Magnetothermoelastic Problem of a Half-Space

S.K. Bhullar and J.L.Wegner

Abstract - **This paper is concerned to study temperature distribution, thermal stresses and displacement components for a magnetothermoelastic problem of a half-space subjected to (i) moving heat source and (ii) moving load. Classical Dynamical Coupled, Lord-Shulman and Green Lindsay theories of thermoelasticity are used for mathematical analysis. It is found that the Lord-Shulman theory is more pronounced than coupled theory and Green Lindsay theories. Numerical computations have been performed for computing temperature, stresses and displacement for these theories. The results obtained using these theories are compared and depicted graphically.**

Keywords: **displacement, moving heat source, moving load, temperature field.**

I. Introduction

The classical theory of thermoelasticity is based on Fourier's law of heat conduction, which predicts an infinite speed of heat propagation. Many new theories have been proposed to eliminate this physical absurdity. Lord and Shulman [1] first modified Fourier's law by introducing into the field equations the term representing the thermal relaxation time. This modified theory is known as the generalized theory of thermoelasticity. Later, Green and Lindsay [2] developed a more general theory of thermoelasticity, in which Fourier's law of heat conduction is unchanged, whereas the classical energy equation and the stress-strain temperature relations are modified by introducing two constitutive constants having dimensions of time. In the last five decades another domain has been developed, which investigates the interaction between the strain and electromagnetic fields. This discipline is called magnetoelasticity. The problem of interaction between the elastic or thermoelastic field and the electromagnetic field has been a research topic for a number of investigations in recent years because of its utilitarian aspects in various branches of science and technology, like geophysics for understanding the effect of the Earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emissions at electromagnetic radiation from nuclear devices, development of a highly sensitive super conducting magnetometer, electrical power engineering, optics and plasma physics. A comprehensive review of the earlier contribution to the subject can be found in [3]. The contribution of some authors who had worked in this field is presented in [4-11]. The other studies performed is a coupled magnetothermoelastic problem in elastic half space [12], transient generalized magnetothermoelastic waves in a rotating half-space [13] and a coupled magnetothermoelastic problem in a perfectly conducting elastic half-space with thermal relaxation [14], magnetothermoelastic waves induced by a thermal shock in a infinitely conducting elastic half space [15] and generation of generalized magneto thermoelastic waves by thermal shock in a perfectly conducting half-space[16]. Recently, relaxation effects on thermal shock problems in an elastic half-space of generalized magneto thermoelasticity are studied in [17]. In the present paper we have formulated a two-dimensional magnetothermoelastic problem of a half-space subjected to moving heat source and moving load to study temperature field, thermal stresses and displacement components.

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II. THEORY

Following Othman [17], for generalized thermoelasticity with two relaxation times, the linearized equations in non- dimensional form of electrodynamics in slowly moving medium and the non-vanishing stress components are given by

$$\beta^2 u_{,xx} + u_{,yy} + (\beta^2 - 1)v_{,xy} - \beta^2(\theta_{,x} + t_1\dot{\theta}_{,x}) = \alpha_o \ddot{u} \quad (1)$$

$$(\beta^2 - 1)u_{,xy} + \beta^2 v_{,yy} + v_{,xx} - \beta^2(\theta_{,y} + t_1\dot{\theta}_{,y}) = \alpha_o \ddot{v} \quad (2)$$

$$\nabla^2\theta = \left(\frac{\partial}{\partial t} + t_o\frac{\partial^2}{\partial t^2}\right)\theta + \varepsilon\left(\frac{\partial}{\partial t} + t_o\frac{\partial^2}{\partial t^2}\right)e \quad (3)$$

$$\sigma_{xx} = \beta_0^2 u_{,x} + (\beta^2 - 2)v_{,y} - \beta^2\left(1 + t_1\frac{\partial}{\partial t}\right)\theta \quad (4)$$

$$\sigma_{yy} = (\beta^2 - 2)u_{,x} + \beta_0^2 u_{,y} - \beta^2\left(1 + t_1\frac{\partial}{\partial t}\right)\theta \quad (5)$$

$$\sigma_{xy} = u_{,x} + v_{,x} \quad (6)$$

$$e = u_{,x} + v_{,y} \quad (7)$$

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$$\alpha_0 = \alpha\beta^2,\ \alpha = 1 + \frac{a_0^2}{c^2},\ c^2 = \frac{1}{\mu_0 \varepsilon_0},\ a_0^2 = \frac{\mu_0 H_0^2}{\rho},$$

$$c_0^2 = \frac{\lambda + 2\mu}{\rho} + a_0^2,\ \beta^2 = \frac{c_0^2}{c_2^2},\ c_2^2 = \frac{\mu}{\rho}.$$

t_0 and t_1 are thermal relaxation times and other symbols having their usual meanings. In order to discuss the results from different theories of thermoelasticity, we shall take for:

C-D theory, $t_0 = t_1 = 0$;

L-S theory, $t_0 = 0,\ t_1 \neq 0$;

G-L theory, $t_0 \neq 0, t_1 \neq 0$.

In the above equations, the following non-dimensional quantities are used

$$x' = \frac{\eta_0}{c_0} x,\ y' = \frac{\eta_0}{c_0} y,\ u' = \frac{\rho c_0 \eta_0}{T_0} u,$$

$$v' = \frac{\rho c_0 \eta_0}{T_0} v, t' = \eta_0 t,\ t_1' = \eta_0 t_1, t_0' = \eta_0 t_0,$$

$$\sigma_{ij}' = \frac{\sigma_{ij}}{\mu},\ \theta = \frac{T - T_0}{T_0}, \eta_0 = \frac{\rho C_E}{K}. \tag{8}$$

where, primes denote dimensional variables. If we introduce the function φ defined by, $\varphi = e - \theta$ Equations (1) and (2) take the form

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{1}{\alpha}\left(\nabla^2 \varphi - t_1 \nabla^2 \dot{\theta}\right) - \frac{\partial^2 \varphi}{\partial t^2} \tag{9}$$

