A Normal Cavity-Expansion (NCE) Model based on the Normal Curve Surface (NCS) Coordinate System

Gao Shiqiao, Jin Lei, Liu Haipeng, Liang Xinjian, Han Li, *Member, IAENG*

***Abstract*—To suit the normal cavity expansion of concrete target penetrated by a projectile, a Normal Curve Surface (NCS) coordinate system is constructed. By considering the dynamic behavior of material under high-velocity and high-pressure shock and assuming that the particle velocity, the wave propagation and the pressure are all in the normal direction of outer surface of the projectile nose, a set of dominating equations are established. The analytical solution of resistant forces on the projectile-nose is obtained. Some calculations and comparisons with tests are made.**

***Keywords*—Coordinate system, concrete target, shock wave, cavity expansion.**

I. INTRODUCTION

Much effort has been directed at predicting the penetration of a projectile against concrete targets. In the past century, most of the achievements are on the penetration depth, perforation thickness and ballistic limit. Recently, a lot of interests have been focused on the analytical model of the resistant force of target on the projectile. Typical work has been done by Forrestal, et al[1]-[5]. Based on the cavity-expansion theory, they derived a series of analytical penetration formulas of resistant force for soil, rock, and concrete material. Li, et al[6]-[8] summarized and developed some work of analytical formula. All these theoretical achievements are based on the cylindrical cavity and spherical cavity analysis. In the Forrestal's dynamic cavity-expansion theory, a constant propagation velocity of the interface between plastic and elastic response regions, a constant expanding velocity of the cavity and a spherically symmetric shape of cavity were assumed. Assuming that the propagation of stress wave, the displacements of medium and the particle velocities are all in the normal direction of outer surface of projectile, Gao et al[9]-[12] presented an idea of normal expansion theory (NET). In this NET model, the propagation velocity of stress wave may be not constant, the expanding velocity of cavity may be not constant and the shape of cavity may be not spherical. To describe the normal expansion theory more accurately, it is necessary to establish a more perfect theory system. To suit the analysis of normal expansion, it is necessary to construct a Normal Curve Surface (NCS) coordinate system.

For high-velocity and high-pressure impact, the amplitude of stress waves will greatly exceed the dynamic flow strength of a material. In this case, in comparison with the compressive hydrostatic component of the stress, one can effectively neglect the shear stresses. During impact, a shock wave occurs. It has a steep front. At the shock front, there is a discontinuity in particle velocity, pressure, and density. Based on some phenomenon of experiments and tests, the following assumptions are presented. 1). A shock front is a steep discontinuous surface. 2). The shear modulus of the material is assumed to be zero. 3). In comparison with the compressive strength, the tensile strength can be neglected. 4). Body forces (such as gravitational) and heat conduction at the shock front are negligible. 5). Material does not undergo phase transformations. 6). During impact, the responding medium of concrete expands in the external normal direction of the outer surface of the projectile. The particle velocity and the wave velocity of responding medium are parallel. Their direction is the same as the external normal direction of the projectile surface. 7). During impact against concrete target, the projectile is rigid (is no-deformable).

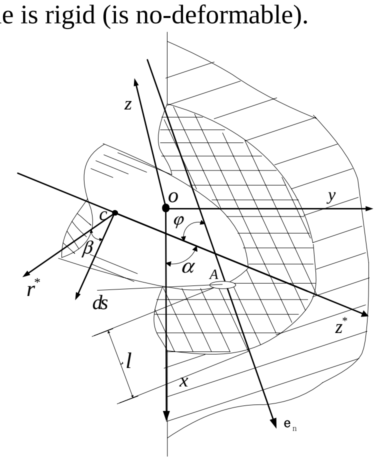

Fig.1 The scheme of penetrating procedure of a projectile against concrete target

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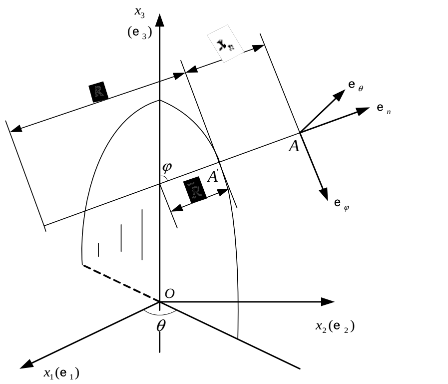

Fig.2 The Cartesian coordinates and NCS coordinates

II. The Normal Curve Surface Coordinate System and the Expression of Field Theory

To suit the normal expansion of particles and stress wave of the responding medium of target material, a Normal Curve Surface coordinate system (NCS) is chosen and constructed, which is shown in Fig.2. Similar to the spherical coordinate system, this normal curve surface coordinate system has two angle coordinates and one line coordinate. The difference is in line coordinate. These two angle coordinates are circumferential angle coordinate θ and meridian angle coordinate φ respectively. The line coordinate is normal coordinate x_n, which is in the external normal direction of the outer surface of projectile, that is, in the direction of curvature radius. In Fig.2, x_1 - x_2 - x_3 is the Cartesian coordinate system and $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ are corresponding unit vectors. A' is a point lied in the surface of projectile. A is a point of target. R is the curvature radius. $\bar{R}$ is a part of curvature radius cut by x_3 axis.ubmit your manuscript electronically for review.

