Axisymmetric Electrogravitational Stability of Fluid Cylinder Ambient With Transverse Varying Oscillating Field

Ahmed E. Radwan, Alfaisal A. Hasan

***Abstract*— The electrogravitational instability of a dielectric fluid cylinder surrounded by medium of negligible motion pervaded by varying transverse oscillating electric field has been investigated in the axisymmetric perturbation. The acting forces on the model are: self-gravitating, pressure gradient and electrodynamic forces. The model is governed by Mathieu second order integro-differential equation. Some limiting cases are recovering from the present general one. The electric field is only destabilizing in few states but it is strongly stabilizing in the remaining states. The self-gravitating force is destabilizing in the domain $0 < x \le 1.0668$ while it is stabilizing in the rest. The oscillating time-dependent electric has strong destabilizing effect.**

***Index Terms*— Hydrodynamic stability, Self-gravitating, Stability of laminar flows, Time-dependent electric field.**

I. Introduction

The stability of self-gravitating fluid cylinder has been studied for first time by Chandrasekhar and Fermi (1953). Later on Chandrasekhar (1981) made several extensions as the fluid cylinder is acted by different forces. See also Reynolds (1965), Yih (1968) Nayyar & Murty (1960) and Baker (1983) as the cylinder subject to forces due to electric fields. The electrogravitational stability of a full fluid cylinder has developed by Radwan (1991). He (1991) considered that the fluids are penetrated by constant and uniform electric fields.

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Here we study the gravitational stability of a fluid cylinder under transverse time-dependent electric field for axisymmetric perturbations. We obtained second order differential equation of Mathieu, cf. Mclachlan (1964), Morse & Feshbach (1953) and Woodson & Melcher (1968). The details and characteristics of the in-stability domains have been obtained with using the normal mode analysis.

II. Formulation of the Problem

Consider a self-gravitating fluid cylinder surrounded by self-gravitating medium of negligible motion. The cylinder of (radius R_o) dielectric constant ε^i while the surrounding medium is being with dielectric constant ε^e . We assume that the quase-static approximation, (see Baker 1983, Mohamed 1986 and Radwan 1991), is valid and initially there is no surface charges at the interfaces so that the surface charge density will be assumed to be zero during the perturbation. The fluid cylinder is pervaded by the longitudinal time-dependent electric field

$$\underline{E}_o^i = (0, 0, E_o) \cos \omega t \tag{1}$$

The surrounding medium is penetrated by the varying transverse time-dependent electric field

$$\underline{E}_o^e = (0, \frac{\beta R_o}{r}, 0)\ E_o \cos \omega t \tag{2}$$

where E_o is the amplitude of the electric field inside the fluid jet, t is the time and ω is the electric field frequency. The components of $\underline{E}_o^{i,e}$ are considered along the cylindrical polar coordinates $(r, 0, z)$ with the z-axis coinciding with the axis of the cylinder. The fluid is acted by the pressure gradient, self-gravitating and electrodynamic forces while

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the surrounding medium is acted by the electrodynamic and self-gravitating forces.

The basic equations for studying the problem under consideration are given as follows

$$\rho\left(\frac{\partial}{\partial t}+\underline{u}^{i}\cdot\nabla\right)u^{i} = -\underline{\nabla}P^{i} + \frac{\varepsilon^{i}}{2}\underline{\nabla}\left(\underline{E}\cdot\underline{E}\right)^{i} + \rho\nabla V^{i} \qquad (3)$$

$$\underline{\nabla}.\,\underline{u}^{i} = 0 \qquad (4)$$

$$\underline{\nabla}.\,\left(\varepsilon\underline{E}\right)^{i,e} = 0 \qquad (5)$$

$$\underline{\nabla}\wedge\underline{E}^{i,e} = 0 \qquad (6)$$

$$\nabla^2 V^{i} = -4\pi\rho G \qquad (7)$$

$$\nabla^2 V^{e} = 0 \qquad (8)$$

where $\rho, \underline{u}$ and P are the fluid density, velocity vector and kinetic pressure, $\underline{E}$ the electric field intensity, V the gravitational potential and G is the gravitational constant.

