Enhancing Structural Health Monitoring by FEM and Laser Doppler Vibrometry

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***Abstract*—A new concept for damage detection in a Euler Bernoulli beam is proposed in this paper. An indirect strategy that yields strain modes as a special derivative of displacement data with respect to a spatial variable is analytically introduced and utilized for the analysis. Though strain modes are sensitive to damage, they are not readily identifiable in practice. The proposed approach provides a prospect for strain mode-based damage detection without involving direct measurement of the strain modes themselves. The derivation is adapted to the FEM analysis and to data measurable by Scanning Laser Doppler Vibrometry (LDV). Various scenarios are considered as an exploration for damage detection. It is analytically demonstrated that the method is useful for damage detection in structures build on Euler Bernoulli beams provided displacement mode data is available.**

***Index Terms*—Damage Detection, Strain mode, Displacement mode, Laser Doppler Vibrometry**

I. Introduction

Prediction of the dynamics of a structure is an integral part of structural health monitoring. Lack of, life cycle evaluation and proper repair management, may be a potential cause of catastrophic failure and accidents in structures. Vibration Analysis (VA) offers a Non-destructive approach, efficient online prediction of structural dynamics and attracts a considerable research effort because of its convenient application and cost effectiveness. Structural damage influences local flexibility, natural frequency, damping ratio and displacement modes [9, 12, 13, 17]. Consequently, vibration-testing data may harbor damage signals.

Damage detection comprises location identification and damage degree evaluation, and residual lifetime (cycle) estimation.

VA of damage could be carried out using the following:

1. *Vibration modes from modal testing [1-3].*

Structures modal testing has a comparatively long history, a systematic theory and well-documented applications. Its relevant experimental results include natural frequencies, modal damping ratios and natural modes. Structural damage reduces natural frequencies and affects natural mode shapes. By comparing modal characteristics of an intact and of a damaged structure, damage identification can be done in principle. Nonetheless, natural frequency is a global characteristic and sensitivity of modal characteristics to changes in local flexibility is low, hence its practical application is limited.

2. *Strain modes from strain modal testing [4-6].*

Strain modes are considerably sensitive to changes in the local flexibility than are displacement modes because the former are first derivatives of the later. In the last two decades, theory and techniques that define the modes have been well established. Multiple experiments reveal that, strain modes carry damage information. Due to practical and technical limitations in their application to engineering structures, for example, setting up and maintaining of strain gauges, hitherto the number of practical reports are limited.

Besides the above categories, the use of SLDV is noted [7, 8]. A Scanning Laser Vibrometry (SLDV) device is a non-contacting measurement instrument and senses at multiple pre-defined points and directions to provide "distribution" information in a particular area. A novel vibration-based (NDT) technique, that integrates (SLDV) and Strain Energy Distribution method has been developed in [7].

In what follows, a damage criterion utilizing the strain response of a structure and vibration data measurable by SLDV is the core of the analysis.

II. Modal Strain Analysis and Underlying Principles

The equation of motion of transverse vibration of a Euler-Bernoulli Beam has the form

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 u(x,t)}{\partial x^2}\right]+\rho(x)\frac{\partial^2 u(x,t)}{\partial t^2}=f(x,t) \quad (1)$$

Where

$u(x,t)$ = the transverse displacement at position x at the instant t

E = the Young's modulus

$I(x)$ = second moment of area of the element

$f(t,x)$ = applied force in the transverse direction

$\rho(x)$ =material density.

Based on vibration theory, the lateral response of the beam under a harmonic excitation has the form:

$$u(x,t)=U(x)\cdot T(t)=\sum_{r=1}^{\infty}Q_{1r}.\varphi_r(x)e^{j\omega t} \qquad (2)$$

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Where

$\varphi_r(x)$ = the rth mass of normalized transverse mode shape under certain boundary conditions

Q_{1r} = the generalized coordinate of rth mode.

Displacement (u)-strain (ε) relationship gives:

$$\varepsilon(x,t) = \frac{\partial u(x,t)}{\partial x}\cdot h_1 = \sum_{r=1}^{\infty} h_1.\frac{\partial \varphi_r(x)}{\partial x} Q_{2r} e^{j\omega t} \quad (3.1)$$

And

$$\frac{\partial \varepsilon(x,t)}{\partial x} = \frac{\partial^2 u(x,t)}{\partial x^2}\cdot h_2 = \sum_{r=1}^{\infty} h_2.\frac{\partial^2 \varphi_r(x)}{\partial x^2} Q_{2r} e^{j\omega t}$$
$$= \sum_{r=1}^{\infty} \Psi_r^{\varepsilon}(x) Q_{2r} e^{j\omega t} \quad (3.2)$$

$\Psi_r^{\varepsilon}(x) = h_2 \dfrac{\partial^2 \varphi_r(x)}{\partial x^2}$ is the rth strain mode corresponding to displacement mode $\varphi_r(x)$. Indeed, Q_{1r}, Q_{2r}, h_1 and h_2 are obviously not the same and are dependent on location along the structure. Equation (3) is valid for thin short plate-like structures in which the shear factors are comparably small or negligible when loaded in their own planes.