To obtain some total derivative, gradient, divergence of scalars, vectors and tensors, the following derivations are made by converting the Cartesian coordinate into the normal curve surface coordinate.

Taking $\mathbf{e}_n$, $\mathbf{e}_\varphi$ and $\mathbf{e}_\theta$ as the unit vectors in the directions x_n , φ and θ respectively, there are the following transformation relations between them and $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$.

$$\begin{cases}\mathbf{e}_n=\mathbf{e}_1\sin\varphi\cos\theta+\mathbf{e}_2\sin\varphi\sin\theta+\mathbf{e}_3\cos\varphi\\ \mathbf{e}_\varphi=\mathbf{e}_1\cos\varphi\cos\theta+\mathbf{e}_2\cos\varphi\sin\theta-\mathbf{e}_3\sin\varphi\\ \mathbf{e}_\theta=-\mathbf{e}_1\sin\theta+\mathbf{e}_2\cos\theta\end{cases} \quad (1)$$

Making the differential of equation (1), leads to

$$\begin{cases}d\mathbf{e}_n=\mathbf{e}_\varphi d\varphi+\mathbf{e}_\theta\sin\varphi d\theta\\ d\mathbf{e}_\varphi=-\mathbf{e}_n d\varphi+\mathbf{e}_\theta\cos\varphi d\theta\\ d\mathbf{e}_\theta=-\mathbf{e}_n\sin\varphi d\theta-\mathbf{e}_\varphi\cos\varphi d\theta\end{cases} \quad (2)$$

From the analysis of spatial geometry, we can obtain the differential of radius vector as

$$d\mathbf{r}=\mathbf{e}_n dx_n+\mathbf{e}_\varphi(R+x_n)d\varphi+\mathbf{e}_\theta(\bar{R}+x_n)\sin\varphi d\theta \quad (3)$$

It can be seen that, the components of $d\mathbf{r}$ are $(dx_n,\ (R+x_n)d\varphi,\ (\bar{R}+x_n)\sin\varphi d\theta)$ respectively.

By means of the relation of $df=\nabla f\cdot d\mathbf{r}$, that is：

$$\frac{\partial f}{\partial x_n}dx_n+\frac{\partial f}{\partial\varphi}d\varphi+\frac{\partial f}{\partial\theta}d\theta$$

, it

$$=(\nabla f)_n dx_n+(\nabla f)_\varphi(R+x_n)d\varphi+(\nabla f)_\theta(\bar{R}+x_n)\sin\varphi d\theta$$

can be known that $(\nabla f)_n=\frac{\partial f}{\partial x_n}$, $(\nabla f)_\varphi=\frac{1}{R+x_n}\frac{\partial f}{\partial\varphi}$ and $(\nabla f)_\theta=\frac{1}{(\bar{R}+x_n)\sin\varphi}\frac{\partial f}{\partial\theta}$. Therefore, the gradient vector operator in NCS coordinate system can be written by

$$\nabla=grad=(\frac{\partial}{\partial x_n},\frac{1}{R+x_n}\frac{\partial}{\partial\varphi},\frac{1}{(\bar{R}+x_n)\sin\varphi}\frac{\partial}{\partial\theta}) \quad (4)$$

Because $\mathbf{v}=v_n\mathbf{e}_n+v_\varphi\mathbf{e}_\varphi+v_\theta\mathbf{e}_\theta$, $(\mathbf{v}\cdot\nabla)=v_n\frac{\partial}{\partial x_n}+\frac{v_\varphi}{R+x_n}\frac{\partial}{\partial\varphi}+\frac{v_\theta}{(\bar{R}+x_n)\sin\varphi}\frac{\partial}{\partial\theta}$ and $\frac{D}{Dt}=\frac{\partial}{\partial t}+(\mathbf{v}\cdot\nabla)$, the total derivative of a scalar variable f can be written by

$$\frac{Df}{Dt}=\frac{\partial f}{\partial t}+v_n\frac{\partial f}{\partial x_n}+\frac{v_\varphi}{R+x_n}\frac{\partial f}{\partial\varphi}+\frac{v_\theta}{(\bar{R}+x_n)\sin\varphi}\frac{\partial f}{\partial\theta} \quad (5)$$

