Efficient Strict Solution Methods for 0-1 Random Fuzzy Programming Problems Based on the Necessity Measure

Takashi Hasuike, Hideki Katagiri, and Hiroaki Ishii

Abstract—This paper considers two general 0-1 random fuzzy programming problems based on the degree of necessity which include some previous 0-1 stochastic and fuzzy programming problems. The proposal problems are not well defined due to including randomness and fuzziness. Therefore, by introducing chance constraint and fuzzy goal for the objective function, and considering the maximization of the aspiration level for total profit and the degree of necessity that the objective function's value satisfies the fuzzy goal, each main problem is transformed into a deterministic equivalent problem. Furthermore, by using the assumption that each random variable is distributed according to a normal distribution, the problem is equivalently transformed into a basic 0-1 programming problem, and the efficient strict solution method to find an optimal solution is constructed.

Keywords—0-1 programming problem, Random fuzzy variables, Degree of necessity, Relaxation problem

I. INTRODUCTION

0-1 programming problems are the most important mathematical models in practical social problems such as project selection, scheduling and facility location problems, and there are many previous academic and practical researches (recent studies; Balev [1], Freville [5]).

In the sense of standard mathematical programming problems involving 0-1 programming problems, the coefficients of objective functions or constraints are assumed to be completely known and dealt with constant values. However, in the real world, they should be treated as rather uncertain values than constant values due to probabilistic situations such as predictions of future profits or some machine troubles derived from historical data and ambiguous situations such as decision makers' substitutions. In order to consider such uncertainty, some models for mathematical programming problems have been introduced; stochastic programming problem (for example, Beale [2], Dantzig [4], Vajda [18]), fuzzy programming problem (for example, Inuiguchi [10], Sakawa [17], Zimmermann [19]) and the applications of fuzzy logics in various real cases (for example, Cheng [3]). Furthermore, Hasuike [6] has considered a 0-1 programming problem considering randomness and flexible goals for objective function and constraints. Katagiri [15] has

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considered a 0-1 programming problem involving both random and fuzzy conditions, i.e., fuzzy random 0-1 programming problem. Then, Katagiri [12] has considered a random fuzzy programming model based on possibilistic programming. Most recently, Huang [9] has proposed a project selection model including random fuzzy variables. However, there are few researches considering a general 0-1 random fuzzy programming problem at this time. Therefore, this paper particularly considers the more general random fuzzy 0-1 programming problem maximizing the objective function involving random fuzzy variables, considering both the objectivity derived from statistical analysis of data and decision maker's subjectivity such as their subjectivities derived from wide-ranging experiences, simultaneously. In this paper, we deal fuzzy numbers with L-R fuzzy numbers and random variables with continuous random distributions, particularly normal distributions.

On the other hand, generally speaking, solution methods of 0-1 programming problems mainly divides into two types; (a) strict solution methods such as dynamic programming and branch-bound method, (b) approximate solution methods such as genetic algorithm, heuristic methods, etc.. Furthermore, in mathematical programming, 0-1 programming problems with uncertainty become more complicate than the previous problems. Then, since these problems are not well defined due to including both random variables and fuzzy numbers, it is almost impossible to solve them analytically. Therefore, we need to set the target values for stochastic and fuzzy constraints and construct its efficient solution method. Most recently, Huang [9] considered a random fuzzy simulation based on approximate solution methods. However, through the development of information technology and improvement of computers, we solve 0-1 programming problems more quickly using not only approximate solution methods but also strict solution methods even if they are little bit large scale problems. Therefore, in this paper, we propose the efficient strict solution method based on a hybrid method with 0-1 relaxation problem and branch-bound method, and show the analytical efficiency comparing with previous strict solution methods. Particularly, considering the case that the decision maker is pessimistic with respect to present practical conditions, we deal with the necessity measure based on the fuzzy programming. Furthermore, introducing the fuzzy goal to the object, we consider the 0-1 random fuzzy programming problem with the degree of necessity based on our previous study [7].

This paper is organized as follows. In Section 2, we formulate a general 0-1 random fuzzy programming problem and introduce the degree of necessity. Using the degree of

necessity, in Section 3, we propose two random fuzzy 0-1 programming problems whose objects are to maximize the aspiration level of total profit and the degree of necessity, respectively. Then, we perform the deterministic equivalent transformation and transform the original problem into a nonlinear 0-1 programming problem. It is difficult to solve this obtained problem analytically using previous strict solution methods, and so in Section 4, we develop the efficient solution method to combine the 0-1 relaxation problem with the strict solution method. Finally, in Section 5, we conclude this paper.

