An EWMA-type Control Chart with Variable Sampling Intervals for Monitoring Process Mean and Variability

Shin-Li Lu; Chen-Fang Tsai; Chi-Jui Huang

***Abstract*—Traditional single exponentially weighted moving average (EWMA) charts for monitoring the process mean and variability are based on taking samples from the process with fixed sampling intervals (FSI). Recent research has demonstrated that control chart with variable sampling intervals (VSI) are quicker than the traditional static ones in detecting process shifts. In this paper, we develop an EWMA-semicircle (EWMA-SC) chart with the VSI feature to enhance the speed of detecting changes in mean and/or variability. Numerical simulations are used to evaluate the average time to signal (ATS) of the VSI EWMA-SC and the corresponding fixed sampling interval (FSI) EWMA-SC charts. Results show that the proposed VSI EWMA-SC chart is considerably more efficient than the FSI EWMA-SC chart.**

Keywords- adaptive control chart; variable sampling interval (VSI); EWMA-SC control chart; average time to signal (ATS)

I. INTRODUCTION

The exponentially weighted moving average (EWMA) chart was first introduced by Roberts (1959) and has been widely used to improve the quality of a product or a manufacturing process when small process shifts are of interest. Traditionally, two control charts are used to simultaneously monitor the shifts in the process mean and variability. In recent years, with the innovation of process control technology, the use of a single EWMA control chart has gradually begun to be taken seriously.

Recent research has demonstrated that adaptive control charts are more effective than traditional static ones in detecting process shifts. Typical adaptive charts can be divided into four types: variable sampling interval (VSI), variable sample size (VSS), variable sample size and variable sampling interval (VSSI), and variable parameter (VP). Relative works are listed as follows:

(1) VSI: Reynolds, Amin, Arnold, and Nachlas (1988) proposed the VSI $\overline{X}$ chart to control the process mean. Various other studies that examine VSI charts include Reynolds (1989); Runger and Pignatiello (1991); Shamma, Amin, and Shamma (1991); Saccucci, Amin, and Lucas (1992); Amin and Miller (1993); Runger and Montgomery (1993); and Reynolds, Arnold, and Baik (1996).

(2) VSS: The VSS $\overline{X}$ chart was first studied by Prabhu, Runger, and Keats (1993) and Costa (1994). Thereafter, some others developed alternative VSS charts, such as Daudin (1992); Annadi, Keats, Runger, and Montgomery (1995); and Zhang and Wu (2007).

(3) VSSI: VSSI charts allow both the sample size and sampling interval to vary depending on the previous value of the control statistic. References on VSSI charts include Prabhu, Montgomery, and Runger (1994); Costa (1997); Arnold and Reynolds (2001); Celano, Costa, and Fichera (2006); and Mahadik and Shirke (2011).

(4) VP: The VP control chart, in which all chart parameters are variable, was developed by Costa (1999b). The idea of adapting all chart parameters was applied to the $\overline{X}$ charts by Lin (2009) and Deheshvar, Shams, Jamali, and Movahedmanesh (2013).

Most control charts found in the literature monitor process mean and variance shifts separately. Traditionally, studies have used two control charts to simultaneously monitor shifts in process mean and variance. Recently, however, considerable attention has been directed to single-chart methods to monitor process mean and variability. The obvious advantages of the method are (a) simple to use and interpret (b) reduced time, resource, work force, money, and effort (c) easily detects shifts in process mean and/or changes in process variability, and (d) identifies the cause of shifts (i.e., mean and/or variation) (Cheng and Thaga, 2006). Domangue and Patch (1991) developed an omnibus EWMA chart to simultaneously detect changes in both the location and spread of a process. Xie (1999) presented several types of EWMA charts, such as Max-EWMA, sum of squares exponentially weighted moving average (SS-EWMA), EWMA-Max, and exponentially weighted moving average semicircle (EWMA-SC). Chen, Cheng, and Xie (2001, 2004) extended Xie's research to the Max-EWMA and EWMA-SC charts.