The total derivative for a vector $\mathbf{v}$ can be written by

$$\frac{D\mathbf{v}}{Dt}=\mathbf{e}_n\frac{Dv_n}{Dt}+v_n\frac{D\mathbf{e}_n}{Dt}+\mathbf{e}_\varphi\frac{Dv_\varphi}{Dt}+v_\varphi\frac{D\mathbf{e}_\varphi}{Dt}+\mathbf{e}_\theta\frac{Dv_\theta}{Dt}+v_\theta\frac{D\mathbf{e}_\theta}{Dt} \quad (6)$$

Noting that $\mathbf{e}_n$ and $\mathbf{e}_\varphi$ are independant of x_n and $\mathbf{e}_\theta$ is independent of x_n or φ, there are

$$\begin{cases}\frac{D\mathbf{e}_n}{Dt}=\frac{v_\varphi}{R+x_n}\frac{\partial\mathbf{e}_n}{\partial\varphi}+\frac{v_\theta}{(\bar{R}+x_n)\sin\varphi}\frac{\partial\mathbf{e}_x}{\partial\theta}=\frac{v_\varphi\mathbf{e}_\varphi}{R+x_n}+\frac{v_\theta\mathbf{e}_\theta}{\bar{R}+x_n}\\ \frac{D\mathbf{e}_\varphi}{Dt}=\frac{v_\varphi}{R+x_n}\frac{\partial\mathbf{e}_\varphi}{\partial\varphi}+\frac{v_\theta}{(\bar{R}+x_n)\sin\varphi}\frac{\partial\mathbf{e}_\varphi}{\partial\theta}=-\frac{v_\varphi\mathbf{e}_n}{R+x_n}+\frac{v_\theta\cot\varphi\mathbf{e}_\theta}{\bar{R}+x_n}\\ \frac{D\mathbf{e}_\theta}{Dt}=\frac{v_\theta}{(\bar{R}+x_n)\sin\varphi}\frac{\partial\mathbf{e}_\theta}{\partial\theta}=-\frac{1}{\bar{R}+x_n}(v_\theta\mathbf{e}_n+v_\theta\cot\varphi\mathbf{e}_\varphi)\end{cases} \quad (7)$$

Substituting equation (7) into equation(6)，leads to

$$\begin{cases}(\frac{D\mathbf{v}}{Dt})_n=\frac{Dv_n}{Dt}-\frac{v_\varphi^2}{R+x_n}-\frac{v_\theta^2}{\bar{R}+x_n}\\ (\frac{D\mathbf{v}}{Dt})_\varphi=\frac{Dv_\varphi}{Dt}+\frac{v_n v_\varphi}{R+x_n}-\frac{v_\theta^2\cot\varphi}{\bar{R}+x_n}\\ (\frac{D\mathbf{v}}{Dt})_\theta=\frac{Dv_\theta}{Dt}+\frac{v_n v_\theta}{R+x_n}+\frac{v_\varphi v_\theta\cot\varphi}{\bar{R}+x_n}\end{cases} \quad (8)$$

The gradient of a vector $\mathbf{v}$, that is $\nabla\mathbf{v}$, is a second-order tensor. By means of the relation

$$d\mathbf{v}=(dv_n)\mathbf{e}_n+v_n d\mathbf{e}_n+(dv_\varphi)\mathbf{e}_\varphi+v_\varphi d\mathbf{e}_\varphi+(dv_\theta)\mathbf{e}_\theta+v_\theta d\mathbf{e}_\theta$$

and equation (2), we can obtain the following relations.

$$\begin{cases}(d\mathbf{v})_n = \dfrac{\partial v_n}{\partial x_n}dx_n + (\dfrac{\partial v_n}{\partial \varphi} - v_\varphi)d\varphi + (\dfrac{\partial v_n}{\partial \theta} - v_\theta \sin\varphi)d\theta \\ (d\mathbf{v})_\varphi = \dfrac{\partial v_\varphi}{\partial x_n}dx_n + (\dfrac{\partial v_\varphi}{\partial \varphi} + v_n)d\varphi + (\dfrac{\partial v_\varphi}{\partial \theta} - v_\theta \cos\varphi)d\theta \\ (d\mathbf{v})_\theta = \dfrac{\partial v_\theta}{\partial x_n}dx_n + \dfrac{\partial v_\theta}{\partial \varphi}d\varphi + (\dfrac{\partial v_\theta}{\partial \theta} + v_n \sin\varphi + v_\varphi\cos\varphi)d\theta \end{cases} \quad (9)$$