II. FORMULATION OF GENERAL 0-1 RANDOM FUZZY PROGRAMMING PROBLEM

A random fuzzy variable is one of the mathematical concepts dealing with randomness and fuzziness, simultaneously. In this paper, we deal with the random fuzzy variable based on the study of Liu [16].

Definition 1 (Liu [16])
A random fuzzy variable is a function ξ from a possibility space $(\Theta, P(\Theta), \mathrm{Pos})$ to collection of random variables R.

Then, we formally introduce the following 0-1 programming problem:

$$\text{Maximize } \overline{\tilde{r}}x$$
$$\text{subject to } \mathbf{A}x \leq \boldsymbol{b}, \, x \in \{0,1\}^n \tag{1}$$

where each notation is as follows:
A: $m \times n$ coefficient matrix
b: m-dimensional column vector
x: n-dimensional decision column vector (Decision variable)

The coefficient vector of objective function is $\overline{\tilde{r}} = (\overline{\tilde{r}}_1, \overline{\tilde{r}}_2, ..., \overline{\tilde{r}}_n)$ and each $\overline{\tilde{r}}_j$ is a random fuzzy variable according to a normal distribution $N(\tilde{m}_j, \sigma_j^2)$ where $\tilde{m}_j$ is a mean value and σ_j^2 is a variance. Then, we represent the ijth element of variance-covariance matrix as σ_{ij}. Furthermore, we assume that $\tilde{m}_j$ is a fuzzy variable characterized by the following membership function:

$$\mu_{\tilde{m}_j}(\omega) = \begin{cases} L\left(\dfrac{m_j - \omega}{\alpha_j}\right) & (m_j - \alpha_j \leq \omega \leq m_j) \\ R\left(\dfrac{\omega - m_j}{\beta_j}\right) & (m_j < \omega \leq m_j + \beta_j) \\ 0 & (\omega < m_j - \alpha_j, m_j + \beta_j < \omega) \end{cases} \tag{2}$$

where $L(x)$ and $R(x)$ are nonincreasing reference function to satisfy $L(0) = R(0) = 1$, $L(1) = R(1) = 0$ and the parameters α_j and β_j represent the spreads corresponding to the left and the right sides, respectively. Problem (1) is a random fuzzy 0-1 programming problem due to including random fuzzy variables. Then, its objective

function $\overline{\tilde{Z}} = \overline{\tilde{r}}x$ is defined as a random fuzzy variable by the following membership function introducing a parameter $\overline{\gamma}_j$ and an universal set of normal random variable Γ:

$$\mu_{\overline{\tilde{Z}}}(\overline{u}) = \sup_{\overline{\gamma}}\left\{\min_{1 \leq j \leq n} \mu_{\overline{\tilde{r}}_j}(\overline{\gamma}_j) \middle| \overline{u} = \sum_{j=1}^{n}\overline{\gamma}_j x_j\right\}, \forall \overline{u} \in Y \tag{3}$$

where $\overline{\gamma} = (\overline{\gamma}_1, \overline{\gamma}_2, ..., \overline{\gamma}_n)$, $\mu_{\overline{\tilde{r}}_j}(\overline{\gamma}_j)$ is defined by

$$\mu_{\overline{\tilde{r}}_j}(\overline{\gamma}_j) = \sup_{s_j}\left\{\mu_{\tilde{M}_j}(s_j) \middle| \overline{\gamma}_j \sim N(s_j, \sigma_j^2)\right\}, \forall \overline{\gamma}_j \in \Gamma$$

and Y is defined by

$$Y = \left\{\sum_{j=1}^{n}\overline{\gamma}_j x_j \middle| \overline{\gamma}_j \in \Gamma, j = 1, 2, ..., n\right\}$$

From these settings, we obtain

$$\mu_{\overline{\tilde{Z}}}(\overline{u}) = \sup_{\overline{\gamma}}\left\{\min_{1 \leq j \leq n} \mu_{\overline{\tilde{r}}_j}(\overline{\gamma}_j) \middle| \overline{u} = \sum_{j=1}^{n}\overline{\gamma}_j x_j\right\}$$
$$= \sup_{s}\left\{\min_{1 \leq j \leq n} \mu_{\tilde{M}_j}(s_j) \middle| \overline{\gamma}_j \sim N(s_j, \sigma_j^2), \overline{u} = \sum_{j=1}^{n}\overline{\gamma}_j x_j\right\} \tag{4}$$
$$= \sup_{s}\left\{\min_{1 \leq j \leq n} \mu_{\tilde{M}_j}(s_j) \middle| \overline{u} \sim N\left(\sum_{j=1}^{n}s_j x_j, V(\boldsymbol{x})\right)\right\}$$

