Mathematical Modeling of Electrochemical Machining Used in the Manufacture of Turbine Engine Parts

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***Abstract*— Electrochemical Machining (ECM) provides an economical and effective method for machining high strength and heat-resistant materials into complex shapes. This paper presents the physical and mathematical models of electrochemical shaping process used in the manufacture of turbine engine parts. The mathematical modeling is based on the assumption that the multi-phase mixture filling the gap is treated as two-phase quasi-homogenous medium The results of computer simulation of electrochemical shaping of airfoil are discussed.**

***Index Terms*— electrochemical machining, ECM, , mathematical modeling, computer simulation**

I. INTRODUCTIONINTRODUCTION

The further development of turbine engines is decreasing unitary fuel consumption, device weight, exploitation costs and negative effects on natural environment. The most important factors, which improve total engine efficiency, is the turbine inlet gas temperature.

Therefore, the temperature increasing is one of the main directions of modern engine development. Advanced aero engine materials are developed to meet high strength level at elevated temperatures, to support higher compression rates and higher gas temperatures Machining of these materials and complex components is really too difficult by mechanical methods.

The unconventional manufacturing technology such as Electrochemical Machining (ECM) is therefore be a cost-effective alternative for these materials and is still important in the manufacture of turbine engine parts.

As shown in Fig. 1 machining based on controlled anodic electrochemical dissolution process in which the workpiece is the anode and the tool is the cathode of an electrolytic cell. In the ECM process, a low voltage (8-30V) is normally applied between electrodes with a small gap size (usually 0.2 to 0.8 mm) producing a high current density of the order of (10 to 100 A/cm^2), and a metal removing rate ranging from an order 0.1 mm/min, to 10 mm/min. Electrolyte (typically NaCl or $NaNO_3$ aqueous solutions) is supplied to flow through the gap with a velocity of 10 to 50 m/s to maintain the electrochemical dissolution with high rate and to flush away the reactions products (usually gases and hydroxides) and heat generated caused by the passage of current and electrochemical reactions.

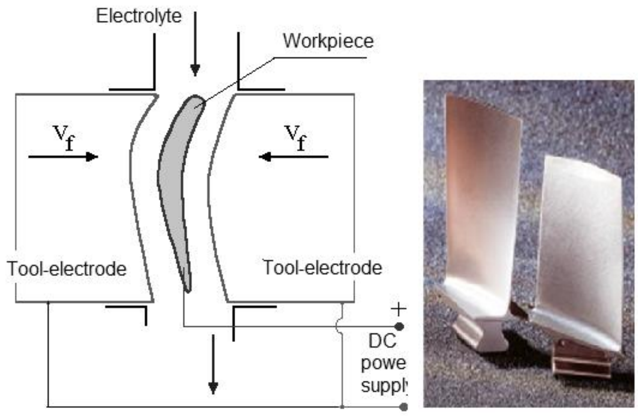

Fig.1. Principal scheme of electrochemical machining (ECM) and example of the machined airfoils.

As electrochemical dissolution proceeds, the tool electrode-cathode can be fed mechanically towards the workpiece - anode in order to maintain the machining action. Under these conditions, the inter-electrode gap width gradually tends to a steady-state value, and a shape, complementary to that of the cathode-tool, is reproduced approximately on the anode-workpiece.

Being a non-mechanical metal removal process, ECM is capable of machining any electrically-conductive material with high stock removal rates regardless of their mechanical properties, such as hardness, elasticity and brittleness.

It has been applied in diverse industries, for example to manufacture airfoils and turbine blades, die and mold, surgical implants and prostheses [1]-[7].

Industrial practices in ECM have revealed some problems impeding its further development and wider acceptance by industrial users. Among them, prediction and control of the local gap width distribution (and hence, the control of dimensional accuracy), along with the design of tool electrodes for complex workpiece shapes and optimization of process, are the major problems encountered by ECM users.

The main objective of ECM is to achieve the required shape of workpiece within a given tolerance on the shape and dimensions. The tasks relating to this purpose can be reduced directly (or indirectly) to a problem of searching for a boundary of the area within which the machining, i.e. to a value boundary problems (moving boundary problem, free boundary problem or inverse boundary problem).

