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# Robust DC Motor System and Speed Control using Genetic Algorithms with Two Degrees of Freedom and H Infinity Control

N. Chitsanga, and S. Kaitwanidvilai

*Abstract*— **This paper presents a new design controller called "Robust and fixed-structure 2DOF control" for a DC motor and speed control system. The controller is designed using the concept of 2DOF control with robust loop shaping. The specification of time domain includes in the controller design by using the reference model. Moreover, the structures of both pre-filter and feedback controllers are fixed as a simple structure to make the system robust and still useful in practical works. Instead of solving mathematically complicated equations, Genetic Algorithm is proposed to solve the process of the design control problem. The proposed control is implemented in a speed control of DC Motor. As seen in the results, the proposed system performs better performance, compared to the 1DOF Fixed Structure control and Robust Loop Shaping Control. When the system payloads are changed from 0 kg to 16 kg, the step response from the conventional 1 DOF controller is oscillated obviously at 16 kg., while the proposed controller performs the smooth and stable response.**

*Index Terms*—**2DOF H infinity Control, DC motor system, genetic algorithm, Fixed-Structure control and Robust Control**

## I. Introduction

IN the design of control, researchers are interested in robust 2DOF control because it retains the robustness and gain better performance comparable to that of the 1DOF control and the robust PID control. The robust 2DOF controller is one of the most solutions for designing well in both time and frequency domains. It is well-known that the conventionality of robust 2DOF control has a high order of controllers [1-2]. Although many methods can reduce the order, the performance of the entire system is deteriorating. The other research in [3] showed that the robust 2DOF control is good enough for overall system, including nonlinear system, but this method is difficult to analyze and synthesize the controllers. Thus, the proposed technique called the fixed structure and robust 2DOF control is presented in this paper to solve this problem. The approach is based on the fundamental control of H infinity and loop shaping control. When the controllers are fixed as low order structure, then the parameters of controllers are optimized by the genetic algorithm (GA). In the experiment, the controllers are designed for the DC motor system at the nominal operating point at the speed of 380 rpm. The changing in payloads is performed to test the robustness of the proposed and conventional systems.

The outline of this article is as follows. Section II shows the linear model of the DC motor system and the model identification. In Section III, the conventional $H_\infty$ loop shaping and the proposed controller are illustrated. Section IV shows the results obtained from the presented control and the fixed structure and 1DOF control with robust loop shaping control. Section V concludes and discusses the results of implementation of the speed control of DC motor system.

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## II. Dynamic Model and Modeling

### A. Linear dynamic model of DC motor system

Typically, the DC motor system has the state space equations as (1) and (2).

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_a \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{k_v}{L_a} \\ \frac{k_t}{J} & -\frac{B}{J} \end{bmatrix}\begin{bmatrix} i_a \\ \omega_a \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}\begin{bmatrix} V_a \\ T_L \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i_a \\ \omega_a \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_a \\ T_L \end{bmatrix} \tag{2}$$

$\omega_a$ is the angular velocity; $V_a$ is the voltage input; $R_a$ is the resistance of DC motor; $L_a$ is the inductance of DC motor; $J$ is the inertia of the rotor; $B$ is the damping coefficient associated with the mechanically rotational system; $k_t$ is the torque constant; $k_v$ is the velocity constant; $i_a$ is the armature current; $T_L$ is the torque of the mechanical load [4].

### B. System identification of DC motor system

System identification is a standard method to find the parameters of dynamic models from the measured data in any systems. In this paper, the linear model of DC motor is modeled as the form of OE (Output Error) method, [5].

Bringing the measured data and setting the number of poles $(n_f)$ with zero plus one $(n_b)$ of the system and delay $(n_k)$, the process of system identification can be computed the model parameter, i.e., $f_1, f_2, \ldots, f_{nf}$ and $b_1, b_2, \ldots, b_{nb}$.

