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Shape Optimization of Glulam Timber Roof Girders

J. Baranski, J. Szolomicki, K. Damian

***Abstract*—This paper presents the development of a reliable and efficient algorithm to analyze and find the optimum shapes of glulam timber roof girders. Glulam girders can be used when designing structures with large spans. For the final solution of the stated problem, girders in three different shapes (pent, gable, curved) and in various timber strength classes were compared. Design conditions in accordance with Eurocode 5 were considered as the optimization constraints. The corresponding ultimate limit state and serviceability limit states were considered in the initial analysis.The optimization was performed by Excel Solver, which can be seen as an accurate and inexpensive tool.**

***Index Terms*—Optimization, Glulam Timber, Girder, Excel**

I. Introduction

Glued laminated timber roof girders are used in a wide range of applications in building construction [1,2,3]. Glulam is made up of wood laminations that are bonded together with adhesives. The grain of all laminations runs parallel with the length of the member. The most critical zone of a glulam bending member, with respect to controlling strength, is the outermost tension zone. Allowable design properties are a key factor in specifying glulam. Bending members are typically specified on the basis of the maximum allowable bending stress of the member. Glulam beams are typically installed with a wide face of laminations that are perpendicular to the applied load. In structural design, it is necessary to obtain an appropriate geometric shape for a structure. This may be achieved with the use of Excel's Solver procedures, in which the shape of the structure is varied in order to achieve a specific objective that satisfies certain constraints. Solver tools can be developed by the efficient integration of structural shape definition, structural analysis, and mathematical programming methods. A homogenous material is assumed in the analysis.

II. Basic elements of the optimization process

The correct formulation of the problem of optimal construction shaping should include four basic elements (Fig. 1):

- optimization criterion,
- optimization variables,
- optimization limitations,
- state equations,

with an objective function defined as the criterion expressed by optimization variables (Fig. 1).

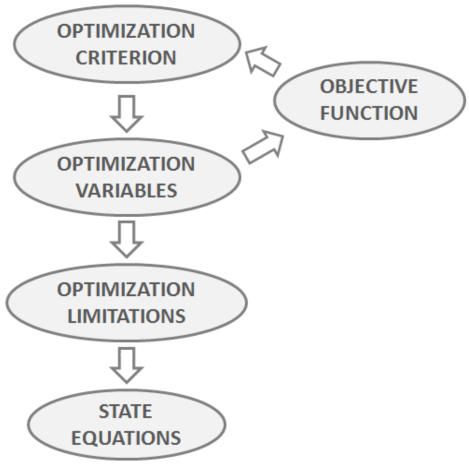

Figure 1 Basic elements of the optimization process

Appropriate choice of optimization criterion, expressing it by optimization variables, which is explicit with acceptance of the objective function, and also selection of appropriate limitations are the most important parts of the formulation of the optimization problem. Adoption of optimization criterion and appropriate restrictions finishes the conceptual work on the problem of optimization. Further steps involve usually:

- creation of the most favourable formal record of the problem,
- selection of optimization methods,
- solution of the problem itself.

A. Optimization criterion

Optimization criterion is a basic tool used to compare individual solutions expressed in terms of mathematic, it is called an objective function. We can distinguish the following examples of optimization criterions:

- reliability,
- safety,
- functionality,
- time of execution,
- cost of execution,
- workload,
- amount of used materials,
- ease of transport,
- maintainability,
- aesthetics.

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The basic groups of criterions are:

- the minimal cost of construction,
- the greatest stiffness or the smallest deformability,
- equalization of the exertions.

Cost of construction is a universal measure of the quality of design solutions, but there is a problem to express it using function parameters.

In strength optimization the total cost of construction usually contains three basic elements:

- cost of material,
- cost of exploitation,
- cost of production.

First of all, if the type of material is established in advance the material costs are proportional to the volume (weight) of the structure.

Secondly, for many types of structures, exploitation costs are proportional to the weight (so also volume) of the structure.

Finally, if the cost of production does not constitute a significant share of the total cost of the construction it can be assumed that the volume (weight) is a substitute optimization criterion. Volume, simultaneously weight, will be used as the optimization criterion in this paper. It is quite basic, but very effective method. In construction, a decrease of weight of an element even around $3 \div 5\%$ becomes crucial when many identical elements are produced. Differences in construction costs might be very high [4,5,6].