The strain mode can be evaluated by:

1. The strain modal transfer function of the vibration system is represented by

$$H_{ij}^{\varepsilon} = \sum_{r=1}^{n} \frac{\psi_{ir}^{\varepsilon}\cdot \phi_{jr}}{k_r - \omega^2 m_r + j\omega c_r} \quad (4)$$

Evaluating modal mass, modal stiffness, modal damping and mode shapes from the displacement mode analysis, the strain modes can be identified using equation (3).

From displacement-strain relationship, the strain modes of the transverse vibration of the beam could be estimated by the central difference solution for equation (3) given by equation (5), though with low accuracy:

$$\psi_{rk}^{\varepsilon} = \frac{\partial^2 \phi_{r(k-1)}}{\partial x^2} \approx \frac{\phi_{r(k-1)} - 2\phi_{rk} + \phi_{r(k+1)}}{\Delta x^2} \quad (5)$$

Here, k is the number of discrete points along the structure, Δx is the finite distance between adjoining discrete points, and ψ_r^{ε} is the rth strain mode corresponding to the displacement mode $\varphi_r(x)$.

III. DAMAGE CRITERIA

Changes in strain modes reflect damage, however, their measurement and evaluation in large engineering structures is technically a formidable task. Time-displacement responses measured at widely and densely scattered points on the structure are considered for the following analysis.

The strain response under the action of an external force $f(t,x) = \delta(x - x_o).e^{j\omega t}$ is:

$$\frac{\partial \varepsilon(t,x)}{\partial x} = \sum_{r=1}^{\infty} \psi_r^{\varepsilon}(x).Q_r.e^{j\omega t} \quad (6)$$

If the strain responses of an intact and a damaged structure are known, then the difference in the two; $Z = \|\varepsilon(t,x) - \varepsilon_d(t,x)\|$ represents damage. (The subscript d indicates "of damaged structure" hereinafter). On the premise of equation (3), strain response is a linear combination of its modes. Therefore, sudden local changes in strain modes at damage locations will result into a local change in the corresponding responses; that is,

$$\varepsilon_d(t,x) = \sum_{x=0}^{x=L} \sum_{r=1}^{\infty} \psi_{d\,r}^{\varepsilon}(x).Q_{d\,r} e^{j\omega t} \Delta x \quad (7)$$

As a result, a partial derivative of strain response with respect to the spatial variable x, would reflect the pertinent change of strain, specifically

$$\frac{\partial \varepsilon(t,x)}{\partial x} = \sum_{r=1}^{\infty} \psi_{d\,r}^{\varepsilon}(x).Q_{d\,r} e^{j\omega t} \quad (8)$$

In view of equations (7-8) and the above considerations, the following functions are proposed for a damage criterion:

$$Z_1(x) = \frac{1}{T}\int_0^T \|\varepsilon(t,x)\| dt \quad (9.1)$$

$$Z_2(x) = \frac{1}{T}\int_0^T \|\varepsilon(x+h,t) - \varepsilon(x,t)\| dt \quad (9.2)$$

To circumvent direct measurement, the transverse displacement of a vibrating beam structure has been used to compute ε from measured displacement signals using equation (10), a solution strategy, specially derived to limit computation errors in solving the equation

$$\frac{\varepsilon(x,t)}{\partial x} = h.\frac{\partial^2 u(x,t)}{\partial x^2} \quad (10)$$

IV. SIMULATION AND ANALYSIS

A FEM program in Matlab to demonstrate damage detection in a cantilever beam by the proposed criterion was developed; $x = 0$, free end; $x = 100$ fixed end, with dimensions L=1 m; W=0.025 m, H=0.0095 m, and material constants $E = 2.0\times 10^{11}$ Pa, μ=0.33, ρ=7850kg/m3. The cantilever was meshed into 100 elements and the damage in an element modeled by a decreased width. The degree of damage is hereby represented by the ratio λ=w/W.