where $V(\boldsymbol{x}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j$ Furthermore, we discuss the probability that the objective function value is greater than or equal to an aspiration level f. Then, we represent the probability as $\Pr\left\{\omega \middle| \sum_{j=1}^{n}\overline{\tilde{r}}_j x_j \geq f\right\}$. Since $\sum_{j=1}^{n}\overline{\tilde{r}}_j x_j$ is represented with a random fuzzy variable, we express the probability $\Pr\left\{\omega \middle| \sum_{j=1}^{n}\overline{\tilde{r}}_j x_j \geq f\right\}$ as a fuzzy set $\tilde{P}$ and defined the membership function of $\tilde{P}$ as follows:

$$\mu_{\tilde{P}}(p) = \sup_{\overline{u}}\left\{\mu_{\overline{\tilde{Z}}}(\overline{u}) \middle| p = \Pr\{\omega|\overline{u}(\omega) \geq f\}\right\}$$
$$= \sup_{s}\min_{1 \leq j \leq n}\left\{\mu_{\tilde{M}_j}(s_j) \middle| \begin{array}{l} p = \Pr\{\omega|\overline{u}(\omega) \geq f\}, \\ \overline{u} \sim N\left(\sum_{j=1}^{n}s_j x_j, V(\boldsymbol{x})\right) \end{array}\right\} \tag{5}$$

where $\boldsymbol{s} = (s_1, s_2, ..., s_n)$. Due to these randomness and fuzziness, problem (1) is not a well-defined problem, and so it is necessary to interpret the problem from some point of view and to transform the problem into the deterministic equivalent problem. In this paper, we consider two cases; (a) the case where a decision maker prefers maximizing aspiration level f more than the degree of necessity for the probability, and (b) the case where a decision maker prefers maximizing the degree of necessity for the probability that the value of objective function satisfies the fuzzy goal, based on previous research Hasuike [8] and Katagiri [12, 13]. A fuzzy goal for the probability is characterized by the following membership function:

$$\mu_{\tilde{G}_P}(p) = \begin{cases} 1 & p_1 < p \\ g(p) & p_0 \leq p \leq p_1 \\ 0 & p < p_0 \end{cases} \tag{6}$$

where $g(p)$ is a monotonous increasing function. Using a concept of necessity measure, the degree of necessity that the objective function value satisfying a fuzzy goal G is as follows:

$$N_{\tilde{p}}(\tilde{G}) = \inf_{p} \max\left\{1 - \mu_{\tilde{P}}(p), \mu_{\tilde{G}_P}(p)\right\} \qquad (7)$$

III. FORMULATION OF PROPOSED MODELS AND DETERMINISTIC EQUIVALNET TRANSFORMATIONS

A. The case maximizing the aspiration level f

First, we consider the fractile optimization model for problem (1) using the degree of necessity (7). In this paper, the proposed fractile optimization model means the maximization of the aspiration level f that the degree of necessity is more than the target level $\bar{h}$. This model is formulated as the following problem:

Maximize f

subject to $N_{\tilde{p}}(\tilde{G}) \geq \bar{h},$

$$\mathbf{A}x \leq \mathbf{b}, x \in \{0,1\}^n \qquad (8)$$

In problem (8), constraint $N_{\tilde{p}}(\tilde{G}) \geq \bar{h}$ is transformed into the following inequality based on the result obtained by Hasuike [8] and Katagiri [12, 13]:

$N_{\tilde{p}}(\tilde{G}) \geq \bar{h}$

$\Leftrightarrow \inf_{p} \max\left\{1 - \mu_{\tilde{P}}(p), \mu_{\tilde{G}_P}(p)\right\} \geq \bar{h}$

$\Leftrightarrow \exists p : 1 - \mu_{\tilde{P}}(p) < \bar{h} \Rightarrow \mu_{\tilde{G}_P}(p) \geq \bar{h}$

$\Leftrightarrow \exists p : \sup_{s} \min_{1 \leq j \leq n} \left\{ \mu_{\tilde{M}_j}(s_j) \left| \begin{array}{l} p = \Pr\{\omega | \bar{u}(\omega) \geq f\}, \\ \bar{u} \sim N\left(\sum_{j=1}^{n} s_j x_j, V(x)\right) \end{array} \right. \right\} > 1 - \bar{h},$

