Approximating Discrete Models for a Two-degree-of-freedom Friction System

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***Abstract* — In this paper, a two-degree-of-freedom mass-on-belt friction dynamical system is considered. Two reconstructed deterministic and stochastic discrete models have been established. The numerical calculations of an example have shown that chaos may occur in this system, and noise can change the non-linear behavior of the system. The example has resulted that the stochastic discrete model may be used in some applications.**

***Index Terms*—Friction system, Stick-slip Phenomenon, Poincaré map, Bifurcation, Chaos**

I. Introduction

In dynamical systems, there are many nonlinear effects caused by friction. One of the most special nonlinear effects is stick-slip phenomenon. Stick-slip phenomenon is firstly found by Den Hartog [1]. But the term “stick-slip” was firstly coined in 1939 by FP Boweden and LL Leben, who had built an apparatus at the University of Cambridge in order to study the phenomenon [2]. Stick-slip phenomenon may be explained that stick motion is in contact with the surface due to static friction; and slip motion is in contact with the surface due to sliding friction. In oscillatory motions both of the phenomena can take place successively, resulting in a stick-slip mode.

Since the friction characteristic consists of two qualitatively different parts with a nonsmooth transition, the resulting motion also has nonsmooth behaviour. Therefore, stick-slip systems belong to the class of nonsmooth systems. Popp and Stelter [3], [4] and Feeny and Moon [5], [6] observed stick-slip chaos in simple oscillators whose nonlinearity is only due to dry friction. In these systems, the motion collapses during stick-slip, leading to a one-dimensional map in the Poincaré section. Popp also pointed out that simulation time by Poincaré mapping model is only 1/1000 of the simulation time by ordinary computer method [7]. For a geophysical fault model, Galvanetto et al [8], [9] studied the chaotic and quasi-periodic behavior of the two-mass system in contact with a moving surface. With similar to the systems, they described a one-dimensional map generated by a two degree-of-freedom mechanical system that undergoes self-sustained oscillations induced by dry friction [10].

Up to now, there are already a lot of papers reported the phenomenon for deterministic friction systems. An important issue which has been paid somewhat less attention in the literature of friction and dynamic systems is the stochastic nature of dynamic surface interactions. The commonly used static rough surface models were proposed by Greenwood and Williamson [11]–[14]. Their observation that friction is a random process is important and it can explain experimental data. A number of recent articles have concluded that random friction force fluctuations of sufficient magnitude can indeed alter the qualitative characteristic of the dynamic response, i.e., change the stability of an equilibrium configuration [15]–[18]. References [19], [20] derived a mean Poincaré map for random systems with one friction interface, and showed that random perturbations may break the limit cycle, leading to chaos. But to derive a discrete model for random systems with two or more friction interfaces has not been investigated.

In this paper, the system used in Reference [10] is considered. Two reconstructed deterministic and stochastic discrete models have been established. The numerical calculations of an example have shown that chaos may occur in this system, and noise can change the non-linear behavior of the system. The example has shown that the stochastic discrete model may be used in some applications.

II. The Dynamical System

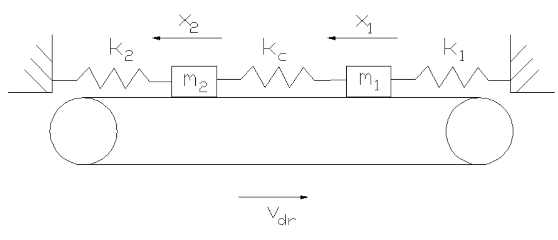

Fig.1 the mechanical system

The mechanical system investigated in the present paper, as shown in Fig. 1, has been the object of attention in several recent publications [10], [21]–[22]. The physical model consists of two blocks supported by a moving belt. Each block is connected to a fixed support by a linear spring and the two blocks are connected together by a third linear spring. The contact surfaces between the blocks and the belt are considered

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rough with different friction coefficients so that two different friction forces are acting on the two slipping surfaces respectively.