By means of the relation $(\nabla\mathbf{v}\cdot d\mathbf{r})_i = (\nabla\mathbf{v})_{ij}(d\mathbf{r})_j$ and substituting equation (3) into it, leads to

$$\begin{cases}(\nabla\mathbf{v}\cdot d\mathbf{r})_n = (\nabla\mathbf{v})_{nn}dx_n + (\nabla\mathbf{v})_{n\varphi}(R+x_n)d\varphi + (\nabla\mathbf{v})_{n\theta}(\bar{R}+x_n)\sin\varphi d\theta \\ (\nabla\mathbf{v}\cdot d\mathbf{r})_\varphi = (\nabla\mathbf{v})_{\varphi n}dx_n + (\nabla\mathbf{v})_{\varphi\varphi}(R+x_n)d\varphi + (\nabla\mathbf{v})_{\varphi\theta}(\bar{R}+x_n)\sin\varphi d\theta \\ (\nabla\mathbf{v}\cdot d\mathbf{r})_\theta = (\nabla\mathbf{v})_{\theta n}dx_n + (\nabla\mathbf{v})_{\theta\varphi}(R+x_n)d\varphi + (\nabla\mathbf{v})_{\theta\theta}(\bar{R}+x_n)\sin\varphi d\theta \end{cases} \quad (10)$$

Because $d\mathbf{v} = \nabla\mathbf{v}\cdot d\mathbf{r}$, comparing equation (9) and (10), we can obtain the following relations.

$$\nabla\mathbf{v} = \begin{pmatrix}(\nabla\mathbf{v})_{nn} & (\nabla\mathbf{v})_{n\varphi} & (\nabla\mathbf{v})_{n\theta} \\ (\nabla\mathbf{v})_{\varphi n} & (\nabla\mathbf{v})_{\varphi\varphi} & (\nabla\mathbf{v})_{\varphi\theta} \\ (\nabla\mathbf{v})_{\theta n} & (\nabla\mathbf{v})_{\theta\varphi} & (\nabla\mathbf{v})_{\theta\theta}\end{pmatrix}$$

$$= \begin{pmatrix}\dfrac{\partial v_n}{\partial x_n} & \dfrac{1}{R+x_n}(\dfrac{\partial v_n}{\partial\varphi} - v_\varphi) & \dfrac{1}{(\bar{R}+x_n)\sin\varphi}(\dfrac{\partial v_n}{\partial\theta} - v_\theta\sin\varphi) \\ \dfrac{\partial v_\varphi}{\partial x_n} & \dfrac{1}{R+x_n}(\dfrac{\partial v_\varphi}{\partial\varphi} + v_n) & \dfrac{1}{(\bar{R}+x_n)\sin\varphi}(\dfrac{\partial v_\varphi}{\partial\theta} - v_\theta\cos\varphi) \\ \dfrac{\partial v_\theta}{\partial x_n} & \dfrac{1}{R+x_n}\dfrac{\partial v_\theta}{\partial\varphi} & \dfrac{1}{(\bar{R}+x_n)\sin\varphi}(\dfrac{\partial v_\theta}{\partial\theta} + v_n\sin\varphi + v_\varphi\cos\varphi)\end{pmatrix} \quad (11)$$

To obtain the divergence of a vector $\mathbf{v}$, that is $\operatorname{div}\mathbf{v}$, the relation $\operatorname{div}\mathbf{v} = \nabla\cdot\mathbf{v} = \operatorname{tr}(\nabla\mathbf{v})$ is used. In term of the characteristic of $\operatorname{tr}(\mathbf{N})$, we can obtain the following result

$$\operatorname{div}\mathbf{v} = \frac{\partial v_n}{\partial x_n} + \frac{v_n}{R+x_n} + \frac{v_n}{\bar{R}+x_n} + \frac{1}{R+x_n}\frac{\partial v_\varphi}{\partial\varphi} + \frac{v_\varphi\cot\varphi}{\bar{R}+x_n} + \frac{1}{(\bar{R}+x_n)\sin\varphi}\frac{\partial v_\theta}{\partial\theta} \quad (12)$$

To obtain a divergence of the second-order tensor $\mathbf{A}$, which expressed by

$$\mathbf{A} = \begin{pmatrix}A_{nn} & A_{n\varphi} & A_{n\theta} \\ A_{\varphi n} & A_{\varphi\varphi} & A_{\varphi\theta} \\ A_{\theta n} & A_{\theta\varphi} & A_{\theta\theta}\end{pmatrix}$$

, that is $\operatorname{div}\mathbf{A}$, the relation $(\operatorname{div}\mathbf{A})\cdot\mathbf{a} = \operatorname{div}(\mathbf{A}\cdot\mathbf{a}) - \operatorname{tr}[\mathbf{A}\cdot(\nabla\mathbf{a})]$ is used, where $\mathbf{a}$ is a vector.