$\qquad \Rightarrow \mu_{\tilde{G}_P}(p) \geq \bar{h}$

$\exists p, \exists s, \exists \bar{u} : \sup_{s} \min_{1 \leq j \leq n} \mu_{\tilde{M}_j}(s_j) > 1 - \bar{h},$

$\Leftrightarrow \qquad p = \Pr\{\omega | \bar{u}(\omega) \geq f\}, \bar{u} \sim N\left(\sum_{j=1}^{n} s_j x_j, V(x)\right),$

$\qquad \Rightarrow \mu_{\tilde{G}_P}(p) \geq \bar{h}$

$\exists p, \exists s, \exists \bar{u} : \min_{1 \leq j \leq n} \mu_{\tilde{M}_j}(s_j) \geq 1 - \bar{h},$

$\Leftrightarrow \qquad p = \Pr\{\omega | \bar{u}(\omega) \geq f\}, \bar{u} \sim N\left(\sum_{j=1}^{n} s_j x_j, V(x)\right), \qquad (9)$

$\qquad \Rightarrow p \geq g^{-1}(\bar{h})$

$\exists s, \exists \bar{u} : \mu_{\tilde{M}_j}(s_j) \geq 1 - \bar{h}, \ (j = 1, 2, ..., n)$

$\Leftrightarrow \qquad \bar{u} \sim N\left(\sum_{j=1}^{n} s_j x_j, V(x)\right),$

$\qquad \Rightarrow \Pr\{\omega | \bar{u}(\omega) \geq f\} \geq g^{-1}(\bar{h})$

$\exists s, \exists \bar{u} : L^*(1 - \bar{h}) \leq s_j \leq R^*(1 - \bar{h}), \ (j = 1, 2, ..., n)$

$\Leftrightarrow \qquad \bar{u} \sim N\left(\sum_{j=1}^{n} s_j x_j, V(x)\right),$

$\qquad \Rightarrow \Pr\{\omega | \bar{u}(\omega) \geq f\} \geq g^{-1}(\bar{h})$

$\exists \bar{u} : \Pr\{\omega | \bar{u}(\omega) \geq f\} \geq g^{-1}(\bar{h}),$

$\Leftrightarrow \qquad \bar{u} \sim N\left(\sum_{j=1}^{n} \left(m_j - L^*(1 - \bar{h})\alpha_j\right) x_j, V(x)\right)$

where $L^*(x)$ is a pseudo inverse function of $L(x)$. From this inequality, problem (8) is equivalently transformed into the following problem:

Maximize f

subject to $\Pr\{\omega | \bar{u}(\omega) \geq f\} \geq g^{-1}(\bar{h}),$

$$\bar{u} \sim N\left(\sum_{j=1}^{n}\left(m_j - L^*(1 - \bar{h})\alpha_j\right) x_j, V(x)\right), \qquad (10)$$

$$\mathbf{A}x \leq \mathbf{b}, x \in \{0,1\}^n$$

Furthermore, with respect to stochastic constraint $\Pr\{\omega | \bar{u}(\omega) \geq f\} \geq g^{-1}(\bar{h})$, by using the property of normal distribution, this constraint is equivalently transformed into the following form:

$\Pr\{\omega | \bar{u}(\omega) \geq f\} \geq g^{-1}(\bar{h})$

$$\Leftrightarrow \frac{\sum_{j=1}^{n}\left(m_j - L^*(1 - \bar{h})\alpha_j\right) x_j - f}{\sqrt{V(x)}} \geq K_{g^{-1}(\bar{h})} \qquad (11)$$

where $F(\cdot)$ is the distribution function of the standard normal distribution and $K_z = F^{-1}(z)$. Furthermore, each decision variable x_j satisfies $x_j \in \{0,1\}$, we obtain $x_j^2 = x_j$ and assume that each variance is independent, i.e.,

$$\sigma_{ij} = \begin{cases} \sigma_j^2 & (i = j) \\ 0 & (i \neq j) \end{cases}$$

Consequently, problem (10) is equivalently transformed into the following problem:

Maximize f

subject to $\dfrac{\sum_{j=1}^{n}\left(m_j - L^*(1 - \bar{h})\alpha_j\right) x_j - f}{\sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j}} \geq K_{g^{-1}(\bar{h})},$

$\qquad \mathbf{A}x \leq \mathbf{b}, x \in \{0,1\}^n$

$\Leftrightarrow$ Maximize f

subject to $\sum_{j=1}^{n}\left(m_j - L^*(1 - \bar{h})\alpha_j\right) x_j - K_{g^{-1}(\bar{h})}\sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j} \geq f,$ $\qquad (12)$

