. Considering the set of EEG data samples, the dimensional sample space is given as. The dimensional class was used to formulate the within-class and between-class scatter matrices. The scatter matrices were modelled as:

$$S_i = \sum_{k=1}^{N_i} \left(x_{i,k} - \bar{x}_i\right)\left(x_{i,k} - \bar{x}_i\right)^T \quad (1)$$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{C} \sum_{k=1}^{N_i} x_{i,k} \quad (2)$$

Where represents the mean of within the EEG dimensional class. The EEG between-class scatter matrix was modelled as:

$$S_B = \sum_{i=1}^{C} N_i \left(\bar{x}_i - \bar{x}\right)\left(\bar{x}_i - \bar{x}\right)^T \quad (3)$$

The EEG within-class scatter matrix is modelled as:

$$S_W = \sum_{i=1}^{C} \sum_{k=1}^{N_i} \left(x_{i,k} - \bar{x}_i\right)\left(x_{i,k} - \bar{x}_i\right)^T \quad (4)$$

Where $x_i$ represents the overall EEG data-cases mean and $\bar{x}_i$ represents each case-mean. The sum of the EEG data matrices and mean vectors were constrained by equation (2). Fisher's discrimination methodology used in the linear transformation of the EEG data. The Linear transformation maximised the ratio between the within-class and the between-class EEG data set. In representing Fisher's discriminant as a linear transformation $\Psi$ of the EEG data, the ratio was modelled as:

$$J(\Psi) = \frac{\Psi^T S_B \Psi}{\Psi^T S_W \Psi} \quad (5)$$

Where $S_B$ represented the "between EEG artefact-classes scatter matrix" and $S_W$ represented the "within EEG artefact-classes scatter matrix", $S_B$ was of rank (c -1) or less. The total scatter matrix for EEG data projections was represented as:

$$S_T = \sum_{\forall x} \left(x - \bar{x}\right)\left(x - \bar{x}\right)^T \quad (6)$$

Where $S_T = S_B + S_W$.The prime Objective of the LDA technique was to perform dimensional reduction on EEG data while preserving the discriminatory EEG artefact class information. For each of the EEG artefact class, their linear function attributes were computed. The EEG class function yielding the highest score represented the predicted EEG artefact. The LDA technique optimized the predicted EEG artefact without multiple passes over EEG data. The LDA provided probability estimates for each of the EEG artefacts. In comparison to principal component analysis (PCA), the LDA produced linear functions required for EEG data reduction. Given that LDA maximized the objective expressed in equation (5) [12]. The EEG data projections having maximum frequency distinct class information represented the eigenvectors corresponding to the largest eigenvalues of $S_W{}^{-1} S_B$. The solution optimized the linear discriminant analysis technique and provided efficient solution to the classification of EEG data [1].

Table 1: EEG Rhythms

| EEG Band | EEG Frequency Range |
|---|---|
| Alpha | 8-13 Hz |
| Beta | 18-25 Hz |
| Mu | 8-12 Hz |
| Theta | 4-7 Hz |
| Gamma | 30-100 Hz |
| Delta | < 4 Hz |

## IV. OPTIMIZING THE INVERSE PROBLEM

In selecting the EEG artefact of choice, there were interferences from other EEG signals. The interferences were as a result of overlapping EEG frequency bands. Optimising the EEG artefact selection and the inverse problem required the minimization of:

$$\left\|\psi x - y\right\|_2{}^2 = \left(\psi x - y\right)_T \left(\psi x - y\right) \quad (7)$$

$$\Phi(x) = \left(\psi x - y\right)^T \left(\psi x - y\right) \quad (8)$$

$$\nabla\Phi(x) = \psi^T \left(\psi x - y\right) + \left(\left(\psi x - y\right)^T \psi\right)^T \quad (9)$$

$\Phi(x)$ can be minimised given that $\psi^T \psi x = \psi^T y x = \left(\psi^T \psi\right)^{-1} \psi^T y$ . This provided the least square solution to the optimisation of the EEG artefact selection algorithm.