In this paper, the EWMA-SC chart is combined with the VSI feature to enhance the speed of detecting changes in mean and/or variability. This novel control chart is called the VSI EWMA-SC chart. In addition, simulations are performed to evaluate the VSI EWMA-SC's adjusted average time to signal (AATS) and the corresponding FSI EWMA-SC charts. An extensive comparison shows that the VSI EWMA-SC control chart is significantly quicker than the FSI EWMA-SC control chart.

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The rest of this paper is structured as follows. Section 2 reviews the EWMA-SC chart. In Section 3, we introduce the VSI feature and propose the VSI EWMA-SC chart. The results of a simulation study are presented in Section 4 to compares the performance of the VSI EWMA-SC and EWMA-SC charts in terms of their average time to signals (ATSs) and conclusion is provided in Section 5.

II. A REVIEW ON THE EWMA-SC CHART

Xie (1999) first introduced the concept of the EWMA semicircle (EWMA-SC) chart, and Chen et al. (2004) extended the research of Xie on the EWMA-SC chart. The EWMA-SC chart not only can monitor both the process mean and the increased process variability simultaneously, but it also clearly indicates the source and direction of a shift.

Let X be a quality characteristic of a process. It has a normal distribution with mean $\mu_0 + \delta\sigma_0$ and standard deviation $\rho\sigma_0$, where μ_0 and σ_0 are defined as standard values of the process. When $\delta = 0$ and $\rho = 1$ indicate that the process is in control, otherwise the process has changed or drifted.

Suppose X_{tj} , $t = 1, 2, \dots$, and $j = 1, 2, \dots, n_t$ be the measurements of the variable X arranged in groups of size n_t with t as the index of the group number. Let $\bar{X}_t$ and $S_t^{\,2}$ denote the sample mean and sample variance of sample t, respectively. Then $\bar{X}_t$, $t = 1, 2, \dots$ are independent normal random variables with mean $\mu_0 + \delta\sigma_0$ and variance $\rho^2\sigma_0^{\,2}/n_t$; $(n_t - 1)S_t^{\,2}/\rho^2\sigma_0^{\,2}$ and $t = 1, 2, \dots$ are independent chi-square random variables with $n_t - 1$ degrees of freedom; and $\bar{X}_t$ and $S_t^{\,2}$ are independent. Define the following two statistics:

$$U_t = (\bar{X}_t - \mu_0)^2 + \frac{n_t - 1}{n_t} S_t^{\,2}, \quad t = 1, 2, \dots . \tag{1}$$

Let $U_t^* = \dfrac{n_t}{\sigma_0^{\,2}} U_t$. The EWMA-SC statistic V_t can be defined from U_t^* as follows:

$$V_t = \lambda U_t^* + (1-\lambda)V_{t-1}, \quad 0 < \lambda \le 1 , \quad t = 1, 2, \dots, \tag{2}$$

where λ is the smoothing constant while $V_0 = n$ is the starting value of V_t . Because $U_t^* \sim \chi^2(n)$ when $\delta = 0$, $\rho = 1$ and $n_1 = n_2 = \cdots = n_t = n$, and so we have the following results:

$$E(V_t) = E(U_t^*) = n , \tag{3}$$

$$Var(V_t) = \frac{2n\lambda\left[\, 1-(1-\lambda)^{2t} \,\right]}{2-\lambda} , \tag{4}$$

In addition, Equation (2) can be rewritten as:

$$V_t = A_t + B_t + n , \tag{5}$$

where $A_t = \lambda\left[\dfrac{(\bar{X}_t - \mu_0)^2}{\sigma_0^2/n} - 1\right] + (1-\lambda)A_{t-1}$ and $B_t = \lambda\left[(n-1)\left(\dfrac{S_t^2}{\sigma_0^{\,2}} - 1\right)\right] + (1-\lambda)B_{t-1}$ with $A_0 = B_0 = 0$. Additionally, it is known that A_t and B_t are also independent because $\bar{X}_t$ and $S_t^{\,2}$ are independent.

