Fuzzy Ideals and Fuzzy Quasi-ideals in Ternary Semirings

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Abstract—**We introduce the notions of fuzzy ternary subsemiring, fuzzy ideal and quasi-ideal in ternary semirings and study some properties of these two ideals. We also study the properties of fuzzy ideal and fuzzy quasi-ideal of ternary semirings.**

Keywords: fuzzy semiring, fuzzy ternary subsemiring , fuzzy ideals, fuzzy quasi-ideals, fuzzy zero divisor free

1 Introduction

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces and the like. This provides sufficient motivation to researchers to review various concepts and results from the realm of abstact algebra in the broader framework of fuzzy setting. The notion of fuzzy subgroup was made by Rosenfeld [16] in 1971. Fuzzy ideals in rings were introduced by W. Liu [14] and it has been studied by several authors [2, 10, 11]. Kim and Park [1] and Jun et. al., [8] have also studied fuzzy ideals in semirings.

The theory of ternary algebraic system was introduced by D. H. Lehmer [12]. He investigated certain ternary algebraic systems called triplexes which turn out to be commutative ternary groups. The notion of ternary semigroups was introduced by S. Banach [cf. 15]. He showed by an example that a ternary semigroup does not necessarily reduce to an ordinary semigroups. In [13], W. G. Lister characterized additive subgroups of rings which are closed under the triple ring product and he called this algebraic system a ternary ring. Dutta and Kar [3] introduced and studied some properties of ternary semirings which is a generealization of ternary ring. Steinfeld [17] introduced the notion of quasi ideal and Good and Hughes [7] introduced the notion of bi-ideal. In [9], Kar studied quasi-ideals and bi-ideals of ternary semirings. Some work on ternary semiring may be found in [4, 5, 6]. Quasi-ideals are generalization of right ideals, lateral ideals and left ideals.

Ternary semiring arises naturally as follows—consider the ring of integers Z which plays a vital role in the theory of ring. The subset Z^+ of all positive integers of Z is an additive semigroup which is closed under the ring product i.e. $Z^+$is a semiring. Now, if we consider the subset $Z^-$of all negative integers of Z, then we see that $Z^-$is an additive semigroup which is closed under the triple ring product (however, Z^- is not closed under the binary ring product), i.e. Z^-forms a ternary semiring. Thus, we see that in the ring of integers Z, Z^+ forms a semiring where as Z^- forms a ternary semiring in the fuzzy settings also. The main purpose of this paper is to introduce the notions of ideals and quasi-ideals in fuzzy ternary semirings and study some properties of the fuzzy ternary semirings.

2 Preliminaries

In this section, we review some definitions and some results which will be used in later sections.

Definition 2.1 A set R together with associative binary operations called addition and multiplication (denoted by + and . , respectively) will be called a semiring provided:

(i) Addition is a commutative operation $(a+b)$

(ii) There exists $0 \in R$ such that $x+0=x$ and $x0=0x=0$ for each $x \in R$,

(iii) Multiplication distributes over addition both from the left and the right. i.e., $a(b+c)=ab+ac$ and $(a+b)c=ac+bc$

Definition 2.2 A nonempty set S together with a binary operation, called addition and a ternary multiplication, denoted by juxtaposition, is said to be a ternary semiring if S is an additive commutative semigroup satisfying the following conditions:

(i) $(abc)de=a(bcd)e=ab(cde)$

(ii) $(a+b)cd=acd+bcd$

(iii) $a(b+c)d=abd+acd$

(iv) $ab(c+d)=abc+abd$, for all $a,b,c,d,e \in S$.

Definition 2.3 Let S be a ternary semiring. If there exists an element $0 \in S$ such that $0+x=x$ and

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$0xy{=}x0y{=}xy0{=}0$ for all$x{,}y{\in}S$, then "0" is called the zero element or simply the zero of the ternary semiring S. In this case we say that S is a ternary semiring with zero.