The dynamics of the mechanical system in Fig. 1 have been defined in a non-dimensional form [21]: X_1, V_1, and X_2, V_2 are the displacements and velocities of the first and the second block, respectively. The limit conditions that indicate the route from stick-motion to slip-motion are given by

$$X_1 + \alpha(X_1 - X_2) = \pm 1 \tag{1a}$$

$$X_2 + \alpha(X_2 - X_1) = \pm \beta \tag{1b}$$

When the blocks are pulled by the belt their velocities are constantly equal to the belt speed, while the blocks slip their motions are described by the equations

$$\ddot{X}_1 + X_1 + \alpha(X_1 - X_2) = \pm \mu(V_{r1}) \tag{2a}$$

$$\ddot{X}_2 + X_2 + \alpha(X_2 - X_1) = \pm \beta\mu(V_{r2}) \tag{2b}$$

In which α is the ratio of the stiffness of the coupling spring to the stiffness of the two other springs, β is the ratio of the maximum static friction forces acting on the second block to one acting on the first block. And $V_{ri} = (V_i - V_{dr})$ (i=1,2) are the relative velocity of the i-th block with respect to the belt. V_{dr} is the velocity of the belt, and is constant. μ indicates friction factor. All these quantities are dimensionless [21].

Consider the corresponding linear homogeneous equations

$$\ddot{\tilde{X}}_1 + \tilde{X}_1 + \alpha(\tilde{X}_1 - \tilde{X}_2) = 0 \tag{3a}$$

$$\ddot{\tilde{X}}_2 + \tilde{X}_2 + \alpha(\tilde{X}_2 - \tilde{X}_1) = 0 \tag{3b}$$

The linear system has two natural frequencies $\omega_i, i = 1,2$, and their displacements and velocities can be expressed as

$$\begin{aligned}\tilde{X}_i &= C_{i1}\cos(\omega_1 - \psi_1) \\ &\quad + C_{i2}\cos(\omega_2 - \psi_2)\end{aligned} \tag{4a}$$

$$\begin{aligned}\tilde{V}_i &= -[C_{i1}\omega_1 \sin(\omega_1 - \psi_1) \\ &\quad + C_{i2}\omega_2 \sin(\omega_2 - \psi_2)]\end{aligned} \tag{4b}$$

Assuming $\alpha = 1.2$ and $\beta = 1.3$, it can be obtained that $\omega_1 = 1$ and $\omega_2 = 1.8439$.

The stretch length is defined by

$$d = X_2 - X_1 \tag{5}$$

When the distance between the blocks is larger than the length of the un-stretched coupling spring, i.e. the coupling spring is in tension, d is positive, whereas negative means that the coupling spring is in compression.

III. Poincaré maps

Here it is assumed that for the considered system, stick-slip motion exists. In following way, the deterministic and stochastic discrete models will be established respectively.

A. *Approximating deterministic discrete model*

(1) At stick-mode

By subtracting (1a) from (1b), it can be obtained that

$$d_i = \frac{\pm\beta \mp 1}{1 + 2\alpha}, \quad i = 1,2,3,4 \tag{6}$$

(2) At slip-mode

The friction factor here is considered as in [10].

$$\mu(V_{ri}) = 1/(1 + \gamma|V_i - V_{dr}|);$$

γ is the shape coefficient of the dynamic friction law. While γ is small, first-order approximation of friction factor can be approached in the following form:

$$\mu(V_{ri}) = 1 - \gamma|V_i - V_{dr}| \tag{7}$$

Subtraction of (2a) from (2b) gives

$$\begin{aligned}&\ddot{d} + (1 + 2\alpha)d = \pm[(\beta \mp 1)(1 + \gamma V_{dr}) \\ &\quad \mp \gamma\beta\dot{d} + (\beta \mp 1)V_1]\end{aligned} \tag{8}$$

Let

$$\begin{aligned}&\gamma\beta = 2n, \quad p^2 = 1 + 2\alpha \\ &c^{\pm} = (\beta \mp 1), \quad h = 1 + \gamma V_{dr}\end{aligned} \tag{9}$$

The stability equation can be obtained as

$$\ddot{d} + 2n\dot{d} + p^2 d = c^{\pm}[h + V_1] \tag{10}$$

Assuming

$$V_1 \cong \tilde{V}_1 + \varepsilon_0 \eta(t) \tag{11}$$

where $\tilde{V}_1 = \sum_1^2 C_{2i}\omega_i \sin(\omega_i t + \psi_i)$ is the solution of the corresponding linear system (3), and the initial conditions for elections are $C_{2i} = \varepsilon_i, \psi_i = 0$. $\eta(t)$ is the random perturbation, and considered in general as a white noise, which satisfies

$$E[\eta(t)] = 0,\ E[\eta(t)\eta(\tau)] = \delta(t - \tau).$$

First consider the deterministic case, in which $\varepsilon_0 = 0$, then

$$\ddot{d} + 2n\dot{d} + p^2 d = c^{\pm}\left(h + \sum_1^2 \varepsilon_i\omega_i \sin\omega_i t\right) \tag{12}$$