Because V_t is non-negative, the EWMA-SC chart only needs an upper control limit (UCL). The UCL corresponding to Equation (2) is given by:

$$\mathrm{UCL}_1 = n + L\sqrt{\frac{2n\lambda\left[\, 1-(1-\lambda)^{2t} \,\right]}{2-\lambda}} , \tag{6}$$

and the UCL corresponding to Equation (5) is given by:

$$\mathrm{UCL}_2 = L\sqrt{\frac{2n\lambda\left[\, 1-(1-\lambda)^{2t} \,\right]}{2-\lambda}} . \tag{7}$$

Note that the term $\left[\, 1-(1-\lambda)^{2t} \,\right]$ in Equations (6) and (7) approaches unity as t gets larger. This means that the UCL will approach the steady state values given by:

$$\mathrm{UCL}_1 = n + L\sqrt{\frac{2n\lambda}{2-\lambda}} , \tag{8}$$

and

$$\mathrm{UCL}_2 = L\sqrt{\frac{2n\lambda}{2-\lambda}} . \tag{9}$$

Here, L is the width of the control limits when the process is in the control state. The process is considered to be out of control whenever V_t exceeds UCL_1 or (A_t, B_t) is outside the control region $\{\, (A_t, B_t):\ A_t + B_t \le \mathrm{UCL}_2 \,\}$, and some action should be taken. When the source of an assignable cause can be directly identified by plotting the location of the sample point of the chart, the latter method of determining is preferable. In addition, to avoid drawing several parallel lines, Chen et al. (2004) plotted (Y_t', Z_t') on a $A' - B'$ coordinate plane and drew the line $A_t' + B_t' = L$ as the boundary of the control region, where $A_t' = A_t\sqrt{\dfrac{2-\lambda}{2n\lambda}}$ and $B_t' = B_t\sqrt{\dfrac{2-\lambda}{2n\lambda}}$.

III. THE VSI EWMA-SC CONTROL CHARTS

A VSI chart is a control chart in which the next sampling interval is allowed to change as a function of the current value of the control statistic. Therefore, the sampling interval of the VSI EWMA-SC chart can be of any form. Assume that a finite number of interval lengths $h_1, h_2, \dots, h_k$, where $h_1 \le h_2 \le \cdots \le h_k$, are employed in the VSI chart proposed in this paper. The control region $\{\, (A_t', B_t'):\ A_t' + B_t' \le L \,\}$ be partitioned into k regions $R_1, R_2, \dots, R_k$, where R_i is the region in which the interval h_i is used when $A_t' + B_t' \in R_i$. Then, the sampling interval function $h(t)$ can be represented as

$$h(t) = \begin{cases} h_1, & if \quad A_t' + B_t' \in R_1 = \{(A_t', B_t') : W_1 < A_t' + B_t' \le L\} \\ h_i, & if \quad A_t' + B_t' \in R_i = \{(A_t', B_t') : W_i < A_t' + B_t' \le W_{i-1}\} \\ h_k, & if \quad A_t' + B_t' \in R_k = \{(A_t', B_t') : A_t' + B_t' \le W_{k-1}\} \end{cases} \tag{10}$$

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where t is the subgroup index, and $A_t' + B_t'$ is the control statistic for the VSI scheme. Note also that L and W_i, $i = 1, 2, \ldots, k-1$ are the UCL and corresponding i^{th} warning limit (WL) of the VSI EWMA-SC chart, respectively, and factor W_i is the width of the i^{th} warning limit $0 < W_{k-1} < W_{k-2} < \cdots < W_1 < L < \infty$ (see Fig. 1). In many applications, it is reasonable to use the shorter sampling interval h_1 for the first sample to avoid problems during startup.

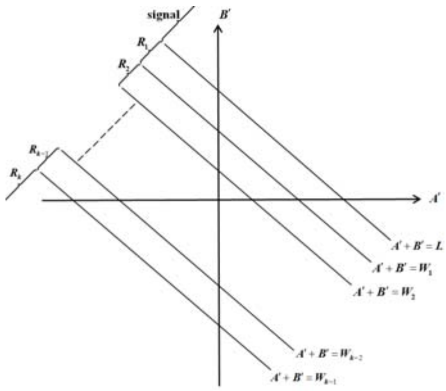

Fig. 1. EWMA-SC chart with variable sampling intervals.