Manuscript received May 04, 2018; revised July 25, 2018.
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Depending on which the electrode surface is to be determined all tasks can be divided into two groups [2]–[5]:
1 - tasks in which for a known shape of the tool electrode and known condition of machining, the evolution of a shape of workpiece surface has to be determined,
2 - tasks in which the tool electrode shape is searched for, which ensures obtaining the required shape of the workpiece.

The first category of the problems is encountered in the analysis of ECM accuracy. The tasks from the second category mainly deal with the tool electrode design and most frequently are encountered in practice.

The paper presents the physical and mathematical models, basis of which a process simulation software of has been developed. The example of results of computer simulation of blade profile shaping are discussed.

II. THE QUASI-HOMOGENEUS MODEL OF ECM PROCESS

During the ECM process, the generation of Joule heat and the liberation of hydrogen gas bubbles on the cathode surface accompany the release anodic dissolution products. Consequently, the electrolyte flowing in the gap becomes a multi-phase medium with rising temperature and void fraction in the flow direction, leading to corresponding changes in the electrical conductivity and the current density along the gap as well. Simultaneously, the changes of other properties of medium such as specific medium density, viscosity, lead to redistribution of flow velocity and pressure in the gap. At the beginning of the ECM in the entire region of the gap, the hydrogen generated appears as a two-phase layer with changing of thickness in time and along flow. It led to difficulties in analysis of ECM process.

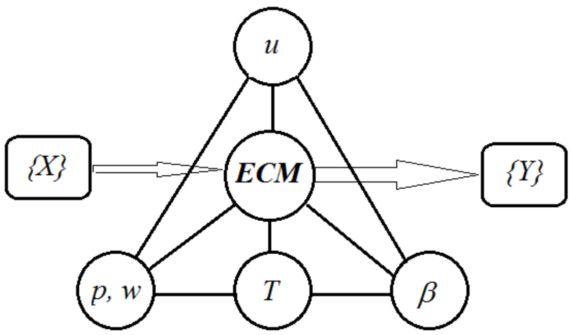

Fig. 2. Schematic diagram of relationships in ECM process.

Figure 2 qualitatively illustrates basic physical fields and machining conditions and their interrelationships having important bearing on the ECM process and summarizes the basic components of process characteristics connected with shaping by electrochemical machining at contoured electrodes. Inspection of the Figure 2 shows complex nature of shaping process and mathematical difficulties in solving field problems. All physical fields in the gap are non-stationary during whole time of machining. Since properties of electrolyte such as electrical conductivity κ are depending on temperature T, and gas phase concentration β, which distributions depend on flow velocity w, and pressure fields p, as well as on current density i. The ECM processes have to be described by set of mass, heat and electric charge transfer equations.

Application of computer simulation techniques based on mathematical modeling is necessary for analysis of machining process and tool electrode design.

The mathematical model of ECM process, referring to the formulated problems consists of sequence of mutual conjugated partial models which describe in the gap:
- distribution of the local gap size, S,
- distribution of the flow parameters such as the static pressure, p, and the velocity, w,
- distribution of the temperature, T,
- distribution of the void fraction, β or the thickness layer, h, with two phase flow (electrolyte and gas),
- distribution of the electrical conductivity, κ.

The more simple mathematical models of the physical conditions and processes in the gap, nevertheless quite adequate in many cases of ECM condition, are the quasi-homogeneous models.

In the quasi-homogeneous models, the multi-phase mixture filling the gap (liquid – gas – solid particle) as well as in layered form, is treated as a uniformly-mixed quasi-continuous medium with constant void fraction in the direction normal to the electrolyte flow (see the distribution β in Fig.3) and equal to the void fraction averaged over the cross-section.