The heat conduction equation given by (3) can be written as

$$\nabla^2 \theta = \left(\frac{\partial}{\partial t} + t_o \frac{\partial^2}{\partial t^2}\right)\left[\theta + \varepsilon(\theta + \varphi)\right] \tag{10}$$

and the stress components given by (4) - (6) are written as

$$\sigma_{xx} = \left[\beta_0^2 - \beta^2\left(1 + t_1 \frac{\partial}{\partial t}\right)\right]\theta + \beta_0^2 \varphi - 2v_{,y} \tag{11}$$

$$\sigma_{yy} = \left[\beta_0^2 - \beta^2\left(1 + t_1 \frac{\partial}{\partial t}\right)\right]\theta + \beta_0^2 \varphi - 2u_{,x} \tag{12}$$

$$\sigma_{xy} = u_{,y} + v_{,x} \tag{13}$$

We change the co-ordinate system moving with input by shifting the origin to the position of input

$$x'' = \frac{\eta_0}{c_0}(x' - pt'),\ y'' = y',\ t'' = t',$$

$$\nabla_1^2 = \frac{\partial^2}{\partial x''^2} + \frac{\partial^2}{\partial y''^2} \tag{14}$$

where $p = \frac{v}{c_0}$, is the dimensionless loading speed and the co-ordinates x'' and y'' move in positive direction with speed p. It follows from (14) that we may use the relation

$$\frac{\partial}{\partial t'} = -p \frac{\partial}{\partial x''} \tag{15}$$

to eliminate time derivatives. In terms of the moving co-ordinates given by (14), (1) and (2) together with (7) and (8) become

$$\left(\beta^2 - 1\right)\theta + \varphi_{,x} + u_{,yy} + u_{,xx}$$
$$-\beta^2\left(\theta_{,x} - pt_1\theta_{,xx}\right) = \alpha_o p^2 u_{,xx} \tag{16}$$

$$\left(\beta^2 - 1\right)\theta + \varphi_{,y} + v_{,yy} + v_{,xx}$$
$$-\beta^2\left(\theta_{,y} - pt_1\theta_{,yy}\right) = \alpha_o p^2 v_{,xx} \tag{17}$$

Equations (9)-(10) together with relation (15), after omitting the primes on x and y are as follows:

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha}\left(\nabla^2 \varphi + pt_1 \nabla^2 \frac{\partial \theta}{\partial x}\right) - p^2 \frac{\partial^2 \varphi}{\partial x^2} \tag{18}$$

$$\nabla^2 \theta = \left(-p\frac{\partial}{\partial x} + p^2 t_o \frac{\partial^2}{\partial x^2}\right)$$
$$\times\left[\theta + \varepsilon(\theta + \varphi)\right] \tag{19}$$

To obtain the expressions for θ, φ, u, v and σ_{ij} let us assume that where, D is the (complex) frequency and a is the wave number in the x - direction and D is unknown quantity. Inserting (20) into (18) and (19) to obtain:

$$\left[\theta, \varphi, u, v, \sigma_{ij}\right](x, y) = \left[\theta_0, \varphi_0, u_0, v_0, \sigma_{0ij}\right](y)$$
$$\times \exp(\iota a x - D y) \tag{20}$$

$$\left[D^2 - a^2 + \alpha p^2 a^2\right]\varphi_0(y)$$
$$= -\left[\left(D^2 - a^2\right)\iota a p t_1 + \alpha p^2 a^2\right]\theta_0(y) \tag{21}$$

$$\varepsilon\omega_1 \varphi_0(y) = \left[\left(D^2 - a^2\right) - (1 + \varepsilon)\omega_1\right]\theta_0(y) \tag{22}$$

Eliminating $\theta_0(y)$ from equations (21)-(22), we obtain

$$\left[D^4 - a_1 D^2 - a_2\right]\theta_0(y) = 0 \tag{23}$$

where,

$$a_1 = 2a^2 + \alpha\omega^2 p^2 + (1+\varepsilon+\iota\varepsilon apt_1)\omega_1$$
$$a_2 = (a^4+\omega_1 a^2)(1-\alpha p^2) + \varepsilon\omega_1 a^2(1+\iota apt_1)$$
$$\omega_1 = -t_0 p^2 a^2 - \iota ap$$

Equation (23) can be factorized as

$$\left[(D^2-k_1^2)(D^2-k_2^2)\right]\theta_0(y) = 0 \tag{24}$$

where,

$$k_{1,2}^2 = a^2 + \omega_2 \pm \omega_3$$
$$\omega_2 = \frac{1}{2}\left[\alpha p^2 a^2 + (1+\varepsilon+\iota\varepsilon\alpha pt_1)\right]\omega_1 \tag{25}$$
$$\omega_3 = \sqrt{\omega_2^2 - \alpha p^2 a^4}$$

The solution of (23) is written as

$$\theta_0 = \sum_{i=1}^{2}\theta_i \exp(\iota ax - k_1 y)$$

Where, θ_i are parameters depending upon a.

Substituting equation (25) in (21) and we get:

$$\varphi_0 = \sum_{i=1}^{2}\left[\frac{(k_i^2-a^2)\iota apt_1 + \alpha p^2 a^2}{k_i^2 - a^2 + \alpha p^2 a^2}\right] \times\theta_i \exp(\iota ax - k_i y) \tag{26}$$

Now, (16) and (17) together with (20) become as follows:

$$(D^2 - a^2 + \alpha_0 p^2 a^2)u_0 + \iota a(\beta^2-1)\varphi_0 - (\iota a + \beta^2 a^2 pt_1)\theta_0 = 0 \tag{27}$$

$$(D^2 - a^2 + \alpha_0 p^2 a^2)v_0 + \iota a(\beta^2-1)D\varphi_0 + (1+\iota a\beta^2 pt_1)D\theta_0 = 0 \tag{28}$$