$\qquad \mathbf{A}x \leq \mathbf{b}, x \in \{0,1\}^n$

$\Leftrightarrow$ Maximize $\sum_{j=1}^{n}\left(m_j - L^*(1 - \bar{h})\alpha_j\right) x_j - K_{g^{-1}(\bar{h})}\sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j}$

subject to $\mathbf{A}x \leq \mathbf{b}, x \in \{0,1\}^n$

This problem is similar to the problem proposed in Hasuike [8], and so we construct the efficient solution method using the proposed approach in Hasuike [8].

B. The case maximizing the degree of necessity

Second, we consider the case maximizing the degree of necessity for problem (1). This model is transformed into the following problem:

Maximize $N_{\tilde{p}}(\tilde{G})$

subject to $\mathbf{A}x \leq \mathbf{b}, x \in \{0,1\}^n$ $\qquad (13)$

This problem is equivalently transformed into the following problem introducing a parameter h :

$$\text{Maximize } h$$

$$\text{subject to } N_{\bar{p}}(\tilde{G}) \geq h, \tag{14}$$

$$\mathbf{A}\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{x} \in \{0,1\}^n$$

In a way similar to equivalent transformations (9), constraint $N_{\bar{p}}(\tilde{G}) \geq h$ is transformed into the following inequality:

$$N_{\bar{p}}(\tilde{G}) \geq h$$

$$\Leftrightarrow \quad \begin{aligned} &\exists \bar{u} : \Pr\{\omega | \bar{u}(\omega) \geq f\} \geq g^{-1}(h), \\ &\bar{u} \sim N\left(\sum_{j=1}^{n}(m_j - L^*(1-h)\alpha_j)x_j, V(\boldsymbol{x})\right) \end{aligned} \tag{15}$$

From this inequality, problem (14) is equivalently transformed into the following problem:

$$\text{Maximize } h$$

$$\text{subject to } \Pr\{\omega | \bar{u}(\omega) \geq f\} \geq g^{-1}(h),$$

$$\bar{u} \sim N\left(\sum_{j=1}^{n}(m_j - L^*(1-h)\alpha_j)x_j, V(\boldsymbol{x})\right), \tag{16}$$

$$\mathbf{A}\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{x} \in \{0,1\}^n$$

Furthermore, we also apply the equivalent transformation (11) to constraint $\Pr\{\omega | \bar{u}(\omega) \geq f\} \geq g^{-1}(h)$, and problem (11) is equivalently transformed into the following problem:

$$\text{Maximize } h$$

$$\text{subject to } \frac{\sum_{j=1}^{n}(m_j - L^*(1-h)\alpha_j)x_j - f}{\sqrt{\sum_{j=1}^{n}\sigma_j^2 x_j}} \geq K_{g^{-1}(h)}, \tag{17}$$

$$\mathbf{A}\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{x} \in \{0,1\}^n$$

It should be noted here that problem (17) is a nonconvex integer programming problem and it is not solved by the linear programming techniques or convex programming techniques. However, since a decision variable h is involved only in the first constraint, we introduce the following subproblem involving a parameter q:

$$\text{Maximize } \frac{\sum_{j=1}^{n}(m_j - L^*(1-q)\alpha_j)x_j - f}{\sqrt{\sum_{j=1}^{n}\sigma_j^2 x_j}} \tag{18}$$

$$\text{subject to } \mathbf{A}\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{x} \in \{0,1\}^n$$

In the case that we fix the parameter q, problem (18) is equivalent to a convex integer programming problem. Furthermore, let $\boldsymbol{x}(q)$ and $Z(q)$ be an optimal solution of problem (18) and its optimal value, respectively. Then, the following theorem is derived from previous study [8, 12, 13].

Theorem 1

For q satisfying $0 < q < 1$, $Z(q)$ is a strictly increasing function of q.

Furthermore, let $\hat{q}$ denote q satisfying $Z(\hat{q}) = g^{-1}(\hat{q})$ and the optimal solutions of main problem (17) be $(\boldsymbol{x}^*, h^*)$. Then the relation between problems (17) and (18) is derived as follows derived from previous study [14].

Theorem 2

Suppose that $0 < \hat{q} < 1$ holds. Then $(x(\hat{q}), \hat{q})$ is equal to $(\boldsymbol{x}^*, h^*)$.

