Backward Stochastic Differential Equation with Monotone and Continuous Coefficient

Xiaoqin Huang[1],Xiaojie Liu[2] *

Abstract—**In this paper, we consider a backward stochastic differential equation(BSDE) with monotone and continuous coefficient and obtain the existence and uniqueness of solution.**

Keywords: backward stochastic differential equation; Yosida approximations; monotone and continuous coefficients

1 Introduction

In 1973, Bismut first introduced the adapted solution for a linear BSDE which is the adjoint process for a stochastic control problem. Later Pardoux and Peng [6] obtained the existence and uniqueness of solution for the following nonlinear BSDE with Lipschitz coefficient f

$$y_t = \xi + \int_t^T f(s, y_s, z_s)ds - \int_t^T z_s dW_s \quad (1)$$

where $(W_s)_{0\leq s\leq T}$ is a standard d-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$, F_t is the natural filtration. $\mathcal{F}_t$ contains all P-null sets of $\mathcal{F}$. ξ is a given $\mathcal{F}_T$ measurable random vector. Since then, many researchers have devoted to obtaining its existence and uniqueness of solution under weaker assumptions on f. For example, Lepeltier [4] obtained the existence for one-dimensional BSDE under the continuous assumption, and Mao [5] obtained the existence and uniqueness under the non-Lipschitz assumption. In this paper, using the Yosida approximations (see for example Da Prato and Zabczyk [1,2] or Hu [3]), we also obtain the existence and uniqueness of solution for BSDE (1).

*Manuscript received May 6, 2009. This work is supported by the science foundation of Hebei University of Science and Technology (XL200824),Dr. fund research of Hebei University of Science and Technology(QD200950) and Hebei province science and technology projects (No.09213508d; No.09213558). Xiaoqin Huang is with the College of Sciences, Hebei University of Science and Technology, Shijiazhuang, Hebei 050018, China(e-mail:sjzhxq@163.com). Xiaojie Liu is with the Institute of Mathematics, Shijiazhuang No.2 middle school, Shijiazhuang 050000,China(e-mail: hxq1977526@sina.com).

2 Preliminary

Let $M^2(0,T;R^n)$ denote the set of all R^n-valued F_t-progressively measurable processes $v(\cdot)$ satisfying $E\int_0^T | v(s)|^2 ds < +\infty$.

In this paper, $(\cdot)$ denotes the usual inner product in R^n; We use the usual Euclidean norm in R^n. For $z \in R^{n\times d}$, its Euclidean norm is defined by $| z |= tr(zz^T)^{\frac{1}{2}}$ and its inner product is $((z^1, z^2)) = tr(z^1(z^2)^T)$. For $u^1 = (y^1, z^1) \in R^n \times R^{n\times d}$, $u^2 = (y^2, z^2) \in R^n \times R^{n\times d}$, we denote $[u^1, u^2] = (y^1, y^2) + ((z^1, z^2))$ and $| u^1 |^2 =| y^1 |^2 + | z^1 |^2$.

Definition 2.1 The solution of Eq.(1) is a couple (y, z) which belongs to $M^2(0,T;\ R^n \times R^{n\times d})$ and satisfies Eq.(1).

Theorem 2.2 [6] If $f' : \Omega \times [0,T] \times R^n \times R^{n\times d} \to R^n$ is a progressively measurable function and satisfies

(i) $f'(t, 0, 0)_{t\in[0,T]}$ belongs to $M^2(0,T;R^n)$;

(ii) There exists a constant $K \geq 0$ s.t.P-a.s., for all y_1, $y_2 \in R^n$, z_1, $z_2 \in R^{n\times d}$, $|f'(t, y_1, z_1) - f'(t, y_2, z_2)| \leq K(|y_1 - y_2| + |z_1 - z_2|)$.

Then $y_t = \xi + \int_t^T f'(s, y_s, z_s)ds - \int_t^T z_s dW_s$ has a unique adapted solution $(y(.), z(.)) \in M^2(0,T;R^n \times R^{n\times d})$.

Theorem 2.3 [6] If $f' : \Omega \times [0,T] \times R^n \times R^{n\times d} \to R^n$ and $g' : \Omega \times [0,T] \times R^n \times R^{n\times d} \to R^{n\times d}$ are progressively measurable functions, and there exist constants $\lambda > 0$ and $\alpha > 0$ such that

$$| f'(t, y_1, z_1) - f'(t, y_2, z_2) | + | g'(t, y_1, z_1) - g'(t, y_2, z_2) | \leq \lambda(| y_1 - y_2 | + | z_1 - z_2 |)$$

$$| g'(t, y, z_1) - g'(t, y, z_2) | \geq \alpha | z_1 - z_2 |$$

for all $y_1, y_2 \in R^n$, $z_1, z_2 \in R^{n\times d}$. Then the following equation

$$y_t = \xi + \int_t^T f'(s, y_s, z_s)ds - \int_t^T g'(s, y_s, z_s)dw_s$$

has a unique adapted solution $(y(.), z(.)) \in M^2(0,T;R^n \times R^{n\times d})$.