As a vector, the components of divergence of the second-order tensor $\mathbf{A}$ can be written as the following forms

$$\begin{cases}(\operatorname{div}\mathbf{A})_n = (\operatorname{div}\mathbf{A})\cdot\mathbf{e}_n = \operatorname{div}(\mathbf{A}\cdot\mathbf{e}_n) - \operatorname{tr}[\mathbf{A}\cdot(\nabla\mathbf{e}_n)] \\ (\operatorname{div}\mathbf{A})_\varphi = (\operatorname{div}\mathbf{A})\cdot\mathbf{e}_\varphi = \operatorname{div}(\mathbf{A}\cdot\mathbf{e}_\varphi) - \operatorname{tr}[\mathbf{A}\cdot(\nabla\mathbf{e}_\varphi)] \\ (\operatorname{div}\mathbf{A})_\theta = (\operatorname{div}\mathbf{A})\cdot\mathbf{e}_\theta = \operatorname{div}(\mathbf{A}\cdot\mathbf{e}_\theta) - \operatorname{tr}[\mathbf{A}\cdot(\nabla\mathbf{e}_\theta)]\end{cases} \quad (13)$$

Substituting $\mathbf{e}_n, \mathbf{e}_\varphi, \mathbf{e}_\theta$ into equation (11), leads to

$$\nabla\mathbf{e}_n = \begin{pmatrix}0 & 0 & 0 \\ 0 & \dfrac{1}{R+x} & 0 \\ 0 & 0 & \dfrac{1}{\bar{R}+x}\end{pmatrix}, \quad \nabla\mathbf{e}_\varphi = \begin{pmatrix}0 & -\dfrac{1}{R+x} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dfrac{\cot\varphi}{\bar{R}+x}\end{pmatrix},$$

$$\nabla\mathbf{e}_\theta = \begin{pmatrix}0 & 0 & -\dfrac{1}{\bar{R}+x} \\ 0 & 0 & -\dfrac{\cot\varphi}{\bar{R}+x} \\ 0 & 0 & 0\end{pmatrix} \quad (14)$$

Therefore there are

$$\begin{cases}\operatorname{tr}[\mathbf{A}\cdot(\nabla\mathbf{e}_n)] = \dfrac{A_{\varphi\varphi}}{R+x_n} + \dfrac{A_{\theta\theta}}{\bar{R}+x_n} \\ \operatorname{tr}[\mathbf{A}\cdot(\nabla\mathbf{e}_\varphi)] = -\dfrac{A_{\varphi n}}{R+x_n} + \dfrac{A_{\theta\theta}\cot\varphi}{\bar{R}+x_n} \\ \operatorname{tr}[\mathbf{A}\cdot(\nabla\mathbf{e}_\theta)] = -\dfrac{1}{\bar{R}+x_n}(A_{\theta n} + A_{\theta\varphi}\cot\varphi)\end{cases} \quad (15)$$

Due to $(\mathbf{A}\cdot\mathbf{e}_k)_i = A_{ik}$, there are

$$\begin{cases}\mathbf{A}\cdot\mathbf{e}_n = A_{nn}\mathbf{e}_n + A_{\varphi n}\mathbf{e}_\varphi + A_{\theta n}\mathbf{e}_\theta \\ \mathbf{A}\cdot\mathbf{e}_\varphi = A_{n\varphi}\mathbf{e}_n + A_{\varphi\varphi}\mathbf{e}_\varphi + A_{\theta\varphi}\mathbf{e}_\theta \\ \mathbf{A}\cdot\mathbf{e}_\theta = A_{n\theta}\mathbf{e}_n + A_{\varphi\theta}\mathbf{e}_\varphi + A_{\theta\theta}\mathbf{e}_\theta\end{cases} \quad (16)$$

Substituting equation (15) and (16) into equation (13) and using equation (12), leads to

$$(\operatorname{div}\mathbf{A})_n = \frac{\partial A_{nn}}{\partial x_n} + \frac{1}{R+x_n}\frac{\partial A_{\varphi n}}{\partial\varphi} + \frac{1}{(\bar{R}+x_n)\sin\varphi}\frac{\partial A_{\theta n}}{\partial\theta} + \frac{1}{R+x_n}(A_{nn} - A_{\varphi\varphi}) + \frac{1}{\bar{R}+x_n}(A_{nn} - A_{\theta\theta} + A_{\varphi n}\cot\varphi] \quad (17)$$

$$(\operatorname{div}\mathbf{A})_\varphi = \frac{\partial A_{n\varphi}}{\partial x_n} + \frac{1}{R+x_n}\frac{\partial A_{\varphi\varphi}}{\partial\varphi} + \frac{1}{(\bar{R}+x_n)\sin\varphi}\frac{\partial A_{\theta\varphi}}{\partial\theta} + \frac{1}{R+x_n}(A_{n\varphi} + A_{\varphi n}) + \frac{1}{\bar{R}+x_n}[A_{n\varphi} + (A_{\varphi\varphi} - A_{\theta\theta})\cot\varphi] \quad (18)$$

