

Frequency Analyze of Multifunctional Robotic Complexes of Modular Type with Industrial Manipulators

A. Alyukov

Abstract— The article deals with the issues related to the development of high-precision multifunctional robotic complexes of modular type that ensure the fulfillment of tasks related to processing operations (cutting, welding, assembling and painting) of large-sized products with high accuracy and productivity. The results of the research can be used for transport engineering, shipbuilding, oil and gas and chemical industries. The content of the article develops the vibration model of mobile platform with serial manipulator mounted. Two main sources of vibration in mobile platform manipulation systems are the ground uncertainties and manipulator motors stiffness. Vibration can affect the behavior of the manipulator control system and decrease its dynamic properties. One of effective solutions to decrease vibration is to design vibration isolation system. To design isolators it is strictly important to perform frequency analysis of the system “mobile platform-manipulator” and to estimate its natural frequencies. Developed vibration model is based on the Denavit-Hartenberg convention. Platform and manipulator are assumed as one kinematic chain. This allows to decrease the number of system’s degrees of freedom. Motors in manipulator joints are modeled by their stiffness and damping. Simulation and frequency analysis is performed. Based on simulation results, the model of the platform with vibration isolators (modeled by their stiffness and damping) is developed.

Index Terms— Manipulator platform, suspension, vibration isolation.

I. INTRODUCTION

CURRENTLY, due to the increasing number of tasks for which the manipulators are used, wheeled mobile platforms with robots mounted are gaining more propagation. Due to the extremely high restrictions on the mobility and maneuverability of these systems, spherical wheels with high stiffness are often used in the construction. However, this

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A. S. Alyukov is with the South Ural State University, 76 Prospekt Lenina, Chelyabinsk, 454080, Russian Federation (corresponding author, home phone: +7-351-267-97-81; sell phone: +79000716743; e-mail: alyukovalexandr@gmail.com).

solution has a major drawback – high level of vibration from the ground is transferred on manipulator and its end-effector.

The main elements of manipulator - actuating motors and gears connecting the motors with the manipulator links. When designing gears, as well as manipulator links, engineers usually strive to meet the requirements of high stiffness for these elements. However, it is not always possible to reduce the elastic compliance of the elements to the point where its effect becomes negligible, primarily because of the severe restrictions on the weight and sizes of the elements of the manipulator [1].

Elements flexibility and vibration transmitted from the wheels leads to the appearance of elastic vibrations in the dynamics of manipulator and its working body. Elastic vibrations adversely affect the operation of the robot, causing an increase of dynamic loads on components, reducing speed and accuracy while implementing the motion control law, the emergence of non-damping vibrations, dangerous resonance phenomena, etc. Therefore, at the phase of design of the mobile platform, and the manipulator control system, elastic properties of the elements must be taken into account. It is important to perform full frequency analysis of the system "platform-manipulator", which should be based on the entire system of differential equations, taking into account the structural features of the manipulator. This is due not only to the need to harmonize the range of operating frequencies of the designed control system and the frequency properties of the system, but also with the definition of achievable accuracy of the manipulator. The fact that the elastic vibrations are one of the causes of dynamic errors in motion leads us to the importance to estimate the contribution of the elastic elements to the total dynamic error [2].

One of the possible solutions for preventing the vibrations arising in the robot's motions on uncertain surfaces is the use of vibration isolators system. To design such a system is necessary to assess system's main frequencies. On the basis of this it is possible to choose the parameters of vibration isolators. The developed mathematical model allows us to do so without carrying out experimental research on real hardware.

II. DENAVIT-HARTENBERG CONVENTION AND HOMOGENEOUS TRANSFORMATIONS

Denavit-Hartenberg (D-H) convention [3] is a commonly used approach to describe homogeneous transformations of the spatial kinematic chain, which is the kinematic model for the typical manipulation robot. In order to define position of a rigid body, six parameters are generally needed: three

rotational and three translational. D-H convention allows to reduce the number of parameters from six to four. In this convention, coordinate frames are attached to the links and joints of the kinematic chains such that one transformation is for the joint, and the second one is for the link. Consider we have a serial manipulator consisting of n joints. Each homogeneous transformation A_i can be written as

$$A_i = Rot_{z_i, \theta_i} Trans_{z_i, d_i} Trans_{x_i, a_i} Rot_{x_i, \alpha_i} =$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$i = \overline{1, n},$$

where $\theta_i, a_i, d_i, \alpha_i$ – are the four D-H parameters, joint angle, link length, link offset and link twist. There are four constraints on the relationship between the axes:

1. The axis x_i is the axis of revolution of joint j_{i+1} .
2. The axis x_i is perpendicular to the axis z_{i-1} and z_i .
3. The axis x_i intersects the axis z_{i-1} .
4. Coordinate frames are right-handed.