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The transfer function of the standard OE system is as follows:

$$G(s) = \frac{B(s)}{F(s)} = \frac{b_{nb}s^{(nb-1)} + b_{nb-1}s^{(nb-2)} + \cdots + b_1}{s^{nf} + f_{nf}s^{(nf-1)} + \cdots + f_1} \quad (3)$$

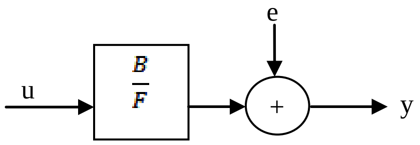


Fig.. 1 Diagram of OE model

Based on the model in (2), the linear model of a DC motor using OE (Output Error) method is presented by $n_b = 1$ ,$n_f = 2$ ,$n_k = 1$.

The result of identification is demonstrated in Fig. 2. and the resulting model is demonstrated in (4).

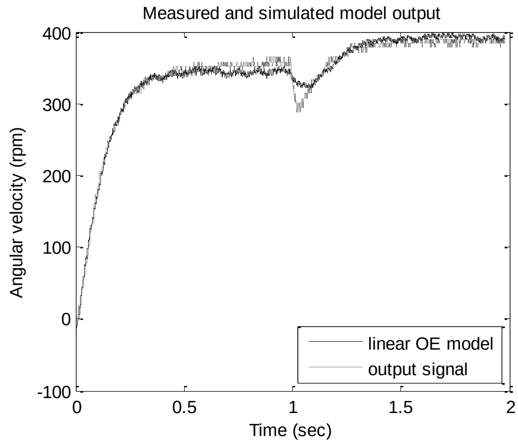


Fig.. 2 The simulated and measured output response of the linear model

$$G_s = \frac{1.198{\times}10^{4}\ s + 5.669{\times}10^{7}}{s^2 + 1.333{\times}10^{4}\ s + 5.476{\times}10^{7}} \quad (4)$$

## III. The Fixed structure and 1DOF control with $H_\infty$ Loop Shaping and the Proposed Control Applied by Genetic Algorithm

### A. *Fixed structure and 2DOF control with $H_\infty$ loop shaping [1]*

The 2DOF H infinity control is a method which can combine both time and frequency domain specifications into a single index, called stability margin. The 2DOF H infinity control consists of a feed-forward or pre-filter controller, $K_1$, and a feedback controller, $K_2$. In considering the loop shaping design, $Gs$ is co-prime factor of the shaped plant [6] which consists of a nominator factor, $N_s$, and a denominator factor, $M_s$. Fig. 3 illustrates the uncertainty model of the system and the robust control of systems.

A loop shaping system can be described in the following equation.

$$G_s = GW_1 = M_s^{-1}N_s \quad (5)$$

Equation (6) is accomplished with the uncertainty in the system, and it can be shown as the following equation.

$$G_\Delta = (N_s + \Delta_{Ns})(M_s + \Delta_{Ms})^{-1} \quad (6)$$

$G_\Delta$ is the uncertain system.
$\Delta_N$ is the uncertain transfer function of the nominator.
$\Delta_{Ms}$ is the uncertain transfer function of the denominator.

$$|\Delta_{Ns}, \Delta_{Ms}|_\infty \le \varepsilon \quad (7)$$

$\varepsilon$ is stability margin.

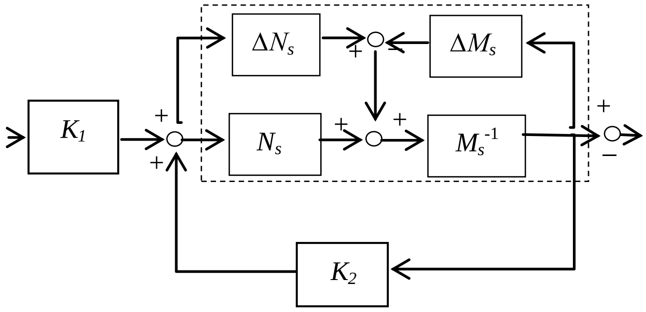


Fig. 3 the uncertainty model in co-prime factor

It is well known that the controller from the conventional H infinity control has high order, and normally the high order controller cannot be implemented easily in practice [1]. To solve this problem, the proposed technique formulates the fixed structure control in the 2DOF H infinity control and loop shaping design problem, and searches the parameters by genetic algorithm.