B. *Optimization variables*

In the strength optimization, there may appear many optimization variables. A substantial number of these is connected to the geometry of the structure. They describe the shape of the construction, size, and shape of the cross-section, shape of the edges etc. In this paper, variables describing cross-section are very essential.

A cross-section of a simple beam, in the most general case, can be described as a function of one variable, defined as a characteristic dimension of the cross-section along the axis of the beam.

The shape of the cross-section can be formally described as a function in polar coordinates:

$$r = r(\theta). \quad (1)$$

If a variation of the cross-section along the axis is allowed, the beam is described by a function of two variables:

$$r = r(\theta, x). \quad (2)$$

In analysis assumed the length of the beam as a constant, while height and width of the cross-sections are optimization variables.

C. *Optimization limitations*

Classification of optimization limitations can be carried out due to different criteria. Considering physical restrictions, so from the point of view of engineering strength optimization, the limitations may be divided into two categories. The first one is a group of restrictions that could have a significant impact on the behavior of the optimal construction, its work under the influence of load and exploitation properties. Such conditions are sometimes called 'behavioral' constraints. This group includes such basic constraints as strength, stiffness, susceptibility, natural or forced vibration, structural stability under applied loading, etc. The second group includes limitations, defining possibilities of producing the structure, called technological constraints. Mostly they are associated with the restrictions imposed on the maximum and minimum values of geometrical parameters of the structure.

Another classification is connected with dividing construction by its properties:

- Economical (cost of the construction, amortization, and so on),
- Dimensional (connected with strength, technology, geometry, and so on),
- Qualitative (connected with durability, technical parameters to function properly),
- Operational (reliability, use of energy, ability to work in different conditions).

D. *State equations*

In optimization problems are the following constituents:

- the components of the stress state (distribution of stresses),
- the components of the strain state,
- the components of the displacement vector.

These variables are associated with the optimization variable and are related to each other with equations of state.

III. Optimization Design Examples

This paper presents a comparative analysis of three shape representatives of a girder: pent, gable (pitched cambered) and arch (Figs. 2-4). It is assumed that the beams are simply supported with a roof slope α<5° and that overturning or buckling out of their plane is prevented. The analyzed roof girders are made of various kinds of glulam (GL22h, GL24h, GL26h, GL28h, GL30h, and GL32h) with a constant rectangular cross-section designed for a given span (14 m) and an assumed design uniform load (roofing, installation, purlins). Due to recommendations according the proper dimensions and proportions of the elements following limitations were used for all analyzed roof girders.

A. *Pent girder*

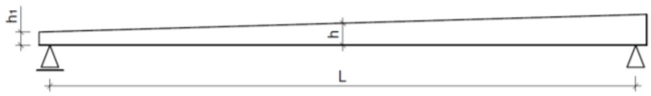

Figure 2 - Scheme of a pent girder

$$h = \frac{L}{15} \div \frac{L}{10}, \qquad h_1 = \frac{L}{30} \div \frac{L}{20}, \qquad L = 10 \div 30\ m$$

$$L = 14m \rightarrow h = \frac{14\ m}{15} \div \frac{14\ m}{10} = 0{,}93m \div 1{,}40\ m$$

$$\rightarrow taken\ value{:}\ h = 1{,}00\ m = 1000\ mm$$

$$L = 14\ m \rightarrow h_1 = \frac{14\ m}{30} \div \frac{14\ m}{20} = 0{,}47\ m \div 0{,}70\ m$$

Height of the shorter edge:

$$h_1 = 1000\ mm - 0{,}5 \cdot \tan(3°) \cdot 14000\ mm = 633\ mm$$

The height of the longer edge:

$$h_2 = 1000\ mm + 0{,}5 \cdot \tan(3°) \cdot 14000\ mm = 1367mm$$

$$b = 200\ mm \rightarrow \frac{h}{b} = \frac{1000\ mm}{200\ mm} = 5 < 10$$

Assumptions for this example:

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- tilt roof slope: 3°,
- value of the snow load: 3,6 kN/m,
- value of the roofing load: 4,0 kN/m,
- value of the installation load: 2,5 kN/m,
- purlins every 4m,
- girder is placed on two columns with cross-sections: $h_s \times b_s$ =400 x 200 mm each.