V. DISPLACEMENT AND STRAIN MODAL ANALYSIS

Eigenvalues were analyzed for intact and damaged structures. Table 1 shows the natural frequencies for λ= 0.5 when the damaged element is varied from 10th to 90th element position by a step 10.

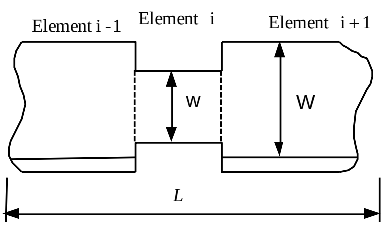

Fig. 1: A damaged beam element (damage symbolized by a reduced diameter- w)

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Table 1: Natural frequencies for a damaged beam, damage degree λ=0.5

Mode	1st	2nd	3rd	4th
Not damaged	7.746	48.5441	135.925	266.359
Damaged 10	7.635	48.314	135.908	266.767
Damaged 20	7.667	48.589	136.000	265.774
Damaged 30	7.695	48.580	135.576	266.117
Damaged 40	7.718	48.436	135.737	266.032
Damaged 50	7.738	48.306	135.919	265.195
Damaged 60	7.755	48.271	135.479	266.192
Damaged 70	7.771	48.333	135.011	265.758
Damaged 80	7.787	48.467	135.192	264.586
Damaged 90	7.805	48.685	135.901	265.676

Table 1 shows that, the changes in natural frequencies are comparatively small; the change in first frequency is less than 0.2 Hz. Non-dimensional strain modes for known displacement modes were computed based on equation (3) for the case of a constant a h as a first order derivative of displacement modes with respect to x. Figure 2 (a)-(c) shows displacement and strain modes. There is no discernible damage symptom in the displacement modes, nonetheless, peaks appear at the damage location in the strain mode curves. Results in Fig 2c were experimentally obtained on a SLDV.

Figure 2 shows the differences in first strain modes of undamaged and damaged beam for λ=0.5. The results show how the peak heights vary with the damage location between elements 10 through 90.

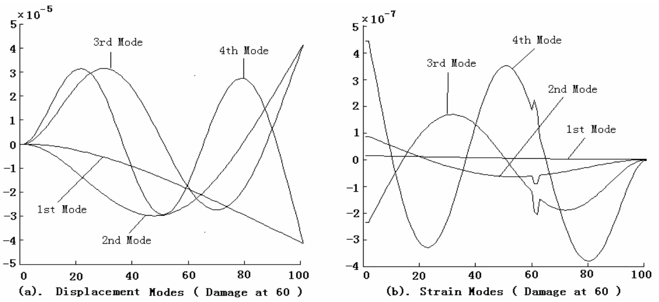
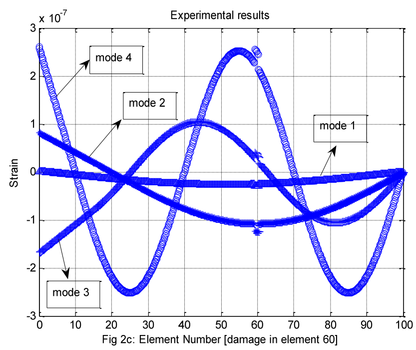


Fig. 2: Displacement and strain modes for undamaged and damaged beams at element 60



(c)

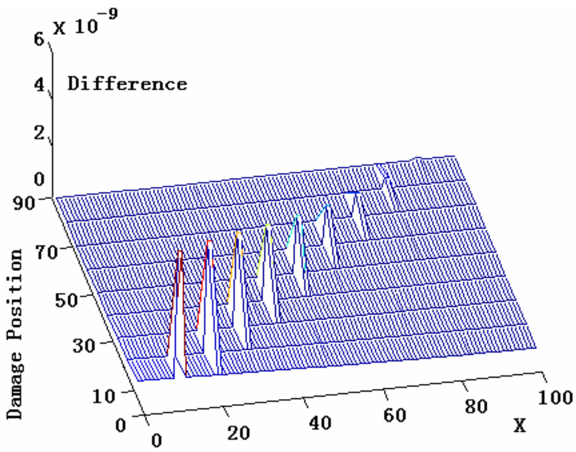
Fig.2: Displacement modes for undamaged and damaged beams at element 60

VI. RESPONSE AND ANALYSIS OF CRITERIA

The cantilever was excited by a harmonic force $f(t) = A\sin(\omega t)$ at the free end and the displacement response computed by FEM. The strain response was evaluated by equation (3). The time interval [0, 0.4s] with step 0.001s was used. Figures 4-5 show displacement and strain responses at node 50, whereas figures 6-7 show displacement and strain mode responses at node 100. Using the responses, the damage criteria z_1 and z_2 were evaluated for various damage locations. Figure 8 illustrates z_1 and z_2 as smooth curves in an intact beam. Figures 8-10 depict scenarios of the damage at elements 10 and 90 respectively.