III. Ground State

In this state, we have

$$\underline{\nabla}\prod_{o}^{i} = 0 \qquad (9)$$

$$\prod_{o}^{i} = P_o^{i} - \rho V_o^{i} - \frac{1}{2}\varepsilon^{i}\left(\underline{E}\cdot\underline{E}\right)^{i} = \text{const}. \qquad (10)$$

$$\underline{\nabla}\cdot\underline{u}_o^{i} = 0 \qquad (11)$$

$$\underline{\nabla}.\left(\varepsilon\underline{E}_o\right)^{i,e} = 0 \qquad (12)$$

$$\underline{\nabla}\wedge\underline{E}^{i,e} = 0 \qquad (13)$$

$$\nabla^2 V_o^{i} = -4\pi\rho G \qquad (14)$$

$$\nabla^2 V_o^{e} = 0 \qquad (15)$$

where the subscript $_o$ here and henceforth indicates unperturbed quantities.

The equations (9)--(15) are simplified, (with $\frac{\partial}{\partial z} = 0$) and solved and moreover the solutions are matched across the fluid cylinder interface at $r = R_o$.

The non-singular solution in the unperturbed state is given by

$$V_o^{i} = -\pi\rho G\, r^2 \qquad (16)$$

$$V_o^{e} = 2\pi\rho G\, R_o^{2}\ln\left(\frac{R_o}{r}\right) - \pi\rho G\, R_o^{2} \qquad (17)$$

$$P_o^{i} = \pi G\,\rho^2\left(R_o^{2} - r^{2}\right) + C \qquad (18)$$

IV. Linearization

For a small wave disturbance on the boundary interface of the fluid, the surface deflection at time t is assumed to be of the form

$$r = R_o + R_1 \qquad (19)$$

with

$$R_1 = \gamma\left(t\right)\exp\left(i\left(kz\right)\right) \qquad (20)$$

where $\gamma(t)$ is the amplitude of the perturbation while k (a real number) the longitudinal wave number. In equation (19), the second term R_1 in the right side is the surface-wave elevation measured from the unperturbed position.

Here each physical quantity $Q\left(r,0,z,t\right)$ may be expanded as

$$Q\left(r,0,z,t\right) = Q_o(r) + \gamma(t)Q_1(r,0,z) + ... \qquad (21)$$

where Q is pertaining to $P, \underline{u}, V^{i,e}$ and $\underline{E}^{i,e}$ where the suffix $_1$ characterizes the perturbed quantities. Consequently the linearized equations in the fluid, see (2)---(8), are given by

$$\frac{\partial \underline{u}_1^{i}}{\partial t} = -\underline{\nabla}\prod_{1}^{i} \qquad (22)$$

$$\prod_{1}^{i} = \left(\frac{P_1^{i}}{\rho}\right) - V_1^{i} - \left(\frac{\varepsilon^{i}}{2\rho}\right)\left(\underline{E}^{i}\cdot\underline{E}^{i}\right) \qquad (23)$$

$$\underline{\nabla}.\,\left(\varepsilon\underline{E}_1\right)^{i} = 0 \qquad (24)$$

$$\underline{\nabla}\wedge\underline{E}_1^{i} = 0 \qquad (25)$$

$$\underline{\nabla}.\,\underline{u}_1^{i} = 0 \qquad (26)$$

$$\nabla^2 V_1^{i} = 0 \qquad (27)$$

In the surrounding medium of negligible motion

$$\underline{\nabla}.\,\left(\varepsilon^{e}\,\underline{E}_1^{e}\right) = 0 \qquad (28)$$

$$\underline{\nabla}\wedge\underline{E}_1^{e} = 0 \qquad (29)$$

$$\nabla^2 V_1^{e} = 0 \qquad (30)$$

It may be noted, in the present state, that $\rho\left(\prod_1^{i} + V_1^{i}\right)$ (in view of (23)) represents the total electrohydrodynamic pressure, which is the sum of the kinetic and electrodynamic pressures.

By combining (22) and (26) via vector analysis, we have

$$\nabla^2\prod_1^{i} = 0 \qquad (31)$$

Equations (25) and (29) mean that $\underline{E}_1^{\,i,e}$ can be derived from scalar (electrical potentials) functions $\psi_1^{i,e}$ such that

$$\underline{E}_1^{\,i,e} = -\underline{\nabla}\,\psi_1^{i,e} \qquad (32)$$

Combining (32) together with (24) and (28), we get

$$\nabla^2 \psi_1^{i,e} = 0 \qquad (33)$$

As we see the perturbed linearized variables could be obtained if Laplace's equations (27), (30), (31) and (33) are solved for the given scalar functions.

