Interior Point Cutting Plane Method for Discrete Decoupled Optimal Power Flow

Ding Xiaoying , Wang Xifan, and Liu Lin

***Abstract*—In this paper, the traditional Π equivalent circuit used to model the transformer is replaced by an ideal model in discrete optimal power flow (DOPF) formulation, which introduces a fictitious bus to express the power and voltage converting relations of the tap-changing transformer, so the admittance matrix is fixed during iterations to reduce computational efforts. Furthermore, this representation of transformer helps DOPF problem to be decoupled into two subproblems, which can improve computational efficiency greatly. Interior point method is used to solve continuous active power subproblem, and interior point cutting plane method (IPCPM) is adopted to solve discrete reactive power subproblem. Unfortunately, we find that the convex combination solution appears with great probability when solving DOPF problem, so in this paper IPCPM is improved to repair this shortcoming. Numerical simulations on IEEE14~300 test systems show that the improvement of IPCPM is efficient, and the proposed method is suitable for solving DOPF problems for large-scale systems.**

***Index Terms*— Discrete Optimal Power Flow; Decoupled Optimal Power Flow; Interior Point Method; Interior Point Cutting Plane Method** [1]

I. INTRODUCTION

A key requirement of any modern society is the economic and secure operation of its electric power system. Such an important objective naturally demands the use of advanced large-scale system analysis, optimization, and control technologies. As a most attractive one of these technologies, optimal power flow (OPF) was proposed by Capentier in 1960s based on economic dispatch (ED) problem. Unlike ED that allocates load to the generating units only, the OPF integrates active and reactive power operation perfectly into one mathematical model via the AC load flow constraints around all buses, in which the economic and secure aspects of the concerned system are considered.

In recent decades, several classes of solution algorithms have been proposed to overcome OPF limitations in terms of flexibility, reliability and performance for real-world applications. However, these algorithms do not deal with the discrete step controls satisfactorily. On the other hand, such discrete controls are widely used by the power industry. For example, transformers are used for voltage control, shunt capacitors and reactors are switched on or off in order to correct the voltage profile and reduce transmission losses, and phase shifters are used to regulate the MW flow of transmission lines. So an efficient and effective OPF discretization procedure is needed to help the operators utilize these discrete controls in realistic and optimal manner. Exact modeling of discrete controls together with continuous control variables makes the OPF become a mixed integer nonlinear programming problem. The combinatorial-search approaches, branch-and-bound and cutting-plane method are usually used to solve this kind of mixed-integer programming model [1,2], but these methods are "nonpolynomial", and all suffer from the so-called problem of "curse of dimensionality" for large-scale applications, making them unsuitable for larger-scale discrete OPF (DOPF) problems. Global optimization techniques, such as genetic algorithm (GA)[3,4], simulated annealing (SA)[5], tabu search (TS)[6], evolutionary programming and evolutionary strategy [7,8] have been applied to DOPF problem, which improve solutions but have relatively slow performance and unstable optima. Recently, due to the basic efficiency of interior point methods, which offer fast convergence and convenience in handling inequality constraints in comparison with other methods, interior point (IP) linear programming [9], quadratic programming [10], and nonlinear programming [11] methods have been widely used to solve OPF problem of large-scale power systems. However, up to now the interior point methods cannot directly solve the mixed-integer programming because gradient information is necessary. Liu et al. [12] extends the primal-dual IP algorithms to handle the discreteness of switchable shunt capacitors/reactors and tap-changing transformers in solving nonlinear reactive-power optimization by incorporating a positive-curvature quadratic penalty function in iterations. In 2004, interior point cutting plane method (IPCPM) was first applied to solve high-dimension nonlinear mixed-integer OPF problems [13]. All these improvements in IP algorithm encourage the successful implementation for rigorous solution of DOPF problem.

The traditional Π equivalent circuit used to model the transformer is replaced by an ideal model in this paper. The limitation of Π transformer model is that the change of tap setting requires repeated calculation of admittance matrix in iterations, which can greatly increase the computational efforts of algorithm. In the proposed formulation, the fictitious buses are added to express the power and voltage converting relations of the tap-changing transformer. So the admittance matrix is fixed in iterations to reduce computational efforts. Furthermore, the new representation of transformer helps DOPF problem to be decoupled into two subproblems

[1] Manuscript received Novenber 9, 2006. This work was supported by special fund of the national priority basic research of China (2004CB217905) .