Self-weight of the girder:
$\rho_{g,k} = 425kg/m^3 = 4{,}25kN/m^3$
Self-weight characteristic load:
$G_{k,self} = b \cdot h \cdot \rho_{g,k} = 200mm \cdot 1000mm \cdot 4{,}25kN/m^3 = 0{,}85kN/m$
Collation of loads:
dead loads (roofing + installations + girder self-weight):
$G_k = 4{,}0kN/m + 2{,}5kN/m + 0{,}85kN/m = 7{,}35kN/m$
live loads (snow):
$Q_k = 3{,}6kN/m$
The calculated value of the load in accordance with a combination:

$$E_d = 1{,}35 \cdot G_k + 1{,}50 \cdot Q_k = 1{,}35 \cdot \frac{7{,}35\ kN}{m} + +1{,}50 \cdot \frac{3{,}6\ kN}{m} \cong 15{,}32\ kN/\text{m}$$

B. Gable girder

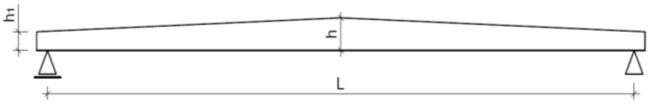

Figure 3 - Scheme of a gable (pitched cambered) girder with a straight bottom flange

$$h = \frac{L}{15} \div \frac{L}{10}, \qquad h_1 = \frac{L}{30} \div \frac{L}{20}, \qquad L = 10 \div 40m$$

$$L = 14m \rightarrow h = \frac{14\ m}{15} \div \frac{14\ m}{10} = 0{,}93\ m \div 1{,}40\ m \rightarrow taken\ value: h = 1{,}00\ m = 1000\ mm$$

$$L = 14m \rightarrow h_1 = \frac{14\ m}{30} \div \frac{14\ m}{20} = 0{,}47\ m \div 0{,}70\ m$$

$$h_1 = h - \tan(\propto) \cdot \frac{L}{2} = 1000\ mm - \tan(3°) \cdot \frac{14000\ mm}{2} = 633mm$$

$$b = 200\ mm \rightarrow \frac{h}{b} = \frac{1000\ mm}{200\ mm} = 5 < 10$$

Assumptions for this example:
- tilt roof slope: 3°,
- value of the snow load: 3,6 kN/m,
- value of the roofing load: 4,0 kN/m,
- value of the installation load: 2,5 kN/m,
- purlins every 4m,
- girder is placed on two columns with cross-sections: $h_s \times b_s$ =400 x 200 mm each.

Self-weight of the girder:
$\rho_{g,k} = 425kg/m^3 = 4{,}25kN/m^3$
Self-weight characteristic load:

$$G_{k,self} = b \cdot \frac{h + h_1}{2} \cdot \rho_{g,k} = 200 \cdot \frac{1000m + 633mm}{2} \cdot 4{,}25\ kN/m^3 \cong 0{,}69\ kN/m$$

Collation of loads:
- dead loads (roofing + installations + girder self-weight):

$$G_k = \frac{4{,}0\ kN}{m} + 2{,}5kN/m + 0{,}69kN/m = 7{,}19kN/m$$

- live loads (snow):

$Q_k = 3{,}6\ kN/m$
The calculated value of the load in accordance with a combination:

$$E_d = 1{,}35 \cdot G_k + 1{,}50 \cdot Q_k = 1{,}35 \cdot \frac{7{,}19\ kN}{m} + +1{,}50 \cdot 3{,}6 \frac{kN}{m} \cong 15{,}11\ kN/\text{m}$$