The solutions of (12) are given by

$$\begin{cases} d = A\exp(-nt)\sin(p_n t + \theta) \\ \quad + \sum_1^2 B_i^{\pm}\sin(\omega_i t - \varphi_i) + B_0^{\pm} \\ \dot{d} = A\exp(-nt)[-n\sin(p_n t + \theta) \\ \quad + p_n\cos(p_n t + \theta)] \\ \quad + \sum_1^2 B_i^{\pm}\omega_i\cos(\omega_i t - \varphi_i) \end{cases} \tag{13}$$

where

$$B_0^{\pm} = \frac{c^{\pm} h}{(1+2\alpha)}$$

$$B_{1i}^{\pm} = \frac{c^{\pm}\varepsilon_i \omega_i}{\sqrt{\left(p^2-\omega_i^2\right)^2 + 4n^2\omega_i^2}} \tag{14}$$

$$\varphi_i = arctg\frac{2n\omega_i}{p^2-\omega_i^2}$$

$$p_n = \sqrt{p^2-n^2}$$

(3) The deterministic Poincaré map

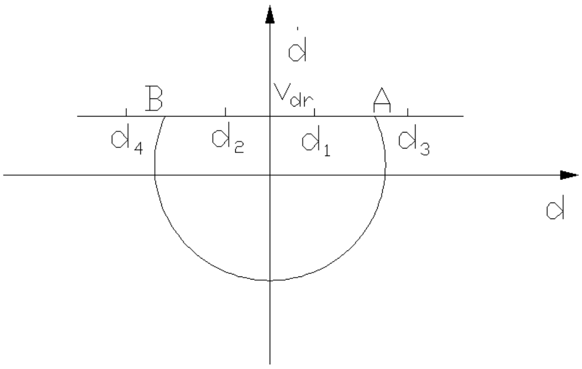

Fig.2 A probable limit cycle of stick-slip motion

When $\dot{d} \le V_{dr}$, the limit cycle is shown in Fig. 2.

At Stick-mode from B to A, it has

$$d_{Ak} - d_{Bk} = V_{dr}\Delta t_{k1} \tag{15}$$

$$\Delta t_{k1} = \min\left|d_i - d_{Bk}\right|/V_{dr} \tag{16}$$

$$d_{Ak} = d_{Bk} + V_{dr}\Delta t_{k1}$$

At transition point A, it has

$$\begin{cases} d_{Ak} = A_k \sin(p_n t_{k1} + \theta_k) \\ \quad + \sum_1^2 B_i^{\pm} \sin(\omega_i t_{k1} - \varphi_i) + B_0^{\pm} \\ V_{dr} = A_k[-n\sin(p_n t_{k1} + \theta_k) \\ \quad + p_n \cos(p_n t_{k1} + \theta_k)] \\ \quad + \sum_1^2 B_i^{\pm}\omega_i \cos(\omega_i t_{k1} - \varphi_i) \\ t_{k1} = t_k + \Delta t_{k1} \end{cases} \tag{17}$$

The solutions of (17) are found to be

$$\theta_k = arcctg\left[\frac{n}{p_n} + \frac{V_{dr} - \sum_1^2 B_i^{\pm}\omega_i \cos(\omega_i t_{k1} - \varphi_i)/p_n}{d_{Ak} - \sum_1^2 B_i^{\pm}\sin(\omega_i t_{k1} - \varphi_i) - B_0^{\pm}}\right] - p_n t_{k1} \tag{18}$$

$$A_k = \frac{d_{Ak} - \sum_1^2 B_i^{\pm}\sin(\omega_i t_{k1} - \varphi_i) - B_0^{\pm}}{\sin(p_n t_{k1} + \theta_k)}$$

At transition point B, it has

$$t_{k+1} = t_{k1} + \Delta t_{k2} = t_k + \Delta t_{k1} + \Delta t_{k2}$$

$$d_{Bk+1} = A_k \exp(-n\Delta t_{k2})\sin(p_n t_{k+1} + \theta_k) + \sum_1^2 B_i^{\pm}\sin(\omega_i t_{k+1} - \varphi_i) + B_0^{\pm} \tag{19}$$

In which Δt_{k2} can be solved by the following equation

$$A_k \exp(-n\Delta t_{k2})\{-n\sin\left[p_n\left(t_{k1}+\Delta t_{k2}\right)+\theta_k\right] + p_n\cos\left[p_n\left(t_{k1}+\Delta t_{k2}\right)+\theta_k\right]\} + \sum_1^2 B_i^{\pm}\cos\left[\omega_i\left(t_{k1}+\Delta t_{k2}\right)+\varphi_i\right] = V_{dr} \tag{20}$$