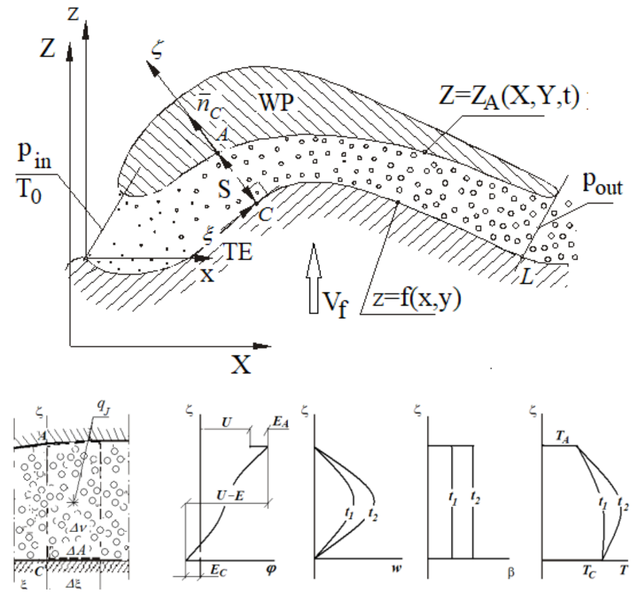

Fig.3. Schematic of quasi-homogeneous two-phase model of medium in the ECM gap.

The following condition are assumed for above models:
- The bubble layers are assumed to extend completely across the channel.
- Ohm's law is applicable in the whole gap to relate the current density i with medium electrical conductivity κ, pulse voltage U and gap size S.
- The electrical conductive of the two-phase medium can be determined by

$$\kappa = \kappa_o \left[1+\alpha_T \theta\right]\left(1-\beta\right)^{3/2},$$

where $\theta = T - T_0$ is temperature increment of electrolyte.
- Electrochemical and diffusion processes are accounted for by the characteristic of anodic dissolution $k_V(i)$ and introducing the total overpotential E (i) into the boundary condition.

ISBN: 978-988-14049-0-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

- The surface tension effect on the gas bubbles is neglected, and so is the bubble formation time.
- The effective density of pseudo-homogeneous medium is determined as following:

$$\rho = \rho_e\left(1-\beta\right)+\rho_g\beta,$$

taking into account that liquid density, ρ_e , is nearly 1000 times more than gases density, ρ_g , we can set on without loss of accuracy:

$$\rho = \rho_e(1-\beta)$$

To simplify the calculations let us introduce a curvilinear coordinate system (ξ ,ζ), connected with the tool-electrode (TE) in which a coordinate ξ lies on the given electrode and is measured from the inlet of the electrolyte and let axis ζ overlap its normal n_C (Fig.3).

To formulate the mathematical model, a general case describing change in shape of the surface of the workpiece can be examined using coordinate system {X,Y,Z} attached to the workpiece (WP), which is immovable during machining (Fig. 3).The surface of the workpiece at a given moment in time can be described by: $Z = Z_A (X, Y, t)$.

According to electrochemical shaping theory, the evolution of the shape of the workpiece, can be described as follows [5] – [7]:

$$\frac{\partial Z_A}{\partial t} = k_V(i_A)i_A\sqrt{1+\left(\frac{\partial Z_A}{\partial X}\right)^2+\left(\frac{\partial Z_A}{\partial Y}\right)^2} \tag{1}$$

where: k_V is the coefficient electrochemical machinability, which is defined as the volume of material dissolved per unit electrical charge.

At the beginning of machining: $t = 0$, $Z = Z_0(X, Y)$, where: $Z_0(X, Y)$ describes an initial shape of the workpiece surface.

Calculating the current density distribution is generally the crucial step in solving an electrochemical moving boundary problem.

Current density distribution in the gap.

Taking into account that the gap size is much less than the length and the width and the radius of surfaces, we can use one-dimensional approximation for the distribution of electrical potential *u* along *the distance, S,* of a given point of the anode from the tool-electrode surface is described by equation:

$$\frac{\partial}{\partial\varsigma}\left(\kappa\frac{\partial u}{\partial\varsigma}\right)=i \tag{2}$$

with boundary condition:

$u(\zeta = 0) = 0$ and $u(\zeta = S) = U - E$

where $E = E_a - E_c$ and E_a, E_c are potential of the workpiece - anode and the tool - cathode, respectively.