ISBN: 978-988-14048-4-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

### *A. Inverse problem development and EEG Signal Analysis*

The models presented in the paper provided the critical solutions embedded in the unique properties of EEG data as the choice control signal in modern robotics [14]. In order to elaborate on the relationship for robotic application, relevant physical parameter associated between the inverse problem, EEG data and control signal with recorded EEG signal d and the connection function $G$ provided the fundamental characterization model expressing the inverse problem. In discretizing EEG analysis system as a linear inverse problem, the EEG data linear system was modelled as:

$$Gm = d \tag{10}$$

It was worth noting that EEG signal $d$ contained noise η in order to accurately represent the actual EEG analysis process. The assumption that the forward representation of the exact system was true and the recorded EEG data model were modelled as:

$$d = G(m_{true}) + \eta \tag{11}$$

The forward EEG signal analysis problem was identified as the identification of EEG artefact of choice in the neighbourhood of neural or cognitive activity $m$. The inverse problem was stated as the identification of the neural or cognitive activity m given the artefact of choice $d$. The sections describing EEG signal analysis were critical in the EEG model identification as it provided the tools necessary for determining the connecting function $G$ provided there were prior evidences or information on the artefact of choice and the cognitive activity. Considering the conditionality of EEG data sequencing, it was useful to model the EEG signal analysis system as a linear system given that linear systems can be scaled and super positioned. In considering EEG signal analysis as a continuous linear inverse problem, the connecting function $G$ between EEG artefact and neural activity can be represented as the linear integral operator with and internal kernel function $g(x,\xi)$ expressed as:

$$\int_a^b g(x,\xi)m(\xi)d\xi = d(x) \tag{12}$$

The linearity of the EEG signal analysis system can easily be expressed if data from each EEG electrode are represented as:

$$\int_a^b g(x,\xi)(m_1(\xi) + m_2(\xi)... + m_n(\xi))d\xi$$

$$= \int_a^b g(x,\xi)m_1(\xi)d\xi + \int_a^b g(x,\xi)m_2(\xi)d\xi \tag{13}$$

$$+ ...\int_a^b g(x,\xi)m_n(\xi)d\xi$$

And the system scalability was modelled as:

$$\int_a^b g(x,\xi)\alpha m(\xi)d\xi = \alpha \int_a^b g(x,\xi)m(\xi)d\xi \tag{14}$$

Considering the neural activity or cognitive activity $m(x)$ as the unknown function, the Fredholm integral equations of the first kind (IFK) was introduced in the EEG data management inverse problem [10]. Useful results were obtained by the discretization of the IFK into tractable techniques. The discretization was executed using linear algebra.

### *B. EEG Signal Convergence –The Inverse Problem Solution*

EEG signal generation was considered to be infinite given that human beings are living subjects. The analysis of EEG signal required the data management system for the EEG signal to handle thousands of rows and column of EEG data. In considering such high-level data management demand and computing power for brain computer interfaces, the section presents an iterative technique which allowed for the convergence of EEG data extraction and also allowed for the classification of EEG artefacts [13].

Consider the sparse matrix $G$ containing EEG data and having only the nonzero component of the EEG data as stored elements in $G$. The density of $G$ represented the function of the stored nonzero elements of $G$. The iterative techniques discussed in this section used m sequence model which converged to the optimal solution rather than using dense matrices for solution convergence. The iterative algorithm multiplied the time vectors of $G$ and $G^T$. This computation required no further storage and computing power with respect to the matrix size. The iterative process took advantage of the matrix sparsity property inherent in large data matrix such as EEG data [10]. The iterative algorithms implemented were the Kaczmarz's algorithm, and the algebraic reconstruction technique (ART). The advantage derived from these iterative techniques enabled the microprocessors managing the EEG data to analyse and extract EEG artefacts for robotic control applications.