The Poincaré map for transition point B can be written in the following form:

$$\begin{cases} d_{Bk+1} = [d_{Bk} + V_{dr}\Delta t_{k+1} - \sum_1^2 B_i^{\pm}\sin(\omega_i t_{k1} - \varphi_i) - B_0^{\pm}] \\ \quad \exp(-n\Delta t_{k2}) \times \sin(p_n t_{k+1} + \theta_k)/\sin(p_n t_{k1} + \theta_k) \\ \quad + \sum_1^2 B_i^{\pm}\sin(\omega_i t_{k+1} - \varphi_i) + B_0^{\pm} \\ \qquad t_{k+1} = t_k + \Delta t_{k1} + \Delta t_{k2} \end{cases} \tag{21}$$

in which Δt_{k1}, Δt_{k2} can be obtained by (16) and (18).

B. Approximate stochastic discrete model

(1) During slip-mode from A to B

Adding a random perturbation in equation (12), it becomes

$$\ddot{d}_s + 2n\dot{d}_s + p^2 d_s = c^{\pm}\left(h + \sum_1^2 \varepsilon_i\omega_i \sin\omega_i t\right) + \varepsilon_0\eta(t) \tag{22}$$

where $\eta(t)$ is white noise; ε_0 is a small parameter.

Supposing $x_1 = d_s$; $x_2 = \dot{d}_s$ and the state vector $x = [x_1, x_2]^T = [d_s, \dot{d}_s]^T$

The state equation of (12) is given by

$$\dot{x} = Ax + R^{\pm} + g\eta(t) \tag{23}$$

$$A = \begin{bmatrix} 0 & 1 \\ -p^2 & -2n \end{bmatrix}$$

$$R^{\pm} = \begin{Bmatrix} 0 \\ \sum_1^2 B_i^{\pm} \sin \omega_i t + B_0^{\pm} \end{Bmatrix} \tag{24}$$

$$g = \begin{Bmatrix} 0 \\ \varepsilon_0 \end{Bmatrix}$$

Let $m_i = E[x_i];$
$K_{ij} = E[(x_i - m_i)(x_j - m_j)]; \quad i, j = 1,2$

The first and the second moment equations of (23) can be expressed as

$$\dot{m} = Am + R^{\pm}$$
$$\dot{K} = AK + KA + gg^T \tag{25}$$

It can be seen that the first equation of (25) is similar to (12). It means that (23) has the same mean value as the deterministic equation (12).

$$E[d_s] = m_1 = d$$
$$E[\dot{d}_s] = m_2 = \dot{d} \tag{26}$$

Considering the second equation in (23), it has

$$\sigma_1 = \sqrt{K_{11}} = \frac{\varepsilon_0}{2p\sqrt{n}}$$
$$\sigma_2 = \sqrt{K_{22}} = \frac{\varepsilon_0}{2\sqrt{n}} \tag{27}$$

$$[d]_{\max} = [x_1]_{\max} = m_1 + 3\sigma_1 \tag{28a}$$

$$[\dot{d}]_{\max} = [x_2]_{\max} = m_2 + 3\sigma_2 \tag{28b}$$

(2) During stick-mode from B to A

$$\Delta t_{k1} = \min \left| d_i - [d_{sBk}]_{\max} \right| / V_{dr} \tag{29}$$

$$[d_{sAk}]_{\max} = [d_{sBk}]_{\max} + V_{dr} \Delta t_{k1}$$
(30)

(3) The random Poincaré map of maximal value

For the transition point B, the map of maximal value of relative displacement can be obtained by

$$\begin{cases} [d_{sBk+1}]_{\max} = [d_{sBk}]_{\max} + V_{dr} \Delta t_{k+1} \\ -\sum_1^2 B_i^{\pm} \sin(\omega_i t_{k1} - \varphi_i) - B_0^{\pm} - 3\sigma_1] \exp(-n\Delta t_{k2}) \times \\ \sin(p_n t_{k+1} + \theta_k) / \sin(p_n t_{k1} + \theta_k) \\ + \sum_1^2 B_i^{\pm} \sin(\omega_i t_{k+1} - \varphi_i) + B_0^{\pm} + 3\sigma_1 \\ t_{k+1} = t_k + \Delta t_{k1} + \Delta t_{k2} \end{cases} \tag{31}$$

where Δt_{k1} is obtained from (29), and Δt_{k2} can be obtained from the following relation