Integration of Eq. (2) with respect of the assumption, led to:

$$\kappa_o\left[1+\alpha_T\theta\right]\left(1-\beta\right)^{3/2}\frac{\partial u}{\partial\varsigma}=i \tag{3}$$

After integration with respect of the boundary conditions, is obtained

$$i\int_0^S[\kappa_o(1+\alpha_T\theta)\left(1-\beta\right)^{3/2}]^{-1}d\varsigma = U-E$$

Hence,

$$i=\kappa_o\varphi_{TG}\frac{U-E}{S} \tag{4}$$

where:

$$\varphi_{TG}=\left[\frac{1}{S}\int_o^S\frac{d\zeta}{(1+\alpha_T\theta)(1-\beta)^{3/2}}\right]^{-1}$$

In one dimensional mathematical model of ECM, the Eq.(4) is simplified to:

$$i=\kappa_o(1+\alpha_T\theta)\left(1-\beta\right)^{3/2}\frac{U-E}{S} \tag{5}$$

The averaged across the gap size, values of *β* and *θ,* is depend on time and coordinate *ξ.*

Distribution of the void faction in the gap.

Let us assume, that initial time needed for formation of a gas bubble is much smaller than the machining time, and can be neglected. The equation of mass conservation for the hydrogen generation in the curvilinear coordinates can be obtained from mass balance for the moving control volume, Δv, in Fig. 3, as:

$$d(\int_v \rho_g\beta\, dv) = (\int_{A(v)} \eta_H k_H i dA)dt \tag{6}$$

After transformation, becomes

$$\frac{\partial\left(\rho_g\beta\right)}{\partial t}+div\left(w\rho_g\beta\right)=\eta_H k_H i \tag{7}$$

where: $\rho_g = \frac{p}{RT}$ is the specific gas density of hydrogen, *R* = gas constant for 1 kg of hydrogen, η_H = current efficiency of the hydrogen generation, k_H is the electrochemical equivalent of hydrogen.

In the curvilinear coordinates (ξ, ζ), the Eq. (7) is obtained the following form:

$$\frac{\partial\left(\beta Sp/T\right)}{\partial t}+\frac{\left(w\beta Sp/T\right)}{\partial\xi}=\frac{10^5}{273}\eta_H K_V^H i \tag{8}$$

with the boundary condition $\beta(0, t) = 0$.

Where $K_V^H = 1.25\times10^{-7}$ m^3/As is the volume of the hydrogen generated per unit electrical charge (1 A·s), at pressure $p = 10^5$ Pa and temperature T = 273 K.

The Temperature Distribution

Generally, the temperature distribution in the gap with respect to Joule's heat generation and heat transfer through the electrode surfaces, is described by:

$$\frac{\partial(1-\beta)\theta}{\partial t}+W(\xi,\zeta,t)\frac{\partial(1-\beta)\theta}{\partial\xi}=\frac{\partial}{\partial\zeta}\left[(a+a_T)\frac{\partial(1-\beta)\theta}{\partial\zeta}\right]+ \frac{i^2}{\kappa_0\rho_e C_p(1-\beta)^{5/2}(1+\alpha_T\theta)} \quad (9)$$

where: a – electrolyte thermal diffusivity, a_T - turbulent thermal diffusivity by turbulence pulses (for laminar flow $a_T = 0$), $\rho=\rho_e(1-\beta)$ - specific medium density and C_p - specific heat of electrolyte, $W(\xi,\zeta,t)$ is a distribution of flow velocity in the the gap,

for laminar flow:

$$W(\xi,\varsigma,t)=2w(\xi,t)\left[1-\left(\frac{S-2\zeta}{S}\right)^2\right],$$

for turbulent flow:

$$W(\xi,\varsigma,t)=1.143w(\xi,t)\left[1-\left|\frac{S-2\zeta}{S}\right|\right]^{1/7}$$

and $w(\xi, t)$ is averaged velocity across the gap size, which depends on ξ and t.

The boundary conditions are as follows: T(x=0) = To, T(x,0) = T_A, T(x, S) = T_C
where: T_A and T_C are temperatures of the anode and cathode, respectively.

Modeling of Hydrodynamic Conditions in the Gap

To complete the mathematical description, the formulation the pressure and the flow rate must be included.