## V. THE KACZMARZ'S AND ART ITERATIVE ALGORITHM

Considering the linear model for the EEG data analysis system Gm=d having n-dimensional hyper-plane in $R^m$. The Kaczmarz's algorithm initialized the data management iterative process with $m^{(0)}$ and then transferred the solution $m^{(1)}$ through the projection of the initial solution onto the hyper-plane defined by the first row in the matrix $G$. Similarly, the solution $m^{(1)}$ was projected onto the hyper-plane defined by the second row in the matrix $G$. The process continued until all the data were projected onto their respective hyper-planes. For a particular solution, there were m hyper-planes defined by the EEG data management system. The EEG signal analysis system defined the number of equations required for the solution projection. New sequences of projections were started once the initial

ISBN: 978-988-14048-4-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

solution has been projected on its number of hyper-planes. The cycles were repeated until the solution convergence was attained [10]. Given that the hyper-plane for projection of the EEG data management systems solution for each $i$ projection was modelled as:

$$G_{i+1} m = d_{i+1} \tag{15}$$

The vector perpendicular to the hyper-plane defined by $G_{i+1}^{T}$ and was required for updating $m^i$. Thus:

$$m^{i+1} = m^i + \beta G_{i+1}^{T} \tag{16}$$

Solving for $\beta$ given that $G_{i+1} m^{i+1} = d_{i+1}$ yields:

$$G_{i+1}(m^i + \beta G_{i+1}^{T}) = d_{i+1} \tag{17}$$

$$\beta = -\frac{G_{i+1} m^i - d_{i+1}}{G_{i+1} G_{i+1}^{T}} \tag{18}$$

The update model is given as:

$$m^{i+1} = m^i - \frac{G_{i+1} m^i - d_{i+1}}{G_{i+1} G_{i+1}^{T}} G_{i+1}^{T} \tag{19}$$

There existed a unique solution from EEG data management system which converged to the point closest to the initialisation data $m^0$. If there was no exact solution from the data management system, an approximate solution was considered as the solution. The Kaczmarz's algorithm for managing EEG data converged with respect to the orthogonality of the EEG data. The algorithm converged quickly when the EEG data were orthogonal to the discriminant kernel hyper-plane. If there was more than one hyper-plane, the algorithm converged slowly. Conditions in which there were more than one hyper-plane were resolved by ensuring that hyper-planes were nearly orthogonal to each other through re-ordering of the EEG data. The rough approximation of the Kaczmarz update was implemented using the algebraic reconstruction technique (ART) and it presented a useful methodology in saving memory space in the microcontroller. The approximation to the Kaczmarz's algorithm update replaced all nonzero elements in the $i+1$ row of $G$ matrix with ones. The approximation to the travel time along ray path $i+1$ was modelled as:

$$q_{i+1} = \sum_{cell\, j\, in\, signal path\, i+1} m_j l \tag{20}$$

Where $l$ represented the cell dimension. For EEG signal path $i+1$ having $N_{i+1}$ number of cells, the ART approximate model was modelled as:

$$m_j^{i+1} = \begin{cases} m_j^i - \dfrac{q_{i+1} - d_{i+1}}{l N_{i+1}} & cell\ j\ in\ EEG\ signal\ path\ i+1 \\ m_j^i & cell\ j\ not\ in\ EEG\ signal\ path\ i+1 \end{cases} \tag{21}$$

Given that the EEG signal path length for each of the cells in the matrix varied, the enhanced approximation for the length of signal $L_{i+1}$ in path $i+1$ was modelled as:

$$m_j^{i+1} = \begin{cases} m_j^i + \dfrac{d_{i+1}}{L_{i+1}} - \dfrac{q_{i+1}}{l N_{i+1}} & cell\ j\ in\ EEG\ signal\ path\ i+1 \\ m_j^i & cell\ j\ not\ in\ EEG\ signal\ path\ i+1 \end{cases} \tag{22}$$

The solution from the kaczmarz's algorithm was considered to be converged when there existed no sub-iterations in the algorithm. In this condition, there exist an orthogonal and orthonormal basis for the cell dimension.