$$\begin{aligned} &A_k \exp(-n\Delta t_{k2}) \{ -n \sin[p_n (t_{k1} + \Delta t_{k2}) + \theta_k] \\ &+ p_n \cos[p_n (t_{k1} + \Delta t_{k2}) + \theta_k] \} \\ &+ \sum_1^2 B_i^{\pm} \omega_i \cos[\omega_i (t_{k1} + \Delta t_{k2}) + \varphi_i] + 3\sigma_2 = V_{dr} \end{aligned} \tag{32}$$

in which

$$\varphi_k = arcctg \left\{ \frac{n}{p_n} + \frac{[V_{dr} - \sum_1^2 B_i^{\pm} \cos(\omega_i t_{k1} - \varphi_i) + 3\sigma_2] / p_n}{d_{Ak} - \sum_1^2 B_i^{\pm} \sin(\omega_i t_{k1} - \varphi_i) - B_0^{\pm} - 3\sigma_1} \right\} \tag{33}$$
$$- p_n t_{k1}$$

When the noise intensity equals to zero, the random discrete model (31) degenerates into the deterministic discrete model (19).

IV. NUMERICAL EXAMPLES

This section will give the numerical analysis of the deterministic and stochastic models in terms of Poincaré maps and bifurcation diagrams, and the main aim is to explore the influence of noise intensity on the dynamic characteristic of the systems.

A. Poincaré maps

Taking the parameters of the mechanical system shown in Fig.1 to be as: $\alpha = 1.2$, $\beta = 1.3$, $\gamma = 0.6$, $V_{dr} = 2$, $\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.5$. Fig.3 shows the Poincaré maps, the abscissa is $d_{B,k}$ or $[d_{sBk}]_{\max}$ and ordinate is $d_{B,k+1}$ or $[d_{sBk+1}]_{\max}$. When the noise intensity is in the range of $0 \le \varepsilon_0 \le 0.58$, the map appears to be a ring structure. This means that the system makes a quasi-periodic vibration. Since the approximate model has two excitation frequencies, it can be understandable. Fig.3 (a) shows the case, $\varepsilon_0 = 0$. When $\varepsilon_0 = 0.59$ in Fig. 3 (b), the ring has been broken, and the graph structure has two-cycles. When $\varepsilon_0 = 0.6$, the structure is confusing, seen in Fig.3 (c). When ε_0 increases up to 0.7, Fig.3 (d) displays the same changes in the structure again. When $\varepsilon_0 = 0.71$ there are only a few points in Fig.3 (e). When $\varepsilon_0 = 0.81$, the graph structure is forming a cycle, seen in Fig.3 (f). When $\varepsilon_0 \ge 0.87$, the ring breaks once again, and chaos occurs, seeing Fig.3 (g, h).

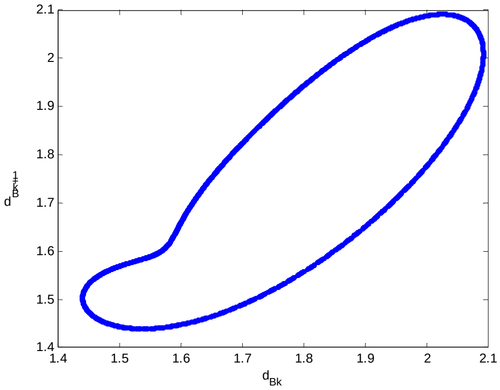

Fig.3 (a) Poincaré Map

$V_{dr} = 2, \gamma = 0.6, \varepsilon_1 = 0.7, \varepsilon_2 = 0.5, \ \varepsilon_0 = 0$

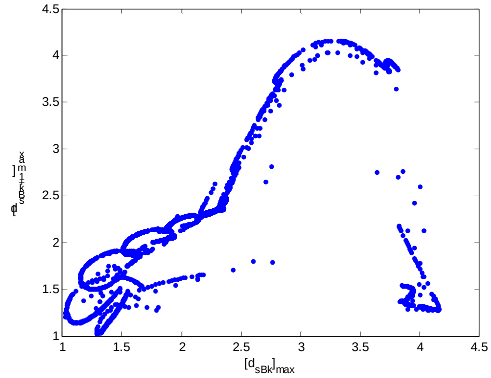

Fig.3 (d) Poincaré Map

$V_{dr} = 2, \gamma = 0.6, \varepsilon_1 = 0.7, \varepsilon_2 = 0.5, \ \varepsilon_0 = 0.7$

