Robust Model Following Load Frequency Sliding Mode Controller Based on UDE and Error Improvement with Higher Order Filter

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Abstract—**A robust control strategy for design of robust model following sliding mode load frequency controller for single area power system based on uncertainty and disturbance estimator (UDE) has been presented where simple first order lowpass filter is used. The control strategy for elimination of reaching phase uses Ackermann's formula and UDE for the estimation of uncertainty and disturbances. The control strategy proposed does not require the knowledge of bounds of uncertainty and disturbances and is continuous. The simulation results of the control strategy for load frequency control is presented with uncertainties of 40% in plant parameters from their nominal values. The error with simple first order low pass filter has been improved with the help of higher order filter. The error in estimation with first order filter is proportional to filter time constant $O(\tau)$. This error can be improved by using higher order filter. We have proposed a UDE with second order low pass filter to improve this error. The analysis shows that, in first order filter error varies with $O(\tau)$, which further can be improved to the $O(\tau^2)$ by using second order filter estimation. As the τ is very small the error can be reduced sufficiently small with the help of this filter.**

Keywords: load frequency controller, uncertainty and disturbance estimator, sliding mode control

1 Introduction

Electric power systems consist of a number of control areas, which generate power to meet the power demand. However, poor balancing between generated power and demand can cause the system frequency to deviate away from the nominal value and create inadvertent power exchanges between control areas. To avoid such situation, load frequency controllers are designed and implemented to automatically balance generated power and demand in each control area [18][19]. In power systems, one of the most important issues is the load frequency control (LFC), which deals with the problem of how to deliver the demanded power of the desired frequency with minimum transient oscillations [11]. Whenever any small load perturbations resulted from the demands of customers occur in any areas of the power system, the changes of hiline power exchanges and the frequency deviations will occur. Thus, to improve the stability and performance of the power system, it is necessary that generator frequency should be setup under different loading conditions. For this reason, many control approaches have been developed for the load frequency control. Among them, PID controllers [10], optimal controllers [12], nonlinear controllers [13] and robust control strategies [5], neural and/or fuzzy [15][16][17] etc. approaches have been proposed in the past. In an industrial plant, such as a power system, one of the problems always encountered is the parametric uncertainties. The usual design approach for load frequency controller employs the linear control theory to develop control law on the basis of the linearized model with fixed system parameters. As the operating point of a power system and its parameter changes continuously, a fixed controller may no longer be suitable in all operating conditions. In order to take this parametric uncertainties into account, several papers have been published using the concept of variable structure system [1], various adaptive control techniques [4] to the design of load frequency control. The VSMFC technique presented in [6] fails to provide robustness if the parameter variations are more than 50%. This limitations has been overcome in [14], but still the accuracy of estimation error was more due to use of first order filter. In this paper the design of robust model following sliding mode load frequency controller for single area power system based on uncertainty and disturbance estimator (UDE) [7][8] with second order filter for error improvement has been presented. In the method proposed in this paper, we have used Ackermann's formula [9] for reaching phase elimination while uncertainty and disturbance estimation (UDE) method [7] for estimation of uncertainty and disturbances. The control proposed does not require the knowledge of bounds of uncertainty and disturbance and is continuous. The simulation results of the control strategy for load frequency control is presented by changing all parameters by up to 40% which can be ex-

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tended to 98% [14] from their nominal values where some other existing method fails. The results shows that system performance is robust to parameter variations and disturbance. Initially we used first order filter for the estimation where the error is varying with $O(\tau)$ where τ is filter time constant which is very small. This result has been extended using second order filter for estimation of error which is $O(\tau^2)$ without causing any disturbance. As 'τ' is very small, error also becomes very small. This result can be generalized for n^{th} order filter, where it can be easily proved that error is of the order $O(\tau^n)$.
This paper is organized as follows: Section II describes dynamic model for load frequency control, while Section III explains the concept of model following and UDE based sliding mode control with first and second order filter. The model following and UDE based control law for load frequency controller is proposed in Section IV. Simulation results and discussions are presented in Section V which is followed by conclusion in Section VI.