C. Arch girder

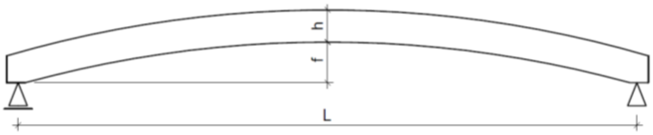

Figure 4 - Scheme of an arch girder

$$h = \frac{L}{15} \div \frac{L}{8}, \qquad f = \frac{L}{20} \div \frac{L}{10}, \qquad L = 6 \div 30$$

$$L = 14m \rightarrow h = \frac{14\ m}{15} \div \frac{14\ m}{8} = 0{,}93\ m \div 1{,}75m \rightarrow taken\ value: h = 1{,}00m = 1000\ mm$$

$$L = 14m \rightarrow f = \frac{14\ m}{20} \div \frac{14\ m}{10} = 0{,}7m \div 1{,}4\ m$$

$$f = R - \sqrt{R^2 - c^2} = 25\ m - \sqrt{(25m)^2 - \left(\frac{14m}{2} - 0{,}2m\right)^2} = 0{,}94\ m$$

$$b = 0{,}2\ m \rightarrow \frac{h}{b} = \frac{1000mm}{200mm} = 5 < 10$$

Assumptions for this example:
- value of the snow load: 3,6k N/m,
- value of the roofing load: 4,0 kN/m,
- value of the installation load: 2,5 kN/m,
- purlins every 4m,
- girder is placed on two columns with cross-sections: $h_s \times b_s$ =400 x 200mm each.

Self-weight of the girder:
$\rho_{g,k} = 425 \frac{kg}{m^3} = 4{,}25\ kN/m^3$
Self-weight characteristic load:

$$G_{k,self} = b \cdot h \cdot \rho_{g,k} = 200mm \cdot 1000m \cdot \frac{4{,}25\ kN}{m^3} = 0{,}85\ kN/m$$

Collation of loads:
- dead loads (roofing + installations + girder self-weight):

$$G_k = 4{,}0 \frac{kN}{m} + \frac{2{,}5kN}{m} + 0{,}85kN/m = 7{,}35kN/m$$

- live loads (snow):

$Q_k = 3{,}6\ kN/m$
The calculated value of the load in accordance with a combination:

$$E_d = 1{,}35 \cdot G_k + 1{,}50 \cdot Q_k = 1{,}35 \cdot \frac{7{,}35\ kN}{m} + +1{,}50 \cdot \frac{3{,}6\ kN}{m} \cong 15{,}32\ kN/\text{m}$$

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IV. Mathematical Formulation of the Optimization Problem

A. Pent girder

To design pent girders, the following aspects have to be analyzed [7,8]:

- bending stresses B,
- shearing stresses S,
- serviceability limit state L.

In theory, the general goal of every construction is to fulfill the condition (a 10% safety margin will be applied):

$$F_D \leq R_D \rightarrow \frac{F_D}{R_D} \leq 0{,}9), \quad (3)$$

where:

$F_D -$ *the design value of the force,*
$R_D -$ *the design value of the load carrying capacity.*

This condition has to be fulfilled for all of these aspects, which can be presented as a set of inequalities:

$$\frac{F_{B,D}}{R_{B,D}} \leq 0{,}9, \quad \frac{F_{S,D}}{R_{S,D}} \leq 0{,}9, \quad \frac{F_{L,D}}{R_{L,D}} \leq 0{,}9. \quad (4)$$

These equations can be extended to the following forms:

$$\begin{cases} \frac{F_{B,D}}{R_{B,D}} = 0{,}9 \\ \frac{F_{S,D}}{R_{S,D}} = 0{,}9 \\ \frac{F_{L,D}}{R_{L,D}} \leq 0{,}9 \end{cases} or \begin{cases} \frac{F_{B,D}}{R_{B,D}} = 0{,}9 \\ \frac{F_{S,D}}{R_{S,D}} \leq 0{,}9 \\ \frac{F_{L,D}}{R_{L,D}} = 0{,}9 \end{cases} or \begin{cases} \frac{F_{B,D}}{R_{B,D}} \leq 0{,}9 \\ \frac{F_{S,D}}{R_{S,D}} = 0{,}9 \\ \frac{F_{L,D}}{R_{L,D}} = 0{,}9 \end{cases} \quad (5)$$

or if they would be impossible to achieve, to find one of those:

