New Solution of Induced L_∞ Optimal Control

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***Abstract*-The main propose of this paper, is performing a new solution on the basis of Linear Matrix Inequality (LMI) for designing induced l_∞ optimal controllers. Induced l_∞ optimal control allows directly time-domain specifications into the controller synthesis procedure and furnishes a complete solution to the robust performance problem. The new technique, which is proposed as an algorithm, combines the original concept of peak-to-peak gain of designed system with optimal control theory and employs a free design parameter allowing for a flexible management of the tradeoff between robustness to disturbance signals and magnitude of the worst peak-to-peak gain of the designed system. For the convergence of this algorithm, a scope is found on the basis of the $H_\infty-$norm. If the length of this interval is small, we have a good estimate of the actual optimal peak-to-peak gain that is achievable by control.**

***Index Terms*— Induced l_∞ optimal control, $H_\infty-$norm, Linear Matrix Inequality.**

I. Introduction

Standard induced l_∞ optimal controller synthesis aim at minimizing the worst case peak-to-peak gain of system disturbed by unknown persistent signals bounded in magnitude. They apply to a large variety of control problems owing to their ability to deal efficiently with time-domain performance objectives. See [1], [2], [3] for examples of customary induced l_∞ oriented design techniques and [4] for a comprehensive summary of result and references. The L_1 optimal control problem was formulation by Vidyasagar [5]. The problem is to synthesize a controller that minimizes the worst case amplification from disturbance signals to regulated signals, where the signal size (norm) is taken to be the signals peak value. Using interpolation ideas, the discrete-time problem has been studied in [6,7,8] and some references therein. In [1], Diaz-Bobillo and Dahleh show that there is a sequence liner programs of increasing size, the solutions to which yield controllers of increasing McMillan degree whose performance converges to the optimal achievable L_1 cost. Linear matrix inequalities (LMIs) have emerged as a powerful formulation and design technique for a variety of liner control problems[9]. Since solving LMI`s is a convex optimization problem, such formulations offer a numerically tractable means of attacking problems that lack an analytical solution. In addition, efficient interior-point algorithms are now available to solve the generic LMI problems with a polynomial-time worst-case complexity. Consequently, induced l_∞ optimal control or l_1-optimal control problem to an LMI can be considered as a practical solution to this problem.

The rest of this paper is organized as follows: Section II gives the problem statement and motivation. The background material concerning induced l_∞ optimal control or l_1-optimal control synthesis is presented in section III. Section IV shows how the problem can be formulated as an LMI problem. Furthermore, the stages of achieving l_1-Optimal control algorithm are presented in section V. And finally, section VI draws conclusions and gives some suggestions for the future work.

The notation is fairly standard. The compact notation $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is used to denote the transfer function $G(s) = D + C(sI - A)^{-1}B$.

II. Problem Statement and Motivation

Consider a multi-input/multi-output (MIMO) liner time invariant (LTI) systems. This section gives a formal statement of the problem and defines the relevant notation. The LTI Plant is given by the state-space equations

$$P\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{12} w \end{cases} \tag{1}$$

Where $u \in R^{n_u}$ is the control input, u is a vector of exogenous inputs (such as reference signals, disturbance signal, Sensor noise), $y \in R^{n_y}$ is the measured output, z is a vector of output signals related to the performance of the control system. Let T denote the closed-loop transfer function from w to z for some dynamical output-feedback control law $u = Ky$. Our goal is to compute a dynamical output-feedback controller

$$K\begin{cases} \dot{\varepsilon} = A_K \varepsilon + B_K y \\ u = C_K \varepsilon + D_K y \end{cases} \tag{2}$$

Henceforth, all specifications and objectives are expressed in terms of the transfer function T_j, keeping in mind that T_j refers to any particular I/O channel in the closed loop mapping. Since our approach is state-space based, we first provide a state-space realization for T_j and introduce some useful

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shorthand notation. With the plant P and controller K defined as above, the closed loop system admits the realization

$$G_{cl}(s)=\begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl} & D_{cl} \end{bmatrix}=\begin{bmatrix} A+B_2D_KC_2 & B_2C_K & B_1+B_2D_KD_{21} \\ B_KC_2 & A_K & B_KD_{21} \\ C_1+D_{12}D_KC_2 & D_{21}C_K & D_{11}+D_{12}D_KD_{21} \end{bmatrix} \quad (3)$$