Substituting (25) - (26) in (27) (28), we get

$$u_0 = \sum_{i=1}^{2}\frac{1}{k_i^2-m^2}\left\{\begin{array}{l} -\dfrac{ia(\beta^2-1)(k_i^2-a^2)\iota apt_1 + \alpha p^2 a^2}{k_i^2-a^2+\alpha p^2 a^2} \\ +(\iota a + pt_1 a^2\beta^2)\end{array}\right\} \times\theta_i \exp(\iota ax - k_i y) \tag{29}$$

$$v_0 = \sum_{i=1}^{2}\frac{1}{k_i^2-m^2}\left[\begin{array}{l}\dfrac{(\beta^2-1)(k_i^2-a^2)\iota apt_1 + \alpha p^2 a^2}{k_i^2-a^2+\alpha p^2 a^2} \\ -(1-\iota apt_1\beta^2)\end{array}\right] \times\theta_i \exp(\iota ax - k_i y) \tag{30}$$

where, $m = a^2 + \alpha_0 a^2 p^2$

In terms of the moving co-ordinates (14) and by making use of relation (15) the stress components given by (11)-(13) become as follows

$$\sigma_{xx} = \left[\beta_0^2 - \beta^2\left(1 - pt_1\frac{\partial}{\partial x}\right)\right]\theta + \beta_0^2\varphi - 2v_{,y} \tag{31}$$

$$\sigma_{yy} = \left[\beta_0^2 - \beta^2\left(1 - pt_1\frac{\partial}{\partial x}\right)\right]\theta + \beta_0^2\varphi - 2u_{,x} \tag{32}$$

$$\sigma_{xy} = u_{,y} + v_{,x} \tag{33}$$

Upon using (20), (25), (26) and (29) into equations (31)-(33), we get

$$\sigma_{0xx} = \sum_{i=1}^{2}\left\{\begin{array}{l}\beta_0^2\left[\begin{array}{l}\dfrac{(k_i^2-a^2)\iota apt_1 + \alpha p^2 a^2}{k_i^2-a^2+\alpha p^2 a^2} \\ +\beta_0^2 - \beta^2(1-\iota apt_1)\end{array}\right] - \dfrac{k_i^2}{k_i^2-m^2} \\ \times\left[\begin{array}{l}-(1-\iota apt_1\beta^2) \\ -\dfrac{(\beta^2-1)(k_i^2-a^2)(\iota apt_1+\alpha p^2 a^2)}{k_i^2-a^2+\alpha p^2 a^2}\end{array}\right]\end{array}\right\} \times\theta_i \exp(\iota ax - k_i y) \tag{34}$$

$$\sigma_{0yy} = \sum_{i=1}^{2}\left\{\begin{array}{l}\beta_0^2\left[\begin{array}{l}\dfrac{(k_i^2-a^2)\iota apt_1 + \alpha p^2 a^2}{k_i^2-a^2+\alpha p^2 a^2} \\ +\beta_0^2 - \beta^2(1-\iota apt_1)\end{array}\right] - \dfrac{2\iota a}{k_i^2-m^2} \\ \times\left[\begin{array}{l}-(1+\iota apt_1\beta^2) \\ -\dfrac{(\beta^2-1)(k_i^2-a^2)(\iota apt_1+\alpha p^2 a^2)}{k_i^2-a^2+\alpha p^2 a^2}\end{array}\right]\end{array}\right\} \times\theta_i \exp(\iota ax - k_i y) \tag{35}$$

$$\sigma_{0xy} = \sum_{i=1}^{2} \frac{k_i^2}{k_i^2 - m^2} \beta_0^2 \left[\frac{\left(k_i^2 - a^2\right) \iota apt_1 + \alpha p^2 a^2}{k_i^2 - a^2 + \alpha p^2 a^2} + \beta_0^2 - \beta^2 \left(1 - \iota apt_1\right) \right] - \frac{\iota ak}{k_i^2 - m^2} \sum_{i=1}^{2} \frac{k_i^2}{k_i^2 - m^2} \times \left[-\left(1 + \iota apt_1 \beta^2\right) + \frac{\left(\beta^2 - 1\right)\left(k_i^2 - a^2\right)\left(\iota apt_1 + \alpha p^2 a^2\right)}{k_i^2 - a^2 + \alpha p^2 a^2} \right] \times \theta_i \exp\left(\iota ax - k_i y\right) \tag{36}$$

PROBLEM **I**

Consider a homogeneous isotropic thermoelastic solid occupying the region

$y \geq 0, -\infty < x < \infty, -\infty < z < \infty$

of the xy-plane and displacement $\vec{\mathbf{u}} = (u, v, 0)$ and the temperature T are function of x, y and time t which is subjected to moving heat source with following boundary conditions,

$$\theta(x, y, t) = f(x - yt), \sigma_{xy} = (x, y, t) = 0, \quad \frac{\partial \theta}{\partial x} + h\theta = 0 \tag{37}$$

where, h is the surface heat transfer coefficient and f is arbitrary function and be the velocity of motion of heat source. Equations (37) together with (25) and(36) gives following expression:

$$\sum_{i=1}^{2} \theta_i \exp\left(\iota ax - k_1 y\right) = \sum_{i=1}^{2} a_k \exp\left(\iota ax\right) \tag{38}$$

$$\sigma_{0xy} = \sum_{i=1}^{2} \frac{k_i^2}{k_i^2 - m^2} \left\{ \beta_0^2 \left[\frac{\left(k_i^2 - a^2\right) \iota apt_1 + \alpha p^2 a^2}{k_i^2 - a^2 + \alpha p^2 a^2} + \beta_0^2 - \beta^2 \left(1 - \iota apt_1\right) \right] - \frac{\iota ak}{k_i^2 - m^2} \times \left[-\left(1 + \iota apt_1 \beta^2\right) + \frac{\left(\beta^2 - 1\right)\left(k_i^2 - a^2\right)\left(\iota apt_1 + \alpha p^2 a^2\right)}{k_i^2 - a^2 + \alpha p^2 a^2} \right] \right\} \times \theta_i \exp\left(\iota ax\right) = 0 \tag{39}$$

$$\sum_{i=1}^{2} \left(k_i + h\right) \theta_i \exp\left(\iota ax\right) = 0 \tag{40}$$

where

, $a_k = \dfrac{1}{\pi} \displaystyle\int_{i=1}^{2} f(x) \exp\left(\iota ax\right) dx$ and $f(x) = \exp\left(-x^2\right)$

$$\theta_1 = \frac{a_{13} a_{22}}{\nabla}, \ \theta_2 = \frac{-a_{13} a_{21}}{\nabla}, \nabla = -a_{21} + a_{22}$$

$$a_{11} = a_{12} = 1, \ a_{13} = \frac{1}{\pi} \exp\left(-b\left(x - vt\right)^2\right)$$

$$a_{21} = \frac{k_1^2}{k_1^2 - m^2} \left[\left(\iota a + pt_1 a^2 \beta^2\right) - \frac{ia\left(\beta^2 - 1\right)\left(k_1^2 - a^2\right) \iota apt_1 + \alpha p^2 a^2}{k_1^2 - a^2 + \alpha p^2 a^2} \right] + \frac{\iota a k_1}{k_1^2 - m^2} \left[\frac{\left(\beta^2 - 1\right)\left(k_1^2 - a^2\right) \iota apt_1 + \alpha p^2 a^2}{k_1^2 - a^2 + \alpha p^2 a^2} - \left(1 - \iota apt_1 \beta^2\right) \right]$$