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completely. The advantages of the decoupled OPF formulation include: (1) decoupling greatly improves computational efficiency, especially for larger systems. It is because that each subproblem has approximately half the dimension of the original problem; (2) decoupling makes it possible to use different optimization techniques to solve the active power and reactive power OPF subproblems. In this paper, IP method is used to solve continuous active power subproblem (P-subproblem), and IPCPM is adopted to solve discrete reactive power subproblem (Q-Subprobelm). Numerical simulations on IEEE14~300 test systems show that the proposed method is efficient in solving OPF problems for large-scale power systems.

IPCPM is a hybrid of cutting plane method and IP method, which obtain good performance from these two methods. Although there are some papers on IPCPM application to several well known combinatorial optimization problems, such as linear ordering problem [14], flowshop scheduling problem [15] etc. IPCPM is first applied to solving high-dimension nonlinear mixed-integer OPF problems in 2004. Numerical simulations on IEEE 14~300 buses test systems show that the IPCPM is suitable for solving discrete optimization problems of large-scale systems. Currently, our observation is that IPCPM cannot obtain the correct information to identify optimal basis when the linear programming relaxation of original integer programming is a multiple-optima problem. Thus ambiguous information may increase the iteration numbers and computational time, even makes IPCPM completely fail. In this paper, IPCPM used in [13] is improved, the optimal solution shifting helps to ensure the generation of effective cutting plane constraints in the solution of the multiple-optima problems.

II. The Formulation of DOPF

A. *The ideal model of Tap-changing Transformer*

OPF problem is a difficult problem in mathematical programming area due to its large dimension, discrete and nonlinear characteristics when discrete controls are considered, such as tap-changing transformers, switchable shunt capacitors/ reactors, feasible AC transmission systems (FACTS) etc. In paper [13], an efficient integer programming approach, IPCPM is used to solve DOPF problems, and its calculation flow shows that the admittance matrix must be calculated with the improved tap setting in each iteration, which consists main computational burden of algorithm. This suggests that a good part of the computational work could be bypassed if the relationship between the transformer tap setting and the admittance matrix are eliminated. The best way we found to do so is to introduce a fictitious bus into the transformer model, which would be used to express the power and voltage converting relations of the tap-changing transformer.

As an example, tap-changing transformer branch is used to show a mathematical interpretation of the two different model (see Fig.1). Where i and j are the head and end bus of transformer branch separately, and non-standard voltage ratio side is j side. The traditional Π equivalent circuit model is illustrated in Fig. 1 (a), and the model we used in this paper is shown in Fig.1 (b), where j, i', i are high voltage bus, fictitious bus and low voltage bus separately.

In Fig.1 (b), for fictitious bus i', $P_{Ti'i}$ can be described as flowing:

$$P_{Ti'i} = V_{i'}^2 g_T - V_{i'}V_i (g_T \cos\theta_{i'i} + b_T \sin\theta_{i'i}) \tag{1}$$

Due to $P_{Ti'i} = P_{Tji'}$, equation (1) can be rewritten as:

$$P_{Tji'} = V_{i'}^2 g_T - V_{i'}V_i (g_T \cos\theta_{i'i} + b_T \sin\theta_{i'i}) \tag{2}$$

In addition, we obtain the following relations as the (1) analogue:

$$Q_{Tji'} = V_{i'}V_i (b_T \cos\theta_{i'i} - g_T \sin\theta_{i'i}) - V_{i'}^2 b_T \tag{3}$$

$$P_{Tii'} = V_i^2 g_T - V_{i'}V_i (g_T \cos\theta_{i'i} - b_T \sin\theta_{i'i}) \tag{4}$$

$$Q_{Tii'} = V_{i'}V_i (b_T \cos\theta_{i'i} + g_T \sin\theta_{i'i}) - V_i^2 b_T \tag{5}$$

Furthermore, we know:

$$\theta_j = \theta_{i'} \tag{6}$$

$$k = V_j / V_{i'} \tag{7}$$

From the expression collected in Table 1, one can see that the action of transformer tap is replaced by the voltage of fictitious bus when using the model in Fig1 (b). We can omit the computational burden for admittance matrix in iterations.