Where

$$\tilde{K}=\begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix},$$

$$A_{cl}=\bar{A}+\underline{B}\tilde{K}\underline{C}, \quad B_{cl}=\bar{B}+\underline{B}\tilde{K}\underline{D}_{21},$$
$$C_{cl}=\bar{C}+\underline{D}_{12}\tilde{K}\underline{C}, \quad D_{cl}=D_{11}+\underline{D}_{12}\tilde{K}\underline{D}_{21} \quad (4)$$

With

$$\bar{A}=\begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \bar{B}=\begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \underline{D}_{12}=\begin{bmatrix} 0 & D_{12} \end{bmatrix},$$
$$\underline{D}_{21}=\begin{bmatrix} 0 \\ D_{21} \end{bmatrix} \quad (5)$$

$$\underline{B}=\begin{bmatrix} 0 & B_2 \\ I & 0 \end{bmatrix}, \bar{C}=\begin{bmatrix} C_1 & 0 \end{bmatrix}, \underline{C}=\begin{bmatrix} 0 & I \\ C_2 & 0 \end{bmatrix} \quad (6)$$

III. L1 OPTIMIZATION

The main objective of robust control is to ensure good performance in the presence of uncertainty in the model, external disturbance and measurement noise. To solve this problem a technique based on the optimization of the L_1 norm has been proposed [10]. As with other robust control techniques, the design specifications are transformed to conditions on the input and output signals. To obtain a control system which fulfills these requirements, it is necessary to describe the magnitude of the input and output signals in the system relative to a certain norm (cost measure), and to then optimize the transfer function for the set of possible inputs.

In the case of L_1 analysis and design method, it concerns input and output signals that are magnitude bounded (that is, described using the *peak-to-peak norm*). Optimization in this case aims to minimize the peak-to-peak norm of the desired outputs when the inputs are magnitude bounded [11]. This minimization can be transformed into a constrained linear programming problem, which can be solved by linear matrix inequality (LMI) [9], [12]. Additional constraints can be added to the linear programming problem to allow for design specification that cannot be expressed as a peak-to-peak norm minimization problem [11]. The L_1 optimization method deals with input and output signals which have magnitude (that is, described using the peak-norm$\| \ \|_\infty$, which is equal to the maximum amplitude of the signal: $\|u\|_\infty = \max|u(t)|$).The L_1 design method minimizes the peak-to-peak norm of output signals can be described as the set of magnitude bounded signals.

Suppose, instead, that the input signal w_j is only bounded in amplitude. To bound the peak amplitude of z_j, we then need to consider the so- called peak-to-peak gain of T_j defined by

$$\|T_j\|_{peak} := \sup\{\|z_j(T)\| : x_{cl}(0)=0, T\geq 0, \quad \|w(t)\|\leq 1 \quad for \quad t\geq 0\}. \quad (7)$$

These measures the peak norms of the output signal $z_j(t)$ for inputs $w_j(t)$ whose amplitude does not exceed one. Note that $\|T_j\|_\infty \leq \|T_j\|_{peak}$ as is easily seen by considering a sinusoidal input whit frequency w such that $\sigma_{\max}(T_j(jw))=\|T_j\|_\infty$.

To date there is no exact characterization of the peak-to-peak norm in the LMI framework. However, it is possible to derive upper bounds for $\|T_j\|_{peak}$ along the lines of [13].

The controller (2) renders mat rice **A** stable and the bound

$$\|w_j\|_\infty \leq \gamma_j \|z_j\|_\infty \quad for \quad all \quad z_j \in L_\infty \quad (8)$$

Satisfied if there exist a symmetric X and real parameter λ, μ with [10]

$$\lambda \succ 0, \quad \begin{pmatrix} A_{cl}X+XA_{CL}+\lambda X & XB_{cl} \\ B_{cl}^TX & 0 \end{pmatrix}+\begin{pmatrix} 0 & I \\ C_{cl} & D_{cl} \end{pmatrix}^T\begin{pmatrix} -\mu I & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & I \\ C_{cl} & D_{cl} \end{pmatrix}\prec 0 \quad (9)$$

$$\begin{pmatrix} 0 & I \\ C_{cl} & D_{cl} \end{pmatrix}^T\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{\gamma_j}I \end{pmatrix}\begin{pmatrix} 0 & I \\ C_{cl} & D_{cl} \end{pmatrix}\prec\begin{pmatrix} \lambda X & 0 \\ 0 & (\gamma_j-\mu)I \end{pmatrix} \quad (10)$$