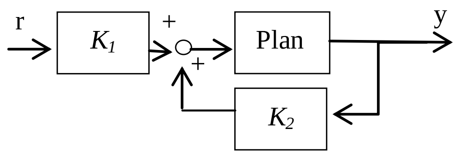


Fig. 4 2 DOF control is fixed-structure controllers.

The fixed-structure robust 2DOF control with $H_\infty$ loop shaping is the proposed control which the structures of the controllers are able to be specified. The design procedures of this method are summarized as follows [1].

**Step 1** The structure of $T_{ref}$, $W_1$, $K_1$ and $K_2$ is selected as:

$$K_1 = \frac{1}{K_f s + 1} \quad (8)$$

$$K_2 = K_p + \frac{K_i}{s} + \frac{K_d s}{T_d s + 1} \quad (9)$$

$$W_1 = K_w \frac{s + \alpha}{s + \delta} \quad (10)$$

**Step 2** Reference model ($T_{ref}$) is designed to identify the desired time domain of the closed loop system and this technique uses $\rho$ to select the ratio between the desired time and robust performance. The range of $\rho$ is from 0 to 1. If $\rho = 0$, the 2DOF and H infinity control become the 1DOF and H infinity control. Calculating the $\varepsilon_{opt}$ shows in (11) which is the maximum value of stability margin. It can be obtained from the optimization problem.

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$$\gamma_{opt}\rho^{-2} = \varepsilon_{opt}^{-1}\rho^{-2} = \left|(I - G_s K_2)^{-1} G_s K_1 - T_{ref}\right|_\infty \quad (11)$$

$K_1$ and $K_2$ are synthesized by solving this inequality equation.

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} \rho(I - K_2 G_s)^{-1} K_1 & K_2 (I - G_s K_2)^{-1} M_s^{-1} \\ \rho(I - G_s K_2)^{-1} G_s K_1 & (I - G_s K_2)^{-1} M_s^{-1} \\ \rho^2 [(I - G_s K_2)^{-1} G_s K_1 - T_{ref}] & \rho(I - G_s K_2)^{-1} M_s^{-1} \end{bmatrix} \right\|_\infty \le \varepsilon^{-1} \quad (12)$$

Finally, $W_i$ is computed by

$$W_i = \left[W_o\left(I - G_s(0)K_2(0)\right)^{-1} G_s(0) K_1(0)\right]^{-1} T_{ref}(0) \quad (13)$$

$W_o = 1$, and $K_1$ and $K_2$ are the synthesized controllers.

**Step 3** Genetic Algorithm searches the control parameters of $K_1$, $K_2$, and $W_1$ simultaneously.

*B. The Fixed structure and1DOFcontrol with $H_\infty$ loop shaping [2]*

The fixed structure and 1DOF control with $H_\infty$ loop shaping are a technique, using GA, [7]. Following steps are used for this method design.

**Step 1** The structure of $W_1$, $K_{PID}$ is selected as.

$$K_{PID}(p) = \left[K_p + \frac{K_i}{s} + \frac{K_d s}{T_d s + 1}\right] \quad (14)$$

$$W_1 = K_w \frac{s+\alpha}{s+\delta} \quad (15)$$

**Step 2** Genetic Algorithm searches the parameters of $K_1$ and $W_1$ simultaneously. Thus, the system is robust and obtains the specified performance at the same time.

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_{PID\infty} \end{bmatrix} (I + G_s K_{PID\infty})^{-1} M_s^{-1} \right\|_\infty \le \varepsilon^{-1} \quad (16)$$

By $K_{PID\infty} = W_1^{-1} K_{PID}(p_2)$

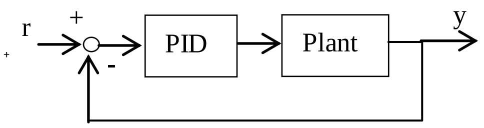


Fig. 5. 1DOF control

## IV. TESTING SYSTEM AND EXPERIMENTAL RESULTS

When the plant is identified by the OE system identification at the operating speed, 380 rpm, the 2DOF controllers, $K_1$, $K_2$ and weighting function are searched by the Genetic Algorithm. The reference model is selected as (17). In the Genetic Algorithm, the boundaries of parameters are selected as shown in Table 1 and the result of convergence of the solution is obtained as shown in Fig. 6.