In figures 9 and 10, there exist peaks at the damage sites. z_1 and z_2 has one peak and two peaks respectively centered at damage sites. This property could hold a premise for isolating a real damage from noisy data.

Figure 11 shows a z_2 curve when damage position is varied from 10 to 90 in a step of 12; the peaks' sites well coincide with the damage location. The curve reflects sensitivity of z_2 to damage location. Comparing Figures 3 and 12, it is obvious that the abrupt change in strain modes results in the peaks of z_2. Figure 12 shows z_2 for increasing degree of damage with a fixed damage location, i.e. sensitivity of z_2 with respect to degree of damage. z_2 Increases exponentially with linear increment in the damage degree.



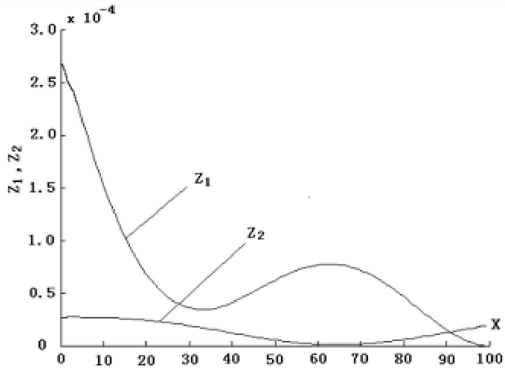


Fig. 3: Influence of damage position on strain mode

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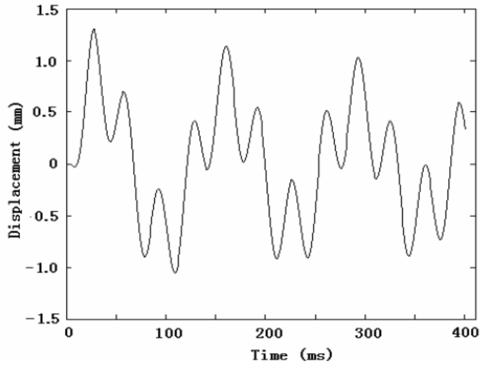

Fig. 4: Time displacement response at node 50

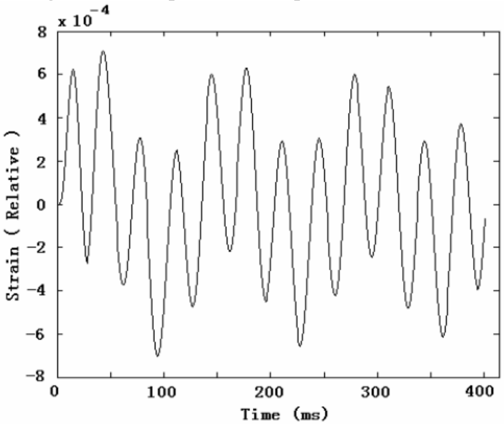

Fig. 5: Relative strain response at node 50 (damaged)

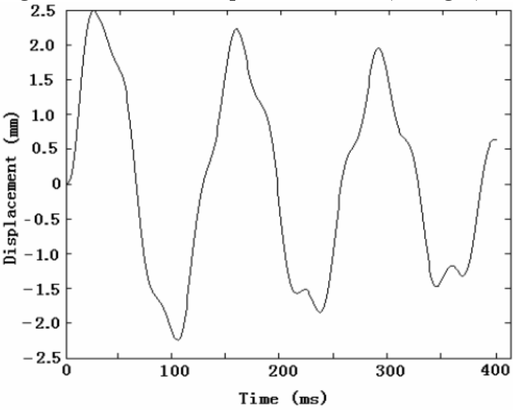

Fig. 6: Displacement response at node 100 (damaged)