From these theorems, by using bisection algorithm for parameter q and comparing objective function $Z(q)$ with $g^{-1}(q)$, we repeatedly solve problem (18) for each q using branch-bound method, and finally obtain the optimal solution. This solution method is assured that its calculation times are infinite. However, it is not efficient due to increasing computational times voluminously with the increase of parameters and decision variables. Therefore, we need to construct the more efficient solution method.

IV. CONSTRUCTION OF THE EFFICIENT STRICT SOLUTIOM METHOD

In order to construct the efficient strict solution method for problem (17), first of all, we introduce the following 0-1 relaxation problem of problem (17):

$$\text{Maximize } h$$

$$\text{subject to } \frac{\sum_{j=1}^{n}(m_j - L^*(1-h)\alpha_j)x_j - f}{\sqrt{\sum_{j=1}^{n}\sigma_j^2 x_j}} \geq K_{g^{-1}(h)}, \tag{19}$$

$$\mathbf{A}\boldsymbol{x} \leq \boldsymbol{b}, 0 \leq x_j \leq 1, \ (j=1,2,...,n)$$

In a way similar to problem (17), this problem is also a nonconvex programming problem and it is not solved by the linear programming techniques or convex programming techniques. Subsequently, we introduce the following subproblem:

$$\text{Maximize } \frac{\sum_{j=1}^{n}(m_j - L^*(1-q)\alpha_j)x_j - f}{\sqrt{\sum_{j=1}^{n}\sigma_j^2 x_j}} \tag{20}$$

$$\text{subject to } \mathbf{A}\boldsymbol{x} \leq \boldsymbol{b}, 0 \leq x_j \leq 1, \ (j=1,2,...,n)$$

In this paper, it is assumed that there exists a feasible solution satisfying $\sum_{j=1}^{n}(m_j - L^*(1-q)\alpha_j)x_j > f$. This means that the probability that total future profit is more than target value f is greater than 1/2. Furthermore, problem (20) is equivalent to the following problem:

Minimize $\dfrac{\sqrt{\sum\limits_{j=1}^{n}\sigma_j^2 x_j}}{\sum\limits_{j=1}^{n}\left(m_j - L^*(1-q)\alpha_j\right)x_j - f}$ (21)

subject to $\mathbf{A}\boldsymbol{x} \le \boldsymbol{b}, 0 \le x_j \le 1, j=1,2,...,n$

In the case we fix the parameter q, since problem (21) is a nonlinear fractional programming problem due to including a square root term $\sqrt{\sum\limits_{j=1}^{n}\sigma_j^2 x_j}$ in the objective function, it is difficult to solve this original problem directly. Therefore, we introduce the following parameters;

$$t = \dfrac{1}{\sum_{j=1}^{n}\left(m_j - L^*(1-q)\alpha_j\right)x_j - f}, \boldsymbol{y} = t\boldsymbol{x}$$

and we do the transformation into the following deterministic equivalent problem:

Minimize $\sqrt{\sum\limits_{j=1}^{n}\sigma_j^2 y_j}$

subject to $\sum\limits_{j=1}^{n}\left(m_j - L^*(1-q)\alpha_j\right)y_j - ft = 1,$ (22)

$\mathbf{A}\boldsymbol{y} \le \boldsymbol{b}t, 0 \le y_j \le t, j=1,2,...,n$

Since objective function $\sqrt{\sum\limits_{j=1}^{n}\sigma_j^2 y_j}$ is a monotonous increasing function, this problem is equivalently transformed into the following problem:

Minimize $\sum\limits_{j=1}^{n}\sigma_j^2 y_j$

subject to $\sum\limits_{j=1}^{n}\left(m_j - L^*(1-q)\alpha_j\right)y_j - ft = 1,$ (23)

$\mathbf{A}\boldsymbol{y} \le \boldsymbol{b}t, 0 \le y_j \le t, j=1,2,...,n$

Problem (23) is a linear programming problem in the case that q is fixed, and so we efficiently obtain the optimal solution using linear programming approaches and bisection algorithm for parameter q.

Furthermore, let the optimal value of parameter h in problem (17) be h^*. Then, the following lemmas hold:

Lemma 1

With respect to problem (17), there exists ranges $\left[h_k, h_{k+1}\right]$ $(k=1,2,...)$ that the optimal solution of problem (17) is unique for any h including in $\left[h_k, h_{k+1}\right]$.

Proof

From the continuity of parameter h and discreteness of decision variable $\boldsymbol{x}^*$, this lemma clearly holds.

