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On New Solutions of Impulsive Quantum Stochastic Differential Inclusion

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***Abstract*— This paper addresses the existence of solution of Impulsive Quantum Stochastic Differential Inclusion (also known as impulsive non-classical ordinary differential inclusion) with an additional bounded linear operator. The multivalued maps are necessarily compact. By showing that the multivalued maps can actually satisfy some conditions of appropriate fixed-point theorems, the major result is established.**

***Index Terms*—QSDI, Stochastic processes, Bounded Linear Operator, Fixed point.**

I. Introduction

Some interesting results concerning the following quantum stochastic differential equation (QSDE) and inclusion (QSDI) introduced by Hudson and Parthasarathy [12]:

$$dy(t) \in E\big(t, y(t)\big)d\Lambda_\pi(t) + F\big(t, y(t)\big)dA_f(t)$$
$$+G\big(t, y(t)\big)dA_g^+(t) + H\big(t, y(t)\big)dt,$$
$$y(0) = y_0 \qquad (1)$$

have been studied. See [1, 4-6, 10]. However, It has shown by Ekhaguere [10], that the following nonclassical ordinary differential inclusion (NODI):

$$\frac{d}{dt}\langle \eta, y(t)\xi\rangle \in P(t, y)(\eta, \xi)$$
$$\langle \eta, y(0)\xi\rangle = \langle \eta, y_0\xi\rangle \qquad (2)$$

is equivalent to inclusion (1) where the map P in (2) is well defined in [1]. The definitions and notations of some spaces such as $L_\gamma^2(\mathbb{R}_+)$, $L_{loc}^p(\tilde{A})$, $L_{loc}^\infty(\mathbb{R}_+)$, $L_{loc}^p(I \times \tilde{A})$, $\Gamma(L_\gamma^2(\mathbb{R}_+))$ are adopted from the reference [6] and the references therein.

(i) The stochastic processes are densely defined linear operators on the complex Hilbert space $\mathcal{R} \otimes \Gamma(L_\gamma^2(\mathbb{R}_+))$. We denote by $\langle .,. \rangle$ the inner product of $\mathcal{R} \otimes \Gamma(L_\gamma^2(\mathbb{R}_+))$ and $\|.\|$ the norm induced by $\langle .,. \rangle$.

(ii) U, V, W, H, are coefficients which are in $L_{loc}^2(\tilde{\mathcal{A}} \times [0,T])_{mvs}$, $\tilde{\mathcal{A}}$ is a locally convex space. The elements of $\tilde{\mathcal{A}}$ consists of linear maps from $\mathbb{D} \underline{\otimes} \mathbb{E}$ into $\mathcal{R} \otimes \Gamma(L_\gamma^2(\mathbb{R}_+))$. For the definitions of the integrators Λ_π, A_g^+, A_f in (1.1), see the references [1, 8, 10].

(iii) $\mathbb{D}$ is an inner product space, $\mathcal{R}$ is the completion of $\mathbb{D}$ and $\mathbb{E}$ a linear space in $\Gamma(L_\gamma^2(\mathbb{R}_+))$.

Differential inclusions with impulsive effects have found more practical applications than differential equations without impulse effect [2-3, 9, 11], hence the need to extend some of these results to this noncommutative quantum setting. For more on impulsive ordinary differential inclusions and their applications, see the references [2-4]. Bishop et. al. [4] studied results on impulsive (QSDE) with nonlocal conditions. The method used was simply an adoption of the methods employed in the classical setting. Similar results are established in [6] with strict conditions on the maps. In this paper, the conditions on the maps are weakened with an additional bounded linear operator.

To the best of our knowledge within the consulted literature, there are no results concerning impulsive QSDI in the sense of this paper. However, in the classical setting, several results on differential equations/inclusions with impulse effects have been established. See the references [2, 3] and the references therein. This work generalizes existing results in the literature on impulsive quantum stochastic differential equations since some of these results can be recovered from the result in this paper. The remaining part of this paper will consist of two sections: Section 2 will contain some preliminaries which we adopt from the references [1, 4, 5, 10], while in section 3, the main result will be established.

In this paper, the fixed-point method is used to establish the major result. In recent times, this method has proven to be very efficient in establishing existence of solutions of differential equations and approximate solutions.