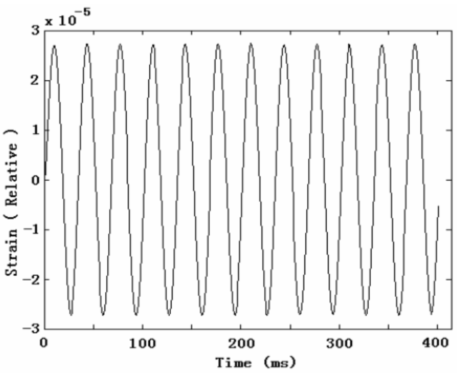

Fig. 7: Relative strain response at node 100

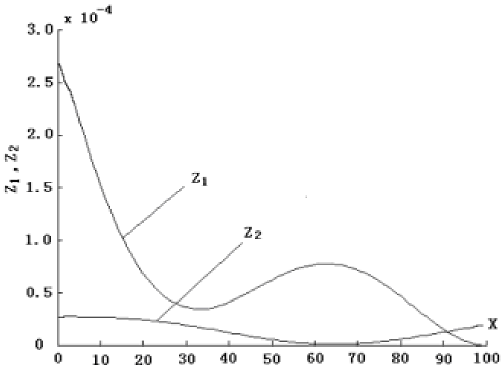

Fig. 8: Sensitivity of damage parameter to location

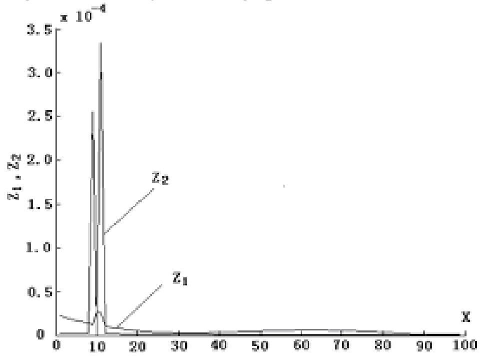

Fig. 9: Case of a damaged element 10

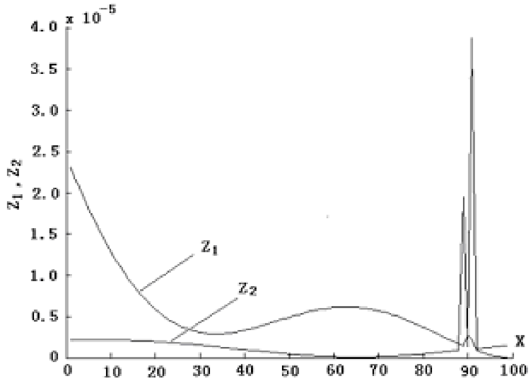

Fig. 10: Case of a damaged element 90

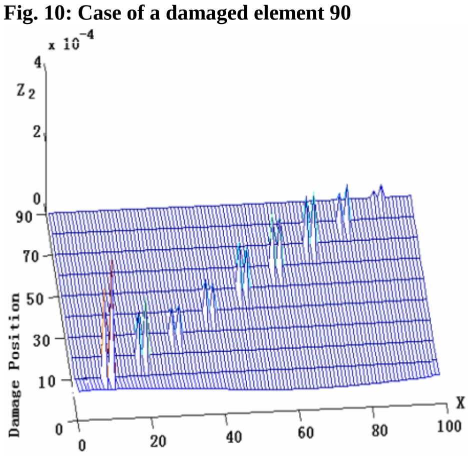

Fig. 11: Z2nder different damage locations (λ=0.5)

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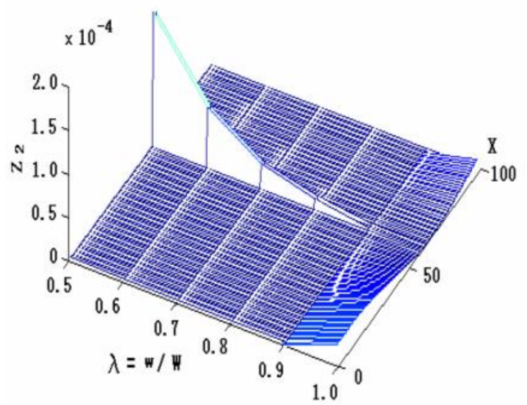

Fig. 12: Under different degrees of damage (damaged element 60)

VII. Conclusion

A criterion based on derivative of displacement modes responses with respect to a spatial variable has been established by analysis for structural damage detection and its validity and properties demonstrated by numerical simulation. The two functions; z_1 and z_2 reflect damage position and damage degree. z_2 is more sensitive to damage than z_1. In the same damage condition, z_1 has one peak and z_2 has two. Only measured displacement response of the structure is sufficient for availability of the criterion. The measurement points for displacement response have to be sufficiently dense to curb on noise error in the strain responses evaluated as a second order derivative of displacement. The approach is good for plate-like structures loaded in their own planes when the shear factors are negligibly small.

Further analysis of the proposed criterion for various cases of excitations; impact, random loading and identification of degree of damage by the author is underway.

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