B. *The Formulation of DOPF Problem*

The DOPF problem can be decomposed into two subproblems, the continuous P-subproblem and discrete Q-subproblem with the transformer model in Fig2 (b). These two suproblems can be stated as follows.

i. P-subproblem

Objective function (operation cost minimization):

$$\min \sum_{i\in SG} (a_{2i} P_{Gi}^2 + a_{1i} P_G + a_{0i}) \tag{8}$$

Equality constrains:

$$P_{Gi} - P_{Di} = \sum_{ij\in Sl} P_{Lij} + \sum_{ij\in St} P_{Tij} \tag{9}$$

$$\theta_j = \theta_{i'} \tag{10}$$

Inequality constraints:

(a) Upper and lower bounds on the active power:

$$P_{Gi\min} \le P_{Gi} \le P_{Gi\max} \; (i \in SG) \tag{11}$$

(b) Upper and lower bounds on the flow of transmission line:

$$P_{Lij\min} \le P_{Lij} \le P_{Lij\max} \; (i, j \in Sl) \tag{12}$$

ii. Q-subproblem

Objective function (transmission losses minimization):

$$\min \Delta P_S = \sum_{sj\in Sl} P_{Lsj} + \sum_{sj\in St} P_{Tsj} \tag{13}$$

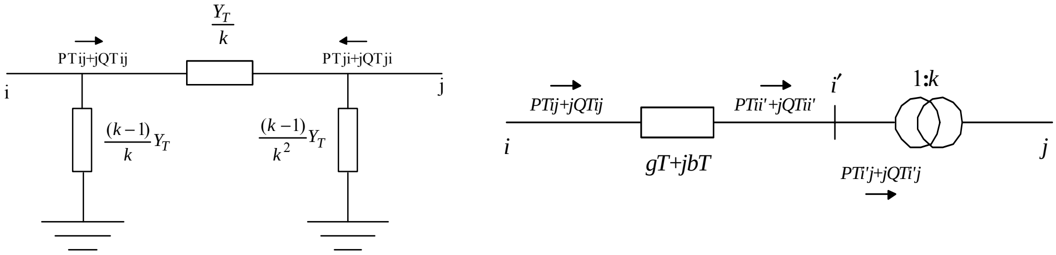

Fig.1 The equivalent circuit of transformer
(a) The traditional Π equivalent circuit model (b) The model used in this paper

Equality constrains:

$$Q_{Gi} - Q_{Di} = \sum_{ij \in Sl} Q_{Lij} + \sum_{ij \in St} Q_{Tij} \tag{14}$$

$$1 + \gamma t_k = V_j / V_{i'} \quad (k \in ST) \tag{15}$$

Inequality constraints:

(a) Upper and lower bounds on the reactive power:

$$Q_{Gi\min} \le Q_{Gi} \le Q_{Gi\max} \ (i \in SG) \tag{16}$$

(b) Upper and lower bounds on the magnitude of voltage in bus i:

$$V_{i\min} \le V_i \le V_{i\max} \ (i \in SB) \tag{17}$$

(c) Upper and lower bounds on the tap position i:

$$t_{i\min} \le t_i \le t_{i\max} \ (i \in ST) \tag{18}$$

Where: SB: the set of all buses, Sl : the set of all branches, St : the set of all transformer branches, S_G : the set of all active sources, ST : the set of all tap-changing transformers, a_{0i} , a_{1i} , a_{2i} : the fuel cost coefficients of unit i , P_{Gi} ， Q_{Gi} : active and reactive power generation at bus i , P_{Di} ， Q_{Di} : active and reactive demand at bus i , P_{Lij} ， Q_{Lij} : active and reactive flow in general branch ij , P_{Tij} ， Q_{Tij} : active and reactive flow in transformer branch ij , t_i : tap setting of tap-changing transformer i , V_i , θ_i : the magnitude and angle of voltage in bus i , $\theta_{ij} = \theta_i - \theta_j$. $\bullet_{\max}$ ， $\bullet_{\min}$: the upper and the lower bounds on variables. γ : the adjust step of the kth tap-changing transformer. The other variables are described in sector 2.1.