Definition 2.4 An additive subsemigroup I of S is called a left (resp., right, and lateral) ideal of S if s_1s_2i (resp. is_1s_2, s_1is_2))$\in I$, for alls_1, $s_2 \in S$ and i$\in I$. If I is both left and right ideal of S, then I is called a two-sided ideal of S. If I is a left, a right, a lateral ideal of S, then I is called an ideal of S.

An ideal I of S is called a proper ideal if $I \neq S$.

Definition 2.5 An additive subsemigroup Q of a ternary semiring S is called a quasi ideal of S if $QSS \cap (SQS + SSQSS) \cap SSQ \subseteq Q$

We now review some fuzzy logic concepts. A function A from a non-empty set X to the unit interval [0, 1] is called a fuzzy subset of X [18].

Definition 2.6 A fuzzy ideal of a semiring R is a function $A{:}R \longrightarrow$[0,1] satisfying the following conditions:

(i) A is a fuzzy subsemigroup of (R,+); i.e., $A(x-y) \geq \min\{A(x), A(y)\}$,

(ii) $A(xy) \geq \max\ \{A(x), A(y)\}$, for all $x{,}y \in R$

Definition 2.7 A and B be any two subsets of S. Then $A{\cap}B$, $A{\cup}B$, $A{+}B$ and $A{\circ}B$ are fuzzy subsets of S defined by

$$(A \cap B) = min\{A(x), B(x)\}$$

$$(A \cup B) = max\{A(x), B(x)\}$$

$$(A+B)(x) = \begin{cases} sup\{min\{A(a), A(b)\}, & \text{if } x = y + z, \\ 0, & \text{otherwise} \end{cases}$$

$$(AB)(x) = \begin{cases} sup\{min\{A(a), A(b)\}, & \text{if } x = yz, \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.8 For any $x{\in}S$ and $t{\in}(0, 1]$, define a *fuzzy point* x_t as

$$x_t(y) = \begin{cases} t, & \text{if } y = x \\ 0, & \text{if } y \neq x \end{cases}.$$

If x_t is a fuzzy point and A is any fuzzy subset of S and $x_t{\leq}A$, then we write $x_t{\in}A$. Note that $x_t{\in}A$ if and only if $x{\in}A_t$ where A_t is alevel subset of A. If x_r and y_s are fuzzy points, than $x_ry_s{=}(xy)_{min\{r,s\}}$.

3 Fuzzy ideals in Ternary Semiring

In this section, we introduce the fuzzy ideal in the ternary semiring S. Throughout this paper unless stated otherwise S is a commutative semiring with unity.

Definition 3.1 A fuzzy subset A of a fuzzy subsemigroup of S is called a fuzzy ternary subsemiring of S if

(i) $A(x-y) \geq min\{A(x), A(y)\}$, for all $x, y \in S$

(ii) $A(-x){=}A(x)$

(iii) $A(xyz){\geq}min\{A(x){,}A(y){,}A(z)\}$, for all $x, y, z{\in}S$.

Definition 3.2 A fuzzy subsemigroup A of a ternary semiring S is called a fuzzy ideal of S is a function $A : S \longrightarrow$[0,1] satisfying the following conditions:

(i) $A(x-y) \geq min\{A(x), A(y)\}$, for all x, $y \in S$

(ii) $A(xyz) \geq A(z)$

(iii) $A(xyz) \geq A(x)$ and

(iv) $A(xyz) \geq A(y)$, for all $x, y, z \in S$

A fuzzy subset A with conditions (i) and (ii) is called an fuzzy left ideal of S. If A satisfies (i) and (iii), then it is called an fuzzy right ideal of S. Also if A satisfies (i) and (iv), then it is called an fuzzy lateral ideal of S.
A fuzzy ideal a ternary semiring of S, if A is both of a fuzzy left, a fuzzy right and a fuzzy lateral ideal of S. It is clear that A is a fuzzy ideal of a ternary semiring S if and only if $A(xyz) \geq max\ \{A(x), A(y), A(z)\}$ for all $x, y, z \in S$, and that every fuzzy left (right, lateral) ideal of S is a fuzzy subsemigroup of S.