$$(\operatorname{div}\mathbf{A})_\theta = \frac{\partial A_{n\theta}}{\partial x_n} + \frac{1}{R+x_n}\frac{\partial A_{\varphi\theta}}{\partial\varphi} + \frac{1}{(\bar{R}+x_n)\sin\varphi}\frac{\partial A_{\theta\theta}}{\partial\theta} + \frac{1}{R+x_n}(A_{n\theta} + A_{\theta n}) + \frac{1}{\bar{R}+x_n}[A_{n\theta} + (A_{\theta\varphi} + A_{\varphi\theta})\cot\varphi] \quad (19)$$

III. The Governing Equations for Continuum Mechanics

As a region of responding medium, behind the shock front, all the physical variables of the material in this region are continuous. There is no discontinuity. By means of the theory of continuum mechanics, for the case of no-shear stress and no-shear strain, the relations of conservation of mass, conservation of momentum and conservation of energy in general Cartesian coordinate system can be respectively written as

$$\frac{D\rho}{Dt}+\rho\,\mathrm{div}\mathbf{u}=0 \tag{20}$$

$$\rho\frac{D\mathbf{u}}{Dt}=-\mathrm{div}\mathbf{p} \tag{21}$$

$$\rho\frac{DE}{Dt}=-p_{ik}\dot{\varepsilon}_{ik} \tag{22}$$

where $\frac{D}{Dt}$ denote the total derivative symbol in Lagrangian coordinate, ρ is the density of material, div( ) stands for divergence of a tensor field, $\mathbf{u}$ and $\mathbf{p}$ are particle velocity tensor and pressure tensor respectively, E is the internal energy per unit mass, and $\dot{\varepsilon}_{ik}=\frac{1}{2}(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i})$ is strain rate tensor. The derivative transformation between Eulerian and Lagrangian is $\frac{D}{Dt}=\frac{\partial}{\partial t}+u_i\frac{\partial}{\partial x_i}$.

At the shock front, there is a discontinuous surface at which there is a discontinuity in variables $\mathbf{p}$, ρ and $\mathbf{u}$. The former equations are not appropriate. But by means of further analysis based on the conservation of mass, momentum and energy, the following leaping equations can be obtained.

$$\Delta\rho v=0 \tag{23}$$

$$\Delta\rho v\mathbf{u}-\mathbf{e_n}\cdot\Delta\mathbf{p}=0 \tag{24}$$

$$\Delta\rho v(\frac{1}{2}\mathbf{u}\cdot\mathbf{u}+E)-\mathbf{e_n}\cdot\Delta(\mathbf{p}\cdot\mathbf{u})=0 \tag{25}$$

Where Δ is the difference of some physical variables (scalar, vector or tensor) between ahead of and behind the shock front surface, $\cdot$ denotes the dot product of two tensors, $v=c_n-u_n$, $\mathbf{e_n}$ is unit vector in normal direction of shock front surface, c_n is the wave velocity, u_n is the particle velocity.

IV. The Normal Expanding Theory

In terms of the assumptions mentioned above. During impact, the responding medium of concrete expands in the external normal direction of the surface of the projectile (especially including the nose part). The particle velocity, the velocity of expanding wave and the pressure have the same direction as the normal direction of projectile-nose surface, hence there are

$$\begin{cases}\mathbf{u}=u_n\mathbf{e}_n\\ \mathbf{c}=c_n\mathbf{e}_n\\ \mathbf{p}=\begin{bmatrix}p_n & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\end{cases} \tag{26}$$

where $\mathbf{c}$ is wave velocity vector.

In this case, the total derivative of density (as a scale) and velocity (as a vector) can be written by

$$\begin{cases}\frac{D\rho}{Dt}=\frac{\partial\rho}{\partial t}+u_n\frac{\partial\rho}{\partial x_n}\\ \frac{D\mathbf{u}}{Dt}=(\frac{\partial u_n}{\partial t}+u_n\frac{\partial u_n}{\partial x_n},\ 0,\ 0)\end{cases} \tag{27}$$

The divergence of velocity (as a vector) and pressure (as a tensor) can be written by

$$\begin{cases}\mathrm{div}\mathbf{u}=\frac{\partial u_n}{\partial x_n}+\frac{u_n}{R+x_n}+\frac{u_n}{\overline{R}+x_n}\\ \mathrm{div}\mathbf{P}=\left(\frac{\partial P_n}{\partial x_n}+\frac{P_n}{R+x_n}+\frac{P_n}{\overline{R}+x_n},0,0\right)\end{cases} \tag{28}$$

where x_n is the coordinate in direction $\mathbf{e_n}$.

