Comparative Analysis of Deterministic and Stochastic Scheduling Model: A Review Article

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Abstract—**The basic assumptions of deterministic scheduling problems includes; job descriptors are fixed and known in advance, machines are continuously available (no breakdowns occur) among others. However, in some scenario, these two assumptions may be relaxed. Under this condition, stochastic scheduling exists. Stochastic scheduling models appear less efficient but more effective. Thus, there is an established distinction between the optimal values of the two models as well as the required computational time for the same performance measure. In this paper, comparative analysis of stochastic and deterministic models is carried out. An illustrative problem was considered for six different performance measures. The percentage errors in the computed values for each of the objectives were calculated.**

***Index Terms*— deterministic scheduling, stochastic scheduling, job descriptors, performance measures**

I. INTRODUCTION

SCHEDULING can be difficult from a technical as well as from an implementation point of view. The difficulties on the implementation side are of a completely different kind. They may depend on the accuracy of the model used for the analysis of the actual scheduling problem and on the reliability of the input data that are needed. (Pinedo, 2008).

Among the model type are the deterministic and stochastic scheduling models. In deterministic scheduling (or off-line scheduling model), all the problem data are known with certainty in advance. The solution of such a problem is a schedule – a set of start times for all the jobs. With respect to their practical implications, however, deterministic models have often been criticized. The reason is obvious: In many practical situations, uncertainty about the future may be inevitable. There are many real world problems in which parameters like the arrival time of new jobs, failure of resources, and completion time of jobs change continuously (Tapan, 2012). If the variations are significant, it is better to design solutions, which are robust to these changes (*Ghulam et al,* 2010). Rolf Möhring (2015) stated that uncertainty is imminent in practical scheduling problems and there are good tools available to analyze risks and implement policies. Different models have been proposed where this restrictive assumption is relaxed to a certain extent. Stochastic scheduling is one of these models. (Vorgelegt von, 2001).

The field of stochastic scheduling is motivated by the design and operational problems arising in systems where scarce resources must be allocated over time to jobs with random and varies features such as job processing time distributions (Jose (2005)). Some common examples of stochastic scheduling include, the case of a manufacturing workstation processing different part types, where part arrival and processing times are subject to random variability, an automobile servicing firm with varying standard time for the same jobs depending on the technician working experience among others.

II. LITERATURE REVIEW

Historically, stochastic scheduling analysis has focused on the same performance measures considered in deterministic scheduling; flowtime (F), tardiness (T), lateness, (L) among others and has sought to optimise their expected values. These models are called stochastic counterparts of the corresponding deterministic problems. For example, the stochastic counterpart of the total flowtime (Ftot) problem is a stochastic scheduling problem in which the objective function is the expected total flowtime E(Ftot). More generally, for deterministic models that seek to minimize the total cost or the maximum cost, stochastic counterparts seek to minimize the expected total cost or the expected maximum cost.

Balseiro et al (2017) studied the problem of non-preemptively scheduling a set of J jobs on a set of M unrelated machines when job processing times are stochastic. Each job j has a positive weight (wj) and must be processed non-preemptively by one machine. Machines operate in parallel and at any given time a machine can process at most one job. Processing times for a job depend both on the job as well as the machine that processes the job, and the processing time of a job is not fully known until

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a job is completed. The objective is to minimize the weighted sum of expected completion times E (WCt). Hoogeveen et al. (2001) study the deterministic version of this problem. Skutella et al. (2016) study static routing policies for stochastic scheduling on unrelated machines. The static routing policies considered are based on a novel time-indexed linear programming relaxation. Moreover, Skutella et al. (2016) derive strong constant factor approximation results for versions of the problem both with and without release dates.

However, Baker and Trietsch (2009) suggested that there could exists a significant difference in the results of a deterministic and stochastic counterpart problem.

The authors also showed that for total flowtime problem, the result of the two approaches could be equal. However, literature is sparse that evaluates the differences for some performance measure for the two scheduling problem approaches. This work addresses this problem.

III. PROBLEM DEFINITION

For a number of scheduling problems, there exists equivalency relationship between deterministic and stochastic counterpart. For such a problem, finding the optimal stochastic policy is equivalent to solving a deterministic scheduling problem. Usually, when such an equivalence relationship exists, the deterministic counterpart can be obtained by replacing all random variables with their means. The optimal schedule for the deterministic problem then optimizes the objective of the stochastic version in expectation. One such case is when the objective in the deterministic counterpart is linear. However, Portougal and Trietsch (2003) stated that the mean approach of approximating stochastic model to deterministic model have failed on many attempts. However, the author did not give any case study of such failure. The failure may due to the uncertainty or error associated with such approximation. The error is a function of various parameters including the performance measure of interest, the problem parameter distribution as well as the nature of the problem; is it static or dynamic. In static policy, the decision-maker decides at time, t = 0 (Pinedo 1981).