When the coordinate frames are assigned, D-H parameters values can be obtained as:

a_i is the distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i .

d_i is the distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes.

α_i is the angle from z_{i-1} to z_i measured about x_i .

θ_i is the angle from x_{i-1} to x_i measured about z_{i-1} .

Position and orientation of the link i -th in the inertial frame can be obtained by multiplication of the homogeneous transformation:

$$T_i^0 = A_0 \dots A_i = \begin{bmatrix} R_i^0 & o_i^0 \\ 0 & 1 \end{bmatrix},$$

where R_n^0 – rotation matrix, o_n^0 – translation vector.

III. MANIPULATOR JACOBIAN AND EQUATIONS OF MOTION

The manipulator Jacobian $J \in \mathbb{R}^{6 \times n}$ is used in the task of velocity kinematics of the robot, and in derivation of the equations of motion. Jacobian represents the relation between the joint velocities \dot{q} (q – vector of generalized coordinates of the system. If the robot has n joints, then q is a $n \times 1$ vector of joint angles) and the angular and linear velocities of the robot's end-effector:

$$\xi = J \dot{q},$$

where $\xi = \begin{bmatrix} v_n^0 \\ \omega_n^0 \end{bmatrix}$ – vector of the linear and angular velocities.

Then, it possible to define two parts of the Jacobian – linear and angular:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}.$$

Jacobian for the center of mass of the link j can be written as [3]

$$J_{c,i} =$$

$$= \begin{bmatrix} z_0^0 \times (r_{c,j}^0 - o_0^0) & \dots & z_{j-1}^0 \times (r_{c,j}^0 - o_{j-1}^0) & 0 & \dots & 0 \\ z_0^0 & \dots & z_{j-1}^0 & 0 & \dots & 0 \end{bmatrix},$$

if the link j is revolute, and

$$J_{c,i} = \begin{bmatrix} z_0^0 \times (r_{c,j}^0 - o_0^0) & \dots & z_{j-1}^0 & 0 & \dots & 0 \\ z_0^0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix},$$

if the link j is translational.

Here, $r_{c,j}^0$ – is the vector pointing to the center of gravity of the link j , expressed in the inertial coordinate frame

$$o_0 \ x_0 \ y_0 \ z_0, \ z_0^0 = k, \ z_{j-1}^0 = R_{j-1}^0 k, \ k = [0, 0, 1]^T.$$

Commonly, vectors $r_{c,j}^j$ are known, which are pointing to the center of gravity of the link j , expressed in the body attached frame. The transformation to $r_{c,j}^0$ is simply denoted by:

$$r_{c,j}^0 = o_j^0 + R_j^0 r_{c,j}^j.$$

Kinetic energy of the rigid body is the sum of the translational and rotational kinetic energies:

$$K = \frac{1}{2} m v^T v + \frac{1}{2} \omega^T J \omega,$$

where m – total mass of the object, v and ω are the linear and angular velocity vectors, respectively, J – inertia tensor, expressed in the inertial frame. It can be defined as:

$$J = R I R^T.$$

Then, the total kinetic energy of the system equals

$$K = \frac{1}{2} \dot{q}^T \left(\sum_{i=1}^n \left(J_{\omega mi}(q)^T R_i(q) I_{mi} R_i(q)^T J_{\omega mi}(q) + m_i J_{v mi}(q)^T J_{v mi}(q) \right) \right) \dot{q}.$$

Inertia matrix of the system is obtained as

$$M = \sum_{i=1}^n \left(J_{\omega mi}(q)^T R_i(q) I_{mi} R_i(q)^T J_{\omega mi}(q) + m_i J_{v mi}(q)^T J_{v mi}(q) \right).$$