$$\begin{cases} \frac{F_{B,D}}{R_{B,D}} = 0{,}9 \\ \frac{F_{S,D}}{R_{S,D}} \leq 0{,}9 \\ \frac{F_{L,D}}{R_{L,D}} \leq 0{,}9 \end{cases} or \begin{cases} \frac{F_{B,D}}{R_{B,D}} \leq 0{,}9 \\ \frac{F_{S,D}}{R_{S,D}} = 0{,}9 \\ \frac{F_{L,D}}{R_{L,D}} \leq 0{,}9 \end{cases} or \begin{cases} \frac{F_{B,D}}{R_{B,D}} \leq 0{,}9 \\ \frac{F_{S,D}}{R_{S,D}} \leq 0{,}9 \\ \frac{F_{L,D}}{R_{L,D}} = 0{,}9 \end{cases} \quad (6)$$

which can be expressed as a target function:

$$F = max\left(\frac{F_{B,D}}{R_{B,D}}, \frac{F_{S,D}}{R_{S,D}}, \frac{F_{L,D}}{R_{L,D}}\right) = 0{,}9, \quad (7)$$

using these optimization variables:

$$F = F(b, h, \alpha). \quad (8)$$

$$B = \frac{F_{B,D}}{R_{B,D}} \cdot 100\%, \;\; S = \frac{F_{S,D}}{R_{S,D}} \cdot 100\%, \;\; L = \frac{F_{L,D}}{R_{L,D}} \cdot 100. \quad (9)$$

It could be assumed that fulfilling all of the conditions evenly on the highest possible level is more important than complying with one of them "perfectly". In that case, the condition would be (Tab. I).

$$F = MAX(B + S + L), \quad (10)$$

where:

$$\sum = B + S + L - relative\ quantitative\ parameter.$$

The next step is an optimization, and due to the cost of the material required for the production of the beam, the objective function has to be modified.

For the optimization, considering the usage of resistance safety margin, the objective function was:

$$F = max\left(\frac{F_{B,D}}{R_{B,D}}, \frac{F_{S,D}}{R_{S,D}}, \frac{F_{L,D}}{R_{L,D}}\right) = 0{,}9. \quad (11)$$

In this case, it can be expressed as:

$$F = min\left(b \cdot h \cdot L \cdot C_{for\ 1m^3}\right). \quad (12)$$

Or in two steps:

$- F = min(V_i) = min(b_i \cdot h_i \cdot L)\ where:\ _{i=GL22h,GL24h\ldots}$
$- Comparison\ of\ obtained\ beams\ with\ minimum\ cost\ as\ a\ parameter.$

The optimal solution would be presented in the form of a function (Fig. 5):

F = min(A)=min(b x h).

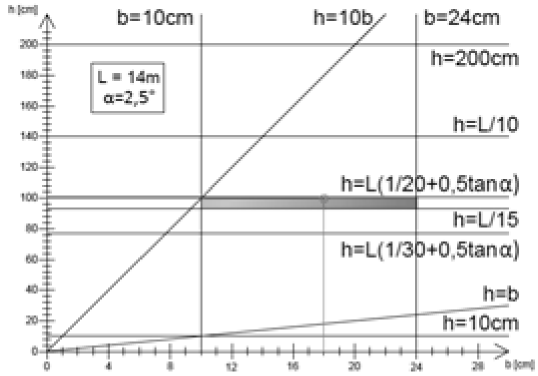

Figure 5 An optimal solution included in the range of possible solutions for $\alpha = 2{,}5°$ presented on graphs

TABLE I
Constraints for the optimization of the pent girder

No	Constraint		Form adjusted to Solver
1	$h \geq \frac{L}{15}$	w1	$h - \frac{L}{15} \geq 0$
2	$h \leq \frac{L}{10}$	w2	$h - \frac{L}{10} \leq 0$
3	$h \geq \frac{L}{30} + 0{,}5 \cdot tan\alpha \cdot L$	w3	$h - \frac{L}{30} - 0{,}5 \cdot tan\alpha \cdot L \geq 0$
4	$h \leq \frac{L}{20} + 0{,}5 \cdot tan\alpha \cdot L$	w4	$h - \frac{L}{20} - 0{,}5 \cdot tan\alpha \cdot L \leq 0$
5	$\frac{h}{b} \leq 10$	w5	$\frac{h}{b} - 10 \leq 0$
6	$b \leq h$	w6	$b - h \leq 0$
7	$b \geq 0{,}1m$	w7	$b - 0{,}1m \geq 0$
8	$b \leq 0{,}24m$	w8	$b - 0{,}24m \leq 0$
9	$h \geq 0{,}1m$	w9	$h - 0{,}1m \geq 0$
10	$h \leq 2{,}0m$	w10	$h - 2{,}0m \leq 0$