The inequalities are obviously equivalent to [10]

$$\lambda \succ 0, \quad \begin{pmatrix} A_{cl}X+XA_{CL}+\lambda X & XB_{cl} \\ B_{cl}^TX & -\mu I \end{pmatrix}\prec 0 \quad (11)$$

$$\begin{pmatrix} \lambda X & 0 & C_{cl}^T \\ 0 & (\gamma_j-\mu)I & D_{cl} \\ C_{cl} & D_{cl} & \gamma_j I \end{pmatrix}\succ 0 \quad (12)$$

If these inequalities are feasible, one can construct a stabilizing controller which bounds the peak-to-peak norm of $w_j = T_j z_j$ by γ_j. We would like to stress that the converse of this statement is not true since the analysis result involves conservatism. Note that the synthesis inequalities are formulated in terms of the variables $X, \lambda,$ and μ ; hence they are *non-linear* since λX depends quadratically on λ and X. This problem can be overcome as follows: for fixed $\lambda \succ 0$, test whether the result linear matrix inequalities are feasible; if yes, one can stop since the bound γ_j on the peak-to-peak norm has been assured; if the LMIs are infeasible, one has to pick another $\lambda \succ 0$ and repeat the test. In practice, it might be advantageous to find the best possible upper bound on the peak-to-peak norm that can be assured with the present analysis result. This would lead to the problem of minimizing γ_j under the synthesis inequality Constraint as follow: perform a line-search over $\lambda \succ 0$ to minimize $\gamma_j^*(\lambda)$, the minimal value of γ_j if $\lambda \succ 0$ is held fixed; note that calculation of $\gamma_j^*(\lambda)$ indeed amount to solving a genuine LMI problem. The line search leads to the best achievable upper bound

$$\gamma_j^u = \inf_{\lambda \succ 0} \gamma_j^*(\lambda) \tag{13}$$

To estimate the conservatism, let us recall that $\|T_j\|_\infty$ is a lower bound on the peak-to-peak norm of T_j. If we calculate the minimal achievable $H_\infty -$norm, say γ_j^l, of T_j, we know that the actual optimal peak-to-peak gain must be contained in the interval $[\gamma_j^l \quad \gamma_j^u]$. If the length of this interval is small, we have a good estimate of the actual optimal peak-to-peak gain that is achievable by control, and if the interval is large, this estimate is poor [10].

IV. LMI Formulation of L_1 Optimization

Suppose that the system with the equations (1) is controllable and the controller forms the *K* output feedback which forms the closed loop G_{cl}. This system Renders A stable and bound $\|w_j\|_\infty \le \gamma_j \|z_j\|_\infty$ if inequalities (11) and (12) exist. The following theorem plays an important role in the subsequent sections.

Theorem 1- Consider matrices *P* and *Q* as well as the symmetrical matrix *H.*

Matrices N_Q and N_P have got complete ranks in a way that

$$\operatorname{Im} N_P = KerP, \qquad \operatorname{Im} N_Q = KerQ \tag{14}$$

Now matrix *J* exists in a way that

$$H + P^T J^T Q + Q^T JP \prec 0 \tag{15}$$

if and only if

$$N_P^T H N_P \prec 0, \qquad N_Q^T H N_Q \prec 0 \tag{16}$$

Proof- Refer to reference [14].

Notice that [13] solved H_2 and H_∞ based on LMI but nonlinear equalities (11) and (12), only were mentioned and any method for solving these equations have not presented [10]. The drawback of this approach is that we need the new parameters for controller designing. Therefore, the designing of controller was complicated. In this paper, for removing drawback, we use qualified transformations for linearization in order to inequities modify for LMI toolbox.