Lemma 2

Let the optimal value of problem (19) be $\overline{h}$, the optimal solution of problem (17) be $\boldsymbol{x}^*$ and the optimal value be h^*.

Then in the case that we set a range $\left[h_L, h_U\right]$ satisfying $\overline{h} \in \left[h_L, h_U\right]$, $h^* \in \left[h_L, h_U\right]$ holds.

Proof

We consider the case that $h^* \notin \left[h_L, h_U\right]$ and $h^* \in \left[h'_L, h'_U\right]$. If $h'_U < h_L$, there exists the optimal solution $\boldsymbol{x}'$ and the optimal solution h' satisfying $h' \in \left[h_L, h_U\right]$. This contradicts the optimality of parameter h^*. In a way similar to $h'_U < h_L$, if $h'_L > h_U$, we obviously find that $h'_L > h^*$. This means that the optimal value of discrete problem is larger than that of continuous problem, and contradicts the optimality of parameter $\overline{h} > h^*$. Consequently, this lemma holds.

From these lemmas, the following theorem to the relation between problems (17) and (19) holds:

Theorem 3

Let the optimal solution of problem (19) be $\boldsymbol{x}^*\left(\overline{h}\right)$ and the optimal value of parameter h be $\overline{h}$. Then, the optimal solution of the following problem;

Maximum $\dfrac{\sum\limits_{j=1}^{n}\left(m_j - L^*(1-\overline{h})\alpha_j\right)x_j - f}{\sqrt{\sum\limits_{j=1}^{n}\sigma_j^2 x_j}}$ (24)

subject to $\mathbf{A}\boldsymbol{x} \le \boldsymbol{b}, \boldsymbol{x} \in \{0,1\}^n$

is equivalent to that of problem (17).

Consequently, in the case that we solve 0-1 relaxation problem (19) and obtain its optimal solution h^*, we obtain an optimal solution more efficiently than previous parametric approaches due to not using branch-bound method every value of parameter h repeatedly. However, since the objective function of problem (24) is a nonlinear function, it is not easy to deal with several efficient solution methods for integer programming approaches. Therefore, in order to have the more general versatility for our proposed model, we consider the other deterministic equivalent transformations for main problem.

First, we equivalently transform main problem (17) into the following problem;

Maximum h

subject to $\sum\limits_{j=1}^{n}\left(m_j - L^*(1-h)\alpha_j\right)x_j - K_{g^{-1}(h)}\sqrt{\sum\limits_{j=1}^{n}\sigma_j^2 x_j} \ge f,$ (25)

$\mathbf{A}\boldsymbol{x} \le \boldsymbol{b}, \boldsymbol{x} \in \{0,1\}^n$

and introduce this 0-1 relaxation problem as follows:

Maximum h

subject to $\sum\limits_{j=1}^{n}\left(m_j - L^*(1-h)\alpha_j\right)x_j - K_{g^{-1}(h)}\sqrt{\sum\limits_{j=1}^{n}\sigma_j^2 x_j} \ge f,$ (26)

$\mathbf{A}\boldsymbol{x} \le \boldsymbol{b}, 0 \le x_j \le 1, j=1,2,...,n$

This problem (26) is a nonlinear programming problem. However, this problem is much similar to problem (12). Therefore, in order to solve problem (26) analytically, we

introduce the following subproblem in a way similar to the transformation from problem (19) into problem (20):

$$\text{Maximum} \sum_{j=1}^{n}\left(m_j - L^*\left(1-h\right)\alpha_j\right)x_j - K_{g^{-1}(h)}\sqrt{\sum_{j=1}^{n}\sigma_j^2 x_j} \qquad (27)$$

subject to $\mathbf{A}x \le \mathbf{b}, 0 \le x_j \le 1, j = 1, 2, ..., n$

Then, we consider the following auxiliary problem:

$$\text{Maximum} \ \gamma\sum_{j=1}^{n}\left(m_j - L^*\left(1-h\right)\alpha_j\right)x_j - K_{g^{-1}(h)}\left(\sum_{j=1}^{n}\sigma_j^2 x_j\right) \qquad (28)$$

subject to $\mathbf{A}x \le \mathbf{b}, 0 \le x_j \le 1, j = 1, 2, ..., n$

In the case that we fix parameter h, with respect to the relation between problems (27) and (28), the following theorem holds based on the previous research of Ishii [11].

Theorem 4

Let the optimal solution of problem (27) be $x\left(h\right)$. Then, in the case $\gamma = 2\sqrt{\sum_{j=1}^{n}\sigma_j^2 x_j^*}$, the optimal solution of problem (28) is equal to $x\left(h\right)$.