In the case of developed model, there are two sources of potential energy in the system: gravity and torsional stiffness in the manipulator joints. We consider the joint configuration is fixed, so that gravity will have no effect on the vibration as it is a constant force. Potential energy of the joint i due to the motor stiffness can be expressed:

$$P_i = \frac{1}{2} k_i (\theta_i - \theta_{i0})^2,$$

where k_i – joint stiffness value, θ_i – joint angle, θ_{i0} – initial joint angle. For the generalized forces of the system, it is proposed to include the damping of the motors in the joints, which represents to each generalized coordinate i :

$$F_i = -c_i(\theta_i - \theta_{i0}), \quad (13)$$

where c_i – joint stiffness value, θ_i – joint angle, θ_{i0} – initial joint angle.

Equations of motion are obtained in the form of ordinary Lagrange equations:

$$\frac{d}{dt} \frac{\partial K}{\partial \dot{q}_i} - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i} = F_i, i = \overline{1, n}.$$

As we consider a fixed robot configuration set, expression for the kinetic energy will contain no terms depending on generalized coordinates q_i . Thus,

$$\frac{\partial K}{\partial q_i} = 0.$$

IV. VIBRATION MODEL OF THE PLATFORM WITH KUKA LIGHTWEIGHT ARM

Mobile manipulator consists of a mobile platform and a KUKA lightweight arm. The arm has 6 joints (and 6 DOF).

We use D-H parameters to describe the vibration behavior of the system “platform-arm”. It is proposed to model the platform as a single degree-of-freedom rigid body, with its parameters: m_s, I_s – mass and inertia tensor, correspondingly. As the main source of the vibration in the manipulator is from the irregularities in the wheels contact points, we introduce one degree-of-freedom for the platform – its vertical displacement z_s . Figure 1 shows a sketch of the platform with assigned coordinate frames. We introduce several coordinate transformations:

1. $o_1 x_1 y_1 z_1 - o_s x_s y_s z_s$ - transformation from the inertial coordinate frame to the frame assigned to the platform center of gravity.
2. $o_s x_s y_s z_s - o_1 x_1 y_1 z_1$ - transformation from the platform frame to the frame attached to the first link. Next, there are 5 coordinate transformations for each following link.

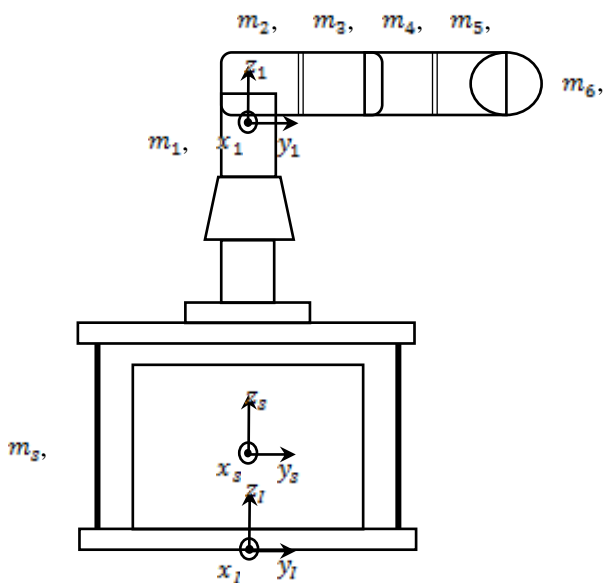


Fig. 1. Sketch of the mobile manipulator with the coordinate frames assigned.

Then, we obtain the D-H parameters, which are summarized in Table 1.

As can be seen from Table 1, the system has 7 DOF.

Then, we can define the vector of the generalized coordinates as:

$$q = [z_s \theta_1 \theta_2 \theta_3 \theta_4 \theta_5 \theta_6]^T. \quad (14)$$

TABLE I
D-H PARAMETERS AND THEIR VALUES FOR THE MOBILE MANIPULATOR

Transformation	θ_i	d_i (m)	a_i (m)	α_i (rad)
I-S	0	$0.2 + z_s$	0	0
S-1	θ_1	0.08916	0	$\pi/2$
1-2	θ_2	0	-0.425	0
2-3	θ_3	0	-0.39225	0
3-4	θ_4	0.10915	0	$\pi/2$
4-5	θ_5	0.09465	0	$-\pi/2$
5-6	θ_6	0.0823	0	0

Then, seven Jacobians are obtained as described in the previous sections, one for each coordinate transformation.