For solving (11), we must be replaced matrices C_{cl}, B_{cl}, A_{cl}, and D_{cl} from relation (5), Hence, inequality (11) can be rewritten as it follows,

$$\begin{bmatrix} (\bar{A}+\underline{B}\tilde{K}\underline{C})^T X_{cl} + X_{cl}(\bar{A}+\underline{B}\tilde{K}\underline{C}) + \lambda X_{cl} & X_{cl}(\bar{B}+\underline{B}\tilde{K}\underline{D}_{21}) \\ (\bar{B}+\underline{B}\tilde{K}\underline{D}_{21})^T X_{cl} & -\mu I \end{bmatrix} \prec 0 \tag{17}$$

As it is seen, this inequality is a linear function in comparison to each of $\tilde{K}$ and X_{cl} variables alone. But in comparison with the two variables together, it is not a linear function. In the following, it is tried to solve this problem.

By defining matrices $P_{X_{cl}}$, Q and $H_{X_{cl}}$ in the following way,

$$P_{X_{cl}} = [\underline{B}^T X_{cl} \quad 0] \tag{18}$$

$$Q = [\underline{C} \quad \underline{D}_{21}] \tag{19}$$

$$H_{X_{cl}} = \begin{bmatrix} \bar{A}X_{cl} + X_{cl}\bar{A} + \lambda X_{cl} & X_{cl}\bar{B} \\ \bar{B}^T X_{cl} & -\mu I \end{bmatrix} \tag{20}$$

Inequality (14) is rewritten in the following way

$$H_{X_{cl}} + Q^T \tilde{K}^T P_{X_{cl}} + P_{X_{cl}} \tilde{K} Q \prec 0 \tag{21}$$

According to theorem 1, this inequality is equal to the following two inequalities

$$N_{P_{X_{cl}}}^T H_{X_{cl}} N_{P_{X_{cl}}} \prec 0, \qquad N_Q^T H_{X_{cl}} N_Q \prec 0 \tag{22}$$

These inequalities exist in matrices of the state space of open loop $G(s)$ and variable X_{cl} but since X_{cl} appear both in $H_{X_{cl}}$ and $N_{P_{X_{cl}}}$, the inequality on the left is not a linear matrix from X_{cl}.

To solve this problem, matrices $T_{X_{cl}}$ and P are defined as it follows

$$T_{X_{cl}} = \begin{bmatrix} \bar{A}X_{cl}^{-1} + X_{cl}^{-1}\bar{A}^T + \lambda X_{cl}^{-1} & B \\ \bar{B}^T & -\mu I \end{bmatrix} \tag{23}$$

$$P = \lfloor \underline{B}^T \quad 0 \rfloor \tag{24}$$

Theorem 2- for $X_{cl} \succ 0$, inequality $N^T_{P_{X_{cl}}} H_{X_{cl}} N_{P_{X_{cl}}} \prec 0$ is equal to:

$$N_P^T T_{X_{cl}} N_P \prec 0 \tag{25}$$

Proof: matrices P and $P_{X_{cl}}$ Are linked together in the following way.

$$P_{X_{cl}} = PS \tag{26}$$

In which

$$S = \begin{bmatrix} X_{cl} & 0 \\ 0 & I \end{bmatrix} \tag{27}$$

So it could be written that

$$N_{P_{X_{cl}}} = S^{-1} N_P \tag{28}$$

By replacing $N_{P_{X_{cl}}}$ from the above relation in inequality

$N^T_{P_{X_{cl}}} H_{X_{cl}} N_{P_{X_{cl}}} \prec 0$ we have

$$N^T_P (S^{-1}) H_{X_{cl}} S^{-1} N_P \prec 0 \tag{29}$$

With regard to the definition of $H_{X_{cl}}$ and replacement in the above mentioned relation, we can have

$$N^T_P (S^{-1}) \begin{bmatrix} \overline{A} X_{cl} + X_{cl} \overline{A} + \lambda X_{cl} & X_{cl} \overline{B} \\ \overline{B}^T X_{cl} & -\mu I \end{bmatrix} S^{-1} N_P \prec 0 \tag{30}$$

By replacing S^{-1} in the above relation, we can have

$$S^{-1} = \begin{bmatrix} X_{cl}^{-1} & 0 \\ 0 & I \end{bmatrix} \tag{31}$$

$$N^T_P \begin{bmatrix} \overline{A} X_{cl} + X_{cl} \overline{A} + \lambda X_{cl} & X_{cl} \overline{B} \\ \overline{B}^T X_{cl} & -\mu I \end{bmatrix} N_P \prec 0 \tag{32}$$

And with regard to the definition of $T_{X_{cl}}$, this inequality is equal to inequality $N_P^T T_{X_{cl}} N_P \prec 0$, and proving this theorem is accomplished in this way.