By the use of the linear perturbation technique for cylindrically symmetric configurations and time-space dependence, each relevant perturbation quantity $Q_1\left(r,0,z,t\right)$ may be expressed as

$$Q_1\left(r,0,z,t\right) = \gamma(t)\, Q_1^*(r)\, \exp\left(i\left(kz\right)\right) \qquad (34)$$

Consequently, Laplace's equations (27), (30), (31) and (33) :

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial}{\partial z}\right)\right] Q_1\left(r,0,z,t\right) = 0 \qquad (35)$$

could be simplified and turned to ordinary total second order differential equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}Q_1^*\left(r\right)\right) - k^2 Q_1^*\left(r\right) = 0 \qquad (36)$$

Apart from the singular solutions as $r \to 0$ interior the fluid cylinder and as $r \to \infty$ exterior the cylinder in the surrounding medium, the non-singular solutions are identified:

$$V_1^{\,i} = A^i(t)\gamma(t) I_0\left(kr\right)\exp\left[i\left(kz\right)\right] \qquad (37)$$

$$V_1^{\,e} = A^e(t)\gamma(t) K_o\left(kr\right)\exp\left[i\left(kz\right)\right] \qquad (38)$$

$$\psi_1^{i} = B^i(t)\gamma(t) I_o\left(kr\right)\exp\left[i\left(kz\right)\right] \qquad (39)$$

$$\psi_1^{e} = B^e(t)\gamma(t) K_o\left(kr\right)\exp\left[i\left(kz\right)\right] \qquad (40)$$

$$\Pi_1^{i} = C^i(t)\gamma(t) I_o\left(kr\right)\exp\left[i\left(kz\right)\right] \qquad (41)$$

where $A^i\left(t\right), A^e\left(t\right), B^i\left(t\right), B^e\left(t\right)$ and $C^i\left(t\right)$ are arbitrary functions of integrations to be determined, while $I_o(kr)$ and $K_o(kr)$ are modified Bessel's functions of the first and second kind.

V. Boundary Conditions

The non-singular solutions of the linearized perturbation (22)--(30) of the basic (2)--(8) given by the system (37)--(41) must satisfy certain appropriate boundary conditions.

A. Kinematic Condition

The normal component of the velocity vector must be compatible with the velocity of the boundary perturbed surface of the fluid at the initial level $r = R_o$. This condition yields

$$u_{1r} = \frac{\partial r}{\partial t} \qquad at \quad r = R_o \qquad (42)$$

By the use of (19), (22), (23) and (41) for the condition (42) we, after straight forward calculations, get

$$C^i\left(t\right) = \frac{-\rho R_o}{x\,\gamma(t) I_o'(x)}\frac{d^2\gamma}{dt^2} \qquad (43)$$

where $x\left(= kR_o\right)$ is the dimensionless longitudinal wave number.

B. Self-gravitating Conditions

The gravitational potential $V\left(=V_o + \gamma\left(t\right)V_1\right)$ and its derivative are continuous across the perturbed boundary fluid surface at $r = R_o$. By the use of these conditions which are given by

$$V_1^{\,i} - V_1^{\,e} = R_1\left(\frac{\partial V_o^{\,e}}{\partial r} - \frac{\partial V_o^{\,i}}{\partial r}\right) \qquad (44)$$

$$\frac{\partial V_1^{\,i}}{\partial r} - \frac{\partial V_1^{\,e}}{\partial r} = R_1\left(\frac{\partial^2 V_o^{\,e}}{\partial r^2} - \frac{\partial^2 V_o^{\,i}}{\partial r^2}\right) \qquad (45)$$

and on utilizing (16), (17), (20), (37) and (38), we obtain

$$V_1^{\,i} = 4\pi G\rho R_o K_o\left(x\right)\gamma(t) I_o\left(kr\right)\exp[i\left(kz\right)] \qquad (46)$$

$$V_1^{\,e} = 4\pi G\rho R_o I_o\left(x\right)\gamma(t) K_o\left(kr\right)\exp[i\left(kz\right)] \qquad (47)$$

where use has been made of the Wronskian relation (Abramowtiz and Stegun (1970))

$$W_o\left(I_o\left(x\right), K_o\left(x\right)\right) = I_o\left(x\right)K_o'\left(x\right) - I_o'\left(x\right)K_o\left(x\right) = -\frac{1}{x} \qquad (48)$$

in obtaining (46) and (47).