To facilitate the derivation of W_i, p_i is defined as the probability of being in the region R_i when the process is in control:

$$p_i = \begin{cases} P\left\{ W_1 < A_t' + B_t' \le L \,\middle|\, A_t' + B_t' \le L \right\} & i = 1 \\ P\left\{ W_i < A_t' + B_t' \le W_{i-1} \,\middle|\, A_t' + B_t' \le L \right\} & i = 2, 3, \cdots, k-1 \\ P\left\{ A_t' + B_t' \le W_k \,\middle|\, A_t' + B_t' \le L \right\} = & i = k \end{cases}, \tag{11}$$

where $p_1 + p_2 + \cdots + p_k = 1$. Due to the intricacy of the distribution of $A_t' + B_t'$, factor W_i corresponding to different p_i s can be obtained using Monte Carlo simulations. To make the EWMA-SC chart with and without VSI comparable, the same in-control average sampling interval is used; that is,

$$h_1 p_1 + h_2 p_2 + \cdots + h_k p_k = h_0 , \tag{12}$$

where h_0 is the fixed sampling interval (FSI) of the EWMA-SC chart. Note that the VSI EWMA-SC chart is identical to a FSI EWMA-SC chart when $h_1 = h_2 = \cdots = h_k = h_0$. Moreover, for comparison purposes, $h_0 = 1$ is assumed in this paper without loss of generality (Refer to Reynolds, Amin, and Arnold (1990) for a more detailed explanation).

Many researchers (Li & Wang, 2010; Reynolds & Arnold, 2001; Reynolds, Amin, Arnold, & Nachlas, 1988; Runger & Montgomery, 1993) have shown that using two sampling intervals provides good statistical properties in VSI control charts. Therefore, the above Equation (12) can be simplified as

$$h_1 p_1 + h_2 p_2 = h_0 , \tag{13}$$

where $p_1 + p_2 = 1$. Then for given values of (h_1, h_2) and $h_1 \le h_0 \le h_2$, p_1 and p_2 must satisfy

$$p_1 = \frac{h_2 - h_0}{h_2 - h_1}, \quad p_2 = \frac{h_0 - h_1}{h_2 - h_1} . \tag{14}$$

They also suggest that the value of h_1 and h_2 should be spaced far apart. In other words, when $h_0 = 1$, the VSI chart with the smaller p_2 is usually perceived to have better performance. For our proposed VSI chart, these results are demonstrated with computer simulations in Section 4.

IV. Performance measurement and comparisons

Measuring statistical performance of VSI control charts is typically assessed in terms of its average time to signal (ATS). According to He and Grigoryan (2002), the ATS is defined as the expected value of the time between the process starting and chart signaling. The ATS should be sufficiently long to avoid false alarms when the process is under control, and it should also be sufficiently short to detect shifts rapidly when the process is out of control. To compare the performance of the control charts, first, charts with the same in-control ATS (ATS_0) are designed. Then, a shift is introduced during the process, and the out-of-control ATSs (ATS_1s) of these charts for this shift are compared.

A Monte Carlo simulation is performed using FORTRAN 95 to compute the ATS profile for the VSI EWMA-SC and FSI EWMA-SC charts, where each of these values is an average of 100,000 simulation trials. The in-control ATS, $\mathrm{ATS}_0 = 370$, sample size, $n_t = 5$, and the steady state limit are considered in the simulation study. The values X_{tj}, $t = 1, 2, \ldots$, $j = 1, 2, \ldots, n_t$, and $n_t = 5$ follow an independent normal distribution with mean δ ($\mu_0 = 0$) and variance ρ^2 ($\sigma_0^2 = 1$), and the normal random variables are generated by the International Mathematics and Statistics Library (IMSL, 1989). Various mean shifts $\delta \in \{0, 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3\}$ and standard deviation shifts $\rho \in \{1, 1.05, 1.1, 1.25, 1.5, 2, 2.5, 3\}$ with different design parameters $\lambda \in \{0.05, 0.06, \ldots, 1.00\}$ in both charts are also considered.

According to the above-mentioned properties, the following two results need proving for our proposed VSI chart. In Result 1, it is optimal to use only two sampling intervals, and in Result 2, two sampling intervals should be spaced far apart. Tables 1 and 2 demonstrate these two results. Comparisons for $k = 2, 3, 5, 9, 19$ are shown in Table 1 for the special case of sampling intervals symmetric about $h_0 = 1$, and for equal interval probabilities p_i ($i = 1, 2, \ldots, k$). Here, the λ value is 0.05 in the comparisons of VSI and FSI charts. All the ATS_1s in the VSI chart are less than their corresponding values in the FSI chart, for the detection of the same changes. Furthermore, the ATS_1 values for $\delta > 0$ and/or $\rho > 1$ are uniformly smaller for the VSI chart with $k = 2$ and gradually increasing as k increases. These phenomena can be also found in other values of λ.

