Multi-Stage Optimization for University Examination Proctor Assignments

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***Abstract*—Our previous researches have proposed a mixed integer programming model for the proctor assignments in final examinations at our university. It is time-consuming for the university's registrars to perform the task of the assigning proctors with satisfying several sorts of conditions. The proposed model has gradually been progressed with respect to deal with more practical requirements. On the other hands, the execution time to derive an optimal solution for the model has been increased.**

This paper proposed another formulation for the target proctor assignment problem to assess the solving time. The original models targeted multiple objectives and treated them as a weighting summation. The proposed formulation in this paper adopts a multi-stage optimization with respect to the multiple objectives. Numerical experiments estimate the performance of the proposed formulation compared with the previous model.

***Index Terms*—combinatorial optimization, examination proctor assignment, mixed integer programming, multi-stage optimization, optimization in university**

I. INTRODUCTION

AT the end of each semester in our university, final examinations are administered in classrooms more spacious than used for regular courses because examinees must make their next seats empty. According to the number of examinees in a course, the number of proctors is determined and faculty and staff members are assigned as examination proctors to invigilate. The university's registrar performs the assignments manually with considering several types of requirements for administration of the exams. The manual assignments have been troublesome for registrar.

Proctor assignment is one of the types of timetabling problems, which include course assignments and classroom assignments. Various models and approaches have been proposed for a variety of situations [1–6]. It is one of the well-known application of timetabling problem in university.

We have tackled the assignment problem for a few years as an application of mathematical optimization. The initial proposed model [7] took fundamental requirements into account and has been gradually improved to incorporate more practical requirements. In our previous work [8], the proposed model satisfied with almost all requirements in the practical proctor assignment. The proposed model was incorporated into a system based on Microsoft Excel. On the other hands, the computational time to solve the developed optimization problem has increased in response to the complexity of the model. The previous model had another problem that system user could not judge if the values of multiple objectives are optimal or not for the resulting assignment because the objective function in the previous model was in the form of the weighted summation of three objectives.

In this paper, we revised the proposed model in [8] as a multi-stage optimization problem. The revised model adopted a multiple stage where an objective function was optimized and guaranteed the optimality of the objective, which is an advantage for system users. We estimated the obtained optimums and computational time to derive optimal proctor assignments by each model as well as from the view of the convenience of system users.

II. PRELIMINARIES

A. Target Problem

This study targets the same problem as that discussed in our previous study [8]. The proctor assignment problem is to find an optimal assignment which satisfies various conditions.

The most essential condition is to assign required number of proctors for each examinations. A proctor is assigned per forty examinees. When the number of proctors required for an examination is even, equal numbers of faculty and staff members are assigned. When the number is odd, one more faculty member is assigned. The lecturer of a course is necessarily assigned as the chief proctor of that course's examination.

A proctor's task is physically and mentally tiring; therefore, the number of invigilating should be equalized among faculty members and among staff members. Additionally, being assigned to multiple examinations in a single day is not favored by most personnel, especially for 2 consecutive examinations in one day or 2 examinations separated by more than 2 interval periods.

For most university departments, assigning multiple personnel during the same period on the same day is disruptive. Hence, for each department, a maximum number of proctors who can invigilate simultaneously must be determined. Some staff members work at other campuses but must travel to the main campus for proctoring. Thus, they are limited to 2 days per week for proctoring.

There are other conditions to be considered for proctor assignments: (1) both one female and one male proctor are required to monitor the examinees' use of the bathroom; (2) some classrooms are rather spacious and the task of

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proctoring is relatively more difficult so the number of proctors assigned to such rooms is restricted to one; and (3) personnel unfamiliar with proctoring must be assigned together with experienced personnel.

B. Notations

Sets:

- M set of university personnel
- M_f set of faculty members, a subset of M
- M_s set of staff members, a subset of M
- M_{fe} set of female members, a subset of M
- M_{ma} set of male members, a subset of M
- M_o set of members working at other campuses, a subset of M
- M_k set of members belonging to department k, a subset of M
- E set of examinations
- E_1 set of examinations held in the first period, a subset of E
- E_s set of examinations held in spacious classrooms, a subset of E
- De set of departments
- Da set of days allotted to administering final examinations
- P set of periods, {1, 2, 3, 4, 5, 6}
- Pr set of pairs of periods to which assignment is prohibited in a day for faculty members

Constants and Parameters:

- $Sc_{e,d,p}$ 1 if examination e is conducted on date d in period p; otherwise, 0.
- $Sc_{e,d}$ 1 if examination e is conducted on date d; otherwise, 0.
- $A_{m,e}$ 1 if member m can be assigned to a proctor for examination e; otherwise, 0.
- $C_{m,e}$ 1 if member m is the chief proctor for examination e; otherwise, 0.
- $R_{e,t}$ the required number of faculty members (t = f) and staff members (t = s) for proctoring
- V_m assignment value for member m.
- $V_{m,e}$ assignment value for member m to examination e.
- V required average assignment value for an examination.
- N^-_m, N^+_m lower and upper bounds of the number of proctor assignments for member m
- Nd^+_m upper bounds of the number of proctor assignments in a day for member m
- Nk^+_k upper bounds of the number of proctor assignments in a period for members belonging to department k

Design variables:

- $x_{m,e}$ 1 if member m is assigned to examination e; otherwise, 0.
- $y_{m,d,p}$ 1 if member m is assigned to an examination conducted on date d in period p; otherwise, 0.
- $z_{m,e}$ 1 if member m is assigned to examination e as a chief proctor; otherwise, 0.
- $yy_{m,d}$ 1 if member m is assigned to an examination conducted on date d; otherwise, 0.
- s^-_m, s^+_m slack variables for lower and upper bounds of number of assignments for member m
- $s^+_{m,d}$ slack variables for upper bound of number of assignments for member m on date d
- $s^+_{k,d,p}$ slack variables for upper bound of number of assignments for members belonging to department k on date d in period p

III. FORMULATIONS

A. Constraints

We adopt a multi-stage optimization where in each stage mixed integer programming is executed. The followings are common constraints for all state optimization.

$$x_{m,e} \le A_{m,e}, \quad \forall m \in M,\ \forall e \in E \tag{1}$$

$$\sum_{m \in M_t} x_{m,e} = R_{e,t}, \quad \forall e \in E,\ \forall t \in \{f, s\} \tag{2}$$

$$z_{m,e} = 1, \quad \forall e \in E, for\ C_{m,e} = 1 \tag{3}$$

$$\sum_{m \in M_f} z_{m,e} = 1, \quad \forall e \in E \tag{4}$$

$$x_{m,e} \ge z_{m,e}, \quad \forall m \in M_f,\ \forall e \in E \tag{5}$$

$$\sum_{m \in M_t} x_{m,e} \ge 1,\ \forall e \in E, \forall e \in \{fe, ma\} \tag{6}$$

$$\sum_{m \in M} V_m x_{m,e} \ge \left(R_{e,f} + R_{e,s}\right) V, \quad \forall e \in E \tag{7}$$

$$\sum_{m \in M_s} V_m x_{m,e} \ge R_{e,s} V, \quad \forall e \in E \tag{8}$$

$$N^-_m + s^-_m \le \sum_{e \in E} x_{m,e} \le N^+_m + s^+_m, \quad \forall m \in M \tag{9}$$

$$\sum_{e \in E_1} x_{m,e} \le 1, \quad \forall m \in M_f \tag{10}$$

$$\sum_{e \in E} z_{m,e} \le 2, \quad \forall m \in M_f\ if \sum_{e \in E} C_{m,e} \le 1 \tag{11}$$

$$y_{m,d,p} = \sum_{e \in E} Sc_{e,d,p}\ x_{m,e}, \quad \forall m \in M,\ \forall d \in Da,\ \forall p \in P \tag{12}$$

$$\sum_{p \in P} y_{m,d,p} \le Nd^+_m + s^+_{m,d},\ \forall m \in M, \forall d \in Da \tag{13}$$

$$y_{m,d,p_1} + y_{m,d,p_2} \le 1, \quad \forall m \in M_f, \forall d \in Da, \forall (p_1, p_2) \in Pr \tag{14}$$

$$\sum_{m \in M_k} y_{m,d,p} \le Nk^+_k + s^+_{k,d,p}, \quad \forall k \in De, \forall d \in Da, \forall p \in P \tag{15}$$

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$$\sum_{e\in E_s} x_{m,e} \le 1,\ \forall m \in M_s \qquad (16)$$

$$6yy_{m,d} \ge \sum_{e\in E} Sc_{e,d}\ x_{m,e}\ ,\ \forall m \in M,\ \forall d \in D \qquad (17)$$

$$\sum_{d\in Da} yy_{m,d} \le 2,\ \ \forall m \in Mem_o \qquad (18)$$

$$x_{m,e},\ z_{m,e} \in \{0,\ 1\}, \forall m \in M,\ \forall e \in E \qquad (19)$$

$$y_{m,d,p} \in \{0,\ 1\}, \forall m \in M,\ \forall e \in E,\ \forall d \in Da \qquad (20)$$

$$yy_{m,d} \in \{0,\ 1\}, \forall m \in M,\ \forall d \in Da \qquad (21)$$

$$s_m^-,\ s_m^+ \ge 0\ ,\ \forall m \in M \qquad (22)$$

$$s_{m,d}^+ \in \{0,1\}, \forall m \in M,\ \forall d \in Da \qquad (23)$$

$$s_{k,d,p}^+ \in \{0,1\}, \forall k \in De,\ \forall d \in Da,\ \forall p \in P \qquad (24)$$