B. Gable (pitched cambered) and arch girders

To design gable and arch girders the following aspects have to be analyzed [7,8]:

- bending stresses B,
- bending stresses in the apex zone A,
- tensile stresses perpendicular to the grain in the apex zone T,
- shearing stresses S,
- serviceability limit state L.

As for the pent girder, the general condition is:

$$\frac{F_D}{R_D} \leq 0{,}9 \quad (13)$$

This condition has to be fulfilled for all of these aspects, which can be presented as a set of inequalities:

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$$\frac{F_{B,D}}{R_{B,D}} \leq 0{,}9,\quad \frac{F_{A,D}}{R_{A,D}} \leq 0{,}9,\quad \frac{F_{T,D}}{R_{T,D}} \leq 0{,}9,\quad \frac{F_{S,D}}{R_{S,D}} \leq 0{,}9,\quad \frac{F_{L,D}}{R_{L,D}} \leq 0{,}9. \tag{14}$$

which can be expressed as a target function (Tab. II-III):

$$F = max\left(\frac{F_{B,D}}{R_{B,D}}, \frac{F_{A,D}}{R_{A,D}}, \frac{F_{T,D}}{R_{T,D}}, \frac{F_{S,D}}{R_{S,D}}, \frac{F_{L,D}}{R_{L,D}}\right) = 0{,}9, \tag{15}$$

using these optimization variables:

$$F = F(b, h, \alpha^{*}). \tag{16}$$

*for the gable girder

$$B = \frac{F_{B,D}}{R_{B,D}} \cdot 100\%, A = \frac{F_{A,D}}{R_{A,D}} \cdot 100\%, T = \frac{F_{T,D}}{R_{T,D}} \cdot 100\%, \tag{17}$$

$$S = \frac{F_{S,D}}{R_{S,D}} \cdot 100\%, L = \frac{F_{L,D}}{R_{L,D}} \cdot 100.$$

where:

$$\sum = B + A + T + S - relative\ quantitative\ parameter$$

As for the pent girder, the adequate target function due to the cost of the material required for the production of the beam can be expressed as:

$$F = min\left(b \cdot h \cdot L \cdot C_{for\ 1m^3}\right). \tag{18}$$

Or in two steps:

$- F = min(V_i) = min(b_i \cdot h_i \cdot L)\ where:\ _{i=GL22h,GL24h\ldots}$

$-Comparison\ of\ obtained\ beams\ with\ minimum\ cost\ as\ a\ parameter.$

TABLE II
CONSTRAINTS FOR THE OPTIMIZATION OF THE GABLE GIRDER

No	Constraint		Form adjusted to Solver
1	$h \geq \frac{L}{15}$	w1	$h - \frac{L}{15} \geq 0$
2	$h \leq \frac{L}{10}$	w2	$h - \frac{L}{10} \leq 0$
3	$h \geq \frac{L}{30} + 0{,}5 \cdot tan\alpha \cdot L$	w3	$h - \frac{L}{30} - 0{,}5 \cdot tan\alpha \cdot L \geq 0$
4	$h \leq \frac{L}{20} + 0{,}5 \cdot tan\alpha \cdot L$	w4	$h - \frac{L}{20} - 0{,}5 \cdot tan\alpha \cdot L \leq 0$
5	$\frac{h}{b} \leq 10$	w5	$\frac{h}{b} - 10 \leq 0$
6	$b \leq h$	w6	$b - h \leq 0$
7	$b \geq 0{,}1m$	w7	$b - 0{,}1m \geq 0$
8	$b \leq 0{,}24m$	w8	$b - 0{,}24m \leq 0$
9	$h \geq 0{,}1m$	w9	$h - 0{,}1m \geq 0$
10	$h \leq 2{,}0m$	w10	$h - 2{,}0m \leq 0$
11	$\propto \leq 5°$	w11	$\propto -5° \leq 0$
12	$\propto \geq 0°$	w12	$\propto -0° \geq 0$
14	$\max(B, A, T, S, L) \leq 0{,}9$	w13	$\max(B, A, T, S, L) - 0{,}9 \leq 0$