Example 3.3 Let $\boldsymbol{Z}$ bea ring of integers and $S{=}\boldsymbol{Z}_0^-{\subset}\boldsymbol{Z}$ be the set of all negative integers with zero. Then with the binary addition and ternary multiplication,($\boldsymbol{Z}_0^-$,+, .) forms a ternary semiring S with zero. Define a fuzzy subset $A{:}Z{\longrightarrow}$[0,1], we have

$$A(x) = \begin{cases} 1, & \text{if } x{\in}\boldsymbol{Z}_0^- \\ 0, & \text{otherwise} \end{cases}$$

Then A is a fuzzy ternary subsemiring of S.

Example 3.4 Consider the set integer modulo 5, non-positive integer Z_5^-={0,-1,-2,-3,-4} with the usual addition and ternary multiplication, we have

+	0	-1	-2	-3	-4
0	0	-1	-2	-3	-4
-1	-1	-2	-3	-4	0
-2	-2	-3	-4	0	-1
-3	-3	-4	0	-1	-2
-4	-4	0	-1	-2	-3

·	0	-1	-2	-3	-4
0	0	0	0	0	0
-1	0	1	2	3	4
-2	0	2	4	1	3
-3	0	3	1	4	2
-4	0	4	3	2	1

·	0	1	2	3	4
0	0	0	0	0	0
-1	0	-1	-2	-3	-4
-2	0	-2	-4	-1	-3
-3	0	-3	-1	-4	-2
-4	0	-4	-3	-2	-1

Clearly (Z_5^-, +, .) is a ternary semiring. Let a fuzzy subset A:$Z_5^- \longrightarrow$[0,1] be defined by $A(0)$=t_0 and $A(-1)$ = $A(-2)$ =$A(-3)$= $A(-4)$=t_1, where $t_0 \geq t_1$ and t_0,$t_1 \in$[0, 1]. By routine calculations given that A is a fuzzy ideal of Z_5^-.

Lemma 3.5 Let A be a fuzzy ideal of a ternary semiring S, then $A(xy) \leq A(0)$ for all $x, y \in S$.
Proof: For any $x, y \in S$, $A(0) = A(xy0) \geq A(xy)$.

Theorem 3.6 Let I be an ideal of a ternary semiring S if and only if the characteristic function λ_I is a fuzzy ideal of ternary semiring S
Proof: Let I be an ideal of a ternary semiring S. Then λ_I is a fuzzy ideal in the sense of Definition 3.2. Conversely, let λ_I be a fuzzy ideal of a ternary semiring S. For any x, $y \in I$, we have $\lambda_I(x + y) \geq min$ $\{\lambda_I(x)$, $\lambda_I(y)\}$= $min\{1,\ 1\}$=1. Thus $x + y \in I$. Now let x, $y \in S$ and $a \in I$. Then $\lambda_I(xya) \geq \lambda_I(a)$ =1, which implies that $xya \in I$. Thus I is a left ideal of ternary semiring S. Similarly $axy \in I$ and $xay \in I$ be a right and lateral ideal of ternary semiring S. Consequently, I is an ideal of ternary semiring S.

Definition 3.7 Let A be a fuzzy subset of a ternary semiring S. Then the set A_t= $\{x, y \in S \mid A(xy) \geq A(x) \geq t\}$($t \in$[0,1]) is called the level subset of S with respect to A.

Theorem 3.8 Let A be a fuzzy left (right, lateral) ideal of a ternary semiring S. Then the level set A_t ($t \leq A(0)$) is the left (right, lateral) ideal of ternary semiring S.
Proof: Let x,y,$z \in A_t$. Then $A(x) \geq t$, $A(y) \geq t$ and $A(z) \geq t$. Since $A(x + y) \geq min\{A(x), A(y)\} \geq t$, $x + y \in A_t$. Similarly, $A(y + z) \in min\{A(y), A(z)\} \geq t$, $y + z \in A_t$ and $A(z + x) \geq min\{A(z), A(x)\}$, $z + x \in A_t$.On the other hand, if $x, y \in A_t$,$xy \in A_t$ and $z \in S$, then $A(zxy) \geq A(y) \geq t$ ($A(xyz) \geq A(x) \geq t$, $A(xzy) \geq A(z) \geq t$) so that $zxy \in A_t$ ($xyz \in A_t$, $xzy \in A_t$. Hence A_t is a left (right, lateral) ideal of S.