2 Dynamic Model For Load Frequency Control

Electrical power systems are complex, nonlinear and dynamic. The usual practice is to linearize the model around the operating point and then develop the control laws. Since the system is exposed to small changes in loads during its normal operation, the linearized model will be sufficient to represent the power system dynamics. The dynamic model in state variable form can be obtained from the transfer function model. The state equations can be written as [2, 3],

$$\Delta\dot{f} = \frac{-1}{T_p}\Delta f + \frac{K_p}{T_p}\Delta P_g - \frac{K_p}{T_p}\Delta P_d \tag{1}$$

$$\Delta\dot{P}_g = \frac{-1}{T_t}\Delta P_g + \frac{1}{T_t}\Delta X_g \tag{2}$$

$$\Delta\dot{X}_g = \frac{-1}{RT_g}\Delta f - \frac{1}{T_g}\Delta X_g - \frac{1}{T_g}\Delta P_c + \frac{1}{T_g}\int \Delta f dt \tag{3}$$

We introduce an integral control of Δf

$$\Delta E = K\int \Delta f dt$$

to ensure the regulation property of Δf i.e.

$$\Delta\dot{E} = K\Delta f \tag{4}$$

where K is the integral control gain. The different symbols used in (1-4) are:
The dynamic model in state variable form can be written as:

$$\dot{x} = Ax + bu + F\Delta P_d \tag{5}$$

where,

$$A = \begin{bmatrix} \frac{-1}{T_p} & \frac{K_p}{T_p} & 0 & 0 \\ 0 & \frac{-1}{T_t} & \frac{1}{T_t} & 0 \\ \frac{-1}{RT_g} & 0 & \frac{-1}{T_g} & \frac{-1}{T_g} \\ K & 0 & 0 & 0 \end{bmatrix}; b = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_g} \\ 0 \end{bmatrix}; \tag{6}$$

and the input $u = \int \Delta f dt$.

$x_1 = \Delta f$	Incremental frequency deviation in Hz
$x_2 = \Delta P_g$	Incremental change in generator output power in p.u. MW
$x_3 = \Delta X_g$	Incremental change in governor valve position in p.u. MW
$x_4 = \Delta E$	Incremental change in phase angle of voltage in radians
ΔP_d	Load disturbance in p.u.MW
ΔP_c	Incremental change in speed changer position in p.u.MW
T_g	Governor time constant in seconds
T_t	Turbine time constant in seconds
T_p	Plant time constant in seconds
K_p	Plant gain
R	Speed regulation ratio in Hz p.u. MW^{-1}

3 Model Following and UDE based Control Law

3.1 Problem statement

Consider a LTI single input single output (SISO) system [7][8] defined by

$$\dot{x} = Ax + bu + \Delta Ax + \Delta bu + d(x,t) \tag{7}$$

where x is the state vector, u is the control input, A and b are the known constant matrices, $\Delta A, \Delta b$ are uncertainties in the system and $d(x,t)$ is the unknown disturbance.

Assumption 1 *: The uncertainties ΔA and Δb and disturbance $d(x,t)$ satisfy the matching conditions given by*

$$\Delta A = bD, \quad \Delta b = bE, \quad d(x,t) = bv(x,t) \tag{8}$$

where D and E are unknown matrices of appropriate dimensions and $v(x,t)$ is an unknown function.

The system of equation (7) can be written as

$$\dot{x} = Ax + bu + be(x,t) \tag{9}$$

where $e(x,t) = Dx + Eu + v(x,t)$. The term $e(x,t)$ although contains uncertainty and disturbance will be referred as lumped uncertainty.
Let

$$\dot{x}_m = A_m x_m + b_m u_m \tag{10}$$

Assumption 2 *The choice of a model is such that*

$$A - A_m = bL, \quad b_m = bM \tag{11}$$

where L and M are suitable known matrices.