For the medium behind the shock front, substituting equation (27) and (28) into equations (20), (21) and (22), leads to

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x_n}(\rho u_n)+\frac{\rho u_n}{R+x_n}+\frac{\rho u_n}{\overline{R}+x_n}=0 \tag{29}$$

$$\rho(\frac{\partial u_n}{\partial t}+u_n\frac{\partial u_n}{\partial x_n})=-(\frac{\partial p_n}{\partial x_n}+\frac{p_n}{R+x_n}+\frac{p_n}{\overline{R}+x_n}) \tag{30}$$

$$\frac{\partial}{\partial t}[(E+\frac{1}{2}u_n{}^2)\rho]+(\frac{\partial}{\partial x_n}+\frac{1}{R+x_n}+\frac{1}{\overline{R}+x_n})[\rho(E+\frac{1}{2}u_n{}^2)u_n+p_nu_n]=0 \tag{31}$$

At the shock front, equations (23), (24) and (25) can be written as

$$(\rho_{sf}-\rho_0)c_n-\rho_{sf}u_n^l=0 \tag{32}$$

$$\rho_{sf}(c_n-u_n^l)u_n^l=p_n^l \tag{33}$$

$$\rho_{sf}(c_n-u_n^l)(E+\frac{1}{2}u_n^{l\,2})-p_n^lu_n^l=0 \tag{34}$$

where ρ_{sf} is density of compressed medium near the shock front, ρ_0 is the original density of the material, u_n^l and p_n^l are the particle velocity of responding medium and the pressure near the shock front.

V. Equations of State and Solutions

To obtain further solutions about equation (32), (33) and (34) and to solve the equation (29), (30) and (31) effectively, an additional equation of state (EOS) is required. For the concrete material concerned in this study, a constitutive relationships about the ultimate density model[9,10] (Rankine-Hugoniot equations) is suggested. In ultimate density model, the density of concrete is constant. In free region of medium, $\rho=\rho_0$, in compressive region of medium undergoing high-pressure impact, $\rho=\rho^*$ where ρ^* is the ultimate density, whose Hugoniot curve is shown in Fig.3.

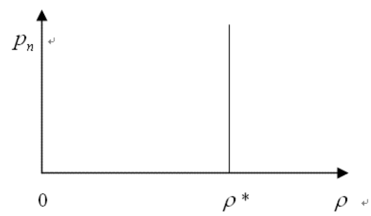

Fig.3. Hugoniot curve of concrete

By means of this model, at the shock front, substituting

$\rho_{sf} = \rho^*$ into equations (32), (33) and (34), leads to

$$c_n = \frac{\rho^*}{\rho^* - \rho_0} u_n^l \quad (35)$$

$$p_n^l = \rho^*(c_n - u_n^l)u_n^l = \rho_0 c_n u_n^l \quad (36)$$

$$E = \tfrac{1}{2} {u_n^l}^2 \quad (37)$$

Behind the shock front, substituting $\rho_{sf} = \rho^*$ into equations (29), (30) and (31), one can obtain the solutions as

$$u_n(x_n, t) = \frac{R\overline{R}}{(R + x_n)(\overline{R} + x_n)} u_n \quad (38)$$

$$P_n(x_n,t) = \frac{\rho^* R\overline{R}}{(R+x_n)(\overline{R}+x_n)}\left\{[\frac{\rho^*}{\rho^*-\rho_0}\frac{R\overline{R}}{(R+l)(\overline{R}+l)} - \frac{R\overline{R}}{(R+x_n)(\overline{R}+x_n)}]u_n^{\ 2} + (l - x_n)\frac{du_n}{dt}\right\} \quad (39)$$

$$\rho^* \frac{DE}{Dt} = -p_n \frac{\partial u_n}{\partial x_n} \quad (40)$$

where l is the propagating distance of the wave front surface relevant to the surface of projectile.

On the surface of projectile-nose, that is for $x_n = 0$, there are

$$u_n = u_n(0,t) \quad (41)$$

$$p_n = A_p u_n^{\ 2} + B_p \frac{du_n}{dt} \quad (42)$$

where $A_p = \rho^*[\frac{\rho^*}{\rho^* - \rho_0}\frac{R\overline{R}}{(R+l)(\overline{R}+l)} - 1]$ and $B_p = \rho^* l$.