After the Jacobians are calculated, it is possible to define the kinetic energy of the system, and its inertia matrix. Kinetic energy is defined as the sum of 7 kinetic energies – of the platform and of the 6 joints:

$$K = \frac{1}{2} \dot{q}^T \left(\begin{aligned} & \left(m_s J_{vms}(q)^T J_{vms}(q) + \right. \\ & \left. + J_{\omega ms}(q)^T R_s(q) I_{ms} R_s(q)^T J_{\omega ms}(q) \right) + \\ & + \sum_{i=1}^6 \left(m_i J_{vmi}(q)^T J_{vmi}(q) + \right. \\ & \left. + J_{\omega mi}(q)^T R_i(q) I_{mi} R_i(q)^T J_{\omega mi}(q) \right) \end{aligned} \right) \dot{q}.$$

7×7 Inertia matrix is derived.

We use the following values of the joint stiffness, as identified in [4]:

$$k_1 = k_2 = \dots = k_5 = 10000 \frac{N}{m}, k_6 = 7500 \frac{N}{m}.$$

For joint i , consider the damping ratio $\xi=0.1$. Then, we obtain damping coefficients values as

$$c_i = \xi 2 \sqrt{k_i I_i}.$$

After substitution equations (11)-(13) into (14) we obtain linearized system in the form [5,6]

$$M \ddot{q} + C \dot{q} + K q = 0,$$

where M is a 7×7 inertia matrix of the system, C – diagonal dissipative matrix of the joints damping, K – diagonal stiffness matrix of the system.

For the purpose of numerical simulation the model can be rewritten in the form of its highest derivative:

$$\ddot{q} = -M^{-1}(C \dot{q} + K q).$$

As the system has 7 DOF, M is 7×7 matrix. We define 1 DOF as an input of the system (vertical movement of the

platform). We can derive two matrices from M: $M_1 - 6 \times 6$ matrix, that is

$$M_1 = \begin{bmatrix} M_{2,2} & \dots & M_{2,7} \\ \dots & \dots & \dots \\ M_{7,2} & \dots & M_{7,7} \end{bmatrix},$$

and $M_2 - 6 \times 1$ vector

$$M_2 = \begin{bmatrix} M_{2,1} \\ \dots \\ M_{7,1} \end{bmatrix}.$$

Then, the system can be rewritten as

$$\ddot{q}_\theta = -M_1^{-1}(C\dot{q} + Kq + M_2\ddot{x}),$$

where $q_\theta = [\theta_1, \dots, \theta_6]$.

V. VIBRATION MODEL OF THE PLATFORM WITH KUKA LIGHTWEIGHT ARM WITH VIBRATION ISOLATION SYSTEM

To model the vibration isolator system it is proposed to introduce one more degree of freedom in the system. Thus, the platform consists of bodies: sprung mass m_s with its inertia I_s , and unsprung mass m_u , with its inertia I_u . Figure 2 show the sketch of the manipulator.

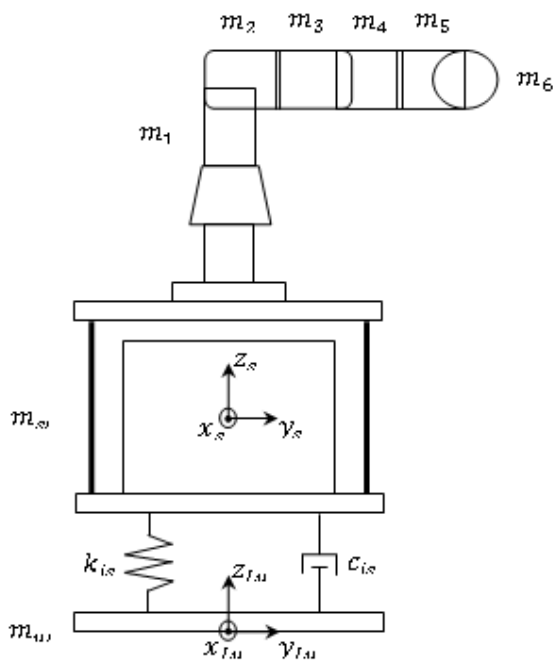


Fig. 2. Sketch of the mobile manipulator with vibration isolation system.