TABLE 1 THE POWER FLOW OF TRANSFORMER BRANCH IN TWO DIFFERENT MODELS

Power flow	Π equivalent circuit model	The ideal model
$P_{Tij}(P_{Tii'})$	$V_i^2 g_T - \frac{1}{k} V_i V_j (g_T \cos\theta_{ij} + b_T \sin\theta_{ij})$	$V_i^2 g_T - V_{i'} V_i (g_T \cos\theta_{ii'} + b_T \sin\theta_{ii'})$
$Q_{Tij}(Q_{Tii'})$	$\frac{1}{k} V_j V_i (b_T \cos\theta_{ij} - g_T \sin\theta_{ij}) - V_i^2 b_T$	$V_{i'} V_i (b_T \cos\theta_{ii'} - g_T \sin\theta_{ii'}) - V_i^2 b_T$
$P_{Tji}(P_{Tji'})$	$\frac{1}{k^2} V_j^2 g_T - \frac{1}{k} V_j V_i (g_T \cos\theta_{ij} - b_T \sin\theta_{ij})$	$V_{i'}^2 g_T - V_{i'} V_i (g_T \cos\theta_{ii'} - b_T \sin\theta_{ii'})$
$Q_{Tji}(Q_{Tji'})$	$\frac{1}{k} V_j V_i (b_T \cos\theta_{ij} + g_T \sin\theta_{ij}) - \frac{1}{k^2} V_j^2 b_T$	$V_{i'} V_i (b_T \cos\theta_{ii'} + g_T \sin\theta_{ii'}) - V_{i'}^2 b_T$

III. THE CALCULATION FLOW OF ALGORITHM

The active power subproblem is described as (8)~(12), which is a typical nonlinear programming problem. In this paper, it is solved by IP method (see [16], for the algorithm flow and formulations). IPCPM (see [13], for the principle and formulations of algorithm) is applied to solve the reactive power subproblem, which is a mixed integer nonlinear programming problem.

The proposed approach for DOPF problem is described in Fig 2 within the context of two optimization modules: the P-subproblem optimization and Q-subproblem optimization.

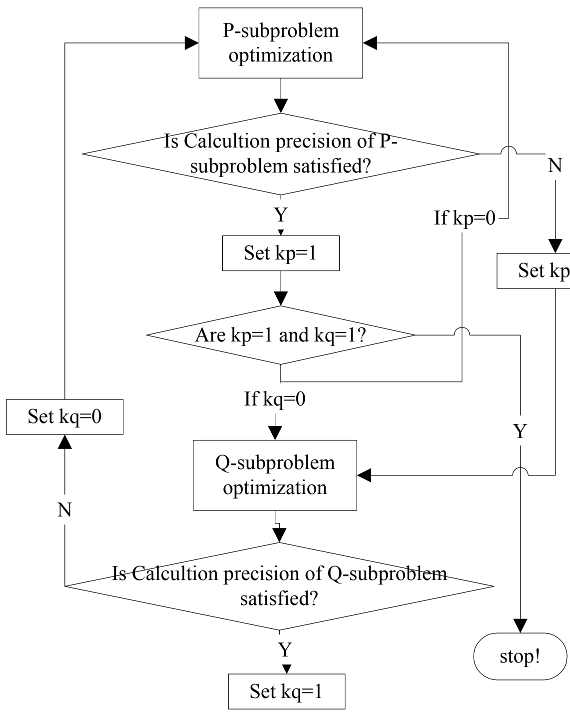

Fig.2 The calculation flow of DOPF problem

IV. THE IMPROVEMENT OF IPCPM

In cutting plane method, cutting plane constrains are available by the information of basis variables. An advantage of the conventional simple cutting plane method (SCPM) in getting a cut is that its optima of linear programming relaxation implicitly converges to the vertex of feasible region of problem in any cases. However, if the linear programming relaxation is multiple-optima problem, it is easy to prove that IPCPM cannot collect the correct information of optimal basis because its optima of linear programming relaxation converges to an edge of feasible region of problem with a great probability. As a result, ambiguous basis information may increase the iteration numbers and computational time of IPCPM, even makes IPCPM completely fail.