Constraint (1) indicates that members cannot be assigned to their unavailable time. The required number of faculty/staff proctors for each examination is assigned by (2). The lecturer of a course is necessarily assigned to its examination as the chief proctor. This condition is expressed by (3). Some examinations are held in multiple classrooms and students are assigned to each classroom. The lecturer of the course is a chief proctor of one of the classrooms. For the other classrooms, chief proctors should be determined in the assignment task. Constraint (4) assures the chief assignment. When a faculty is a chief proctor for an examination, he/she is also a proctor for the examination, expressed by constraint (5). Constraints (6) indicates the conditions that at least one female and one male proctor should be assigned for each examination. The proctoring experience for a member m is represented as assignment value V_m. The average of assignment values for assigned members in an examination should not be below a required level V, expressed by constraint (7). The requirement is for all assigned members as well as assigned staff members because proctoring procedures are mainly administrated by staff members, given by constraint (8). The above-mentioned are constraints for each examination.

The followings are constraints for university personnel. The respective numbers of required faculty and staff members are determined separately. The total number of assignments for each member is bounded by N^-_m and N^+_m. The hard constraint is relaxed into a soft constraint by the introduction of the slack variables s^-_m and s^+_m in (9). The number of assignments for faculty members to examinations held in the first period is restricted to once by (10). The number of chief proctors is also limited to twice by (11) unless it is determined to be more than once before assignments. The design variable $y_{m,d,p}$ in (12) is defined as a conditional sum of the main design variable $x_{m,e}$. The number of assignments in a day is also restricted by (13) and the constraint is also relaxed by the slack variables. Assignments for 2 consecutive examinations and 2 examination periods separated by more than 2 interval periods are prohibited. The prohibited pairs of two periods are given by a set Pr and the condition is represented by (14). The number of staff members of a certain department with simultaneous assignments is limited and the constraint is relaxed as shown in (15). The proctoring task in more spacious classroom is harder because of the increasement of travel distance. Constraint (16) express the conditions for staff members to assign to examinations conducted in some specified spacious classrooms. Another design variable $yy_{m,d}$ is determined by (17) and is introduced as the constraints of the number of working days for proctoring. Constraint (18) limits the number of proctoring days for staff members working at other campuses to two days.

The remaining constraints, (19) through (24), are for variable types and definition domains for the design variables.

B. Multi-Stage Optimization

The target problem has the following objectives:

(a) minimize the violation penalty of lower/upper bound of the number of assignments for each member
(b) minimize the violation penalty of upper bound of the number of assignments in a day for each member
(c) minimize the violation penalty of upper bound of the number of assignments in a period for each department
(d) maximize the total assignment value

As a result of repeated interviews to university's registrar, the objectives are emphasized in the order of (a), (b), (c), and (d), while the objectives were simultaneously optimized as a weighted summation in our previous study [8].

The target problem, examination proctor assignment (EPA), is formulated as a multi-stage optimization, shown as follows. The common constraints (1) through (24) are omitted.

Problem EPA1:

$$Minimize \quad \varphi_1 = \sum_{m\in M} (s_m^- + s_m^+) \qquad (25)$$

Problem EPA2:

$$Minimize \quad \varphi_2 = \sum_{m\in M} \sum_{d\in Da} s_{m,d}^+ \qquad (26)$$

subject to

$$\sum_{m\in M} (s_m^- + s_m^+) = \varphi_1 \qquad (27)$$

Problem EPA3:

$$Minimize \quad \varphi_3 = \sum_{k\in De} \sum_{d\in Da} \sum_{p\in P} s_{k,d,p}^+ \qquad (28)$$

subject to

$$\sum_{m\in M} (s_m^- + s_m^+) = \varphi_1$$

$$\sum_{m\in M} \sum_{d\in Da} s_{m,d}^+ = \varphi_2 \qquad (29)$$

Problem EPA4:

$$Maximize \quad \varphi_4 = \sum_{m\in M} \sum_{e\in E} V_{m,e} x_{m,e} \qquad (30)$$

subject to

$$\sum_{m\in M} (s_m^- + s_m^+) = \varphi_1$$

TABLE I
VALUE OF PARAMETERS

Parameter	2018 fall	2019 spring
$\lvert M_s \rvert$	167	167
$\lvert M_f \rvert$	484 (219)	469 (218)
$\lvert E \rvert$	910	929
N^-_m for staffs	3	2
N^+_m for staffs	4	3
N^-_m for faculties	2	3
N^+_m for faculties	3	4
Nd^+_m for staffs	1	1
Nd^+_m for faculties	2	2
Nk^+_k	1	1