II. Preliminaries

Let $\tilde{\mathcal{A}}$ be a topological space and clos($\tilde{\mathcal{A}}$), comp($\tilde{\mathcal{A}}$) be as defined in [6]. Define $I = [0, b]$, $I' = I \backslash \{t_1, t_2, \ldots, t_m\}$, $0 < t_1 < \ldots < t_m < t_{m+1} = b$.

Definitions 2.1

(i) $PC(I, \tilde{\mathcal{A}}) = \{y : I \to \tilde{\mathcal{A}} : y(t)$ is continuous and $y(t_k^-) = y(t_k)\}$

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We define the sesquilinear equivalent forms $PC(I, A(\eta, \xi))$ associated with PC($I, sesq(\mathbb{D} \underline{\otimes} \mathbb{E}$)) in the same way with the above, where A is a subset of $\tilde{\mathcal{A}}$.
(ii) $PC(I, A(\eta, \xi))$ is a Banach space whose norm is defined by
$\|y\|_{\eta\xi, PC} = \sup\{|y(t)(\eta, \xi)|: t \in I\}$, where $A \subseteq \tilde{\mathcal{A}}$
(iii) $\rho(B, C) \coloneqq \max(\delta(B, C), \delta(C, B)),\ B, C \in clos(\mathbb{C})$
and

$$d(y, B) = inf_{y \in B}|x - y|,$$
$$\delta(B, C) \equiv sup_{y \in A} d(y, B),$$

where $y \in \mathbb{C}$, is a complex number, $\rho(B, C)$ the Hausdorff distance, and ρ a metric on $clos(\mathbb{C})$.
Note: (i) in definition 2.1 holds everywhere except for some t_k.

Definition 2.2
For the rest of this paper, Φ indexed by $I = [0, T] \subseteq \mathbb{R}_+$ is multivalued stochastic process.

(i)Φ is L^1- measurable in the sense of [2]
(ii) A selection of Φ is a stochastic process $y: I \to \tilde{\mathcal{A}}$ such that $y(t) \in \Phi(t)$ for almost all $t \in I$.
(iii) Φ is said to be lower semicontinuous if for every open set $V \subset \tilde{\mathcal{A}}$, $\Phi^-(V)$ is open.
(iv) Φ is adapted if $\Phi(t) \subseteq \tilde{\mathcal{A}}_t$, $t \in I$ and its denoted by $\mathrm{Ad}(\tilde{\mathcal{A}})$
(v) A member $y \in \mathrm{Ad}(\tilde{\mathcal{A}})$ is said to be weakly absolutely continuous (AC) if $t \to \langle \eta, y(t)\xi \rangle, t \in I$ is AC for $\eta, \xi \in \mathbb{D} \underline{\otimes} \mathbb{E}$. Denote all such sets by $\mathrm{Ad}(\tilde{\mathcal{A}})_{wac}$
(vi) If $t \to \|\Phi(.)\|_{\eta\xi} \in L^p_{loc}(\tilde{\mathcal{A}}), p \in (0, \infty)$, then y(t) is locally absolutely p-integrable.
Denote this set by $L^p_{loc}(\tilde{\mathcal{A}})_{mvs}$.

Also define the set S^{Φ} by

$$S^{\Phi} = \{g \in L^1(\tilde{\mathrm{A}}, I): g(t) \in \Phi(y(t), t), a.e.\, t \in I\} \neq \emptyset$$

Note: Subsequently, $\eta, \xi \in \mathbb{D} \underline{\otimes} \mathbb{E}$ is arbitrary.
Next, we introduce the following QSDI with an additional bounded linear operator:

$$dy(t) - A(t)y(t) \in \mathrm{B}(y(t)) + U(y(t), t)d\Lambda_\pi(t)$$
$$+V(y(t), t)dA_f(t) + W(y(t), t)dA_g^+(t) + H(y(t), t),$$
$$\Delta y(t) = J_k(y(t_k^-)), t = t_k, k = 1, \dots, m \qquad (3)$$

Observe that, the following inclusion is equivalent to inclusion (3).