C. Electrodynamic Conditions

i) The normal component of the electric displacement current must be continuous across the perturbed boundary interface at $r = R_o$. This condition read

$$\underline{n}.\left(\varepsilon^i \underline{E}^{\,i} - \varepsilon^e \underline{E}^{\,e}\right) = 0 \qquad (49)$$

with

$$\underline{E} = \underline{E}_o + R_1\frac{\partial \underline{E}_o}{\partial r} + \underline{E}_1 \qquad (50)$$

while $\underline{n}$ is , the outward unit vector normal to the interface (19) at $r = R_o$, given by

$$\underline{n} = \underline{\nabla} F\left(r,0,z,t\right) / \left|\underline{\nabla} F\left(r,0,z,t\right)\right| \quad (51)$$

$$F\left(r,0,z,t\right) = r - R_o - R_1 \quad (52)$$

so that

$$\underline{n}_o = \left(1,0,0\right)\ ,\ \underline{n}_1 = \left(0,0,-ik\right)\gamma(t)\exp[i\left(kz\right)] \quad (53)$$

Consequently, the condition (49) yields

$$x\,\varepsilon^i I_o'\left(x\right)B^i - x\,\varepsilon^e B^e K_o'\left(x\right) = \left(\beta - ix\,\varepsilon^i\right)E_o\cos\omega t \quad (54)$$

ii) The electric potential ψ must be continuous across the perturbed boundary surface of the fluid cylinder at the initial level $r = R_o$, i.e.

$$B^i(t)I_o\left(x\right) = B^e(t)K_o\left(x\right) \quad (55)$$

Solving (54) and (55), we finally obtain

$$B^i(t) = \left(K_o\left(x\right)/I_o\left(x\right)\right)B^e(t) \quad (56)$$

and

$$B^e(t) = \left(\frac{\left(\beta - ix\,\varepsilon^i\right)E_o I_o\left(x\right)}{x\left[\varepsilon^i I_o'\left(x\right)K_o\left(x\right) - \varepsilon^e I_o\left(x\right)K_o'\left(x\right)\right]}\right)\cos\omega t \quad (57)$$

D. The Dynamical Stresses Condition

The stresses across the cylindrical fluid interface are due to the fluid kinetic pressure, surface tension (neglected here), self-gravitating and electrical forces.

For the problem under consideration the jump of the normal component, yields

$$\Pi^i - \frac{1}{2}\varepsilon^i\left(\underline{E}^i \cdot \underline{E}^i\right) + \rho^i V^i = \frac{1}{2}\varepsilon^e\left(\underline{E}^e \cdot \underline{E}^e\right) \quad (58)$$

which is applicable across the displaced interface $r = R_o + \gamma(t)\exp\left(i\left(kz\right)\right)$.

Substituting for $P_1, P_o, \underline{E}_o^{i,e}, \underline{E}_1^{i,e}$ and R_1 , after some algebraic calculations, we finally obtaine

$$\frac{d^2\gamma}{dt^2} - 4\pi G\rho\left(\frac{xI_o'\left(x\right)}{I_o\left(x\right)}\right)\left(I_o\left(x\right)K_o\left(x\right) - \frac{1}{2}\right)\gamma(t) + \frac{E_o^2\gamma(t)}{\rho R_o^2}\left(\frac{xI_o'\left(x\right)}{I_o\left(x\right)}\right)\left[\frac{xI_o'\left(x\right)K_o\left(x\right)\left(\varepsilon^i\right)^2}{\left[\varepsilon^i I_o'\left(x\right)K_o\left(x\right) - \varepsilon^e I_o'\left(x\right)K_o'\left(x\right)\right]} - \varepsilon^e\beta^2\right]\cos^2\omega t = 0 \quad (59)$$