The objective of this work is to evaluate the error associated with the deterministic-stochastic equivalence for some performance measures under static policy.

Basic Notation

The following notations are employed in the illustrative example

D_j = Due time of job, J
P_J = Processing time of job,J
⟦E(P⟧_J) = Expected Processing time of job, J
E(Lmax) = Expected maximum lateness
E(Tmax) = Expected maximum tardiness
E(U) = Expected number of late jobs
E(F) = Expected total flowtime
E(T) = Expected total tardiness
$C_{\_max}$ = the makespan
F_max = the maximum flow time
PM = Performance Measure
OS = Optimal Solution
SAVE = Stochastic approach value in Expectation
DAVE = Deterministic Approach value in certainty
PD = Percentage Deviation
SPT = Shortest Processing Time
EDD = Early due date

IV. ILLUSTRATIVE EXAMPLE

Consider a problem containing, n = 5 jobs with stochastic processing times. The due date and expected processing time for each job are shown in the following table.

TABLE I
A 5 jobs with expected processing times.

Suppose that two factors influence these processing times, the weather and the quality of raw materials. Each factor has two equally likely conditions (Good and Bad), so together they define four states of nature: GG (when both conditions are Good), GB, BG, and BB. Each job has a different processing time under each state of nature as follows:

TABLE II
THE NEW PROCESSING TIME

State	Jobj	1	2	3	4	5
GG	Pj	2.6	3.5	3.8	3.2	6.4
GB	P_j	2.8	3.9	4.4	5.5	6.6
BG	P_j	3.2	4.1	5.6	6.5	7.4
BB	P_j	3.4	4.5	6.2	8.8	7.6

(Source: Baker and Trietsch (2013)

We are interested in computing, E(F), E(T), E(Lmax), E (Ltot), E(Tmax), E(U) as well as the deterministic counterpart. Then determine the error incurred in each performance measure.

Solution

Stochastic Approach: In this approach, the effect of the weather and the quality of raw materials on the processing time will be taken into consideration.
Table III shows the optimal solution against each of the performance measure

TABLE III
THE OPTIMAL SOLUTION AGAINST THE OBJECTIVES

P M	E(*F*)	E(*T*)	E(L_{max})	E(T_{max})	E(L_{tot})
O S	SPT	EDD	EDD	EDD	SPT

The performance measures can be classified into the

Jobj	1	2	3	4	5
E(Pj)	3	4	5	6	7
D(j)	8	5	15	20	12

Processing time based as well as the due date based.

For the due date based performance measures; (E(Lmax), E(Tmax), E(U)), we explore EDD to obtain the values of the objectives.
EDD Sequence = 2, 1, 5, 3, 4

For the processing time based performance measures; E(Ftot), E(Ltot), we explore SPT to obtain the values of the objectives.
SPT Sequence = 1, 2, 3, 4, 5
Forthe EDD sequence, Table IV shows the completion time.

TABLE IV
COMPLETION TIME TABLE FOR EDD SEQUENCE

State	Seq	2	1	5	3	4
GG	Pj	3.5	6.1	12.5	16.3	19.5
GB	P_j	3.9	6.7	13.3	17.7	23.2
BG	P_j	4.1	7.3	14.7	20.3	26.8
BB	P_j	4.5	7.9	15.5	21.7	30.5

The tardiness of job, i is defined as:

$$T_i = \max\ \{0,\ (C_i - d_i)\} \quad (1)$$

The maximum tardiness is defined as:

$$T_{max} = \max\ (T_1, T_2, \ldots, T_n) \quad (2)$$

The total tardiness

$$T_{tot} = \sum_{i=1}^{n} \{(\max\ \{0,\ (C_i - d_i)\}\} \quad (3)$$

The lateness of job i is defined as;

$$L_i = (C_i - d_i) \quad (4)$$

The maximum lateness is defined as:

$$L_{max} = \max(C_i - d_i) \quad (5)$$

The minimum lateness is defined as

$$L_{min} = \min(C_i - d_i) \quad (6)$$

From the completion time computed in Table IV, the tardiness as well as the lateness of each of the conditions is computed.
Table V and Table VI show the tardiness and the lateness, respectively.