The objective is to design a control u so as to force the plant (9) to follow the model (10) inspite of the parameter variations. The equation (8) and (11) are well known matching conditions required to guarantee invariance and are explicit statements of the structural constraints stated in [7].

3.2 Design of control

In this section a model following control is designed with help of method suggested in [9][8]. Define a sliding surface

$$\sigma = b^T x + z \tag{12}$$

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where

$$\dot{z} = -b^T A_m x - b^T b_m u_m \quad z(0) = -b^T x(0) \tag{13}$$

Equation (13) for the auxiliary variable z defined here is different from that given in [9]. By virtue of the choice of the initial condition on z, $\sigma = 0$ at $t = 0$. If a control u can be designed ensuring sliding then $\dot{\sigma} = 0$ implies

$$\dot{x} = A_m x + b_m u_m \tag{14}$$

and hence fulfills the objective of the model following. Differentiating equation (12) and using (9) and (13) gives

$$\dot{\sigma} = b^T A x + b^T b u + b^T b e(x,t) \tag{15}$$

$$- b^T A_m x - b^T b_m u_m \tag{16}$$

$$= b^T b L x - b^T b M u_m + b^T b u + b^T b e(x,t) \tag{17}$$

Let the required control be expressed as

$$u = u_n + u_{eq} \tag{18}$$

Selecting

$$u_{eq} = -Lx + M u_m - (b^T b)^{-1} k \sigma \tag{19}$$

where k is a positive constant. From (15) and (19) we get

$$\dot{\sigma} = b^T b u_n + b^T b e(x,t) - k\sigma \tag{20}$$

Now we will design the component u_n. The lumped uncertainty $e(x,t)$ can be estimated as given in [7]. Rewriting above equation

$$e(x,t) = (b^T b)^{-1}(\dot{\sigma} + k\sigma) - u_n \tag{21}$$

It can be seen that lumped uncertainty $e(x,t)$ can be computed from (21). This cannot be done directly. Let the estimate of the uncertainty be defined as

$$\hat{e}(x,t) = [(b^T b)^{-1}(\dot{\sigma} + k\sigma) - u_n] G_f(s) \tag{22}$$

where $G_f(s)$ strictly proper first order low pass filter with unity gain steady-state gain and have enough bandwidth. With such filter

$$\hat{e}(x,t) \cong e(x,t) \tag{23}$$

Error in the estimation is

$$\widetilde{e}(x,t) = e(x,t) - \hat{e}(x,t) \tag{24}$$

3.2.1 UDE with first order filter

If $G_f(s)$ is proper first order low pass filter with unity gain defined as

$$G_f(s) = \frac{1}{\tau s + 1} \tag{25}$$

where τ is small positive constant. With the above $G_f(s)$ and in view of (21), (22) and (24)

$$\begin{aligned} \widetilde{e}(x,t) &= (1 - G_f(s))[(b^T b)^{-1}(\dot{\sigma} + k\sigma) - u_n] \\ &= \tau \dot{e}(x,t) G_f(s) \end{aligned} \tag{26}$$

The error in estimation varies with τ, enabling design of u_n as

$$\begin{aligned} u_n &= -\hat{e}(x,t) \\ &= -(b^T b)^{-1}(\dot{\sigma} + k) G_f(s) + G_f(s) u_n(s) \end{aligned} \tag{27}$$