Equation (59) is an integro-differential equation governing the surface displacement $\gamma(t)$. By means of this relation we may identify the (in-) stability states and also the self-gravitating and electrodynamic forces influences on the stability of the present model. However in order to do so, it is found more convenient to express this relation in the simple form

$$\left[\frac{d^2}{d\eta^2} + \left(b - h^2\cos^2\eta\right)\right]\gamma(t) = 0\ ,\ \eta = \omega t \quad (60)$$

where

$$b = \frac{-4\pi G\rho}{\omega^2}\left(\frac{xI_o'\left(x\right)}{I_o\left(x\right)}\right)\left(I_o\left(x\right)K_o\left(x\right) - \frac{1}{2}\right) \quad (61)$$

$$h^2 = \frac{-E_o^2}{\rho R_o^2\omega^2}\left(\frac{xI_o'\left(x\right)}{I_o\left(x\right)}\right)\left[\frac{\left(x\,\varepsilon^i\right)^2 I_o'\left(x\right)K_o\left(x\right)}{x\left[\varepsilon^i I_o'\left(x\right)K_o\left(x\right) - \varepsilon^e I_o'\left(x\right)K_o'\left(x\right)\right]} - \varepsilon^e\beta^2\right] \quad (62)$$

Equation (60) has the canonical form

$$\left[\frac{d^2}{d\eta^2} + \left(a - 2q\cos 2\eta\right)\right]\gamma(t) = 0 \quad (63)$$

where

$$q = \frac{h^2}{4}\quad ,\quad a = b - \left(\frac{h^2}{2}\right) \quad (64)$$

Equation (63) is Mathieu differential equation. The properties of the Mathieu functions are explained and investigated by Mclachlan (1964). The solutions of (63), under appropriate restrictions, could be periodic and consequently the considered model will be stable and vice versa. The conditions required for periodicity of Mathieu functions is mainly dependent on the correlation between the parameters a and q . However it is well known, see Mclachlan (1964), that $\left(a,q\right) -$ plane is divided essentially into two stable and unstable domains separated by the characterstic curves of Mathieu functions. Thence we can state generally that a solution of Mathieu integro-differential equation is unstable if the point $\left(a,q\right)$ say, in the $\left(a,q\right) -$ plane lies interior an unstable domain, otherwise it is stable.

VI. Discussions and Limiting Cases

The appropriate solutions of (63) are given in terms of what called ordinary Mathieu functions which, indeed, are periodic in time t with period π and 2π.

Corresponding to extremely small values of q, the first region of instability is bounded by the curves

$$a = \pm q + 1 \tag{65}$$

The conditions for oscillation lead to the problem of the boundary regions of Mathieu functions where Mclachian (1964) gives the condition of stability as

$$\left|\Delta(0)\sin^2(\pi a/2)\right|^{\frac{1}{2}} \le 1 \tag{66}$$

where $\Delta(0)$ is the Hill's determinant.

An approximation criterion for the stability near the neighbourhood of the first stable domains of the Mathieu stability domains given by Morse and Feshbach (1953) which is valid only for small values of h^2 or q i.e. the frequency ω of the electric field is very large.

This criterion, under the present circumstances, states that the model is ordinary stable if the restriction

$$h^4 - 16(1-b)h^2 + 32b(1-b) \ge 0 \tag{67}$$

is satisfied where the equality is corresponding to the marginal stability state. The inequality (67) is a quadratic relation in h^2 and could be written in the form

$$\left(h^2 - \alpha_1\right)\left(h^2 - \alpha_2\right) \ge 0 \tag{68}$$

where α_1 and α_2 are, the two roots of the equality of the relation (67), being

$$\alpha_1 = 8(1-b) - \Delta \tag{69}$$

$$\alpha_2 = 8(1-b) + \Delta \tag{70}$$

$$\Delta^2 = 32(1-b)(2-3b) \tag{71}$$

The magnetogravitational stability and instability investigation analysis should be carried out in the following two cases i) $0 < b < 2/3$ ii) $2/3 < b < 1$

A. The case $0 < b < 2/3$

In this case Δ^2 is positive and therefore the two roots α_1 and α_2 of the equality (67) are real. Now we will show that both α_1 and α_2 are positive. If $\alpha_1 \ne +ve$ then α_1 must be negative and this means that

$$8(1-b) \le b \tag{72}$$

or alternatively

$$64(1-b)^2 \le 32(1-b)(2-3b)$$

From which we get

$$2b \ge 3b \tag{73}$$

and this is contradiction, so α_1 must be positive and consequently $\alpha_2 \ge 0$ as well (noting that $\alpha_2 > \alpha_1$). This means that both the quantities $\left(h^2 - \alpha_1\right)$ and $\left(h^2 - \alpha_2\right)$ are negative and that in turn shows that the inequality (67) is identically satisfied in the axisymmetric disturbance mode.