TABLE 1. THE PARAMETERS OF GENETIC ALGORITHM OPTIMIZATION AND THE BOUNDARY OF CONTROL OPTIMIZATION PROBLEM FOR ROBUST 2 DOF CONTROL

| Parameter | Boundary |
|---|---|
| $K_{p1}$ | [1, 10] |
| $K_{i1}$ | [1, 20] |
| $K_{d1}$ | [0, 0.005] |
| $\tau_{d1}$ | [0, 100] |
| $x_1$ | [0, 100] |
| $x_2$ | [0, 3000] |
| $K_{f1}$ | [0, 1005] |
| Population | 150 |
| the probability of mutation | 0.2 |
| the probability of crossover | 0.7 |

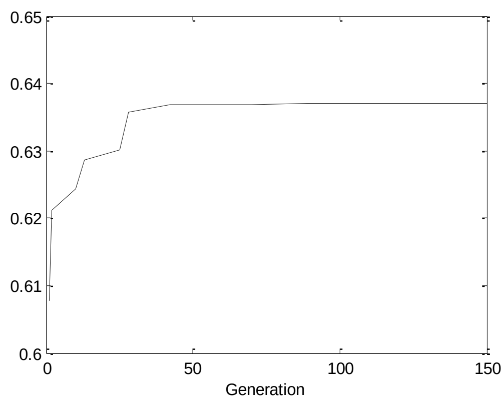


Fig. 6. The stability margin from genetic algorithm for robust 2 DOF control

The resulting controllers from the presented method and conventional method are shown in Table 2. In addition, the weights and controllers can be demonstrated in Table 2.

TABLE 2. THE WEIGHTS, CONTROLLERS AND STABILITY MARGIN OF THE PROPOSED CONTROL AND 1DOF CONTROL

| | 1DOF control | 2DOF control |
|---|---|---|
| Weighting function | $W_1 = \frac{5.1845s + 20.0024}{s + 0.001}$ | $W_1 = \frac{27.4544s + 449.556}{s + 0.001}$ |
| Controller | $K_{PID}(p) = 1.864 + \frac{10.999}{s} + \frac{0.4214s}{80.715s + 1}$ | $K_1(p) = \frac{1}{1002.887s + 1}$ <br> $K_2(p) = 25.776 + \frac{600.433}{s} + \frac{0.266s}{71.806s + 1}$ |
| Stability margin | 0.481 | 0.635 |

Fig. 7 shows the step responses of the presented controller and the robust 1DOF controller, compared with the reference model.

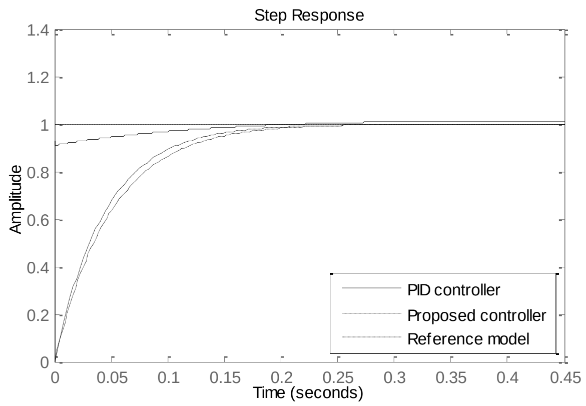


Fig. 7. Step response of control

TABLE 3. THE COMPARISON OF TIME DOMAIN PERFORMANCE OF THE DC MOTOR SPEED CONTROLS.

| | Settling time (s) | Rise time (s) | Overshoot (%) |
|---|---|---|---|
| Proposed controller | 0.181 | 0.0992 | 0 |
| 1DOF controller | 0.133 | 0.0625 | 0 |
| Reference model | 0.196 | 0.1097 | 0 |

$$T_{ref} = \frac{1}{1+0.05s} \quad (17)$$

Table 3 shows the simulation results of step responses from the proposed controller and the 1DOF controller. The response of the proposed controller is close to the reference model, but that of the1 DOF control is much different to the desired response.