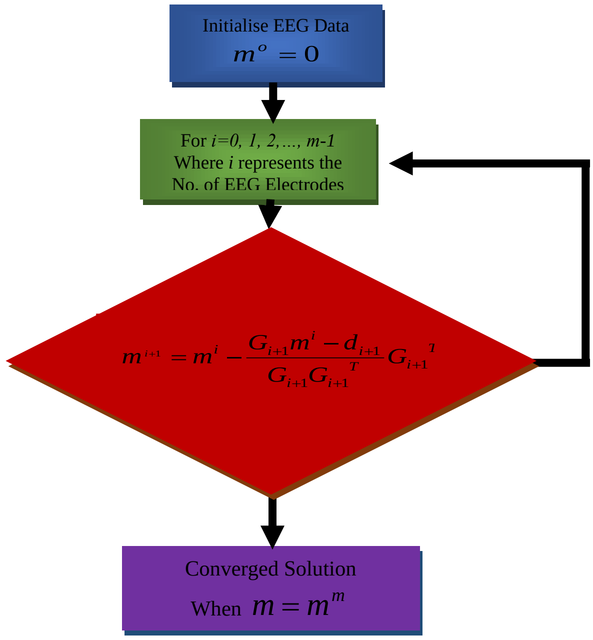


Figure 1: Implementation of Kaczmarz's Algorithm

Given the EEG Signal Analysis Modelled as: $Gm = d$

Step 1. Let $m^0 = 0$ {System Initialization}

Step 2. $For\ i = 0,1,2,\ldots,m$ {Let $N_i$ be the No. of EEG electrodes/Neural nodes Covered by the EEG signal $i$ }

Step 3. $For\ i = 0,1,2,\ldots,m$ {Let $L_i$ be the length of the EEG signal path $i$ }

Step 4. $For\ i = 0,1,2,\ldots,m$ and $j = 0,1,2,\ldots,n$ ; Let

$$m_j^{i+1} = \begin{cases} m_j^i + \dfrac{d_{i+1}}{L_{i+1}} - \dfrac{q_{i+1}}{l N_{i+1}} & cell\ j\ in\ EEG\ signal\ path\ i+1 \\ m_j^i & cell\ j\ not\ in\ EEG\ signal\ path\ i+1 \end{cases}$$

Step 5. $If\ the\ Solution\ != Converged\ Solution$

Let $m^0 = m^m$ {Return to Step 4}

Else

$m = m^m$ {Return the Solution} {Converged Solution}

Figure 2: Implementation Algebraic Reconstruction Algorithm

ISBN: 978-988-14048-4-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

## VI. RESULTS AND DISCUSSION

Facial experiments such as eye blinking, smirking, frowning and smiling were used to record real EEG data sets. The EEG data used in the experiments were recorded from 4 EEG channels and were used to create four-dimensional EEG data sets. The EEG data was used to form a scatter matrix. The scatter matrix was formed using four-dimensional EEG data set in order to test the stability of EEG data as normally distributed points within the identified EEG frequency group. The data sets were used to train the linear discrimination kernels with the aim of selecting EEG artefact of choice. Fig. 3 shows the EEG scatter matrix rotated and modelled to have elliptical shape.

The within-class and between-class scatter matrices for the four-dimensional EEG data sets were computed and projected using the linear discriminant kernel. The within-class data points are indicated by the purple line. The between-class EEG data points are indicated by the blue line and the brown line indicates EEG frequencies which were in both classes. These were usually from overlapping EEG frequencies. When the EEG frequency groups were far apart, the discriminant kernel indicated that overlapping EEG frequencies had less influence on the EEG artefact selection process. The results are shown in Fig. 4_a, Fig. 4_b and Fig. 4_c. The results showed that the linear discriminant kernel maximised the scatter ratio and the discriminant plane. The discriminant kernel plane was perpendicular to the discriminant separator plane. In Fig.4_a, Fig. 4_b and Fig. 4_c, the discriminant kernel plane is indicated by the green line. The blue line in Fig. 4_a to Fig. 4_c represented the discriminant separator plane. Fig. 4_c was zoomed in to show the within-class selection done by the linear discriminant.