B. The case $2/3 < b < 1$

In this case in which $b < 1$ and simultaneously $3b > 2$, it is found that Δ^2 is negative i.e. Δ is imaginary, therefore the two roots α_1 and α_2 are complex. We may prove that the inequality (67) is satisfied as follows.

Let $h^2 = -c$ and $\alpha_{1,2} = c_1 - ic_2$ where c, c_1 and c_2 are real, so

$$\left(h^2 - \alpha_1\right)\left(h^2 - \alpha_2\right) = \left[-c - (c_1 + ic_2)\right]\left[-c - (c_1 - ic_2)\right]$$

$$= c^2 + 2cc_2 + c_1^2 + c_2^2$$

$$= (c + c_1)^2 + c_2^2 = +ve \tag{74}$$

which is positive definite.

By an appeal to the cases (i) and (ii), we deduce that the model is stable under the restrictions

$$0 < b < 1 \tag{75}$$

This means that the model is stable if there exists a critical value ω_o of the electric field frequency ω such that $\omega > \omega_o$ where ω_o is given by

$$\omega_o^2 > 4\pi G\rho\left(xI_o'(x)/I_o(x)\right)\left(I_o(x)K_o(x) - 0.5\right) > 0 \tag{76}$$

One has to mention here that if $\omega = 0,\ \beta = 0,$ and $E_o = 0$ and we suppose that

$$\gamma(t) = (\text{const})\exp(\sigma t), \tag{77}$$

the second order integro-differential equation of Mathieu equation (59), yield

$$\sigma^2 = 4\pi G\rho\left(xI_o'(x)/I_o(x)\right)\left(I_o(x)K_o(x)-0.5\right) \quad (78)$$

where σ is the temporal amplification and note by the way that $(4\pi G\rho)^{-\frac{1}{2}}$ has a unit of time. The relation (78) is identical to the gravitational dispersion relation derived for the first time by Chandrasekhar and Fermi (1953). In fact they (1953) have used a totally different technique rather than that used here. They have used the method of representing the solenoidal vectors in terms of poloidal and toroidal vector fields, which is valid only for the axisymmetric perturbation. To determine the effect of ω it is found more convenient to investigate the eigenvalue relation (78) since the right side of it is the same the middle side of (76).

Taking into account the recurrence relation (cf. Abramowitz and Stegun 1970) of the modified Bessel's functions and their derivatives, we see, for $x \neq 0$, that

$$\left(xI_o'(x)/I_o(x)\right) > 0 \quad (79)$$

and

$$\left(I_o(x)K_o(x)\right) > 0 \quad (80)$$

Now, returning to the relation (78), we deduce that the determining of the sign $\left(\sigma^2/(4\pi G\rho)\right)$ is identified if the sign of the quantity

$$Q_o(x) = \left(I_o(x)K_o(x)-0.5\right) \quad (81)$$

is identified.

Here it is found that the quantity $Q_o(x)$ may be positive or negative depending on $x \neq 0$ values. Numerical investigations and analysis of the relation (78) reveal that σ^2 is positive for small values of x while it is negative in all other values of x. In more details it is unstable in the domain $0 < x < 1.0668$ while it is stable in the domains $1.0668 \le x < \infty$ where the equality is corresponding to the marginal stability state.