Definition 3.9 A fuzzy subset A of a ternary semiring S is called a fuzzy zero divisor free ($FZDF$) if for $x, y, z \in S$, $A(xyz)$=$A(0)$ implies that $A(x)$=$A(0)$ or $A(y)$=$A(0)$ or $A(z)$=$A(0)$

Definition 3.10 A fuzzy subset A of a ternary semiring S is called

(i) fuzzy multiplicatively left cancellative ($FMLC$) if $A(abx)$=$A(aby)$ implies that $A(x - y)$=$A(0)$

(ii) fuzzy multiplicatively right cancellative ($FMRC$) if $A(xab)$=$A(yab)$ implies that $A(x - y)$=$A(0)$

(iii) fuzzy multiplicatively laterally cancellative ($FMLLC$) if $A(axb)$=$A(ayb)$ implies that $A(x-y)$= $A(0)$, for all a, b, x, $y \in S$.

A fuzzy subset A of a ternary semiring S is called a fuzzy multiplicatively cancellation (FMC) if it is $FMLC$, $FMRC$ and $FMLLC$. Clearly, a fuzzy multiplicatively cancellation (FMC) A of a ternary semiring S is $FZDF$.

Definition 3.11 A fuzzy subset A of a ternary semiring S($|S| \geq 2$) is said to be fuzzy ternary division semiring if for any nonzero element x of S and there exists a nonzero element y of S such that $A(xyz)$=$A(yxz)$=$A(zxy)$=$A(zyx)$=$A(z)$ for all $z \in S$.

4 Fuzzy Quasi-ideals in Ternary Semiring

Definition 4.1 Let A be a fuzzy subset of ternary semiring S. We define $SAS + SSASS$

$$= \begin{cases} sup\{min\{A(a), A(b)\}, & \textit{if } z = x(a{+}xby)y, \\ & \textit{for all } x,\ y,\ a,\ b \in S \\ 0, & \textit{otherwise} \end{cases}$$

Definition 4.2 A fuzzy subsemigroup A of a ternary semiring S is called a fuzzy quasi-ideal of S if
$\{ASS \cap (SAS + SSASS) \cap SSA\} \leq A$. that is
$A(x) \geq min\{(ASS))(x), (SAS + SSASS)(x), (SSA)(x)\}$

Example 4.3 Consider the ternary semiring (Z_5^-,+, .) as defined in Example 3.4. Let A = $\{0, -2, -3\}$. Then SSA = $\{-2, -3, -4\}$, $(SAS + SSASS)$ = $\{0, -1, -2, -3\}$ and ASS = $\{-1, -2, -3\}$. Therefore $ASS \cap (SAS{+}SSASS) \cap SSA = \{-2, -3\} \subseteq A$. Hence A is a quasi-ideal of Z_5^-.Define a fuzzy subset A:$Z_5^- \longrightarrow$ [0,1] by $A(0) = A(-2) = A(-3) = 1$ and $A(-1) = A(-4) = 0$. Clearly A is a fuzzy quasi-ideal of Z_5^-.