$$\frac{d}{dt}\langle \eta, [y(t) - A(t)y(t)]\xi \rangle \in By_{\eta\xi}(t) + P(t, y)(\eta, \xi)$$
for some $t \in I, t \neq t_k, k = 1, \dots, m,$
$$\Delta y(t) = J_k(y(t_k^-))(\eta, \xi),\ t = t_k,$$
$$\langle \eta, y(0)\xi \rangle = \langle \eta, y_0 \xi \rangle \qquad (4)$$

where the multivalued map $P: \tilde{\mathcal{A}} \times I \to clos(\tilde{\mathcal{A}})$ is assumed to be compact and convex, $y \to \Phi(y(t), t),\ \Phi \in \{\mu E, \gamma F, \sigma G, H\}$ is a multivalued stochastic process, $y_0 \in \tilde{\mathcal{A}}$ and $B: \tilde{\mathcal{A}} \to \tilde{\mathcal{A}},\ J_k \in C(\tilde{\mathcal{A}}, \tilde{\mathcal{A}})$.

Definition 2.3
(i) $\Delta y(t) = y(t_k^+) - y(t_k^-), t = t_k,$
$$y(t_k^+) = \lim_{h \to 0^+} y(t_k + h)$$
(ii) $y(t_k^-) = \lim_{h \to 0^+} y(t_k - h), k = 1, \dots, m.$
(iii) Let $v \in L^1(I, \tilde{\mathcal{A}}): v(t) \in \Phi(y(t), t)$ then, $y(t) \in (I, \mathrm{Ad}(\tilde{\mathcal{A}})_{wac}) \cap PC(I, \tilde{\mathcal{A}})$ is a solution of (3) if it satisfies the following integral inclusion

$$y(t)(\eta, \xi) \in T(t)y_0(\eta, \xi) +$$
$$+ \int_0^t T(t - s)B(y(s))(\eta, \xi)$$
$$+ \int_0^t T(t - s)v(s)(\eta, \xi)ds$$
$$+ \sum_{0<t_k<t} T(t - s) J_k(y(t_k))(\eta, \xi) \qquad (5)$$

The following conditions are assumed:
(S_1) The stochastic processes $P: \tilde{\mathcal{A}} \times I \to clos(\tilde{\mathcal{A}})$ are compact and convex such that $y \to \Phi(y(t), t)$ satisfies conditions (iii) and (iv) of *Definition 2.2* above.
(S_2) $\|J_k(y)\| \le c_k, k = 1, 2, \dots, m$ for each $y \in \tilde{\mathcal{A}}$, where c_k are constants.
(S_3) A is the infinitesimal generator of a family of semigroup $\{T(t), t \ge 0\}$ and T is compact for $t > 0$, such that $\|T(t)\|_{\eta\xi} \le M$, where M is a constant, $t \ge 0$. B is an additional bounded linear operator with values in $\tilde{\mathcal{A}}$.
(S_4) We define a function Ψ by $\Psi: [0, \infty] \to (0, \infty)$ which is continuous and nondecreasing such that

$$\|\Phi(y, t)\|_{\eta\xi} \le p^{\Phi}(t)\Psi(\|y\|_{\eta\xi})$$

for almost every $\eta, \xi \in (\mathbb{D} \underline{\otimes} \mathbb{E}),\ p^{\Phi} \in L^1(I, R_+),\ y \in \tilde{\mathcal{A}}$ and

$$\int_0^b p^{\Phi}(s)ds < \int_d^\infty \frac{dv}{\Psi(v)} = \|y_0\|_{\eta\xi} + \sum_{k=1}^m c_k \qquad (6)$$

where

$$w^{\Phi}(t) = \max\{M\|B\|,\quad Mp^{\Phi}(t)\}$$
$$d = M[\|y_0\|_{\eta\xi} + \sum_{k=1}^m c_k], \qquad (7)$$

$B \subseteq PC(\tilde{\mathrm{A}}, I)$, and $sup_{t \in I}\|y(t)\|_{\eta\xi} < \infty$.