From Theorem 4, in the case that parameter h is fixed, we obtain the optimal solution $x\left(h\right)$. Furthermore, we consider the following problem to deal with optimal value $\overline{h}$ of problem (19):

$$\text{Maximum} \ \gamma\sum_{j=1}^{n}\left(m_j - L^*\left(1-\overline{h}\right)\alpha_j\right)x_j - K_{g^{-1}(\overline{h})}\left(\sum_{j=1}^{n}\sigma_j^2 x_j\right) \qquad (29)$$

subject to $\mathbf{A}x \le \mathbf{b}, 0 \le x_j \le 1, j = 1, 2, ..., n$

Let this optimal solution be $x\left(h^*\right)$. Subsequently, the following lemma with respect to each optimal solution for problems (17) and (25) holds.

Lemma 3

The optimal solution of problem (25) is equal to that of problem (17).

Proof

Since each problem is the deterministic equivalent problem for main problem (13), this lemma obviously holds.

Therefore, we obtain $x\left(\overline{h}\right) = x^*$. Then, the following theorem holds extending previous research of Hasuike.

Theorem 5

With respect to $\gamma^* = 2\sqrt{\sum_{j=1}^{n}\sigma_j^2 x_j^*}$, the optimal solution of the following problem;

$$\text{Maximum} \ \gamma^*\sum_{j=1}^{n}\left(m_j - L^*\left(1-\overline{h}\right)\alpha_j\right)x_j - K_{g^{-1}(\overline{h})}\left(\sum_{j=1}^{n}\sigma_j^2 x_j\right) \qquad (30)$$

subject to $\mathbf{A}x \le \mathbf{b}, x \in \{0,1\}^n$

is equal to that of problem (13).

From Theorem 5, we finally solve this linear 0-1 programming problem. It is more efficient to obtain its optimal solution of problem (30) using some efficient solution methods for integer programming approaches than that of problem (17). Furthermore, in the case using branch-bound method, we find that upper limited value for main problem becomes

$$\sum_{j=1}^{n}\left(m_j - L^*\left(1-\overline{h}\right)\alpha_j\right)x_j\left(\overline{h}\right) - K_{g^{-1}(\overline{h})}\sqrt{\sum_{j=1}^{n}\sigma_j^2 x_j\left(\overline{h}\right)}$$

substituting optimal solution $x\left(\overline{h}\right)$ and optimal value $\overline{h}$ of problem (19) and lower limited value becomes f. Therefore, by using these values in branch-bound method efficiently, we obtain the optimal solution of main problem more easily and rapidly. Consequently, we develop the following solution method.

Solution method
STEP 1: Elicit the membership function of a fuzzy goal for with respect to the probability and set each parameter.
STEP 2: Solve 0-1 relaxation problem (19), and find the optimal solution x^* and optimal value $\overline{h}$.
STEP 3: Solve 0-1 programming problem (30) by using integer programming approaches such as branch-bound method.

Furthermore, with respect to previous parametric integer programming approach for 0-1 random fuzzy programming problems, integer programming approaches such as the Branch-bound method have been used to every parameter value set by using the bisection method. Therefore, the computational time complexity has been $\left(K_{IPM}\right)\cdot\left(\log n\right)^2$, where K_{IPM} is the computational time complexity of integer programming approach. However, with respect to our proposed approach, the computational time complexity is $n^4\log n + K_{IPM}$ due to not use integer programming approach repeatedly. Consequently, our proposed solution approach may be more useful in the case that the number of decision variables and constraints are larger.

V. CONCLUSION

In this paper, we have proposed two general 0-1 random fuzzy programming problems considering both random and fuzzy conditions based on the degree of necessity. Since our proposed models have been nonlinear 0-1 programming problems by introducing chance constraints and performing the transformation into the deterministic equivalent problems, we have constructed the efficient strict solution method by dealing with some 0-1 relaxation problems. Consequently, we have found that the number of using branch-bound method in our proposed method is much less than that in previous parametric solution methods.

This solution method may be applicable to the general integer programming problems because our proposal model includes some previous models not considering uncertainty. However, in the case there are large number of decision variables and parameters, it takes much computational time to solve this 0-1 random fuzzy programming problem even if

we use this efficient solution method due to the nonpolynomial time algorithm to branch-bound method. Therefore, as future studies, we need to construct its efficient solution method using not only strict solution method such as branch-bound method but also approximation methods such as genetic algorithm and heuristic approaches.

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