Theorem 4.4 Let A be a fuzzy subset of S. If A is a fuzzy left ideal (right ideal and lateral ideal) of S, then A is a fuzzy quasi-ideal of S.
Proof: Let A be a fuzzy left ideal of S. Let $x = as_1s_2 = s_1(b + s_1cs_2)s_2 = s_1s_2d$, where b, y_1, y_2, d, s_1 and s_2 are in S. Consider $(ASS \cap (SAS + SSASS) \cap SSA)(x)$

$$= min\{(ASS)(x), (SAS + SSASS)(x), (SSA)(x)\}$$

$$= min\{sup_{x=as_1s_2}\{A(a)\}, sup_{x=s_1(b+s_1cs_2)s_1} min\{A(b), A(c)\},$$

$sup_{x=s_1s_2d}\{A(d)\}\}$

$\leq$ $\{1, sup_{x=s_1(b+s_1cs_2)s_1} min\{A(s_1(b+s_1cs_2)s_1)\}, 1\}$ (as A is a fuzzy left ideal, $A(s_1(b+s_1cs_2)s_1) \geq min\{A(b), A(c)\} = A(b), (A(c))$, if $A(b) < A(c), (A(b) > A(c)) \leq A(x)$.

Theorem 4.5 For any nonempty subsets A, B and C of S,

(1) $f_A f_B f_C = f_{ABC}$;

(2)$f_A \cap f_B \cap f_C = f_{A\cap B\cap C}$;

(3) $f_A + f_B = f_{A+B}$.

Proof: Proof is straight forward.

Theorem 4.6 Let Q be a additive subsemigroup of S. If Q is a quasi-ideal of S if and only if f_Q is a fuzzy quasi-ideal of S.
Proof: Assume that Q is a quasi-ideal of S. Then f_Q is a fuzzy subsemigroup of S

$$(f_Q SS) \cap (Sf_Q S + SSf_A SS) \cap (SSf_Q)$$

$$= (f_Q f_S f_S) \cap (f_S f_Q f_S + f_S f_S f_Q f_S f_S) \cap (f_S f_S f_Q).$$

$$= f_{QSS} \cap f_{(SQS+SSQSS)} \cap f_{SSQ}$$

$$= f_{QSS\cap(SQS+SSQSS)\cap SSQ} \subseteq f_Q$$

. This means that f_Q is a fuzzy quasi-ideal of S. Conversely, let us assume that f_Q is a fuzzy quasi-ideal of S. Let x be any element of $QSS \cap (SQS + SSQSS) \cap SSQ$. Then, we have

$$f_Q(x) \geq \{(f_Q SS \cap (Sf_Q S + SSf_Q SS) \cap SSf_Q)(x)\}$$

$$= min\{(f_Q SS)(x), (Sf_Q S + SSf_Q SS)(x), (SSf_Q)(x)\}$$

$$= min\{(f_{QSS}(x), f_{(SQS+SSQSS)}(x), f_{SSQ}(x)\}$$

$$= f_{QSS\cap(SQS+SSQSS)\cap SSQ}(x) = 1$$

This implies that $x \in Q$, and so

$$QSS \cap (SQS + SSQSS) \cap SSQ \subseteq Q$$

. This means that Q is a quasi-ideal of S.

Theorem 4.7 Let A be a fuzzy subset of S. If A is a fuzzy quasi-ideal of S, if and only if A_t is an quasi-ideal of S, for all $t\in Im(A)$.
Proof: Let A be a fuzzy quasi-ideal of S. Let $t\in Im(A)$. Suppose x, $y\in S$ such that x, $y,\in A_t$. Then $A(x)\geq t$, $A(y)\geq t$, and $min\{A(x), A(x)\}\geq t$. As A is a fuzzy quasi-ideal, $A(x-y)\geq t$ and thus $x-y\in A_t$. Suppose $a\in A_t SS\cap(SA_t S+SSA_t SS)\cap SSA_t$. Then their exist, x, y_1, y_2, $z\in A_t$ and s_1, $s_2\in S$ such that $a=xs_1s_2$ $=s_1(y_1+s_1y_2s_2)s_2=s_1s_2z$. Then $(ASS\cap(SAS+SSASS)\cap SSA)(a)=min\{(ASS)(a)$, $(SAS+ SSASS)(a)$, $(SSA)(a)\}$. Now,

$$(ASS)(a) = sup_{a=xs_1s_2}\{A(x)\}$$

$$(SAS + SSASS)(a) = sup_{a=s_1(y_1+s_1y_2s_2)s_2}$$
$$\{min\{A(y_1), A(y_2)\}\}$$