The proposed and 1DOF controllers were also tested in the real DC motor speed control system. The results of step responses of the proposed controller and the 1DOF controller indicate that the overshoot is not presented in the step responses of both methods, and the settling time of the proposed controller is close to that of the 1DOF method. The robust performance of the DC motor system was verified by taking the different payloads to the system, which increases the torque load on the system. The loads were increased from 0 kilograms to 16 kilograms. The experimental results indicate that the proposed controller obtains high performance and well robustness. The results shown in Fig. 8(a) and 8 (b) are the responses at the system load 16 kilograms. Clearly, the robust performance of proposed controller is better than that of the 1DOF controller. There is a large steady state error and oscillation in 1DOF control response.

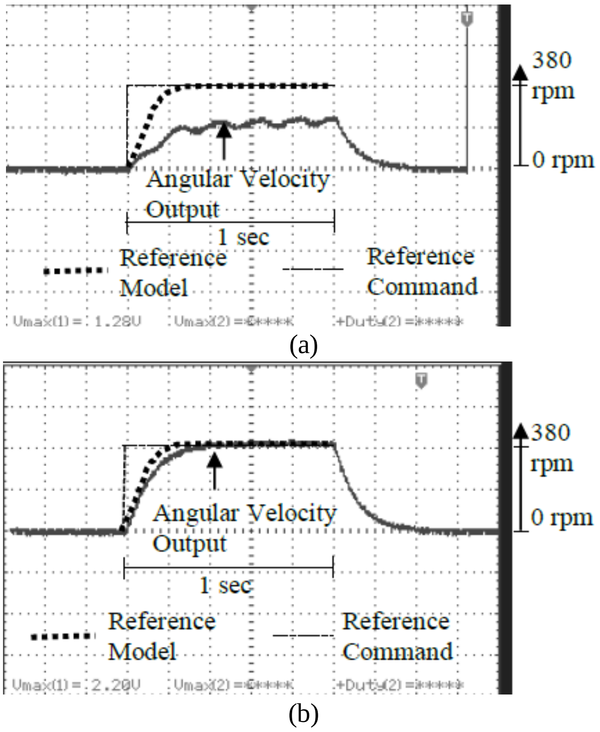


Fig. 8. The step responses of both controllers at 16 kilograms load (a) 1DOF controller and (b) the proposed controller

TABLE 4. THE DYNAMIC RESPONSES OF THE PROPOSED CONTROLLER AND 1DOF AT VARIOUS LOADS IN REAL ON DC MOTOR SYSTEM.

| Weight( kg) | The proposed controller | | | The 1DOF method | | |
|---|---|---|---|---|---|---|
| | Rise time (s) | Settling time (s) | Steady state error (%) | Rise time (s) | Settling time (s) | Steady state error (%) |
| 0 | 0.23 | 0.43 | 0 | 0.29 | 0.47 | 0 |
| 2 | 0.23 | 0.43 | 0 | 0.29 | 0.47 | 0 |
| 4 | 0.23 | 0.43 | 0 | 0.29 | 0.47 | 0 |
| 6 | 0.23 | 0.43 | 0 | 0.29 | 0.47 | 0 |
| 8 | 0.23 | 0.43 | 0 | 0.29 | 0.47 | 0 |
| 10 | 0.23 | 0.43 | 0 | 0.32 | 0.5 | 31 |
| 12 | 0.23 | 0.43 | 0 | N/A | N/A | 36 |
| 14 | 0.23 | 0.43 | 0 | N/A | N/A | 42 |
| 16 | 0.23 | 0.43 | 0 | N/A | N/A | 49 |

## V. CONCLUSION

In this research, the OE system identification is adopted to find the transfer function of the DC motor system. The proposed controllers are designed using the Genetic Algorithms to find the parameters of fixed-structure 2DOF robust controllers which can guarantee the system by the stability margin. Although this technique cannot guarantee by mathematical proof, this solution can be proved by single indicator that is stability margin. This method is simple and easy to be used in real practice. In the experiments, the results indicate that the response of the proposed controller is better than that of the 1DOF controller in terms of robustness. Obviously, at the 16 kilograms load, the time domain performance of the proposed controller is not

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changed from the nominal condition. In contrary, the performance of the 1DOF controller is deteriorated by large steady state error and oscillations.

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