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Table 1 (λ , L, W) combinations for the VSI SS-EWMA chart in a steady state with sample sizes $n_t = 3,\ 4,\ 5$

shift		FSI	VSI				
ρ	δ		k=2	k=3	k=5	k=9	k=19
1.00	0.00	370.01	370.00	370.00	370.00	370.00	370.00
	0.25	137.96	**94.60**	98.39	101.11	103.13	105.79
	0.50	30.68	**9.52**	11.08	12.14	12.98	14.12
	0.75	11.66	**2.29**	2.95	3.31	3.61	4.04
	1.00	6.26	**0.92**	1.27	1.45	1.59	1.80
	1.50	2.88	**0.31**	0.39	0.45	0.50	0.57
	2.00	1.78	**0.18**	0.19	0.20	0.22	0.24
	2.50	1.27	**0.13**	0.13	0.13	0.13	0.13
	3.00	1.04	**0.10**	0.10	0.10	0.10	0.10
1.05	0.00	86.73	**47.72**	51.03	53.39	55.16	57.51
	0.25	50.85	**21.10**	23.43	25.10	26.38	28.10
	0.50	20.12	**5.23**	6.29	6.96	7.51	8.26
	0.75	9.71	**1.81**	2.36	2.66	2.90	3.25
	1.00	5.69	**0.83**	1.14	1.30	1.43	1.61
	1.50	2.78	**0.30**	0.38	0.44	0.48	0.55
	2.00	1.76	**0.18**	0.19	0.20	0.22	0.24
	2.50	1.26	**0.13**	0.13	0.13	0.13	0.14
	3.00	1.05	**0.10**	0.10	0.10	0.11	0.11
1.50	0.00	5.17	**0.83**	1.11	1.25	1.36	1.53
	0.25	4.93	**0.78**	1.04	1.17	1.28	1.44
	0.50	4.35	**0.66**	0.88	0.99	1.08	1.21
	0.75	3.67	**0.52**	0.69	0.77	0.85	0.95
	1.00	3.03	**0.39**	0.52	0.58	0.64	0.72
	1.50	2.11	**0.24**	0.29	0.32	0.35	0.39
	2.00	1.56	**0.16**	0.18	0.19	0.20	0.22
	2.50	1.24	**0.12**	0.13	0.13	0.14	0.14
	3.00	1.08	**0.11**	0.11	0.11	0.11	0.11

Table 2 $(p_0,\ \lambda,\ L,\ W,\ h_1,\ h_2)$ combinations and the corresponding AATSs for optimal VSI SS-EWMA control charts, for various shift combinations of (δ, ρ), based on the steady state $\mathrm{ATS}_0 = 370$ and sample size $n_t = 5$.