Taking into account that the working gap thickness is much less than the length and the width, we can write hydrodynamics equations averaged across the gap in direction normal to the tool-electrode surface. Under the given assumptions, we simulate the medium as a two phases homogeneous mixture of an incompressible fluid (water solution of salt and products of the anode reactions) and gas.

Assume that fluid occupies the part $(1-\beta)$ of a small volume of the electrolyte and the part β is occupied by the gas. Additional, the mass flow rate of dissolved material is much smaller than the mass flow rate of medium in the gap, and can be neglected in transport equations.

Under given assumptions, the continuity equation can be described in curvilinear system coordinates as follows:

$$\frac{\partial[\rho_e(1-\beta)S]}{\partial t}+\frac{\partial[\rho_e(1-\beta)wS]}{\partial\xi}=0 \quad (10)$$

Application of the law of motion to the two – phase mixture flowing in the gap i.e. describing of the macroscopic momentum balance equation in the curvilinear coordinates, gives the equation

$$\frac{\partial[\rho_e(1-\beta)Sw]}{\partial t}+w\frac{\partial[\rho_e(1-\beta)Sw]}{\partial\xi}=-S\frac{\partial p}{\partial\xi}-\tau_A-\tau_C \quad (11)$$

where τ_A and τ_C are the shear stresses on the surface anode and cathode, respectively, which are assumed to be equal ($\tau_A = \tau_C = \tau$).

In general, the shear stress is expressed by:

$$\tau=\lambda\rho\frac{w^2}{8} \quad (12)$$

where $\lambda = C/Re^m$ is friction factor, $Re = 2wS/\nu$ is the Reynolds number, , for laminar flow i.e. at Re < 2300: $C = 96$ and $m = 1$, for turbulent flow: $C = 0.316$ and $m = 0.316$).

The initial conditions is obtained by solution of Eq.(11) at β = 0 and with neglecting the first term of equation, i.e. local derivative on time.

Substituting Eq. (12) into Eq. (11), yields

$$\frac{\partial[\rho_e(1-\beta)Sw]}{\partial t}+w\frac{\partial[\rho_e(1-\beta)Sw]}{\partial\xi}=-S\frac{\partial p}{\partial\xi}- C\left[\frac{\nu_e}{2wS(1-\beta)^{5/2}}\right]^m\frac{\rho_e(1-\beta)w^2}{4} \quad (13)$$

The boundary conditions are described by:

$$p(\xi=0)=p_{in}-\varsigma_1\frac{\rho_e w(\xi=0)^2}{2};$$

$$p(\xi=L)=p_{out}+\varsigma_2\frac{[\rho_e(1-\beta)w]^2_{\xi=L}}{2},$$

where: ς_1,ς_2 are the hydraulic loss pressure in inlet and outlet, respectively.

Basis on presented mathematical models the Simulation Process Module in the computer-aided engineering software CAE-ECM has been developed.

iv. AN EXAMPLE OF SIMULATION OF ELECTROCHEMICAL SHAPING OF BLADE PROFILE

To illustrate effects of processes on shaping, the results for machining of a turbine blade are presented.

Computer simulation and tool electrode design has been performed using the CAE-ECM system for following set of input:

- workpiece material – Inconel 817;
- electrolyte – 13% water solution of $NaNO_3$;
- electrochemical machinability: k_v=1.64-2.13exp(-0.03i) mm^3/Amin, where i is a current density in A/cm^2;
- voltage U = 12 V;
- total overpotential E = 3 V;
- inlet electrolyte velocity w = 10 m/s;
- outlet pressure p_k = 0.1 MPa;

Example of used discretization of 3-D surfaces is shown in Fig.4 and the initial electrode profiles on Fig.5.

Examples of simulation of ECM shaping are shown in Fig.6, where subsequent graphs illustrate anode-workpiece shape evolution in time. Simulation carried out at feed rate V_f = 0.8 [mm/min].