We introduce one more coordinate transformation: $o_u x_u y_u z_u - o_s x_s y_s z_s$ - transformation from the coordinate frame assigned to the center of gravity of the unsprung mass of the platform to the frame assigned to the sprung mass center of gravity.

D-H parameters are summarized in Table 2.

As the system has now 8 DOF, the vector of generalized coordinates is:

$$q = [z_u \ z_s \ \theta_1 \ \theta_2 \ \theta_3 \ \theta_4 \ \theta_5 \ \theta_6]^T.$$

The following procedures are the same, as for the previous model: we obtain 8 Jacobians for each coordinate transformation, write the expression for the kinetic and

potential energies, and derive equations of motion in the form of Lagrange equations [7]. Note, that stiffness and damping values of the isolation system are included in the expressions for potential energy and generalized forces, correspondingly.

(21)

TABLE II

D-H PARAMETERS AND THEIR VALUES FOR THE MOBILE MANIPULATOR				
Transformati on	θ_i	d_i (m)	a_i (m)	α_i (rad)
I-U	0	z_u	0	0
U-S	0	$0.2 + z_s$	0	0
S-1	θ_1	0.08916	0	$\pi/2$
1-2	θ_2	0	-0.4223	0
2-3	θ_3	0	-0.39225	0
3-4	θ_4	0.10915	0	$\pi/2$
4-5	θ_5	0.09465	0	$-\pi/2$
5-6	θ_6	0.0823	0	0

As described above, the system can be written in the form

$$\ddot{q}_\theta = -M_1^{-1}(C\dot{q} + Kq + M_2\ddot{x}),$$

with M_1 is a 7×7 matrix

$$M_1 = \begin{bmatrix} M_{2,2} & \dots & M_{2,8} \\ \dots & \dots & \dots \\ M_{8,2} & \dots & M_{8,8} \end{bmatrix},$$

and $M_2 - 7 \times 1$ vector

$$M_2 = \begin{bmatrix} M_{2,1} \\ \dots \\ M_{8,1} \end{bmatrix}.$$

VI. SIMULATION RESULTS

Developed software can be used to perform frequency response analysis of the system. In order to do this, inputs and outputs of the system have to be assigned. We choose vertical displacement of the platform x as an input variable, and angle θ_6 as an output. An example is shown on Figure 3, where 100 input sinestream signals are generated with frequencies from 1 Hz to 100 Hz.

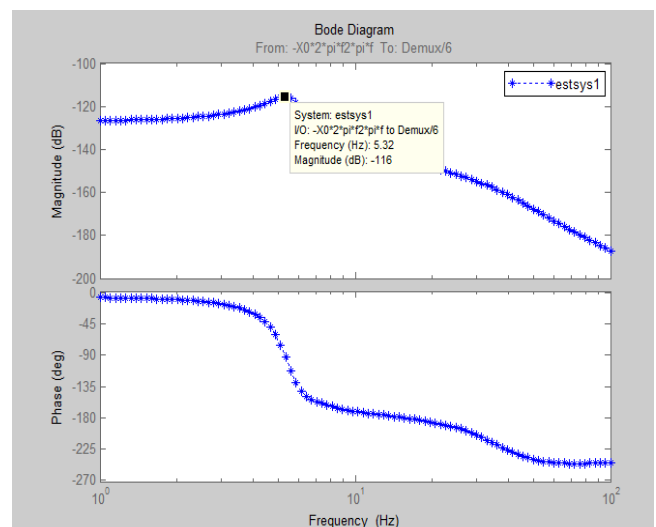


Fig. 3. Bode diagram of the system.

VII. STIFFNESS ESTIMATION

The described approach for designing the vibration isolation system can be used if the robot configuration remains constant during the platform movement. To provide good vibration isolation when the platform moves with manipulator in different configurations, or while performing operations, it is essential to change the characteristics of the system. We can consider the problem of designing active vibration isolation system. To do this, it is strictly important to identify the natural frequencies of the system in certain configuration. This can be achieved by introducing joint stiffness observer.