III. MAIN RESULTS

Theorem 3.1: Assume conditions (S_1) - (S_4) hold, then (3) has at least a weak solution.
Proof: Firstly, (4) has to be transformed into a fixed-point problem.
Define $N: PC(I, A(\eta, \xi)) \to \wp(PC(I, A(\eta, \xi)))$ by

$$N(y(t)(\eta, \xi)) = \langle \eta, \mathrm{T}(t)y_0\xi \rangle + \int_0^t T(t - s)B(y(s))(\eta, \xi)\, ds$$
$$+ \int_0^t T(t - s)g_{\eta\xi}(s)\, ds$$
$$+ \sum_{0<t_k<t} T(t - s) J_k(y(t_k))(\eta, \xi)\} \qquad (8)$$

Let

$$K \coloneqq \{y \in (I, sesq(\mathbb{D} \underline{\otimes} \mathbb{E})): \|y(t)\|_{\eta\xi} \le b(t)\} \quad (9)$$

where

$$b(t) = J^-\left(\int_0^b w^{\Phi}(s)ds\right) \qquad (10)$$

Hence by (9), the set K is: (a) compact and (b) convex.

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Assume $K^* = \sup\{\|y\|_{\eta\xi} : y \in K\}$.
We show that $N(K) \subset K$ and is also relatively compact.
Fix $t \in I$ and let $g \in S^{\Phi}$ such that,

$$h(t)(\eta,\xi) = \langle \eta, T(t)y_0\xi\rangle + \int_0^t T(t-s)B\big(y(s)\big)(\eta,\xi)\,ds + \int_0^t T(t-s)g_{\eta\xi}(s)\,ds + \textstyle\sum_{0<t_k<t} T(t-s)\,J_k\big(y(t_k)\big)(\eta,\xi)$$

(11)

Therefore,

$$h(t)(\eta,\xi) = M\|y_0\|_{\eta\xi} + M\sum_{k=1}^{m} c_k + \int_0^t w^{\Phi}(s)(\|y(s)\|_{\eta\xi} + \Psi\|y(s)\|_{\eta\xi})\,ds$$

$$\le M\|y_0\|_{\eta\xi} + M\sum_{k=1}^{m} c_k + \textstyle\int_0^t w^{\Phi}(s)(b(s) + \Psi(b(s)))\,ds \qquad (12)$$

$$\le M\|y_0\|_{\eta\xi} + M\sum_{k=1}^{m} c_k + \int_0^t b(s)\,ds = b(s)$$

given that

$$\textstyle\int_d^{b(s)} \frac{du}{u+\Psi(u)} = \int_0^s w(\tau)\,d\tau,\ \ d>0 \qquad (13)$$

where $h \in PC\left(I, sesq\left(\mathbb{D} \underline{\otimes} \mathbb{E}\right)\right)$.
Now, K satisfies (a) above, $N(K) \subset K$ and $N: K \to K$. If we let $h \in N(y)$, $y \in K, \tau_1, \tau_2 \in I', 0 < \epsilon \le \tau_1 \le \tau_2$ then for each $t \in I$ there exists a $g \in S^{\Phi}$ such that

$$|h(\tau_2)(\eta,\xi) - h(\tau_1)(\eta,\xi)| \le |h(\tau_2)y_0(\eta,\xi) - h(\tau_1)y_0(\eta,\xi)| + \int_0^{\tau_1-\epsilon} |T(\tau_2-s) - T(\tau_1-s)|B(s)(\eta,\xi)\,ds$$
$$+ \int_{\tau_1}^{\tau_1-\epsilon} |T(\tau_2-s)|B(s)(\eta,\xi)\,ds + \int_{\tau_1}^{\tau_2} |T(\tau_2-s) - T(\tau_1-s)|B(s)(\eta,\xi)\,ds$$
$$+ \int_0^{\tau_1-\epsilon} |T(\tau_2-s) - T(\tau_1-s)|_{PC}|g(s)(\eta,\xi)|\,ds + \int_{\tau_1}^{\tau_1-\epsilon} |T(\tau_2-s) - T(\tau_1-s)||g(s)(\eta,\xi)|\,ds$$
$$+ \int_{\tau_1}^{\tau_2} |T(\tau_2-s)|_{PC}|g(s)(\eta,\xi)|\,ds + Mc_k(\tau_2-\tau_1) + \sum_{0<t_k<\tau_1} c_k|T(\tau_1-t_k) - T(\tau_2-t_k)| \qquad (14)$$