Solving for u_n gives

$$u_n = \frac{(b^T b)^{-1}}{\tau}\Big(1 + \frac{k}{s}\Big)\sigma \tag{28}$$

3.2.2 UDE with second order filter

The accuracy of estimation can be improved as much as desired by an appropriate choice of filter $G_f(s)$. The second order filter is used here with transfer function

$$G_f(s) = \frac{1}{\tau^2 s^2 + 2\tau s + 1} \tag{29}$$

The lumped uncertainties and disturbances can be written as

$$e(x,t) = e(x,t)G_f(s) + e(x,t)(1 - G_f(s)) \tag{30}$$

$$= e(x,t)G_f(s) + e(x,t)(2\tau s + \tau^2 s^2)G_f(s) \tag{31}$$

$$= e(x,t)(1 + 2\tau s)G_f(s) \tag{32}$$

$$+ \tau^2 G_f(s)\ddot{e}(x,t) \tag{33}$$

Now the estimation is

$$\hat{e}(x,t) = e(x,t)(1 + 2\tau s)G_f(s) \tag{34}$$

$$= (1 + 2\tau s)G_f(s) \tag{35}$$

$$((b^T b)^{-1}(\dot{\sigma} + k\sigma) - u_n) \tag{36}$$

$$= (1 + 2\tau s)G_f(s) \tag{37}$$

$$((b^T b)^{-1}(\dot{\sigma} + k\sigma) - u_n) \tag{38}$$

From (30) and (34)

$$\widetilde{e} = \tau^2 G_f(s)\ddot{e}(x,t)$$

which proves error of the estimation is proportional to τ^2 so control u_n

$$u_n = -(1 + 2\tau s)G_f(s)[(b^T b)^{-1}(\dot{\sigma} + k\sigma) - u_n] \tag{39}$$

after simplifying

$$\begin{aligned} u_n(1 - (1 + 2\tau s)G_f(s)) = -(1 + 2\tau s)G_f(s) \\ [(B^T B)^{-1}(\dot{\sigma} + k\sigma)] \end{aligned} \tag{40}$$

put the value of $G_f(s)$ in preceding equation from (29) and simplify

$$u_n = (b^T b)^{-1}\Big[\frac{2}{\tau} + \frac{(2\tau k + 1)}{\tau^2 s} + \frac{k}{\tau^2 s^2}\Big]\sigma \tag{41}$$

4 Model Following and UDE based Load Frequency Controller

The plant considered is as equation (6) with the parameter values as[1, 3],

$$T_p = 20 \text{ sec}, \; T_t = 0.3 \text{ sec}, \; T_g = 0.08 \text{ sec},$$
$$K_p = 120 \text{ Hz p.u. MW}^{-1}, K = 0.6 \text{ p.u. rad}^{-1},$$
$$R = 2.4 \text{ Hz p.u. MW}^{-1}$$

The state space model is given by,

$$A = \begin{bmatrix} -0.05 & 6 & 0 & 0 \\ 0 & -3.333 & 3.333 & 0 \\ -5.208 & 0 & -12.5 & -12.5 \\ 0.6 & 0 & 0 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 12.5 \\ 0 \end{bmatrix};$$

Disturbance $F\Delta P_d = 10sin(10t)$. The reference input u_m is a square wave of unity amplitude. In order to satisfy the model following conditions we will convert above system into phase variable form by using transformation,

$$Z = Tx$$

Then the plant in equation (6) becomes,

$$\dot{Z} = TAT^{-1}Z + Tbu + TF\Delta P_d \tag{42}$$

where,

$$TAT^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -149.985 & -106.2327 & -42.4545 & -15.833 \end{bmatrix}$$

$$Tb = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The model selected was critically damped model such that,

$$\dot{x}_m = A_m x_m + B_m u_m \tag{43}$$

where

$$A_m = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -24 & -50 & -35 & -10 \end{bmatrix}; \quad B_m = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 24 \end{bmatrix}.$$

The initial conditions for plant and model states are given by $x(0) = [1 \;\; 0 \;\; 0 \;\; 0]^T$, $x_m(0) = [0 \;\; 1 \;\; 1 \;\; 1]^T$. Uncertainties in A and b are

$$\Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -43.5 & -55 & -17 & -29 \end{bmatrix}; \quad \Delta b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.4 \end{bmatrix}.$$