$$a_{22} = \frac{k_2^2}{k_2^2 - m^2} \left[\left(\iota a + pt_1 a^2 \beta^2\right) - \frac{ia\left(\beta^2 - 1\right)\left(k_2^2 - a^2\right) \iota apt_1 + \alpha p^2 a^2}{k_2^2 - a^2 + \alpha p^2 a^2} \right] + \frac{\iota a k_1}{k_2^2 - m^2} \left[\frac{\left(\beta^2 - 1\right)\left(k_2^2 - a^2\right) \iota apt_1 + \alpha p^2 a^2}{k_2^2 - a^2 + \alpha p^2 a^2} - \left(1 - \iota apt_1 \beta^2\right) \right]$$

$$a_{23} = 0$$

PROBLEM **II**

Consider a homogeneous isotropic thermoelastic solid occupying the region

$y \geq 0, -\infty < x < \infty, -\infty < z < \infty$ of the xy-plane which is subjected to moving load with following boundary conditions,

$$\sigma_{yy}(x, y, t) = g(x - vt) \tag{41}$$

$$\sigma_{xy}(x, y, t) = 0 \tag{42}$$

$$\frac{\partial \theta}{\partial y} + h\theta = 0 \tag{43}$$

$$\sum_{i=1}^{2}\left\{\begin{array}{l}\beta_0^2\left[\begin{array}{l}\dfrac{\left(k_i^2-a^2\right)\imath apt_1+\alpha p^2a^2}{k_i^2-a^2+\alpha p^2a^2}\\+\beta_0^2-\beta^2(1-\imath apt_1)\end{array}\right]-\dfrac{2\imath a}{k_i^2-m^2}\\ \times\left[\begin{array}{l}-\left(1+\imath apt_1\beta^2\right)\\-\dfrac{\left(\beta^2-1\right)\left(k_i^2-a^2\right)\left(\imath apt_1+\alpha p^2a^2\right)}{k_i^2-a^2+\alpha p^2a^2}\end{array}\right]\end{array}\right\}\times\theta_i\exp(\imath ax)=\sum_{k=1}^{2}\exp(\imath ax) \quad (44)$$

$$\sum_{i=1}^{2}\frac{k_i^2}{k_i^2-m^2}\left\{\begin{array}{l}\beta_0^2\left[\begin{array}{l}\dfrac{\left(k_i^2-a^2\right)\imath apt_1+\alpha p^2a^2}{k_i^2-a^2+\alpha p^2a^2}\\+\beta_0^2-\beta^2(1-\imath apt_1)\end{array}\right]\\-\dfrac{\imath ak}{k_i^2-m^2}\\ \times\left[\begin{array}{l}-\left(1+\imath apt_1\beta^2\right)\\+\dfrac{\left(\beta^2-1\right)\left(k_i^2-a^2\right)\left(\imath apt_1+\alpha p^2a^2\right)}{k_i^2-a^2+\alpha p^2a^2}\end{array}\right]\end{array}\right\}\times\theta_i\exp(\imath ax)=0 \quad (45)$$

$$\sum_{i=1}^{2}(k_i+h)\,\theta_i\exp(\imath ax)=0 \quad (46)$$

where,

$$b_k=\frac{1}{\pi}\int_{i=1}^{2}g(x)\exp(\imath ax)dx \text{ and}$$

$$g(x)=\exp\left(-x^2\right)$$

Solving equations (44)-(46) for unknown constants

$$\theta_1'=\frac{a_{13}'a_{22}'}{\nabla'},\ \theta_2'=\frac{-a_{13}'a_{21}'}{\nabla'},$$

$$\nabla'=-a_{21}'a_{12}'+a_{11}'a_{22}'$$

$$a_{11}'=\left(\beta_0^2-\beta^2(1-\imath apt_1)\right)+\beta_0^2\left[\frac{\left(k_1^2-a^2\right)\imath apt_1+\alpha p^2a^2}{k_i^2-a^2+\alpha p^2a^2}\right]-\frac{2\imath a}{k_1^2-m^2}\left[\begin{array}{l}\left(\imath a+pt_1a^2\beta^2\right)\\-\dfrac{\imath a\left(\beta^2-1\right)\left(k_1^2-a^2\right)\left(\imath apt_1+\alpha p^2a^2\right)}{k_1^2-a^2+\alpha p^2a^2}\end{array}\right]$$

$$a_{12}'=\left(\beta_0^2-\beta^2(1-\imath apt_1)\right)+\beta_0^2\left[\frac{\left(k_2^2-a^2\right)\imath apt_1+\alpha p^2a^2}{k_2^2-a^2+\alpha p^2a^2}\right]-\frac{2\imath a}{k_2^2-m^2}\left[\begin{array}{l}\left(\imath a+pt_1a^2\beta^2\right)\\-\dfrac{\imath a\left(\beta^2-1\right)\left(k_2^2-a^2\right)\left(\imath apt_1+\alpha p^2a^2\right)}{k_2^2-a^2+\alpha p^2a^2}\end{array}\right]$$

$$a_{13}'=0$$

$$a_{21}'=\frac{k_1^2}{k_1^2-m^2}\left[\begin{array}{l}\left(\imath a+pt_1a^2\beta^2\right)\\-\dfrac{ia\left(\beta^2-1\right)\left(k_1^2-a^2\right)\imath apt_1+\alpha p^2a^2}{k_1^2-a^2+\alpha p^2a^2}\end{array}\right]+\frac{\imath ak_1}{k_i^2-m^2}\left[\begin{array}{l}\dfrac{\left(\beta^2-1\right)\left(k_1^2-a^2\right)\imath apt_1+\alpha p^2a^2}{k_1^2-a^2+\alpha p^2a^2}\\-\left(1-\imath apt_1\beta^2\right)\end{array}\right]$$