A. The theoretic analysis

For clarification we assume that the following linear programming is the relaxation problem of mixed integer programming.

$$\max.\quad 2x_1 + 4x_2 \tag{19}$$

$$s.t.\quad x_1 + 2x_2 + x_3 = 8 \tag{20}$$

$$x_1 + x_4 = 8 \tag{21}$$

$$x_2 + x_5 = 8 \tag{22}$$

$$x_1, x_2, x_3, x_4, x_5 \ge 0 \tag{23}$$

The above linear programming is a multiple-optima problem, which have three kinds of solutions:

i. Normal solution: $\boldsymbol{x}'^* = (2,3,0,6,0)$, that is to say, the number of non-zero elements is equal to 3. It equals to the number of equality constraints.

ii. Degenerate solution: $\boldsymbol{x}''^* = (8,0,0,0,3)$, and the number of non-zero elements is less than 3.

iii. Convex combination solution: $\boldsymbol{x}'''^* = \alpha\boldsymbol{x}'^* + (1-\alpha)\boldsymbol{x}''^*$, where $\alpha \in (0,1)$. For example, when $\alpha = \frac{7}{12}$, $\boldsymbol{x}'''^* = (4.5,1.75,0,3.5,1.25)$, and the number of non-zero elements is more than 3.

The example in Fig.3 provides a geometrical interpretation of the three different kinds of solution when IP method is applied to solving problem (19)~(23). In Fig.3, the convex polytope ABDO is the feasible region of problem (19)~(23). Clearly, the constraint edge BD should parallel to the objective function (19). As a result, the maximum of objective function can be found in any point of edge BD, which explains the reason for the appearance of $\boldsymbol{x}'''^*$. From Fig.3 we can see, $\boldsymbol{x}'^* = (2,3,0,6,0)$ is obtained when the algorithm convergences to point B, or $\boldsymbol{x}''^* = (8,0,0,0,3)$ is obtained when the algorithm convergences to point D, or $\boldsymbol{x}'''^* = (4.5,1.75,0,3.5,1.25)$ is obtained when the algorithm convergences to point P stands between the point B and the point D.

For SCPM algorithm, simplex method(SM) is a vertex-searching method. It start at the origin, then it moves along the intersection of the boundary hyper-planes of the constraints, hopping from one vertex to the neighboring vertex, until an optimal vertex is reached. As a result, only two kinds of solution can be found, normal solution $\boldsymbol{x}'^*$ and degenerate solution $\boldsymbol{x}''^*$ (see Fig.3). $\boldsymbol{x}''^*$ can be transformed into $\boldsymbol{x}'^*$ by selecting some zero variable columns to enter the basis in accordance with some column selection criteria. That is to say, there is not much trouble of cut generation in SCPM, even in the presence of $\boldsymbol{x}''^*$. Unlike SCPM, the IP method used in IPCPM crosses the interior of feasible region in search for optima of linear program. Clearly, the probability of optima standing in edge BD is always much more than the probability of optima standing in point B or point D (see Fig.3). In other words, IP method found convex combination solution $\boldsymbol{x}'''^*$ with a great probability in this case. If $\boldsymbol{x}'''^*$ is obtained, then IPCPM cannot collect the correct information of optimal basis. Therefore, ambiguous basis information may increase the iteration numbers and computational time of IPCPM, even makes IPCPM completely fail. Unfortunately, we observe that this phenomenon occures frequently when we attempted to solve DOPF problem with IPCPM. So addressing above issue is key to the successful implementation of IPCPM for solving DOPF problem.

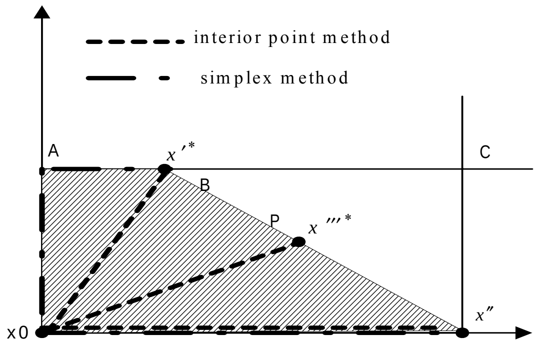

Fig.3 The search route of IPM and SM

B. The improvement of IPCPM

The success of cut generation is dependent on the ability of the method to locate the normal solution $\boldsymbol{x}'^*$ correctly and effectively. Thus, it is obviously that transform $\boldsymbol{x}'''^*$ into $\boldsymbol{x}'^*$ is one of the important factors to solve the problem presented previously. When the optimum converges to an edge of feasible region of problem, it can be moved to the neighboring vertex by pivoting [17]. That is to say, the $\boldsymbol{x}'''^*$ can be transformed into $\boldsymbol{x}'^*$ with the following scheme.