For the pent and gable girder geometrical constraints were functions of:

$$f = f(h, b, L, \propto), where\ L = const, so: f = f(h, b, \propto).$$

For a curved girder:

$$f = f(h, b, L, R, \boldsymbol{f}), where\ L = const, so: f = f(h, b, R, \boldsymbol{f})$$

In order to reduce the number of variables, the relationship between the function f(R) and the radius R of arc (Fig. 6) might be helpful:

$$\begin{cases} \boldsymbol{f} = R - \sqrt{R^2 - c^2} \geq \frac{L}{20} \\ \boldsymbol{f} = R - \sqrt{R^2 - c^2} \leq \frac{L}{10} \end{cases} \tag{19}$$

$$c = 6{,}8m = const \rightarrow \boldsymbol{f} = R - \sqrt{R^2 - (6{,}8m)^2} = R - \sqrt{R^2 - 46{,}24m^2} \rightarrow \boldsymbol{f} = f(R) \tag{20}$$

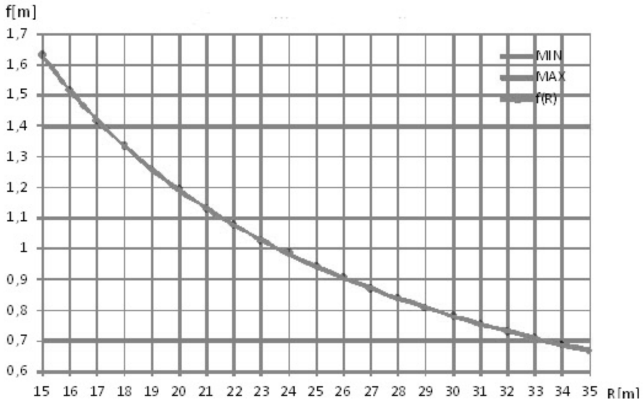

Figure 6 Graphical presentation of f(R) of a circular arc dependent on the radius, limited by geometrical conditions

TABLE III
CONSTRAINTS FOR THE OPTIMIZATION OF THE ARCH GIRDER

No	Constraint		Form adjusted to Solver
1	$h \geq \frac{L}{15}$	w1	$h - \frac{L}{15} \geq 0$
2	$h \leq \frac{L}{8}$	w2	$h - \frac{L}{8} \leq 0$
3	$R - \sqrt{R^2 - c^2} \geq \frac{L}{20}$	w3	$R - \sqrt{R^2 - c^2} - \frac{L}{20} \geq 0$
4	$R - \sqrt{R^2 - c^2} \leq \frac{L}{10}$	w4	$R - \sqrt{R^2 - c^2} - \frac{L}{10} \leq 0$
5	$R \geq L$	w5	$R - L \geq 0$
6	$R \leq 10L$	w6	$R - 10L \leq 0$
7	$\frac{h}{b} \leq 10$	w7	$\frac{h}{b} - 10 \leq 0$
8	$b \leq h$	w8	$b - h \leq 0$
9	$b \geq 0{,}1m$	w9	$b - 0{,}1m \geq 0$
10	$b \leq 0{,}24m$	w10	$b - 0{,}24m \leq 0$
11	$h \geq 0{,}1m$	w11	$h - 0{,}1m \geq 0$
12	$h \leq 2{,}0m$	w12	$h - 2{,}0m \leq 0$
13	$\max(B, A, T, S, L) \leq 0{,}9$	w13	$\max(B, A, T, S, L) - 0{,}9 \leq 0$

V. COMPARISON OF RESULTS

For the final solution of the stated problem, girders in three different shapes (pent, gable, curved) and in various timber strength classes were compared (Tab. IV-VI).