$$E = \frac{1}{2} u_n^{\ 2} \quad (43)$$

VI. Dynamic Equations of the Projectile during Penetration

By use of the pressure acting on the surface of the projectile in equation (42), the dynamic penetration equation of the projectile can be written as

$$(m_p + m_f)\ddot{\xi} - \overline{J}_{add}\ddot{\alpha} = -\iint_{S_A}(A_p u_n^{\ 2} + \sigma_d)\cos\varphi ds \quad (44)$$

$$(J_p + J_f)\ddot{\alpha} - \overline{m}_{add}\ddot{\xi} = \iint_{S_A}(A_p u_n^{\ 2} + \sigma_d)\cdot(z^*\sin\varphi - r^*\cos\varphi)\cos\beta ds \quad (45)$$

where m_p is the mass of projectile, σ_d is dynamic compressive limit stress which is expressed by Holmquist et al[13] as $\sigma_d = \sigma_c[1 + 1.6(p_{lock}/\sigma_c)^{0.61}][1 + 0.007\ln(0.1v_0)]$ in which σ_c is static compressive limit stress, p_{lock} is locked pressure of concrete material and v_0 is striking velocity of projectile, J_p is the moment of inertia of the projectile relative to axes r^*. The variable $\ddot{\xi}$ is tangent acceleration of the centroid trajectory of the projectile and its direction is identical with the projectile axes $z*$, $\dot{\alpha}$ is angular velocity of the projectile relative to axes r^*, S_A is the interface curve surface between projectile and target. $m_f = \iint_{S_A} B_p \cos^2\varphi ds$

$J_f = \iint_{S_A} B_p (z^*\sin\varphi - r^*\cos\varphi)^2\cos^2\beta ds$,

$\overline{J}_{add} = \iint_{S_A} B_p \cos\varphi(z^*\sin\varphi - r^*\cos\varphi)\cos\beta ds$,

$\overline{m}_{add} = \iint_{S_A} B_p \cos\varphi(z^*\sin\varphi - r^*\cos\varphi)\cos\beta ds$, the others parameters were shown in Fig1.

The relationship among the normal velocity u_n, axial velocity u_ξ and angular velocity $\dot{\alpha}$ and the relationship among their acceleration are as follows

$$u_n = u_\xi\cos\varphi - \dot{\alpha}(z^*\sin\varphi - r^*\cos\varphi)\cos\beta \quad (46)$$

$$\frac{du_n}{dt} = \ddot{\xi}\cos\varphi - \ddot{\alpha}(z^*\sin\varphi - r^*\cos\varphi)\cos\beta \quad (47)$$

VII. Calculation and Comparison with Experiments

By means of the method, the calculation and comparison with the experiments are made on the deceleration and depth of penetration characteristics of ogive-nose projectile shown in Fig.4 perpendicularly penetrating into thick concrete target, where R is curvative radius of the meridian arc of nose, $2r$ is the calibre diameter of projectile shank, $\psi = R/(2r)$ is the caliber-radius-head (CRH). We conducted the deceleration and depth of penetration experiments with the ogive-nose steel projectile. In the experiments, two kinds of projectiles were used, one has short nose and the other has long nose.

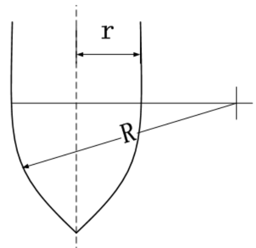

Fig.4 The projectile nose used in experiments

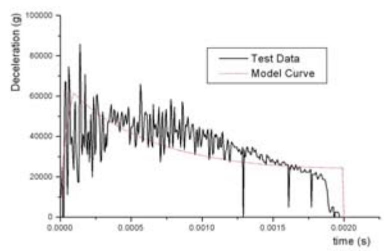

Fig.5 Curves of deceleration from experiment and analytical calculation

The density of concrete target is $\rho = 2400kg/m^3$ and the compressive strength of it is $\sigma_c = 3.0\times10^7 N/m^2$. Referring Reference [13], the locked pressure is token as $p_{lock}/\sigma_c = 16.7$. The limit density of concrete target is $\rho^* = 2640kg/m^3$ which was obtained from experiments[12]. Table 1 summarizes results from 6 experiments for striking velocities range from 538m/s to 763m/s and the lists corresponding calculating results.

Table 1 Data summary for projectiles and results from experiments and calculation

Shot Number	Projectile mass (kg)	R (m)	r (m)	ψ	Striking velocity (m/s)	Penetration depth h(m) from experiment	Penetration depth h(m) from calculation
02-0001	3.777	0.17679	0.031	2.85	763	0.83	0.82
02-0002	3.034	0.09453	0.031	1.53	577	0.34	0.38
02-0003	3.747	0.09453	0.031	1.53	666	0.56	0.57
02-0004	3.022	0.09453	0.031	1.53	538	0.37	0.37
02-0005	3.154	0.09453	0.031	1.53	630	0.46	0.41
02-0006	3.133	0.09453	0.031	1.53	__	0.48	__

The deceleration curves vs time from experiments and analytical calculation for number 02-0003 are shown in Fig.5. From the results, it can be seen that, the results from the analytical calculation by the method in this paper are in good agreement with those from the experiments.

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