$$a_{22}'=\frac{k_2^2}{k_2^2-m^2}\left[\begin{array}{l}\left(\imath a+pt_1a^2\beta^2\right)\\-\dfrac{ia\left(\beta^2-1\right)\left(k_2^2-a^2\right)\imath apt_1+\alpha p^2a^2}{k_2^2-a^2+\alpha p^2a^2}\end{array}\right]+\frac{\imath ak_1}{k_2^2-m^2}\left[\begin{array}{l}\dfrac{\left(\beta^2-1\right)\left(k_2^2-a^2\right)\imath apt_1+\alpha p^2a^2}{k_2^2-a^2+\alpha p^2a^2}\\-\left(1-\imath apt_1\beta^2\right)\end{array}\right]$$

$$a_{23}'=\frac{1}{\pi}\exp\left(-b(x-\nu t)^2\right)$$

III. NUMERICAL RESULTS

In order to study temperature field, thermal stresses and displacement components, we have computed them for a specific model. The material chosen for numerical calculation is Copper. The physical data for such material in SI units is,

$$\rho=8.93\times10^3\,kg/m^3,\ C_E=0.398\times10^3\,J/kg,$$

$$K=381W/m^oC,\ \varepsilon=0.0168,\ \beta^2=3.5,\ \beta_0^2=2.01.$$

To compare the results obtained using classical Dynamic Coupled, Lord-Shulman and Green-Lindsay theories of thermoelasticity. The value of thermal relaxation times have been taken as:

C-D theory, $t_0=t_1=0$;

L-S theory, $t_0=0.5,\ t_1=0$;

G-L theory, $t_0=0.2, t_1=0.5$.

The graphs are drawn for different values of time, $t=0.2, t_1=0.5.$ The values of real part of temperature field and displacement components

$u(x,t)$ and $v(x,t)$ are evaluated on the plane y = 1, for moving heat source and moving load.

Moving heat source: In Fig. 1-3, 3-D graphs shows the variation in temperature for C-D, G-L and L-S theories due to moving heat source at dimensionless time, t=0.2 and Fig.4, three curves, predicted by the three theories, C-D, G-L and L-S for temperature distribution, are shown. The graph in Fig. 5, is drawn to see the variation in temperature at time t=0.5 whereas the comparison for temperature variation, at time t=0.2 and t=0.5 due to moving heat source is shown in Fig. 6. The horizontal displacement for C-D, G-L and L-S theories respectively due to moving heat source at dimensionless time t=0.2 is shown in Fig. 7-9. and comparison of three theories is given in Fig.10. The graph in Fig. 11, is drawn to see horizontal displacement at time t=0.5 whereas the comparison for horizontal displacement, at time t=0.2 and t=0.5 due to moving heat source is shown in Fig. 12. Also, 3-D graphs in Fig. 13-15, shows the vertical displacement due to moving heat source at dimensionless time t=0.2 and their comparison can be seen in Fig.16. The graph in Fig. 17, is drawn to see the vertical displacement at time t=0.5 and the comparison of vertical displacement at t=0.2 and t= 0.5 all the three theories, is shown in Fig. 18.

Moving load: Similarly the results are obtained for the problem of moving load. The variation in temperature, horizontal displacement and vertical displacement at different values of time t=0.2 and t=0.5 and their Comparison is shown in Fig. 19-36, for C-D, G-L and L-S theories due to moving. Comparison, for C-D, G-L and L-S theories due to moving load are shown in Fig.10-18.

IV. CONCLUSION

It is observed that:

i. Temperature variation is more in L-S theory than C-D and G-L theory with distance at small time due to moving heat source.

ii. The same variation is observed in the case of horizontal and vertical displacement distribution.

iii. As well as case of moving load source is concerned the variation in temperature and displacement occurs in same fashion.

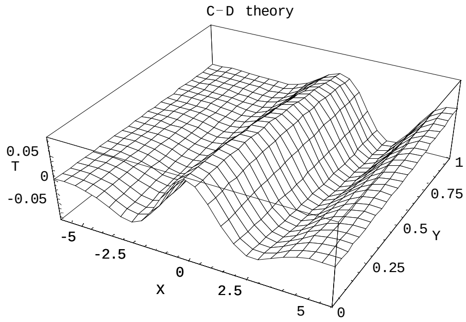

Fig. 1, 3-D graph for temperature distribution for C-D theory, due to moving heat source at t=0.2

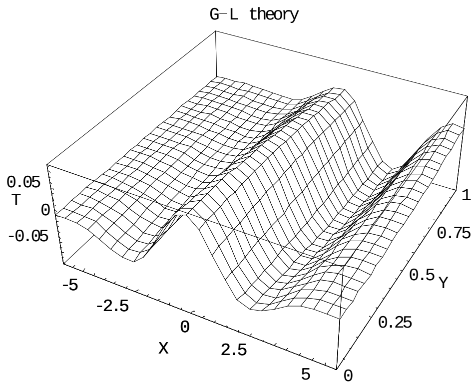

Fig. 2, 3-D graph for temperature distribution for G-L theory, due to moving heat source at t=0.2

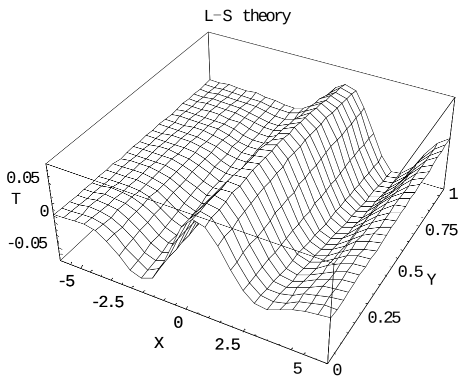

Fig. 3, 3-D graph for temperature distribution for L-S theory, due to moving heat source at t=0.2

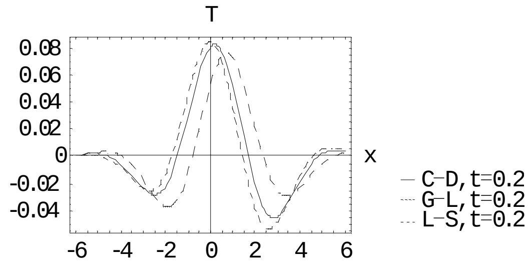

Fig. 4, Temperature distribution for C-D, G-L and L-S theories, due to moving heat source, at t=0.2

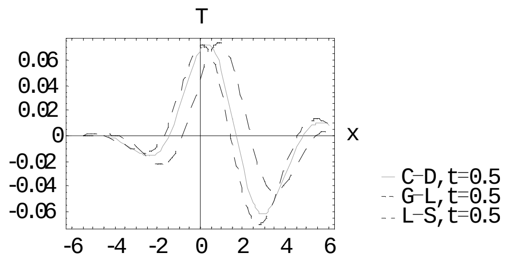

Fig. 5, Temperature distribution for C-D, G-L and L-S theories, due to moving heat source, at t=0.5