We assume that the linear program relaxation $\{\min \boldsymbol{c}^T\boldsymbol{x} | \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}, \boldsymbol{x} \ge \boldsymbol{0}\}$ (where $\boldsymbol{c} \in R^n$, $\boldsymbol{x} \in R^n$, $\boldsymbol{A} \in R^{m \times n}$, $\boldsymbol{b} \in R^m$) is solved by IP method, and $\boldsymbol{x}$ primal and $\boldsymbol{y}, \boldsymbol{s}$ dual optimal solutions are available. Let $\boldsymbol{A} = [\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3]$, $\boldsymbol{x} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3]$, $\boldsymbol{s} = [\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{s}_3]$, $\boldsymbol{c} = [\boldsymbol{c}_1, \boldsymbol{c}_2, \boldsymbol{c}_3]$, where index 1 refers to the coordinates where $\boldsymbol{x}$ is positive, index 2 refers to the coordinates where both of $\boldsymbol{x}$ and $\boldsymbol{s}$ are zero, and finally index 3 refers to the coordinates where $\boldsymbol{s}$ is positive. Then we have: $\boldsymbol{A}_1\boldsymbol{x}_1 = \boldsymbol{b}$, $\boldsymbol{A}_1^T\boldsymbol{y} = \boldsymbol{c}_1$, $\boldsymbol{A}_2^T\boldsymbol{y} = \boldsymbol{c}_2$ and $\boldsymbol{A}_3^T\boldsymbol{y} < \boldsymbol{c}_3$.

Step1：Determine the kind of solution:

(a) If it is $\boldsymbol{x}'^*$, then components of $\boldsymbol{x}'^*$ corresponding to zero element are said to be basic variables. Stop calculation.

(b) If it is $\boldsymbol{x}''^*$, go to Step7.

(c) If it is $\boldsymbol{x}'''^*$, go to Step2.

Step2 : Are the columns of A_1 linearly dependent, if the answer is not go to Step6.

Step3 : Pivoting: set $x_1' = x_1 + tz$.

To guarantee the optimization of x : There must exist one but not only one vector z satisfying $A_1 z = 0$ because the columns of A_1 are linearly dependent, so any z is the one we need. (We can prove: The new objective function is $c_1^T x_1' + c_2^T x_2 + c_3^T x_3 = (A_1^T y)^T (x_1 + tz) + c_2^T x_2 + c_3^T x_3 = c_1^T x_1 + c_2^T x_2 + c_3^T x_3 + ty^T A_1 z = c_1^T x_1 + c_2^T x_2 + c_3^T x_3 = c^T x$, so the optimization of original problem solution does not be affected when x_1 is transformed into x_1'.)

To guarantee the feasibility of x : Compute $t_{\min} \le t \le t_{\max}$ by solving $x_1' = x_1 + tz \ge 0$.

Step4 : Eliminate zero element (say j) from x_1' using $t_{\min}$ or $t_{\max}$. Remove a_j from A_1 and add to A_2, then go to Step2.

Step5: Set $x = [x_1', x_2, x_3]$, and then go to Step1.

Step6: Set $B = A_1$, if $rank(B) > rank([A_1 A_2])$ go to Step8.

Step7: A column a_j of $[A_1 A_2]$ is independent from B, add a_j to B.

Step8: Go to Step11 if $rank(B) = m$.

Step9: Pivoting: Set $y' = y - vu$.

To guarantee the optimization of y : There is more than one vector u satisfying $B^T u = 0$, and any u is the one we need. (We can prove: The new objective function is $b^T y' = b^T y - vb^T u = b^T y - vx^T A^T u$. It is clearly that $x^T A^T u = [x_1' x_2 x_3] \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} u = [x_1' 0 0] \begin{bmatrix} B \\ A_3 \end{bmatrix} u = 0$, so the optimization of dual problem solution does not be affected when y is transformed into y'.).