TABLE II
OPTIMIZATION RESULTS

	2018 fall		2019 spring	
Problem	Time (s)	φ	Time (s)	φ
EPA1	7.5	15	10.2	22
EPA2	8.4	4	11.4	4
EPA3	12.9	26	18.2	45
EPA4	15.2	66595	25.3	73415

$$\sum_{m\in M}\sum_{d\in Da} s^+_{m,d} = \varphi_2$$
$$\sum_{k\in De}\sum_{d\in Da}\sum_{p\in P} s^+_{k,d,p} = \varphi_3 \quad (31)$$

The optimal assignment is derived by solving Problem EPA1 through Problem EPA4 in sequence. Unlike the procedure in the previous study, the optimal values for the objectives are shown and they are useful information for university's registrar.

IV. NUMERICAL EXPERIMENTS

The optimization problem represented by the proposed multi-stage optimization model was implemented in a Microsoft Excel workbook in which VBA macros called a program written in Python to construct the target optimization problem. Th optimization problem itself was solved by Gurobi Optimizer [9], an optimization solver. This section describes the results of numerical experiments using two datasets of the final examination for the fall semester of 2018 and the spring semester of 2019.

Three types of information were entered into the system: examination schedules, examination information, and academic member information. There were 6 days of examinations for both datasets. There were 6 periods per day. The information for each examination contained the date, period, number of examinees, name of the chief proctors, assigned classroom, and the university department administering the examination. The information determined the value of some constants such as $Sc_{e,d,p}$, $Sc_{e,d}$, $C_{m,e}$, and $R_{e,t}$. The faculty member information contained the ID number, affiliation, name, employment type, gender, assignment value. The staff member information was the same as faculties' and positions are added. The information of employment type and position was utilized to compute the value of N^-_m and N^+_m. The schedule for academic members was also inputted into the system and fixed the value of $A_{m,e}$. The value Nd^+_m was computed based on the number of examinations and that of university personnel.

The values of some parameters in the two datasets are figured in Table I where the number of tenure faculties is shown in the parentheses. The assignment values V_m were determined by a system user manually from 0 to 250 based on the examination experiences of member m. The values of $V_{m,e}$ were obtained by V_m and additional preference on the condition if faculty m belonged to the department administering examination e.

The problems defined in the previous section were solved on a computer with Intel Core i7 CPU and 8 GB of memory running on Microsoft Windows 10. The optimal value and executional time in each stage are shown in Table II. The computational times increase in later stages. It is because more conditions are considered in later stages. The optimal values in stages are quite different which means that it is difficult to adjust the weights in a weighted summation of objective function when multiple objectives are simultaneously optimized. The optimal values in stages play an essential role to assess the resulting assignments for university's registrar.

The approach proposed in our previous study was also estimated, namely, a weighted summation of objective function. The objective function was set as follows:

$$\text{Minimize} \quad 10^6\varphi_1 + 10^7\varphi_1 + 10^5\varphi_3 + \varphi_4 \quad (32)$$

The coefficients of φ_i (i=1, 2, 3, 4) were fixed by the optimal values indicated in Table II. The solving time were respectively 18.9 and 21.0 seconds for two datasets. The values of φ_i were completely identical to those in Table II. The results revealed that an adequate setting of coefficients in (32) reduces total solving time and obtains the optimum in every stage in multi-stage model.

V. CONCLUSIONS

This study proposed a multi-stage optimization model for solving an examination proctor assignment problem and deriving a proctor assignment plan for our university. Multiple objectives were ordered based on the preferences of system users and the target proctor assignment problem represented as the multi-stage optimization model. The model derived the optimum values for each stage which provided the system user with useful information on the objectives.

The proposed approach was not as preferable as the weighted summation approach from the view of the solving time. Combining the two approaches are expected to be more practical in solving time as well as quality of resulting assignments. The proposed model can be extended to other types of assignment problem by generalizing the model such as staff scheduling which is more widely demanded.

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