shift		FSI	VSI (h_1, h_2)							
			symmetric			asymmetric				
ρ	δ	$h_0 = 1$	(0.5, 1.5)	(0.3, 1.7)	(0.1, 1.9)	(0.1, 1.1)	(0.1, 1.6)	(0.1, 3.1)	(0.1, 9.1)	(0.1, 90.1)
1.00	0.00	370.01	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
	0.25	137.96	113.87	104.24	94.60	125.69	101.99	78.80	59.37	40.69
	0.50	30.68	18.93	14.22	9.52	21.27	11.89	5.73	3.73	3.16
	0.75	11.66	6.45	4.37	2.29	6.51	3.13	1.32	1.17	1.17
	1.00	6.26	3.29	2.11	0.92	3.01	1.29	0.64	0.63	0.63
	1.50	2.88	1.45	0.88	0.31	0.96	0.37	0.29	0.29	0.29
	2.00	1.78	0.89	0.54	0.18	0.34	0.18	0.18	0.18	0.18
	2.50	1.27	0.63	0.38	0.13	0.15	0.13	0.13	0.13	0.13
	3.00	1.04	0.52	0.31	0.10	0.11	0.10	0.10	0.10	0.10
1.05	0.00	86.73	65.06	56.39	47.72	74.10	53.63	36.00	24.10	15.47
	0.25	50.85	34.32	27.71	21.10	39.50	24.96	14.10	8.77	6.15
	0.50	20.12	11.85	8.54	5.23	12.83	6.76	2.97	2.16	2.02
	0.75	9.71	5.32	3.57	1.81	5.25	2.50	1.07	0.98	0.97
	1.00	5.69	2.99	1.91	0.83	2.69	1.15	0.58	0.57	0.57
	1.50	2.78	1.41	0.85	0.30	0.92	0.36	0.28	0.28	0.28
	2.00	1.76	0.88	0.53	0.18	0.34	0.18	0.18	0.18	0.18
	2.50	1.26	0.63	0.38	0.13	0.15	0.13	0.13	0.13	0.13
	3.00	1.05	0.52	0.31	0.10	0.11	0.10	0.10	0.10	0.10
1.50	0.00	5.17	2.76	1.80	0.83	2.47	1.15	0.53	0.52	0.52
	0.25	4.93	2.63	1.70	0.78	2.33	1.08	0.51	0.49	0.49
	0.50	4.35	2.30	1.48	0.66	1.97	0.91	0.44	0.44	0.44
	0.75	3.67	1.92	1.22	0.52	1.55	0.70	0.37	0.37	0.37
	1.00	3.03	1.57	0.98	0.39	1.16	0.52	0.30	0.30	0.30
	1.50	2.11	1.07	0.65	0.24	0.60	0.28	0.21	0.21	0.21
	2.00	1.56	0.78	0.47	0.16	0.30	0.17	0.16	0.16	0.16
	2.50	1.24	0.62	0.37	0.12	0.17	0.13	0.12	0.12	0.12
	3.00	1.08	0.54	0.32	0.11	0.12	0.11	0.11	0.11	0.11

In Table 2, comparisons are given for three symmetric and five asymmetric VSI charts, all using two sampling intervals matched to a FSI chart with $h_0 = 1$ and $\mathrm{ATS}_0 = 370$. As expected, the more widely spaced intervals yield smaller values of the ATS_1 relative to the FSI chart. From an administrative point of view, it is fortunate that only two sampling intervals perform best.

Next, in order to compare the performances of the charts on an equal footing, the smallest AATS is computed from each combination of δ and ρ in VSI and FSI charts. From the design procedure in the previous section, $p_2 \in \{0.01,\ 0.02,\ \ldots,\ 1.00\}$ and $h_1 \in \{0.1,\ 0.2,\ \ldots,\ 1.0\}$ are utilized for the VSI EWMA-SC. After p_2 and h_1 are determined, h_2 is computed from Eq. (13). For each shift combination of (δ, ρ), we can obtain the smallest ATS_1 and the corresponding optimal combination of $(p_2,\ \lambda,\ L,\ W_1,\ h_1,\ h_2)$ .The $(p_2,\ \lambda,\ L,\ W_1,\ h_1,\ h_2)$ combination, having the smallest ATS_1 value, has been generally viewed as the optimal parameter combination in past work. A similar approach is used to identify the optimal parameter combination for the FSI EWMA-SC chart, wherein $p_2 = 1$ (namely, $L = W_1$) and $h_1 = h_2 = h_0 = 1$.

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V. Conclusions

This study investigates the effectiveness of the VSI EWMA-SC and FSI EWMA-SC control charts in simultaneously detecting shifts in the process mean and/or in the increased process variability. The analytical results show that, by adding the VSI feature, the EWMA-SC control chart has better detection abilities than its FSI form in terms of run-time performance.

Further recommended work is to investigate the robustness of VSI EWMA-SC control charts in terms of the quality characteristics of a process under the assumption of non-normality. To enhance the detection ability of the proposed chart, an analytic study of its performance based on the VSS, VSSI, and VP features should be undertaken. In addition, when designing control charts from a statistical perspective, other viewpoints, such as the costs of sampling, inspection, and defective products when using the VSI EWMA-SC control chart, should be consider in future studies.

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