$$(SSA)(a) = sup_{a=s_1s_2z}\{A(z)\}$$

Therefore, $min\{(ASS)(a)$,$(SAS+SSASS)(a)$,$(SSA)(a)\}$ $\geq t$ and thus $(ASS\cap(SAS+SSASS)\cap SSA)(a)\geq t$. As A is an quasi-ideal of S, $A(a)\geq t$ implies $a\geq A_t$.. Hence A_t is an quasi-ideal in S. Conversely, let us assume that A_t is an quasi-ideal of S, $t\in Im(A)$. Let $p\in S$. Consider

$$(ASS \cap (SAS + SSASS) \cap SSA)(p)$$

$$= min\{(ASS)(p), (SAS + SSASS)(p), (SSA)(p)\}$$

$$= min\{sup_{p=xs_1s_2} min\{A(x), S(s_1), S(s_2)\},$$

$$sup_{p=s_1(y_1+s_1y_2s_2)s_2}\{min\{A(y_1), A(y_2)\}\},$$

$$sup_{p=s_1s_2z} min\{S(s_1), S(s_2), A(z)\}\}$$

$$= min\{sup_{p=xs_1s_2}\{A(x)\}, sup_{p=s_1(y_1+s_1y_2s_2)s_2}\{min$$

$$\{A(y_1), A(y_2)\}\}, sup_{p=s_1s_2z}\{A(z)\}\}$$

. Let $sup_{p=xs_1s_2}\{A(x)\} = t_1$, $sup_{p=s_1(y_1+s_1y_2s_2)s_2} min$ $\{A(y_1)$,$A(y_2)\} = sup\ min\{t_2, t_3\} = t_2$, if $t_2 < t_3$ or t_3, if $t_2 > t_3$, and $sup_{p=s_1s_2z}\{A(z)\} = t_4$ for any x, y_1, y_2, z, s_1, $s_2 \in S$. Let $t_2 < t_3$, assume that $min\{t_1, t_2, t_4\} = t_1$. Then $x, y_1, z \in A_{t_1}$. Since A_{t_1} is a quasi-ideal of S, then $p = xs_1s_2 \in A_{t_1}SS$, $p = s_1(y_1 + s_1y_2s_2)s_2 \in (SA_{t_1}S + SSA_{t_1}SS)$, and $p = s_1s_2z \in SSA_{t_1}$. This implies $p \in A_{t_1}SS \cap (SA_{t_1}S + SSA_{t_1}SS) \cap SSA_{t_1} \subseteq A_{t_1}$. Thus, $A(x) \geq t_1 = min\{t_1, t_2, t_4\}$. Hence, $(ASS \cap (SAS + SSASS) \cap SSA)(p) \leq t_1 \leq A(p)$. Similarly, if we take $min\{t_1, t_2, t_4\} = t_2$ or t_3, we can prove that $(ASS \cap (SAS + SSASS) \cap SSA)(p) \leq t_2$ or $t_3 \leq A(p)$. Let $t_2 > t_3$ assume that $min\{t_1, t_3, t_4\} = t_3$. Then $x, y_2, z \in A_{t_3}$. Since A_{t_3} is a quasi-ideal of S, then $p = xs_1s_2 \in A_{t_3}SS$, $p = s_1(y_1 + s_1y_2s_2)s_2 \in (SA_{t_3}S + SSA_{t_3}SS)$, and $p = s_1s_2z \in SSA_{t_3}$. This implies $p \in A_{t_3}SS \cap (SA_{t_3}S + SSA_{t_3}SS) \cap SSA_{t_3} \subseteq A_{t_3}$. Thus, $A(x) \geq t_3 = min\{t_1, t_3, t_4\}$. Hence, $(ASS \cap (SAS + SSASS) \cap SSA)(p) \leq t_3 \leq A(p)$. Thus $(ASS\cap(SAS+SSASS)\cap SSA)(p) \leq A(p)$, for all $p \in S$. This shows that A is a fuzzy quasi-ideal of S.

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