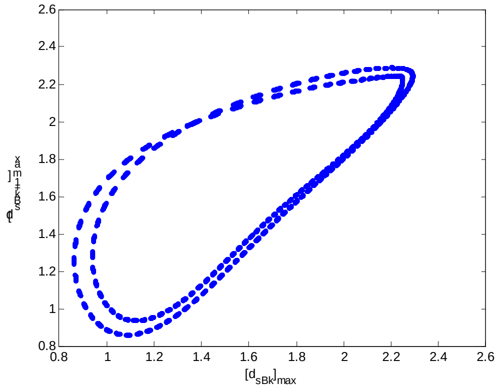

Fig.3 (b) Poincaré Map

$V_{dr} = 2, \gamma = 0.6, \varepsilon_1 = 0.7, \varepsilon_2 = 0.5, \ \varepsilon_0 = 0.59$

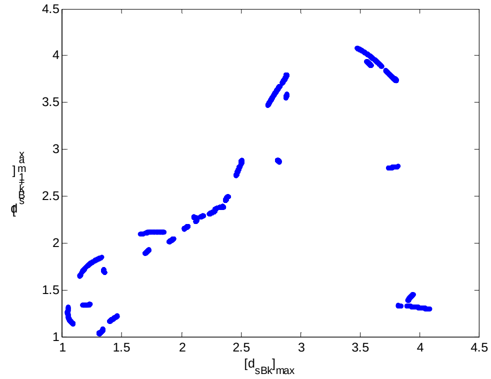

Fig.3 (e) Poincaré Map

$V_{dr} = 2, \gamma = 0.6, \varepsilon_1 = 0.7, \varepsilon_2 = 0.5, \varepsilon_0 = 0.71$

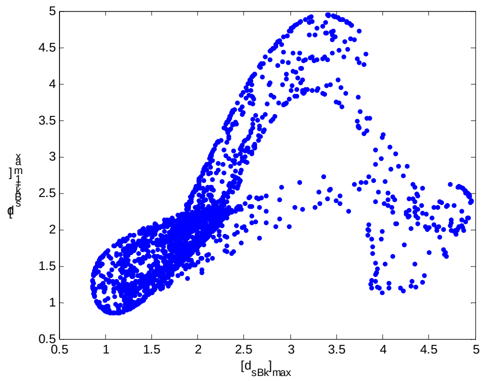

Fig.3 (c) Poincaré Map

$V_{dr} = 2, \gamma = 0.6, \varepsilon_1 = 0.7, \varepsilon_2 = 0.5, \ \varepsilon_0 = 0.6$

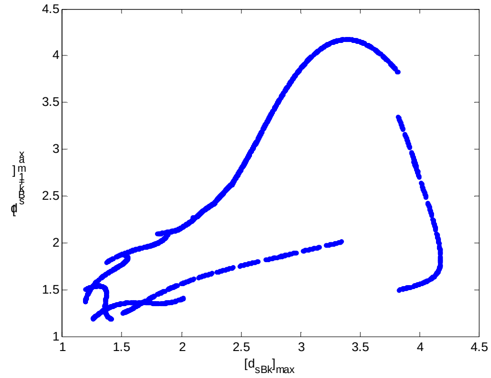

Fig.3 (f) Poincaré Maps,

$V_{dr} = 2, \gamma = 0.6, \varepsilon_1 = 0.7, \varepsilon_2 = 0.5, \ \varepsilon_0 = 0.81$

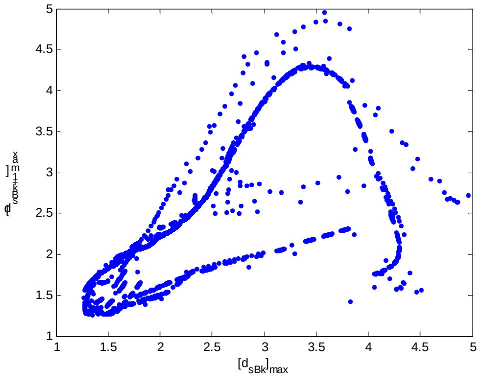

Fig.3 (g) Poincaré Map

$V_{dr}=2, \gamma=0.6, \varepsilon_1=0.7, \varepsilon_2=0.5, \varepsilon_0=0.87$

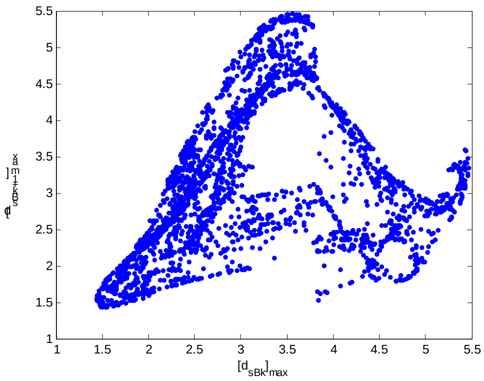

Fig.3 (h) Poincaré Map

$V_{dr}=2, \gamma=0.6, \varepsilon_1=0.7, \varepsilon_2=0.5, \varepsilon_0=1$

B. Bifurcation diagrams

Fig. 4-6 give the bifurcation diagrams, in order to compare the random and deterministic models.