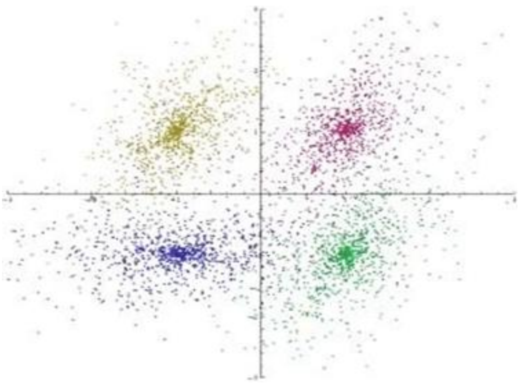

Figure 3: EEG data scatter matrix

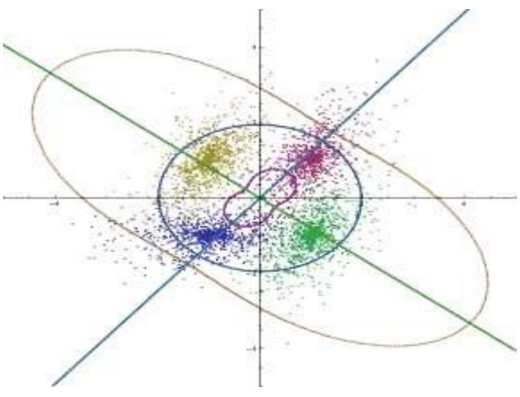

Figure 4_a: Close Interference in EEG frequencies

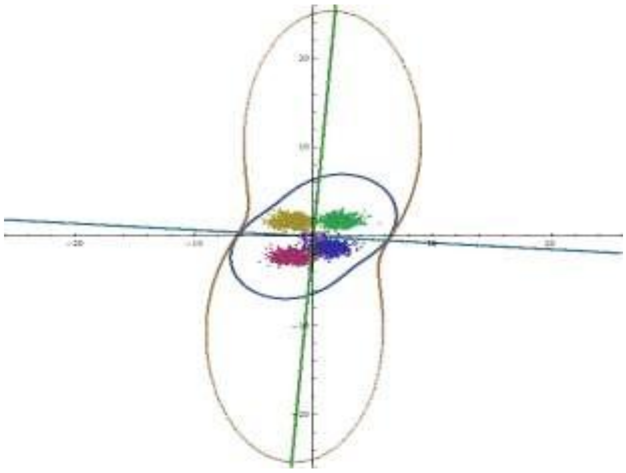

Figure 4_b: Minimal interference in EEG data

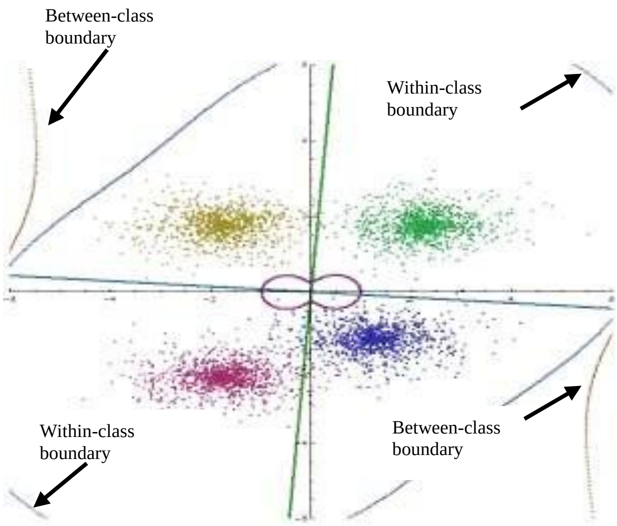


Figure 4_c: Minimal relation in the within-class data group

## VII. CONCLUSION

The study analysed the forward inverse problem in EEG data management by using the Kaczmarz and ART algorithms. A solution suitable for application in microcontroller resource management and robotic control was obtained. The solution was derived from the implementation of the Kaczmarz and ART models in EEG signal analysis and data management. The EEG signal analysis process discretized EEG information and parameterized EEG data using linear discriminant kernels. EEG data management issues were resolved using efficient discretization techniques and compatible kernels in the adaptive EEG artefact selection process. The response of the models affected the solution type derived from the EEG data management system. The system responses were not visible in the statistical solution analysis of the discretised system.

The discretization of the EEG data management system was implemented using adequate approximations to continuous EEG signal. The classical techniques discussed in the paper can be extended to inverse EEG data models. The computation of the complex data management system was performed through resolution of the nonlinear EEG data

ISBN: 978-988-14048-4-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

system having local optimal solution. The optimization was achieved by linearization of the nonlinear EEG signal model in the community of the estimated parameters.