To guarantee the feasibility of y: Compute $v_{\min} \le v \le v_{\max}$ by solving $A_3^T y' \le c_3$.

Step10: Substitute $v_{\min}$ or $v_{\max}$ to $y' = y - vu$, and $a_{3j}^T y' = c_{3j}$ must exist. (where a_{3j} represents the j[th] column of matrix A_3, c_{3j} is the j[th] component of vector c_3). Remove a_{3j} from A_3 and add to A_2 and B, then go to Step8.

Step11: Stop, matrix B is the basis matrix that we need.

V. NUMERICAL SIMULATE AND ANALYSIS

The proposed algorithm was implemented using the Visual C++6.0 language and the software program was executed on an 800-MHz Pentium Pro computer. Numerical simulations on RTS-24 test systems have been done to test the performance of the presented algorithm.

A. *The performance of proposed algorithm*

In the proposed formulation, the fictitious buses are added to express the power and voltage converting relations of the tap-changing transformer. So the admittance matrix is fixed during iterations to reduce computational efforts. Furthermore, the new representation of transformer helps DOPF problem to be decoupled into two subproblems. The advantages of the decoupled OPF formulation include: (1) decoupling greatly improves computational efficiency, especially for larger systems. This is because each subproblem has approximately half the dimension of the original problem; (2) decoupling makes it possible to use different optimization techniques to solve the active power and reactive power OPF subproblems. In this paper, IP method is used to solve continuous P-subproblem, and IPCPM is adopted to solve discrete Q-subproblem.

From Table2, comparing with the algorithm proposed in paper [13], we find that the presented algorithm has attractive performance because its calculation speed enhances obviously during the scale of system becoming larger and larger. Based above analysis, we can conclude that the proposed method is very promising for solving discrete OPF problem, especially for large-scale power systems.

TABLE 2 THE COMPUTATION TIME OF TWO FORMULATION (ms)

Model / Test system	II model	Ideal model
IEEE14	1359	359
RTS-24	2703	1062
IEEE30	5797	2578
IEEE57	10734	2625
IEEE118	26641	7359
IEEE300	77609	57125

B. *The improvement of IPCPM*

Many numerical experiments have indicated that objective function of OPF has a very plain shape for the transformer tap control. Therefore, very similar cost values can be obtained with different settings of the transformer tap. So OPF becomes a multiple-optima problem. The same conclusion can be obtained form the numerical results illustrated in Table3, which shows that the convex combination solution appears with great probability when solving OPF problem (8)~(18) for IEEE14~300 test systems. Furthermore, It is seen in sector 3.1 that IPCPM has bad computational performance for multiple-optima problem. There is a need to extend IPCPM to repair this shortcoming. Table4 compares the performance of the two IPCPM for solving OPF problem, which shows that the proposed method is more efficient than its old version proposed in paper [13]. In summary, the improvements of IPCPM meet the needs of practical application, and it offers a new way to solve complicated discrete optimization problem for large-scale power system, which result in dramatic property and human save.

TABLE.3 THE TYPE OF OPTIMUM OF DOPF PROBLEM

system	The dimension of matrix A	The number of non-zero elements	The number of zero elements	The type of optimum
5	24×28	25	3	x'''^*
14	65×74	65	9	x'^*
24	114×132	118	14	x'''^*
30	132×144	133	11	x'''^*
57	243×258	245	13	x'''^*
118	546×620	550	70	x'''^*
300	1300×1400	1314	86	x'''^*

TABLE.4 THE CALCULATION RESULTS OF TWO ALGORITHM

system		The number of cuts	The value of tap
5	Before improvement	fail	fail
	After improvement	1	5
14	Before improvement	1	-4,-10
	After improvement	1	-4,-10
24	Before improvement	fail	fail
	After improvement	1	-2,-5,5
30	Before improvement	0	-10,-10,5,-5
	After improvement	0	-10,-10,5,-5
57	Before improvement	0	-10,-5,-10,-5,10
	After improvement	0	-10,-5,-10,-5,10
118	Before improvement	fail	fail
	After improvement	1	-5,5,-5,0,-2,-5,5
300	Before improvement	fail	fail
	After improvement	2	-6,-5,-20,10,10,-5,-5,-10, -10,-5,-20

VI. CONCLUSION

The OPF problem becomes a nonlinear mixed integer programming problem when the discrete controllers are considered, such as tap-changers in transformers or switching of capacitor/reactor banks and so on. It is proposed in this paper, that the traditional Π equivalent circuit used to model the transformer be replaced by an ideal model, which provides the following advantages:

(1) The admittance matrix is fixed in iterations to reduce computational efforts.