(1) Fig. 4 displays the bifurcation diagram of the relative displacement versus the belt-velocity. In Fig. 4, the abscissa is V_{dr} and ordinate is $d_{B,k}$ or $[d_{sBk}]_{\max}$ .The parameters are as follows: $\alpha=1.2$, $\beta=1.3$, $\gamma=0.6$, $\varepsilon_1=0.7$, $\varepsilon_2=0.5$. Fig. 4 (a) and (b) give the deterministic case and random one, respectively; In Fig. 4 (a) $V_{dr}\geq 0.7$, the structure comes to the alternating bifurcation. In Fig.4 (b) this situation will arise only when $V_{dr}\geq 1.7$ (the intensity of noise $\varepsilon_0=0.5$).

(2) Fig. 5 shows the changes of the relative displacement according to the shape coefficient γ. The abscissa is γ and ordinate is $d_{B,k}$ or $[d_{sBk}]_{\max}$. The parameters are $\alpha=1.2$, $\beta=1.3$, $V_{dr}=2$, $\varepsilon_1=0.7$, $\varepsilon_2=0.5$. Fig.5 (a) and (b) give the deterministic case and random one, respectively. Fig.5 (a) shows a jump for the value of $d_{B,k}$, while $\gamma=0.2$. When $\gamma>0.2$, an alternating bifurcation arises. For the random case, ($\varepsilon_0=1$), when $\gamma=0.3$, the jump occurs again, and when $0.3<\gamma<0.9$, chaos arises, as shown in Fig.5 (b). When $0.9<\gamma<1.18$, the graphics becomes a single line, this means the motion of the system becomes periodic.

(3) Fig.6 gives the diagram of the relative displacement with the change of the initial parameters ε_1 .In this figure, the abscissa is ε_1 and ordinate is $d_{B,k}$ or $[d_{sBk}]_{\max}$, the parameters are $\alpha=1.2$, $\beta=1.3$, $\gamma=0.6$, $V_{dr}=2$, $\varepsilon_2=0.5$. Fig. 6 (a) gives the deterministic case and Fig. 6 (b) and (c) show the random case. Fig. 6 (a) is the alternating bifurcation. In Fig.6 (b) when $\varepsilon_1>0.8$, the plot shows a single line. This means that there is a periodic motion of the system for these parameters. When $\varepsilon_0=1$, chaos arises again, see Fig.6 (c).