Now with regard to relation (19) and theorem 2 the necessary and sufficient condition for the L_1-Optimal controller is expressed as follows.

$$N_P^T T_{X_{cl}} N_P \prec 0, \qquad N_Q^T H_{X_{cl}} N_Q \prec 0 \tag{33}$$

The inequality on the left is a linear matrix from $X_{cl}{}^{-1}$ and the inequality on the right is a linear matrix inequality from X_{cl}.Therefore these two inequalities are not a linear matrix inequality from X_{cl}.

To solve this problem it is hypothesized that matrices X_{cl} and X_{cl}^{-1} are of the structure below.

$$X_{cl} = \begin{bmatrix} X & X_2 \\ X_2^T & X_3 \end{bmatrix}, \qquad X_{cl}^{-1} = \begin{bmatrix} Y & Y_2 \\ Y_2^T & Y_3 \end{bmatrix} \tag{34}$$

And sub matrices $X$and Y, n as the state of open loop system $G(s)$ and n_K as the state of controller $K(s)$.

The following theorem demonstrates how it is possible to express the inequalities in relation (30) in the form of linear matrix inequalities on the basis of $X$and Y.

Theorem3- inequalities

$$N_P^T T_{X_{cl}} N_P \prec 0, \qquad N_Q^T H_{X_{cl}} N_Q \prec 0 \tag{35}$$

Exists if and only if The linear matrix inequalities

$$\begin{bmatrix} N_C & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} AY + YA^T + \lambda Y & B_1 \\ B_1^T & -\mu I \end{bmatrix} \begin{bmatrix} N_C & 0 \\ 0 & I \end{bmatrix} \prec 0 \tag{36}$$

$$[N_O \quad 0]^T \begin{bmatrix} AX + XA^T + \lambda X & XB_1 \\ B_1^T X & -\mu I \end{bmatrix} [N_O \quad 0] \prec 0 \tag{37}$$

Exist. In (33) and (34), N_C and N_O are matrices of full rank in way that

$$\operatorname{Im} N_C = Ker[B_2^T \quad D_{12}^T], \qquad \operatorname{Im} N_O = Ker[C_2 \quad D_{21}] \tag{38}$$

Proof- First it is demonstrated that inequality $N_P^T T_{X_{cl}} N_P \prec 0$ is equal to the linear matrix inequality in relation (33). By replacing the matrices $\overline{A}$, $\overline{B}$ and $\overline{C}$ from (5) in (20), matrix $T_{X_{cl}}$ is achieved as in the following.

$$T_{X_{cl}} = \begin{bmatrix} AY + YA^T + \lambda Y & AY_2 + \lambda Y_2 & B_1 \\ Y_2^T & \lambda Y_3 & 0 \\ B_1^T & 0 & -\mu I \end{bmatrix} \tag{39}$$

Also by replacing matrix $\underline{B}$ from 5 in relation 21, matrix P is achieved as it follows

$$P = \underline{B}^T = \begin{bmatrix} 0 & I & 0 \\ B_2^T & 0 & 0 \end{bmatrix} \tag{40}$$

Thus, matrix N_P has got a structure like this.

$$N_P = \begin{bmatrix} V_1 & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} \tag{41}$$

In which V_1 is a vector from the empty space of B_2^T. Since the second row of matrix N_P equals zero, the second row and column of matrix $T_{X_{cl}}$ have no effect on condition $N_P^T T_{X_{cl}} N_P \prec 0$, and these two rows and columns could be omitted. Therefore, by selecting $N_C = V_1$, this inequality is rewritten as it follows.

$$\begin{bmatrix} N_C & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} AY+YA^T+\lambda Y & B_1 \\ B_1^T & -\mu I \end{bmatrix} \begin{bmatrix} N_C & 0 \\ 0 & I \end{bmatrix} \prec 0 \quad (42)$$