(2) The new representation of transformer helps DOPF problem to be decoupled into two subproblems, which improves computational efficiency.

On the other hand, in this paper, IPCPM is improved to meet the needs of practical application. Numerical simulations on IEEE14~300 test systems show that the proposed method is efficient in solving OPF problems of large-scale power systems.

REFERENCES

[1] E. C. Finardi, E. L. da Silva. Unit commitment of single hydroelectric plant. Electric Power Systems Research, 2005, vol. 75, pp: 116~123

[2] R.A. Jabr. Homogeneous cutting-plane method to solve the security-constrained economic dispatching problem. IEE Proc.-Gener. Transm. Distrib. 2002, vol. 149, NO. 2, pp: 139~144

[3] Z. H. Wang, X. G. Yin, Z. Zhang, J. C.Yang. Pseudo-parallel genetic algorithm for reactive power optimization, IEEE Power Engineering Society General Meeting, 2003, vol. 2, pp:13~17

[4] S. Durairaj, Devaraj, P. S. Kannan. Improved Genetic Algorithm Approach for Multi-Objective contingency constrained Reactive Power Planning, INDICON, 2005 Annual IEEE, 2005, pp: 510~515

[5] Chen Haoyong, Wang Xifan. A Genetic Algorithm with Annealing Selection for Reactive Power Optimization. ZhongGuo Dianli/Electric Power, 1998, vol. 31, NO. 2, pp: 3~6

[6] Yutian Liu, Li Ma, Jianjun Zhang. GA/SA/TS Hybrid algorithm for reactive power optimization. IEEE Power Engineering Society Summer Meeting, 2000, vol. 1, pp: 245~249

[7] C. Jiang, C. Wang. Improved evolutionary programming with dynamic mutation and metropolis criteria for multi-objective reactive power optimization. IEE Proceedings on Generation, Transmission and Distribution, 2005, vol. 152, NO. 2, pp: 291~294

[8] C. C. A.Rajan, M. R. Mohan. An evolutionary programming-based tabu search method for solving the unit commitment problem, IEEE Trans. on Power Systems, vol. 2004, 19, NO. 1, pp: 577~585

[9] Chen Ji, Wei Hua. Application of an Improved Trust Region Interior Point Algorithm in OPF. Modern Electric Power. 2005, vol. 22, NO. 6, pp: 13~17

[10] Hua Wei, H. Sasaki, R. Yokoyama. An application of interior point quadratic programming algorithm to power system optimization problems. IEEE Transactions on Power Systems. 1996, vol. 11, NO. 1, pp: 260~266

[11] WEI Hua, LI Bin, HANG Nai-shan, et al. An implementation of interior point algorithm for large-scale hydro-thermal optimal power flow problem. Proceedings of the Csee. 2003, vol. 23, NO. 6, pp: 13~18

[12] Mingbo Liu, S. K. Yso, Ying Cheng. An extended nonlinear primal-dual intwerior point algorithm for reactive-power optimization of large-scale power systems with discrete control variables. IEEE Trans. on Power Syetems, 2002, vol. 17, NO. 4, pp: 982~991

[13] X. Y. Ding, X. F. Wang, Y. H. Song. An interior Point Cutting Plane Method for Optimal Power Flow. International Journal of Management Mathematics, 2004, vol. 15, NO. 4, pp: 355~368

[14] J. E. Mitchell, B. Borchers. Solving Real-world Linear Ordering Problems Using a Primal-dual Interior Point Cutting Plane Method. Annals of Operations Research, 1996, vol. 62,pp.: 253~276

[15] Ding Xiao-ying, WANG Xi-fan, CHEN Hao-yong. A combined algorithm for optimal power flow. Proceedings of the CSEE, 2002, vol. 22, NO. 12, pp: 11-15

[16] Nimrod Megiddo, On Finding Primal and Dual Optimal Bases. Theory.Stanford. edu/~megiddo/pdf/primdual.pdf