Based on the above studies and tables, the following general conclusions can be drawn:

- using timber GL22h and GL24h is economically the best solution in this task, independent of the shape of the beam.
- if there is only one type of material, the smallest beam can be, in many cases, regarded as the optimal one from an economical point of view. However, when there are a few materials, the analyzed costs of their production, transportation etc. have to be included in calculations;

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- cross-sections with a proportion $b/_h = 1/_5 \div 1/_6$ appear to be optimal ones,
- the gable shape transpired to be the optimal solution for this task - the smallest volume of timber is needed to produce a proper girder that fulfills all of the requirements,
- choosing the curved shape would be a worse solution from an economical point of view (the cheapest curved girder would be 10% more expensive than the cheapest gable girder),
- the "resistance usage" of the curved girder is more balanced than for the other shapes,
- choosing the pent shape would be the most expensive solution (the cheapest pent girder would be 28% more expensive than the cheapest gable girder).

Aspects that affected the dimensions of the cross-sections were:
1. Shearing stresses (S) and serviceability limit state (L) were decisive for the pent girder. The reason for this is the decrease of the cross-section's area towards the end of the beam where the shearing forces are the highest and the decrease of the height area towards the end of the beam, which increases the deflection.
2. As above, shearing stresses (S) and serviceability limit state (L) were also decisive for the gable girder. The reasons are similar here, but different in that the cross-section's height and area are decreasing in both directions. On the other hand, for the curved girder, bending stresses (B) appeared to be decisive. For this shape, the cross-section is constant and is not increasing towards the middle of the beam, which is the place where bending forces are the highest.

TABLE IV
OPTIMAL PENT-SHAPED GIRDERS FOR DIFFERENT TIMBER STRENGTH CLASSES

Timber	h [m]	b [m]	α [º]	B %	S [%]	L [%]	V [m³]
GL22h	1,16	0,18	3,91	59	87	85	2,9
GL24h	**1,12**	**0,18**	**3,71**	**59**	**89**	**84**	**2,8**
GL26h	1,08	0,18	3,11	61	85	83	2,7
GL28h	1,04	0,18	2,78	62	85	86	2,6
GL30h	1,00	0,18	2,64	63	87	88	2,5
GL32h	1,00	0,18	2,55	60	86	84	2,5
minimum total price – for timber GL24h							

TABLE V
OPTIMAL GABLE-SHAPED GIRDERS FOR DIFFERENT TIMBER STRENGTH CLASSES

Timber	h [m]	b [m]	α [º]	B %	A [%]	T [%]	S L [%]	V [m³]
GL22h	**1,16**	**0,18**	**4,19**	**88**	**73**	**56**	**90 86**	**2,3**
GL24h	1,12	0,18	3,86	85	71	55	90 85	2,2
GL26h	1,08	0,18	3,52	82	70	53	90 87	2,2
GL28h	1,04	0,18	3,19	80	69	51	90 90	2,1
GL30h	1,00	0,18	2,86	79	69	49	90 90	2,1
GL32h	0,96	0,18	2,25	77	69	41	85 90	2,1
minimum total price – for timber GL22h								

TABLE VI
OPTIMAL ARCH-SHAPED GIRDERS FOR DIFFERENT TIMBER STRENGTH CLASSES

Timber	h [m]	b [m]	R [m]	f [m]	B [%]	A T [%]	S L [%]	V [m³]
GL22h	1,00	0,18	30,0	0,78	89	88 63	59 82	2,5
GL24h	**0,96**	**0,18**	**27,0**	**0,87**	**88**	**88 72**	**61 84**	**2,4**
GL26h	0,96	0,18	27,0	0,87	83	82 72	61 80	2,4
GL28h	0,96	0,18	28,4	0,82	79	76 69	61 77	2,4
GL30h	0,96	0,16	25,6	0,92	90	79 83	69 80	2,2
GL32h	0,96	0,16	25,6	0,92	86	74 83	69 77	2,2
minimum total price – for timber GL24h								

VI. CONCLUSIONS

This paper presents an algorithm that enables the finding of the optimal shape of a timber roof girder that can carry imposed loads safely and economically, and also fulfill the requirements resulting from the function of the object. The algorithm can also be used to test and demonstrate the capabilities offered by the Excel Solver tool. The basic criteria are the strength, stability, and rigidity of the structure. Optimal construction shaping included four basic elements: optimization criterion, optimization variables, optimization limitations, and state equations.

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