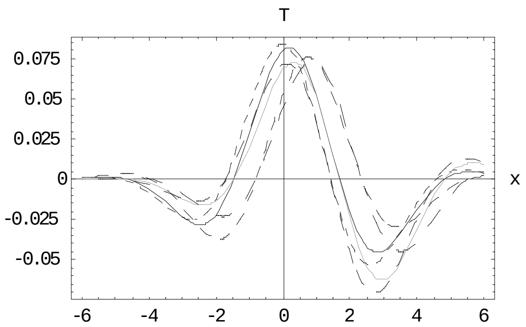

Fig. 6, Comparison for temperature distribution for C-D, G-L and L-S theories, due to moving heat source, times, t=0.2 and t=0.5.

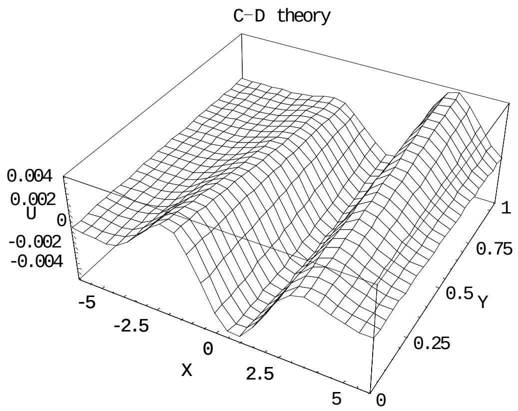

Fig. 7, 3-D graph for horizontal displacement for C-D theory, due to moving heat source at t=0.2

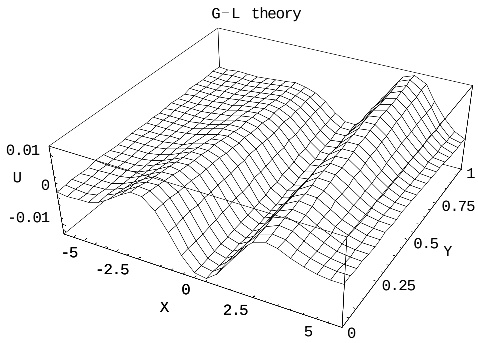

Fig. 8, 3-D graph for horizontal displacement for G-L theory, due to moving heat source at t=0.2

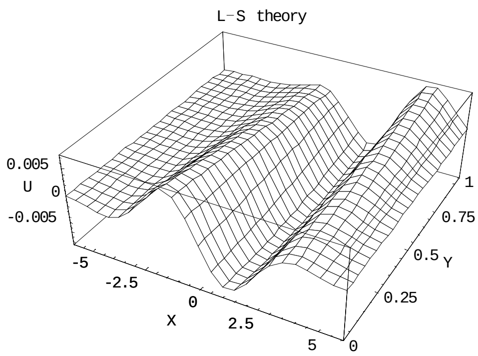

Fig. 9, 3-D graph for horizontal displacement for L-S theory, due to moving heat source at t=0.2

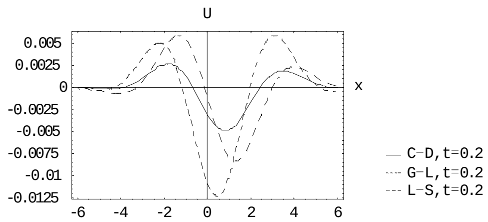

Fig. 10, Horizontal displacement for C-D, G-L and L-S theories, due to moving heat source, at t=0.2

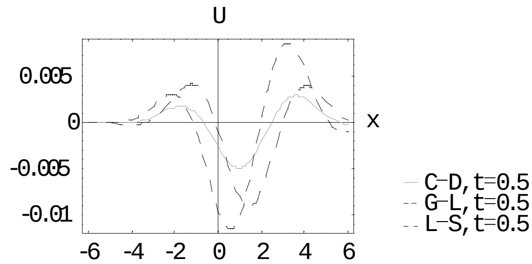

Fig. 11, Horizontal displacement for C-D, G-L and L-S theories, due to moving heat source, at t=0.5

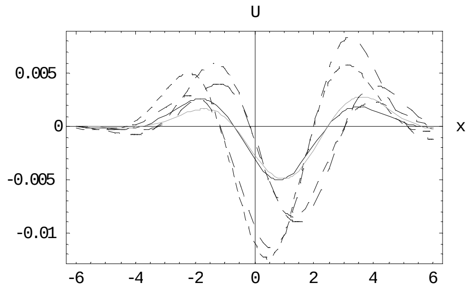

Fig. 12, Comparison for horizontal displacement for C-D, G-L and L-S theories, due to moving heat source, times, t=0.2 and t=0.5.

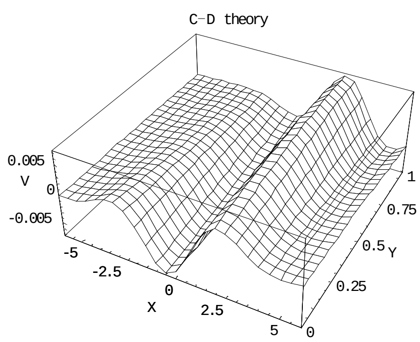

Fig. 13, 3-D graph for vertical displacement for C-D theory, due to moving heat source at t=0. 2

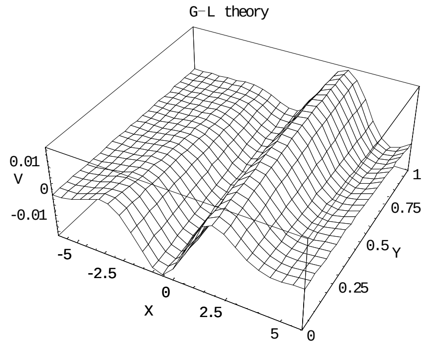

Fig. 14, 3-D graph for vertical displacement for G-L theory, due to moving heat source at t=0. 2