(14) tends to zero as $\tau_2 - \tau_1 \to 0$, for small ϵ. By the definition of $T(t)$, it is continuous, establishing the equicontinuity for the case when $t \ne t_k$. When $t = t_k$ follows from the results in [4, 6, 7].
Using Arzela-Ascoli theorem, it is established that $N: K \to \mathcal{A}$, where N is a multivalued map and $\mathcal{A}$ is a precompact set. Let $0 < \epsilon < t$ and fix $0 < t \le a$.
Now, for $y \in K$, define

$$h(\tau_2)(\eta,\xi) = T(t)y_0(\eta,\xi) + T(\epsilon)\int_0^{\tau_1-\epsilon} |T(t-s-\epsilon)|(B(s)(\eta,\xi))\,ds + T(\epsilon)\int_0^{\tau_1-\epsilon} T(t-s-\epsilon)g(s)(\eta,\xi)\,ds + \sum_{0<t_k<t-\epsilon} T(t-t_k-\epsilon)J_k\big(y(t_k)\big)(\eta,\xi), \qquad g \in S^{\Phi}.$$

Since the operator T(t) is compact, it implies that $H_\epsilon(t) = \{h_\epsilon(t): h_\epsilon \in N(y)\}$ is relatively compact in $\mathcal{A}$, $0 < \epsilon < t$.
Now, $\forall\, h \in N(y)$, we get

$$|h_\epsilon(t)(\eta,\xi) - h(t)(\eta,\xi)| \le \|B\|_{\eta\xi}K^* \int_{t-\epsilon}^{t} \|T(t-s)\|ds + \int_{t-\epsilon}^{t} \|T(t-s)\||a(s)(\eta,\xi)|\,ds + \sum_{t-\epsilon<t_k<t} c_k\|T(t-t_k)\|$$

Showing that there are relatively compact sets close to the set $\{h(t): h \in N(y)\}$ and so, the set $\{h(t): h \in N(y)\}$ is relatively compact in $\mathcal{A}$.
Let $y_n \to y_*$, $h_n \to h_*$. We establish that $h_* \in N(h_*) \Rightarrow g_n \in S^{\Phi}$.
For $t \in I$,

$$h_n(t)(\eta,\xi) = \langle \eta, T(t)y_0\xi\rangle + \int_0^t T(t-s)B(y_n(s))(\eta,\xi)\,ds + \textstyle\int_0^t T(t-s)g_{n,\eta\xi}(s)\,ds + \displaystyle\sum_{0<t_k<t} T(t-t_k)J_k\big(y_n(t_k)\big)(\eta,\xi) \qquad (15)$$

We show that $\exists\ g_* \in S_*^{\Phi}$ from which we get,

$$h_*(t)(\eta,\xi) = \langle \eta, T(t)y_0\xi\rangle + \int_0^t T(t-s)B(y_*(s))(\eta,\xi)\,ds + \int_0^t T(t-s)g_{*,\eta\xi}(s)\,ds + \sum_{0<t_k<t} T(t-t_k)J_k\big(y_*(t_k)\big)(\eta,\xi) \qquad (16)$$

By the continuity of J_k and B, we get

$$|h_n(t)(\eta,\xi) - \langle \eta, T(t)y_0\xi\rangle - \int_0^t T(t-s)B\big(y_n(s)\big)(\eta,\xi)\,ds - \int_0^t T(t-s)g_{n,\eta\xi}(s)\,ds - \sum_{0<t_k<t} T(t-t_k)J_k\big(y_n(t_k)\big)(\eta,\xi)$$
$$-h_*(t)(\eta,\xi) - \langle \eta, T(t)y_0\xi\rangle - \int_0^t T(t-s)B\big(y_*(s)\big)(\eta,\xi)\,ds - \int_0^t T(t-s)g_{*,\eta\xi}(s)\,ds + \textstyle\sum_{0<t_k<t} T(t-t_k)J_k\big(y_*(t_k)\big)(\eta,\xi)| \to 0$$

as n $\to\infty$

Now, by considering the operator

$$\mathcal{F}: L^1(I, \mathcal{A}) \to C(I, \mathcal{A}),$$

$$g_{\eta\xi} \to \mathcal{F}(g_{\eta\xi})(t) = \int_0^t T(t-s) g_{\eta\xi}(s) ds,$$

we get,

$$h_n(t)(\eta, \xi) - \langle \eta, T(t) y_0 \xi \rangle$$
$$- \int_0^t T(t-s) B(y_n(s))(\eta, \xi)\, ds$$
$$- \sum_{0<t_k<t} T(t-t_k) J_k(y_n(t_k))(\eta, \xi) \in \Gamma(S^{\Phi}).$$