ISBN: 978-988-14049-0-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

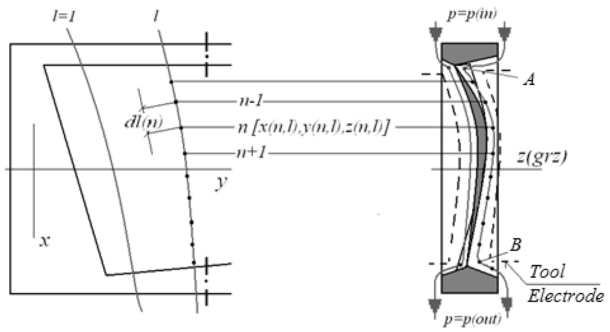

Fig.4. Discretization of the gap domain along flow lines.

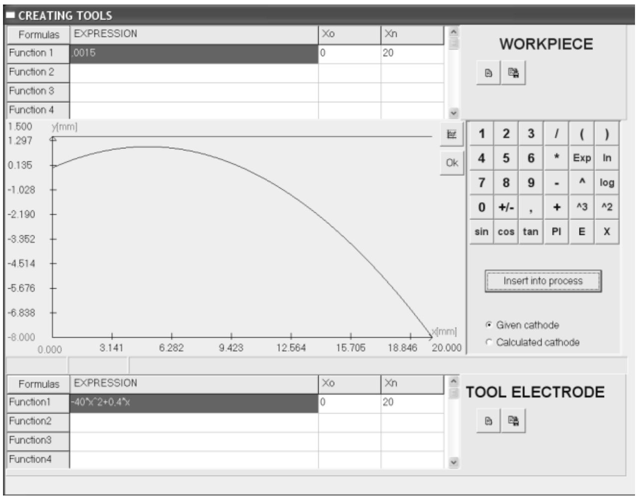

Fig. 5. Screen print-out containing the electrode profiles in initial stage of ECM.

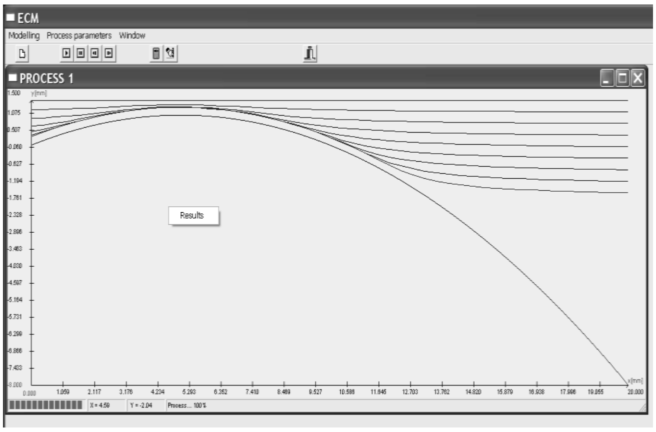

Fig. 6. Screen print-out containing the evolution of the shape of the workpiece during machining.

Calculated distributions of S, w, p, T, β and i as functions of distance along the electrolyte flow (with $x = 0$ at the inlet) are shown in (Fig. 7). The change in inter electrode gap width, S, results from blade's feather profile change as well as from change of physical conditions along electrolyte flow.

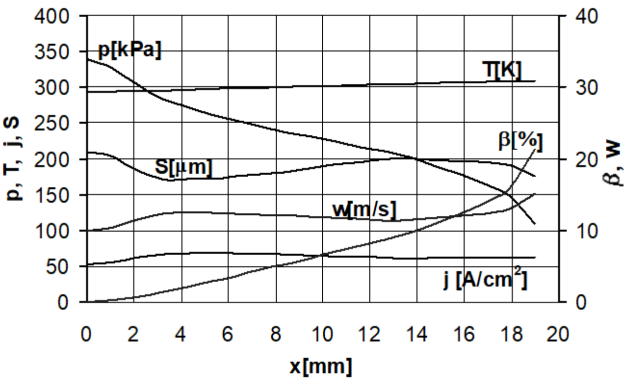

Fig. 7. Calculated distributions of S, w, p, T, β and j as functions of distance along the electrolyte flow.

To evaluate influence of changes of ECM input parameters on inter electrode gap distribution (or, in other words, on workpiece shape error distribution) the computer simulations for different values of voltage, feed rate and velocity of electrolyte at inlet were performed. Following values for these parameters were used:

- inlet electrolyte velocity w = 5, 10 and 15 m/s;
- voltage U = 8, 12, 16 V;
- feed rate V_f = 0.75, 1.00, 1.50 mm/min.