From the foregoing discussions, investigations and analysis, we conclude (on using (81) for (78)) that the quantity

$$M^2 = \left(xI_o'(x)/I_o(x)\right)\left(I_o(x)K_o(x)-0.5\right), \quad M = \sigma\Big/(4\pi G\rho)^{\frac{1}{2}} \quad (82)$$

has the following properties

$$\left.\begin{array}{ll} M^2 \le 0 \text{ in the ranges} & 1.0668 \le x < \infty \\ M^2 > 0 \text{ in the range} & 0 < x < 1.0668 \end{array}\right\} \quad (83)$$

Now returning to the relation (76) concerning the frequency ω_o of the periodic electric field

$$\frac{\omega^2}{(4\pi G\rho)} > \left[\left(\frac{xI_o'(x)}{I_o(x)}\right)\left(\frac{1}{2}-I_o(x)K_o(x)\right)\right] > 0. \quad (84)$$

Therefore, we deduce that the electrodynamic force (with a periodic time electric field) has stabilizing influence and could predominate and overcoming the self-gravitating destabilizing influence of the dielectric fluid cylinder dispersed in a dielectric medium of negligible motion.

However, the self-gravitating destabilizing influence could not be suppressed whatever is the greatest value of the magnitude and frequency of the periodic electric field because the gravitational destabilizing influence will persist.

VII. Numerical Discussions

If we assume that $\omega = 0$ and consider the condition (77), then the second order integro-differential equation of Mathieu equation (59), yield

$$\frac{\sigma^2}{4\pi G\rho} = \left(\frac{xI_o'(x)}{I_o(x)}\right)\left(I_o(x)K_o(x)-\frac{1}{2}\right) - \left(\frac{E_o}{E_s}\right)^2 \left(\frac{xI_o'(x)}{I_o(x)}\right)\left[\frac{xI_o'(x)K_o(x)}{\left[I_o'(x)K_o(x)-\varepsilon I_o'(x)K_o'(x)\right]} - \varepsilon^e\beta^2\right] = 0 \quad (85)$$

where $E_s^2 = \left(4\pi G\rho^2 R_o^2/\varepsilon^i\right)$ and $\varepsilon = \left(\varepsilon^e/\varepsilon^i\right)$.

In order to verify and confirm the foregoing analytical results, the relation (85) has been inserted in the computer and computed. This has been done for several values of β as $\beta < 1$, $\beta = 1$ and $\beta > 1$ in the wide domain $0 \le x \le 0.5$. The numerical data of instability corresponding $\sigma/(4\pi G\rho)^{\frac{1}{2}}$ are collected and tabulated and presented graphically (see figs. (1) and (2))

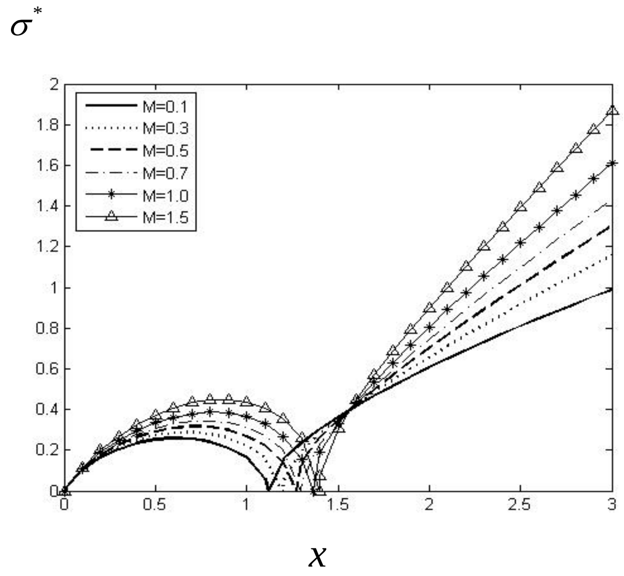

Fig. (1)

Electrogravitational stable and unstable domains for β =0.5

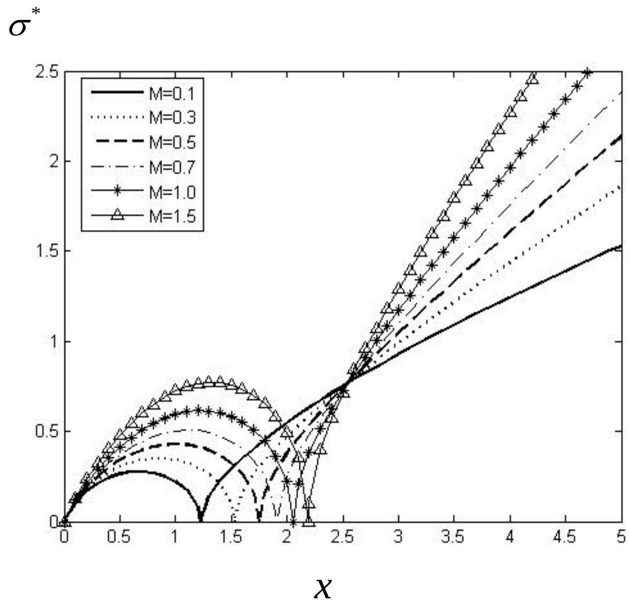

Fig. (2)