As it is shown in [8], consider the case of constant linear impedance in the elementary system

$$f = m\ddot{y} + b\dot{y} + ky,$$

where we assume that m, b, k are unknown but constant, and y and f can be measured, as force and position. To prove the solution exists and is unique, consider the state vector

$$\zeta = \left(y \quad \dot{y} \quad -\frac{k}{m} \quad -\frac{b}{m} \quad \frac{1}{m} \right)^T,$$

And rewrite the system as

$$\dot{\zeta} = \begin{pmatrix} \zeta_2 \\ \zeta_1 \zeta_3 + \zeta_2 \zeta_4 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \zeta_5 \\ 0 \\ 0 \\ 0 \end{pmatrix} f,$$

$$y = \zeta_1.$$

The nonlinear observability problem [9-11] then is to estimate the initial state $\zeta(0)$, from the knowledge of the input f and output y . From the

$$\Omega(\zeta) = span \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \zeta_3 & \zeta_4 & \zeta_1 & \zeta_2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \zeta_3 \zeta_4 & \zeta_2 + \zeta_4^2 & \zeta_2 + \zeta_1 \zeta_4 & \zeta_1 \zeta_3 + 2\zeta_2 \zeta_4 & 0 \\ 0 & 0 & 0 & \zeta_5 & \zeta_4 \end{pmatrix}$$

can be easily seen the, for $m, b, k > 0$, $\Omega(\zeta)$ has codimension zero for all states, except if $\zeta_1 = \zeta_2 = 0$. So, whenever the system is not in stationary equilibrium, the impedance can, in principle, be reconstructed.

To actually estimate the impedance, different methods can be adopted (e.g. extended Kalman filters).

VIII. CONCLUSION

The article describes the approach for developing vibration models of the mobile platforms with manipulators mounted. For this, mobile platform and manipulator are assumed as one kinematic chain with rigid connection. Using Denavit-Hartenberg convention allows to decrease the number of degrees-of-freedom of the system. Frequency analysis of the platform with KUKA lightweight arm mounted was performed, and natural frequency of the system was estimated. Vibration model of the platform with vibration isolators is developed.

REFERENCES

- [1] R. Paul. Robot Manipulators: Mathematics, Programming and Control: the Computer Control of Robot Manipulators, 1981.
- [2] S. Burdakov. Design of the manipulators, industrial robots and robotic systems. Moscow: Higher School, 1986.
- [3] R. Hartenberg, J. Denavit. Kinematic synthesis of linkages. McGraw-Hill series in mechanical engineering. New York: McGraw-Hill. p. 435.
- [4] A. Jubien, M. Gautier and A. Janot. Dynamic Identification of the Kuka LightWeight Robot: comparison between actual and confidential Kuka's parameters. Proc. of 2014 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM). Pages 483-488.
- [5] K. Kufieta. Force Estimation in Robotic Manipulators: Modeling, Simulation and Experiments. Master's Thesis. Norwegian University of Science and Technology, NTNU, 2011.
- [6] L-W. Tsai. "Robot Analysis: The Mechanics of Serial and Parallel Manipulators". NY, 1999, John Wiley & Sons, Inc.
- [7] W. W. Melek. "ME 547: Robot Manipulators: Kinematics, Dynamics, and Control". Waterloo, ON, 2010, University of Waterloo.
- [8] Siciliano, Khatib. Springer Handbook of Robotics. Springer Publishing Company, Incorporated, 2016.
- [9] S. Aliukov, A. Keller, A. Alyukov, "Dynamics of Overrunning Clutches of Relay Type," SAE Technical Paper 2015-01-1130, 2015, doi:10.4271/2015-01-1130.
- [10] S. Aliukov, A. Keller, A. Alyukov, "Method of Calculating of Relay Type Free-Wheel Mechanism," SAE Technical Paper 2015-01-2782, 2015, doi:10.4271/2015-01-2782.
- [11] S. Alyukov, "Improved models of inertial torque transformers," Russian Engineering Research, vol. 30, Issue 6, 2010, pp. 535-542.