Some results for these simulations are shown in (Fig. 8 - 10). The biggest values of gap width, ($S(x)$) occur at the point where electrolyte exits that gap. They result from electrolyte an conductivity change that is caused by temperature and gas fraction increase. Decrease of inlet velocity of electrolyte causes decrease of gap width at the electrolyte outlet. Gap width significantly depends on pressure at the outlet what can be seen from graphs in (Fig.8) for p_k = 0.10 MPa and p_k = 0.15 MPa.

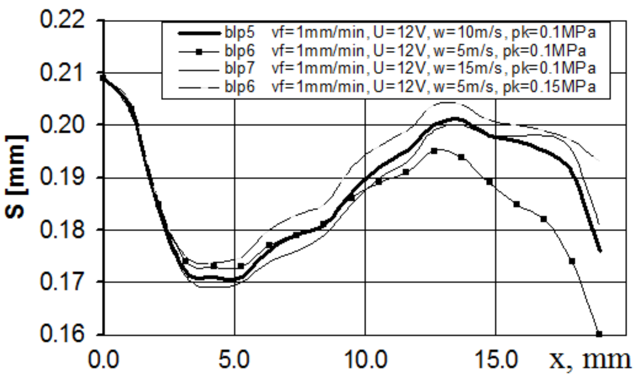

Fig.8. The gap width for different input parameters.

Increase of outlet pressure causes significant increase of gap width what can be explained by decrease of concentration of gas phase.

Changes of profiles with respect to reference profile, *blp5*, for different electrolyte velocities are shown in (Fig. 9).

ISBN: 978-988-14049-0-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

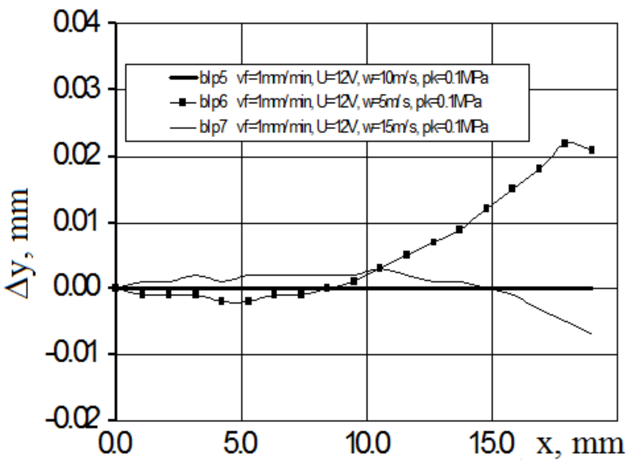

Fig. 9. Distribution of profile deviations with respect to reference profile *blp5* for different electrolyte velocities.

Despite significant changes of inlet velocity the maximum difference between profiles was relatively small (< 0.03 mm).

Much more pronounced influence on the machined profile have working voltage, U, and electrode feed rate, V_f.

As an example, influence of feed rate on shape deviation is shown in (Fig. 10).

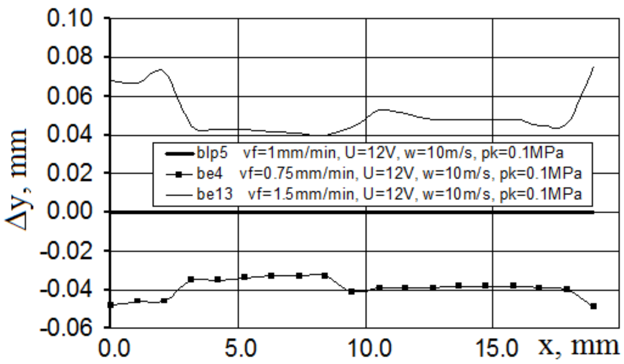

Fig. 10. Distribution of profile deviations with respect to reference profile *blp5* for different electrode feed rates.

Results show that stabilization of these parameters is necessary during machining. During ECM their values should be maintained as close as possible to their nominal values meaning values that were used for tool-electrode design.

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