The Kaczmarz and ART models used linear discriminant kernel to maximize EEG data class discrimination and differentiation. The linear discriminant kernel produced as many linear functions as the number of the required EEG data classes. This facilitated the selection of EEG artefacts from raw EEG data. The linear discriminant kernel utilised correlated EEG data vectors in resolving the forward inverse problem in EEG signal analysis.

The algorithms presented in the paper were effective in selecting useful EEG artefacts for robotic control and brain-computer interface (BCI) applications. The computational efficiency of the proposed algorithms allowed for efficient microcontroller resource management in processing EEG data. The memory space in the microcontroller was not used up during the EEG data analysis while using the algorithms. The algorithms enhanced the computational efficiency of the microcontroller as it provided fast control signals from EEG artefacts.

## REFERENCES

[1] M. Mahmud, D. Hawellek and A. Bertoldo, "EEG Based Brain-Machine Interface for Navigation of Robotic Device," in 3rd International Conference on Biomedical Robotics and Biomechatronics, Tokyo, Japan, 2010.

[2] D. Dwinnell, "Data Mining in MATLAB," [Online]. Available: http://matlabdatamining.blogspot.com/2010/12/linear-discriminant-analysis-lda.html. [Accessed 15 October 2016].

[3] M. Wairagkar, I. Zoulias, V. Oguntosin, Y. Hayashi and S. Nasuto, "Movement Intention Based Brain Computer Interface for Virtual Reality and Soft Robotics Rehabilitation using Novel Autocorrelation Analysis of EEG," in 6th International Conference on Biomedical Robotics and Biomechatronics, UTown, Singapore, 2016.

[4] J. C. Mosher, R. M. Leahy and P. S. Lewis, "EEG and MEG: Forward Solutions for Inverse Methods," IEEE Transaction on Biomedical Engineering, vol. 46, no. 3, pp. 245-259, 1999.

[5] H. Murrell, H. Hashimoto and D. Takatori, "Fisher Discrimination with kernels," The Mathematical Journal, vol. 13, pp. 13-13, 2011.

[6] T. Chakraborti, A. Sengupta, D. Banerjee and A. Konar, "Implementation of EEG Based Control of Remote Robotic Systems," in International Conference on Recent Trends in Information Systems, Kolkata, 2011.

[7] S. J. Li and W. Y. Zhang, "Hadamard Well-Posed Vector Optimization Problems," Journal of Global Optimization, vol. 46, pp. 383-393, 2010.

[8] G. R. Shamsaei, Review of Clinical Electroencephalography, Jundishapour: University of Medical Sciences, 2010.

[9] W. O. Tatum, A. M. HUssain, S. R. Benbadiz and P. W. Kaplan, The Handbook of EEG Interpretation, USA: Demo Medical Publishing, 2008.

[10] S. Balakrishnama and A. Ganapathiraju, "Linear Discriminant Analysis-A Brief Tutorial," 1998. [Online]. Available: http://www.music.mcgill.ca/~ich/classes/mumt611/classifiers/lda_theory.pdf. [Accessed 10 September 2015].

[11] C.-C. Chen and A. Hsu, High Dimensional Data Visualization by PCA and LDA, Hsinchu, Taiwan: National Tsing Hua University, 2015.

[12] M. Welling, Fisher Linera Discriminant Analysis, Toronto, Canada, 2005.

[13] J. Hu, Z.-G. Hou, Y.-X. Chen, N. Kasabov and N. Scott, "EEG-Based Classification of Upper-Limb ADL using SNN for Active Robotic Rehabilitation," in 5th IEEE RAS & EMBS International Conference on Biomedical Robotics and Biomechatronics, Sao Paulo, Brazil, 2014.

[14] S. Comani, L. Velluto, L. Schinaia, G. Cerroni, A. Serio, S. Buzzelli, S. Sorbi and B. Guarnieri, "Monitoring Neuro-Motor Recovery from Stroke with High-Resolution EEg, Robotics and Virtual Reality: A Proof of Concept," IEEE Transactions on Neural Systems and Rehabilitation Engineering, vol. 23, no. 6, pp. 1106-1116, 2015.