5 Simulation Results

The simulation results are shown in Fig.1-Fig.7. System response by using UDE with first and second order filter are shown in Fig.1 and Fig.2. The Fig.1(a)-(d) shows the plant states, while Fig.1(e) indicate control torque required. Fig.1(f) shows error plot, after using first order filter. Similar results using second order filter are shown in Fig.2(a)-(f). Both results are for 40% parameter uncertainties. The figure reveals the ability of the controller to drive the system to follow the reference model. It is observed that the system remains invariant to the imposed parameter variation. This reveals the controller ability to force the plant to follow the model in spite of parameter variations. Fig.3-Fig.7 shows comparison of error plot after using first and second order filter estimation when τ changes differently. Table 1 shows the numerical variation of error for different τ.

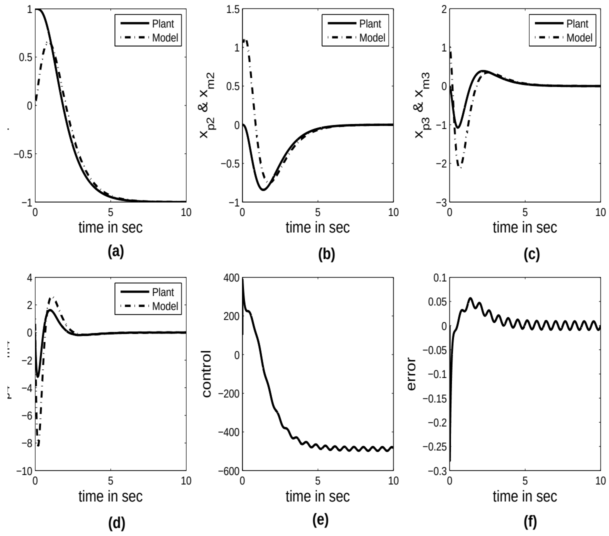

Figure 1: Load frequency response by using UDE with first order filter (a) plant and model state x_1 (b) plant and model state x_2 (c) plant and model state x_3 (d) plant and model state x_4(e) control (f) error

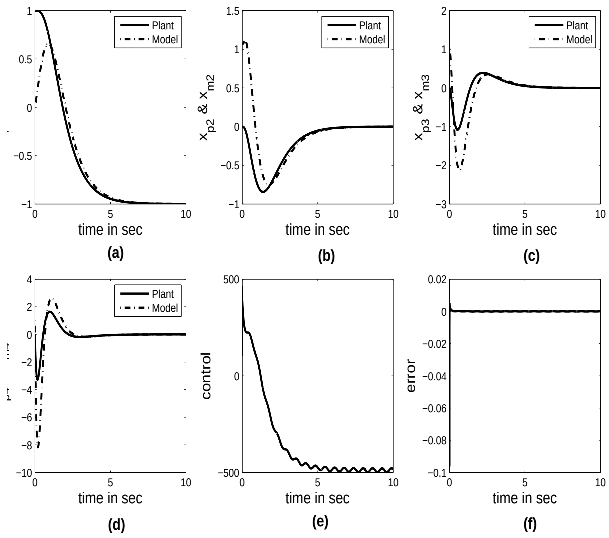

Figure 2: Load frequency response by using UDE with second order filter (a) plant and model state x_1 (b) plant and model state x_2 (c) plant and model state x_3 (d) plant and model state x_4(e) control (f) error

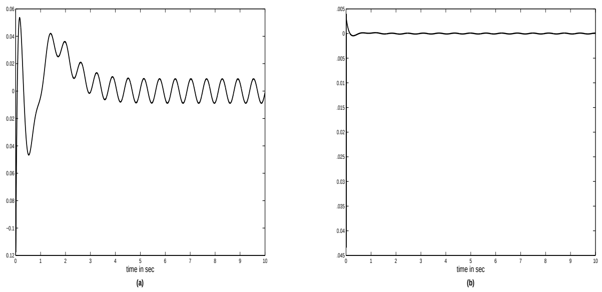

Figure 3: Error when $\tau = 1$ ms for (a) first order filter (b) second order filter