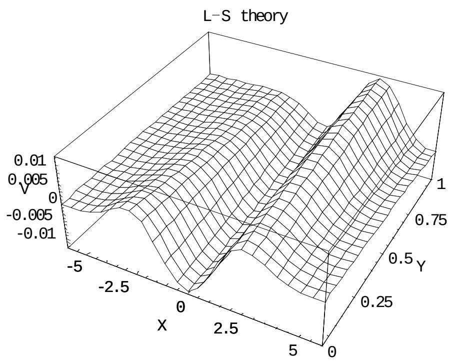

Fig. 15, 3-D graph for vertical displacement for L-S theory, due to moving heat source at t=0. 2

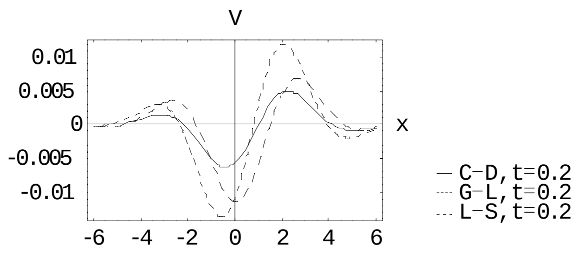

Fig. 16, Vertical displacement for C-D, G-L and L-S theories, due to moving heat source

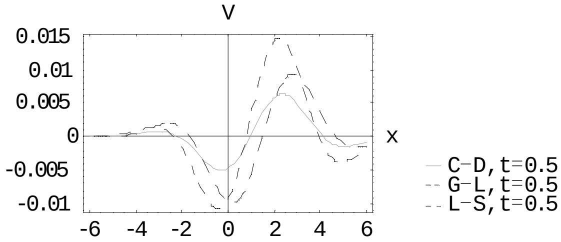

Fig. 17, Vertical displacement for C-D, G-L and L-S theories, due to moving heat source, at, t=0.5

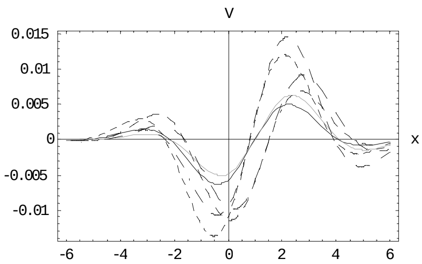

Fig. 18, Comparison for vertical displacement for C-D, G-L and L-S theories, due to moving heat source, times, t=0.2 and t=0.5.

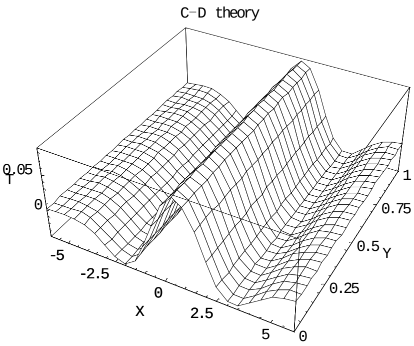

Fig. 19, 3-D graph for temperature distribution for C-D theory, due to moving load at t=0.2

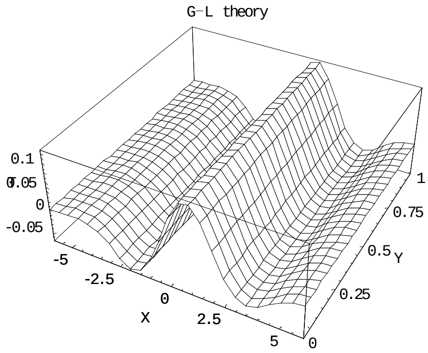

Fig. 20, 3-D graph for temperature distribution for G-L theory, due to moving load at t=0.2

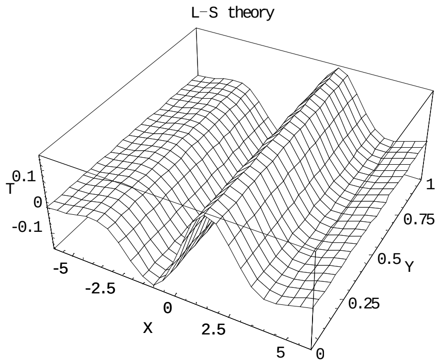

Fig. 21, 3-D graph for temperature distribution for L-S theory, due to moving load at t=0.2

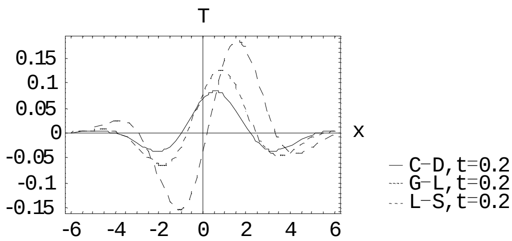

Fig. 22, Temperature distribution for C-D, G-L and L-S theories, due to moving load at t=0.2.

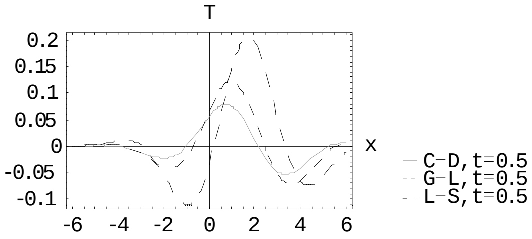

Fig. 24, Comparison for temperature distribution for C-D, G-L and L-S theories, due to moving heat source, times, t=0.2 and t=0.5.

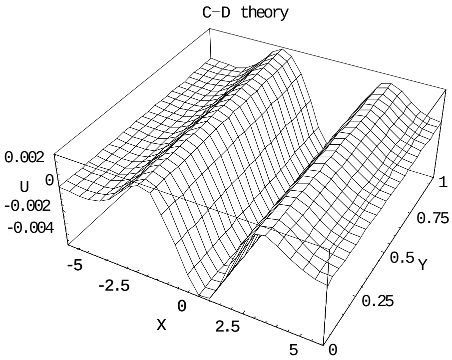

Fig. 25, 3-D graph for horizontal displacement for C-D theory, due to moving load at t=0.2

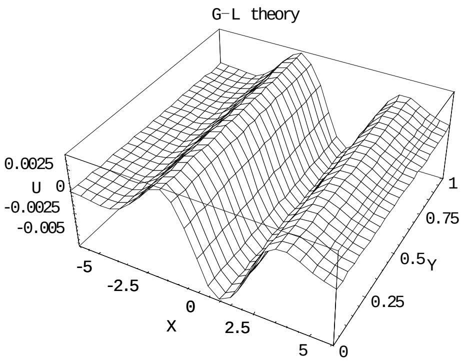

Fig. 26, 3-D graph for horizontal displacement for G-L theory, due to moving load at t=0.2

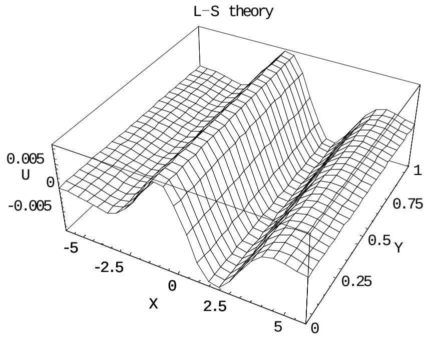

Fig. 27, 3-D graph for horizontal displacement for L-S theory, due to moving load at t=0.2.