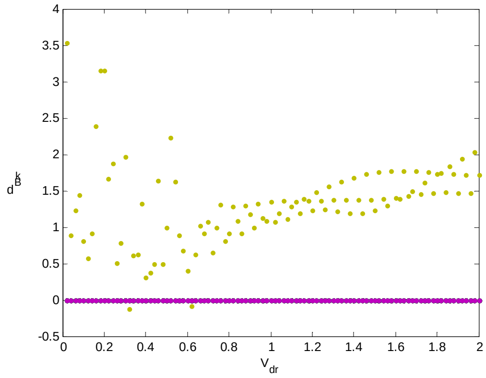

Fig.4 (a) Bifurcation diagram of the deterministic case

$\gamma=0.6, \varepsilon_1=0.7, \varepsilon_2=0.5, \varepsilon_0=0$

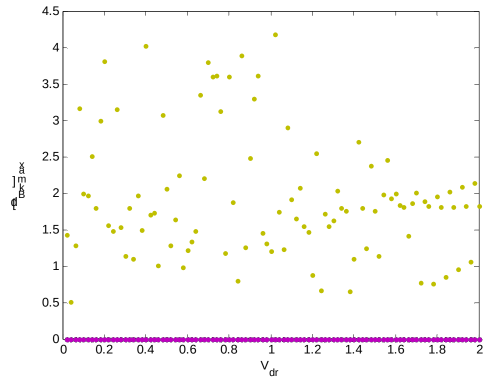

Fig.4 (b) Bifurcation diagram of the random case

$\gamma=0.6, \varepsilon_1=0.7, \varepsilon_2=0.5, \varepsilon_0=0.5$

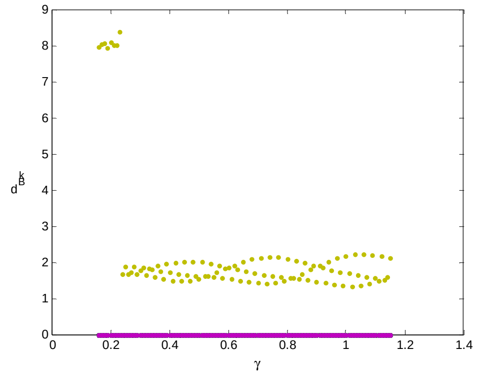

Fig.5(a) Bifurcation diagram of the deterministic case

$V_{dr} = 2, \varepsilon_1 = 0.7, \varepsilon_2 = 0.5, \varepsilon_0 = 0$

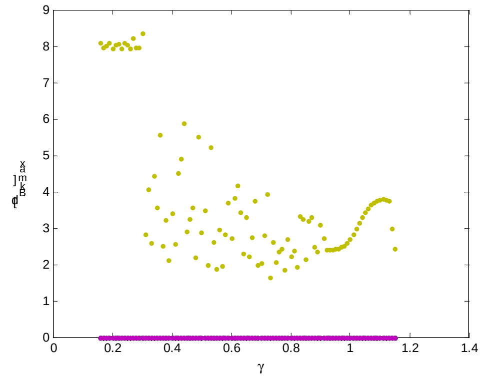

Fig.5 (b) Bifurcation diagram of the random case

$V_{dr} = 2, \varepsilon_1 = 0.7, \varepsilon_2 = 0.5, \varepsilon_0 = 1$

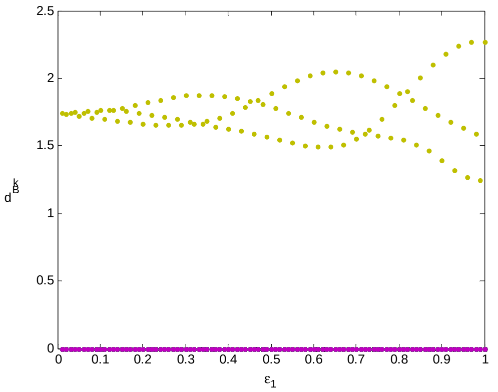

Fig.6 (a) Bifurcation diagram of the deterministic case

$V_{dr} = 2, \gamma = 0.6, \varepsilon_2 = 0.5, \varepsilon_0 = 0$

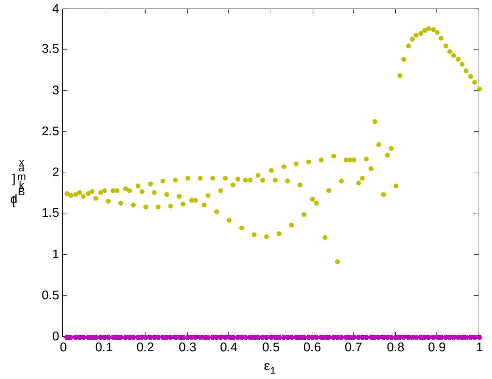

Fig.6 (b) Bifurcation diagram of the random case

$V_{dr} = 2, \gamma = 0.6, \varepsilon_2 = 0.5, \varepsilon_0 = 0.6$

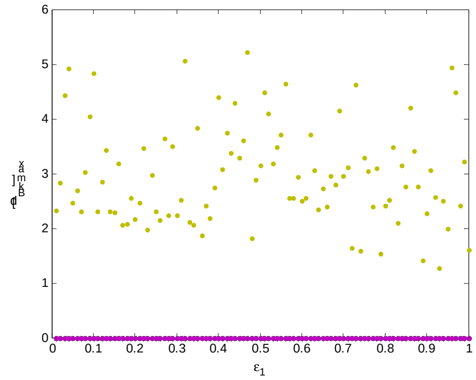

Fig.6 (c) Bifurcation diagram of the random case

$V_{dr} = 2, \gamma = 0.6, \varepsilon_2 = 0.5, \varepsilon_0 = 1$

V. CONCLUSION

Based on the above analysis, some conclusions can be drawn:

1) The 2-DOF friction system can be degraded, in principle, to a one-dimensional discrete model.
2) Using the one-dimensional discrete model to do simulation, we can greatly reduce the computer time;
3) Rich dynamic characteristics may be shown in friction systems.

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