Electrogravitational stable and unstable domains for β =1.0

The numerical data of stability corresponding to $\zeta/(4\pi G\rho)^{\frac{1}{2}}$ are collected and tabulated and presented graphically (see figs. (3) and (4)).

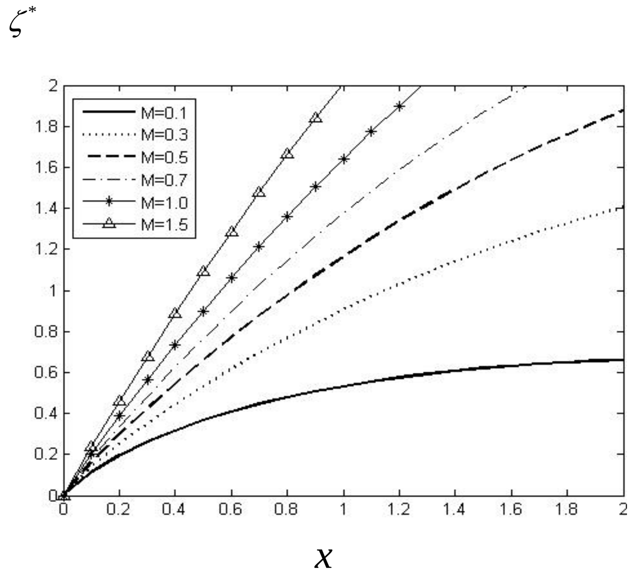

Fig. (3)

Electrogravitational stable domains for β =2.5

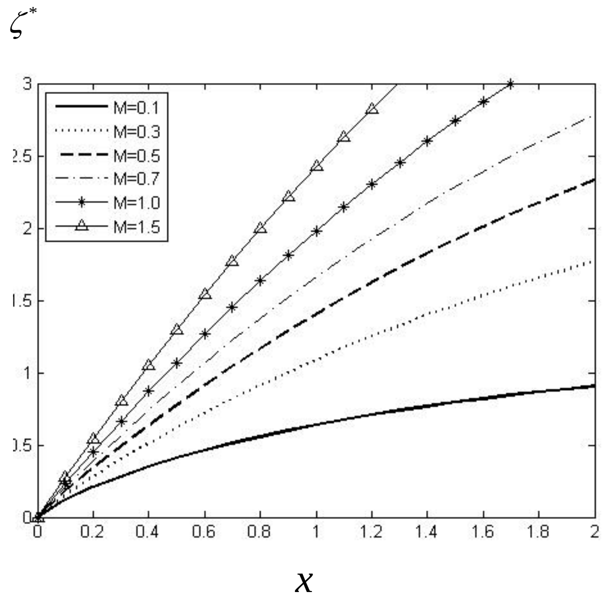

Fig. (4)

Electrogravitational stable domains for β =3.0

From the analytical and numerical (see figs. (1)-- (4)) discussions of the relation (85) it is found that the electric field E_o^e has strong stabilizing effect on the model. In reality this can be realized from the fact that the unstable domains are fastly decreasing with increasing β values. This strong stabilizing effect could be predominant over the destabilizing effect of the self-gravitating force, so it can suppress the stability character of the model and stability arises.

VIII. CONCLUSION

From the foregoing numerical results we may deduce the following.

For the same value of $\beta(0<\beta<2.0)$, it is found that the unstable domains are increasing with increasing M values. This means that the capillary force has a strong destabilizing influence on the self-gravitating instability of the model.

For the same value of M it is found that the unstable domains are increasing with increasing of β values. This means that the electric field has a strong destabilizing influence on the self-gravitating instability of the model. In fact, this confirms the analytical discussions of the general eigenvalue relation (85).

Moreover, as $\beta(2.0\le\beta<\infty)$, it is found that the self-gravitating instability character is disappeared and has been dispersed and the model becomes completely stable.

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