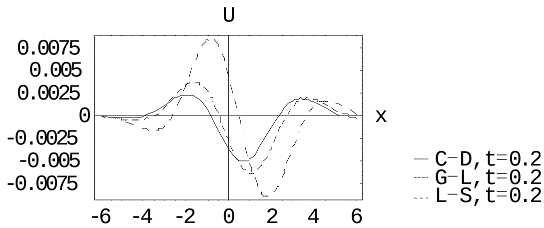

Fig. 28, Horizontal displacement for C-D, G-L and L-S theories, due to moving load at t=0.2

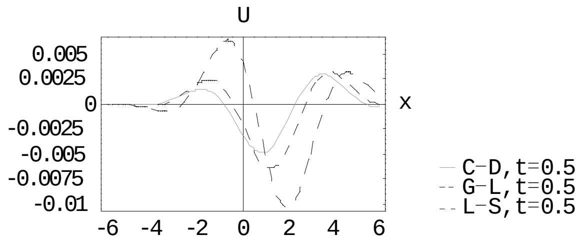

Fig. 29, Horizontal displacement for C-D, G-L and L-S theories, due to moving load at t=0.2

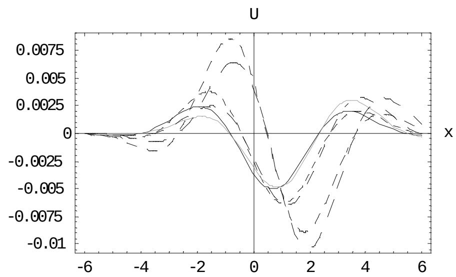

Fig. 30, Comparison for horizontal displacement for C-D, G-L and L-S theories, due to moving load, times, t=0.2 and t=0.5.

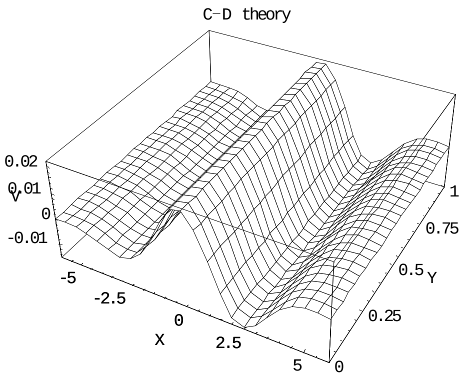

Fig. 31, 3-D graph for vertical displacement for C-D theory, due to moving load at t=0.2.

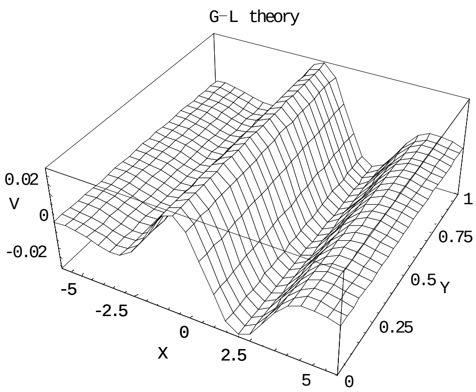

Fig. 32, 3-D graph for vertical displacement for G-L theory, due to moving load at t=0.2

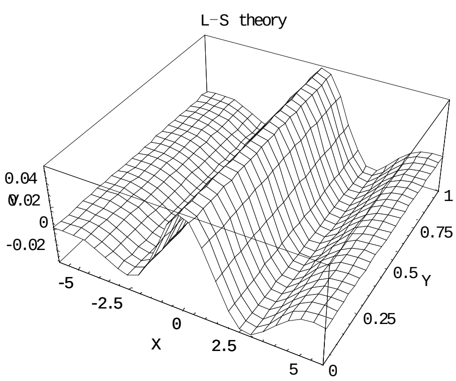

Fig. 33, 3-D graph for vertical displacement for L-S theory, due to moving load at t=0.2

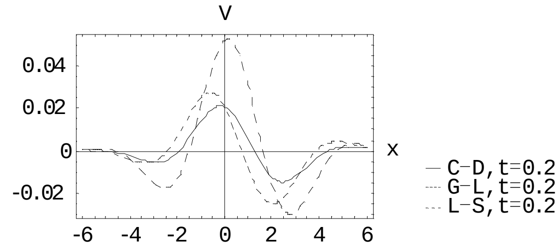

Fig. 34, Vertical displacement for C-D, G-L and L-S theories, due to moving load at t=0.2.

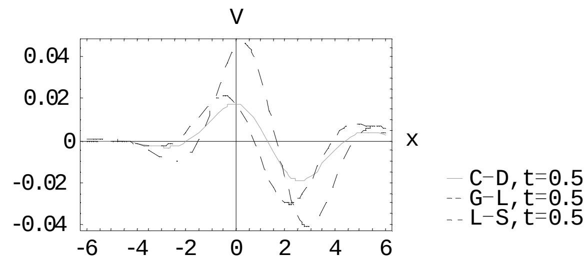

Fig. 35, Vertical displacement for C-D, G-L and L-S theories, due to moving load, at t=0.5.

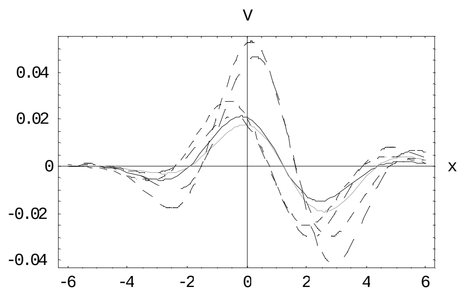

Fig. 36, Comparison for vertical displacement for C-D, G-L and L-S theories, due to moving load at times, t=0.2 and t=0.5.

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