TABLE V
TARDINESS TABLE

State	Seq	2	1	5	3	4	T_{tot}	T_{max}
GG	Pj	0	0	0.5	1.3	0.0	1.8	1.3
GB	P_j	0	0	1.3	2.7	3.2	7.2	3.2
BG	P_j	0	0	2.7	5.3	6.8	14.8	6.8
BB	P_j	0	0	3.5	6.7	10.5	20.7	10.5
		Mean value					11.13	5.45

TABLE VI
LATENESS TABLE

State	Seq	2	1	5	3	4	T_{tot}	T_{max}
GG	Pj	-1.5	1.9	0.5	1.3	-0.5	1.9	-1.5
GB	P_j	-1.1	-1.3	1.3	2.7	3.2	3.2	-1.3
BG	P_j	-0.9	-0.7	2.7	5.3	6.8	6.8	-0.9
BB	P_j	-0.5	-0.1	3.5	6.7	10.5	10.5	-0.5
		Mean value					5.6	-1.1

The total flowtime is defined as sum of the flowtime.

$$F_{tot} = F_1 + F_2 + F_3 + \ldots + F_n \quad (8)$$

The makespan is defined as the maximum completion time.

$$C_{max} = \max(C_1, C_2, C_3 \ldots, C_n) \quad (9)$$

The total flowtime (F_{tot}) as well as the completion time for each job were computed from Table VII.

TABLE VII
COMPLETION TIME TABLE FOR SPT SEQUENCE

State	Job	1	2	3	4	5	F_{tot}
GG	Pj	2.6	6.1	9.9	13.1	19.5	51.2
GB	P_j	2.8	6.7	11.1	16.6	23.2	60.4
BG	P_j	3.2	7.3	12.	19.	26.8	69.6

The lateness for each of the conditions computed from the Table VII is shown in Table VIII

Deterministic Counterpart: In the deterministic model, it would be assumed that the effect of the two factors influencing the processing times (the weather and the quality of raw materials) is negligible. In other words, the processing times are known with certainty. Therefore, the expected condition is eliminated. Table I thus become Table IX as shown.

TABLE VIII
LATENESS TABLE FOR THE SPT SEQUENCE

State	Job j	L1	L2	L3	L4	L5	L_{tot}
GG	Pj	-5.4	1.1	-5.1	-6.9	7.5	-8.8
GB	P_j	-5.2	1.7	-3.9	-3.4	10.8	0
BG	P_j	-4.8	2.3	-2.1	-0.6	13.2	8
BB	P_j	-4.6	2.9	-5.9	2.9	17.5	12.8
		Mean Total Lateness					3

TABLE IX
The deterministic counterpart variable

For the due date based performance measures Lmax, Tmax, U, we explore EDD to obtain the values of the objectives

EDD Sequence = 2, 1, 5, 3, 4

Table X shows the completion time table for the EDD sequence.

TABLE X
COMPLETION TIME TABLE FOR EDD SEQUENCE

Sequence	2	1	5	3	4
Pj	-1.5	1.9	0.5	1.3	-0.5
GB P_j	-1.1	-1.3	1.3	2.7	3.2
Blateness:(Cj-Dj)	-0.9	-0.7	2.7	5.3	6.8
Tardiness(max(0,Cj-Dj)	-0.5	-0.1	3.5	6.7	10.5

From the table X, it can be deduced that;

Lmax = 5 Lmin = -1
Ttot = 11 Tmax = 5

For the processing time based performance measures; E(Ftot), E(Ltot), we explore SPT to obtain the values of the objectives

SPT Sequence = 1, 2, 3, 4, 5

Table XI shows the completion time table for the SPT Sequence

TABLE XI
COMPLETION TIME TABLE FOR SPT SEQUENCE

Job (j)	1	2	3	4	5
(Cj)	3	7	12	18	25
D(j)	8	5	15	20	12
Lateness	-5	2	-3	-2	13

From the Table XI, it can be deduced that;
Ftot = 65, Ltot = 5, Lmax = 15

Job (j)	1	2	3	4	5
(Cj)	3	4	5	6	7
D(j)	8	5	15	20	12

The summary of the results for the two approaches as well as the percentage deviation of the deterministic over the stochastic is shown in Table XII.

V. CONCLUSION AND RECOMMENDATION

The stochastic approach evaluates the performance measure values in expectation while the deterministic approach do so incertainty. However, the difference in result of the two approaches was established in this paper using an illustrative example for some stated objectives. Nevertheless, further work will be carried out on large problem sizes; involving different problem ranges with large number of instances. The solution will be done using a robust computer programming language.

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