This is the very inequality on the left in relation (33).
By applying the same method, it could be demonstrated that inequality $N_Q^T H_{X_{cl}} N_Q \prec 0$ is also equal to linear matrix inequality $\begin{bmatrix} N_O & 0 \end{bmatrix}^T \begin{bmatrix} AX+XA^T+\lambda X & XB_1 \\ B_1^T X & -\mu I \end{bmatrix} \begin{bmatrix} N_O & 0 \end{bmatrix} \prec 0$ and in this way it is concluded that this theorem is proved.
So far it has been demonstrated that the necessary and sufficient condition for the existence of an L_1-Optimal Controller is that sub matrices X and Y from matrices X_{cl} and X_{cl}^{-1} pave the ground for the condition existing in theorem 3.
The following theorem expresses under what circumstances matrix X_{cl} is achieved by having sub matrices X and Y.

Theorem 4- Suppose matrices $X, Y \in R^{n\times n}$ are symmetrical and appointed definite positive. Then matrices $X_2, Y_2 \in R^{n\times n_k}$ and symmetrical matrices $X_3, Y_3 \in R^{n_k \times n_k}$ which form the relation (39) exist if and only if

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \quad (43)$$

$$rank \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \leq n + n_K \quad (44)$$

Proof- Refer to reference (14).

This theorem expresses the conditions of forming matrix X_{cl} from sub matrices X and Y. One of these conditions is in the form of a linear matrix inequality and the other in the form of an inequality condition on the rank of a matrix.
Although the second condition is not linear,

$$rank \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \leq 2n \quad (45)$$

Because of relation (42) this condition is automatically omitted and only condition (40) remains.
Finally, by adding this condition to the conditions of theorem 3 expressing the problem of designing L_1-Optimal controller in the form of linear matrix inequality are accomplished.

V. NEW SOLUTION OF L1 OPTIMAL CONTROL ALGORITHM

We summarized above solution for L_1-Optimal controller with rank $n_K \geq n$ as follows:

Step 1. By considering a fixed amount for real parameter $\lambda \succ 0$, sub matrices X and Y are achieved through simultaneous solving of inequalities (33), (34) and (40).

Step 2. Calculating $X_2 \in R^{n\times n_k}$ from relation

$$X - Y^{-1} = X_2 X_2^T$$

Step 3. Provided that stages 1 and 2 are solved, condition $\|w_j\| \leq \gamma_j \|z_j\|$ exists, but in order for the controller to be optimized by using repletion algorithm on parameter λ, the minimum amount of $\gamma_j^*(\lambda)$ is achieved. If stages 1 and 2 are not possible to be solved, the algorithm is repeated by choosing a new amount for λ.

Step 4. Calculating X_{CL} by the use of relation

$$X_{CL} = \begin{bmatrix} X & X_2^T \\ X_2 & I \end{bmatrix}$$

Step 5. Replacing X_{CL} in $H_{X_{CL}}$ and $P_{X_{CL}}$, and achieving controller $\tilde{K}$ by solving the inequalities below.

$$H_{X_{cl}} + Q^T \tilde{K}^T P_{X_{cl}} + P_{X_{cl}} \tilde{K} Q \prec 0 \quad (46)$$

$$\begin{bmatrix} \lambda X_{CL} & 0 & C_{CL}^T \\ 0 & (1-\mu) & D_{CL}^T \\ C_{CL} & D_{CL} & \gamma_j \end{bmatrix} \succ 0 \quad (47)$$

VI. CONCLUSION

In this paper a new l_1 − optimal control technique on the basis of linear matrix inequality was introduced. Taking advantage of the fact that l_1 − optimal design problem can be restated as LMI problems, a new approach is developed in this paper that combines the original concept of peak-to-peak gain of designed system with optimal control theory. The new methodology employs a free design parameter allowing for a flexible management of the tradeoff between robustness to disturbance signals and magnitude of the worst peak-to-peak gain of the designed system. This nonlinear problem consists of 3 variables, and to solve it a linear construction has been used. This algorithm is on the basis of considering a fixed amount for two variables and finding the optimal amount for the next variable, as well as trial and error. For the convergence of this algorithm, a scope is found on the basis of H_∞ − norm. If the length of this interval is small, we have a good estimate of the actual optimal peak-to-peak gain that is achievable by control, and if the interval is large, this estimate is poor. In the forthcoming paper that will be presented by the authors, a low-order l_1 − optimal controller design will be proposed.

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