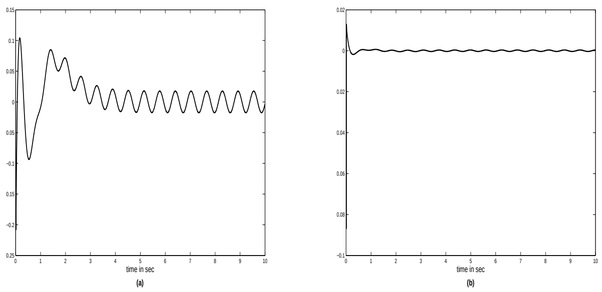

Figure 4: Error when $\tau = 2$ ms for (a) first order filter (b) second order filter

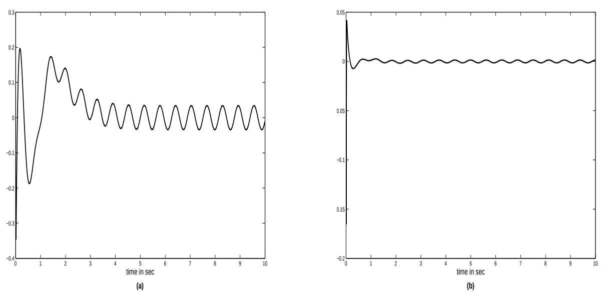

Figure 5: Error when $\tau = 4$ ms for (a) first order filter (b) second order filter

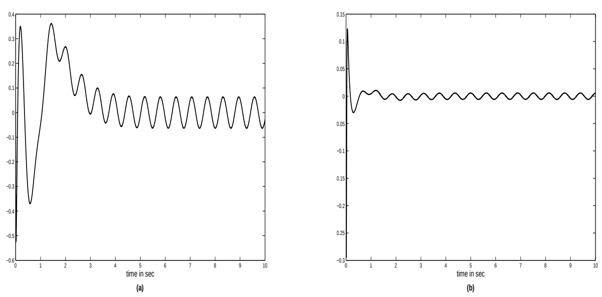

Figure 6: Error when $\tau = 8$ ms for (a) first order filter (b) second order filter

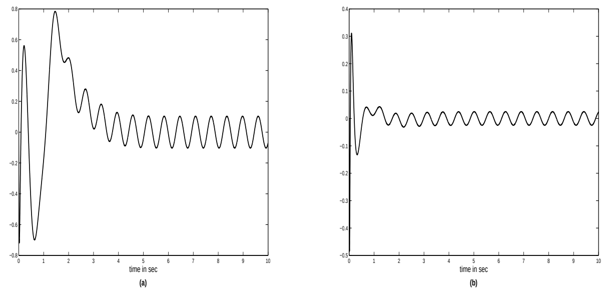

Figure 7: Error when $\tau = 16$ ms for (a) first order filter (b) second order filter

Table 1: First and second order filter error for different τ

τ	First Order Filter	Second Order Filter
0.001	0.017755	0.0001812
0.002	0.03518	0.0007104
0.004	0.0686	0.002817
0.008	0.12676	0.012017
0.016	0.206	0.04908

6 Conclusion

A systematic procedure for design of robust model following sliding mode load frequency controller for single area power system based on uncertainty and disturbance estimator (UDE) has been presented. The control strategy uses Ackermann's formula for elimination of reaching phase and UDE for the estimation of uncertainties and disturbances. The control proposed does not require the knowledge of bounds of uncertainty and disturbances. The simulation results of the control strategy for load frequency control is presented by changing all parameters by 40% from their nominal values. One can easily extend this results up to 98% parameter variation. These results show that system performance is robust to parameter variations and disturbances. The uncertainty and disturbance estimation with second order filter is used for sliding mode control of uncertain plant whose parameters are changing and can be controlled properly and lumped uncertainty can be estimated at a best accurate level with help of UDE with second order filter than first order filter. The error in the estimation can be improved to $O(\tau^2)$ using second order filter.

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