Since $y_n \to y_*$, it follows that

$$h_*(t)(\eta, \xi) - \langle \eta, T(t) y_0 \xi \rangle$$
$$- \sum_{0<t_k<t} T(t-t_k) J_k(y_*(t_k))(\eta, \xi)$$
$$- \int_0^t T(t-s) B(y_*(s))(\eta, \xi)\, ds = \int_0^t T(t-s) g_{*,\eta\xi}(s)\, ds$$

for some $g_* \in S_*^{\Phi}$. And the desired result is obtained.

IV. CONCLUSION

Having satisfied the conditions of Schaefer's theorem, we conclude that *N* has a fixed point which is a solution of (3) respectively (4). If the operator B in equation (3) is zero, i.e., B = 0, then we obtain some results in [4, 6]. Hence, the result in this paper is a generalization of the results in the existing literature on impulsive quantum stochastic differential inclusion (equations).

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REFERENCES

[1] E. O. Ayoola, Lipschitzian Quantum Stochastic Differential Equations and the Associated Kurzweil Equations. Stochastic analysis and applications, **19**(4), 581-603, 2001.

[2] M. Benchohra, J. Henderson and S. K. Ntouyas, Impulsive Differential Equations and Inclusions. Hindawi Publishing Corporation, 2006. http://www.hindawi.com.

[3] M. Benchohra and S. K. Ntouyas, Existence of mild solutions of second order initial value problems for differential inclusions with nonlocal conditions. Atti Sem. Mat. Fis. Univ. Modena. **49**(2), 351-361,2001.

[4] S. A. Bishop, E. O. Ayoola, J. G. Oghonyon, Existence of mild solution of impulsive quantum stochastic differential equation with nonlocal conditions, Analysis and Mathematical Physics, 7(3), pp. 255-265,2017

[5] S. A. Bishop, On Continuous Selection Sets of Non-Lipschitzian Quantum Stochastic Evolution Inclusions. International Journal of Stochastic Analysis, **2015**, 5 pages, Hindawi Publishing Corporation,2015. doi:10.1155/2015/834194.

[6] S. A. Bishop and P. E. Oguntunde, On Existence of solution for Impulsive Quantum Stochastic Differential inclusion. Journal of Engineering and Applied Sciences, **10**(7), 181-185, 2015.

[7] Bishop, S. A., Agarana, M. C., Okagbue, H. I., Oghonyon, J. G., On Unique Solution of Quantum Stochastic Differential Inclusions. World Congress on Engineering and Computer Science, WCECS 2017, VOL I.

[8] T. P. Cardinali and P. Rubbioni, Impulsive Mild solutions for semilinear differential inclusions with nonlocal conditions in Banach spaces. Nonlinear Analysis, **75**, 871-879, 2012.

[9] T. P. Cardinali, F. Portigiani and P. Rubbioni, Local mild solutions and impulsive mild solutions for semilinear Cauchy problems involving lower Scorza- Dragoni Multifunctions, Topological Methods in Nonlinear Analysis. Journal of the Juliusz Schauder Center, **32**, 247-25, 2008.

[10] G.O. S. Ekhaguere, Lipschitzian Quantum Stochastic Differential Inclusions. Int. journal of theoretical physics, **31(**11), 2003-2034,1992.

[11] Z. Fan, Impulsive Problems for Semilinear Differential equations with nonlocal conditions. Nonlinear Analysis, **72,** 1104-1109, 2010.

[12] R. L. Hudson and K. R. Parthasarathy, Quantum Ito's formulae and stochastic evolutions. Comm. math. phys., **93**, 301-324 (1984).