In this paper, we consider the following BSDE

$$y_t = \xi + \int_t^T f(s, y_s, z_s)ds - \int_t^T z_s dW_s$$

Assumption 2.4 f is continuous in (y, z) for almost all (t, ω). $f(., y, z) \in M^2(0, T; R^n)$. For all $u = (y, z) \in R^n \times R^{n\times d}$, there exist constants $\frac{3}{4} \le C \le 1$ and $C_1 > 0$ such that P-a.s.,a.e.

$$\textbf{(H1)} \quad | f(t,u) | \le | f(t,0) | + C_1 | u |$$

$$\textbf{(H2)} \quad (f(t,y^1,z^1) - f(t,y^2,z^2), y^1 - y^2) \le (1-C) | z^1 - z^2 |^2 - C | y^1 - y^2 |^2$$

We denote $g(t, y, z) \equiv z$. Then the equation (1) is equal to

$$y_t = \xi + \int_t^T f(s, y_s, z_s)ds - \int_t^T g(s, y_s, z_s)dW_s \qquad (1')$$

Let $F(t, u) = F(t, y, z) = (f(t, y, z), -g(t, y, z)) = (f(t, u), -g(t, u))$. Then by the Assumption 2.4, $F(t, .)$ is also continuous, and $(H2)$ is equal to

$$\textbf{(H2')} \quad [F(t,u^1) - F(t,u^2), u^1 - u^2] \le -C | u^1 - u^2 |^2$$

Lemma2.5[3]. Let $\Phi : R^n \longrightarrow R^n$ be a continuous function, and there exists a constant $c > 0$ such that $(\Phi(x^1) - \Phi(x^2), x^1 - x^2) \le -c | x^1 - x^2 |^2, \quad \forall x^1, x^2 \in R^n$. Then for the Yosida approximations Φ^α of Φ, $\alpha > 0$, we have

(i) $(\Phi^\alpha(x^1) - \Phi^\alpha(x^2), x^1 - x^2) \le -c | x^1 - x^2 |^2$

$$| \Phi^\alpha(x^1) - \Phi^\alpha(x^2) | \le (\tfrac{2}{\alpha} + c) | x^1 - x^2 |$$

$$| \Phi^\alpha(x) | \le | \Phi(x) | + 2c | x |$$

(ii) For any $\alpha, \beta > 0$, we have

$(\Phi^\alpha(x^1) - \Phi^\beta(x^2), x^1 - x^2) \le (\alpha + \beta)(| \Phi(x^1) | + | \Phi(x^2) | + c | x^1 | + c | x^2 |)^2 - c | x^1 - x^2 |^2$

(iii) For any $\{x^\alpha\}_{\alpha>0} \subset R^n$, $x \in R^n$, if $lim_{\alpha\to 0} x^\alpha = x$, then

$$lim_{\alpha\to 0} \Phi^\alpha(x^\alpha) = \Phi(x)$$

3 Main results

Theorem 3.1 Let Assumption 2.4 hold, then there exists a unique adapted solution $(y(.), z(.)) \in M^2(0, T; R^n \times R^{n\times d})$ for Eq.(1).

Proof. First, we prove the existence. We divide the proof into four steps.

Step 1. There exists a unique adapted solution for the approximating BSDE.

For arbitrary $\alpha > 0$, we consider the approximating BSDE of (1')

$$y_t^\alpha = \xi + \int_t^T f^\alpha(s, y_s^\alpha, z_s^\alpha)ds - \int_t^T g^\alpha(s, y_s^\alpha, z_s^\alpha)dw_s \qquad (2)$$

where $F^\alpha(t, y_t^\alpha, z_t^\alpha) = (f^\alpha(t, y_t^\alpha, z_t^\alpha), -g^\alpha(t, y_t^\alpha, z_t^\alpha))$ is the Yosida approximation of $F(t, y_t^\alpha, z_t^\alpha)$.

Let $v^1 = (y^1, z^1)$ and $v^2 = (y^2, z^2)$, then by Lemma 2.5, we have

$$| F^\alpha(t,v^1) - F^\alpha(t,v^2) |^2 \le (\frac{2}{\alpha} + C)^2 | v^1 - v^2 |^2$$

hence

$$\begin{aligned} & 2 | f^\alpha(t,v^1) - f^\alpha(t,v^2) |^2 + 2 | g^\alpha(t,v^1) - g^\alpha(t,v^2) |^2 \\ \le\ & 2(\frac{2}{\alpha} + C)^2(| y^1 - y^2 |^2 + | z^1 - z^2 |^2) \end{aligned}$$

$$\begin{aligned} & (| f^\alpha(t,v^1) - f^\alpha(t,v^2) | + | g^\alpha(t,v^1) - g^\alpha(t,v^2) |)^2 \\ \le\ & 2(\frac{2}{\alpha} + C)^2(| y^1 - y^2 |^2 + | z^1 - z^2 |^2) \end{aligned}$$

therefore

$$\begin{aligned} & | f^\alpha(t,y^1,z^1) - f^\alpha(t,y^2,z^2) | + | g^\alpha(t,y^1,z^1) - g^\alpha(t,y^2,z^2) | \\ \le\ & \sqrt{2}(\frac{2}{\alpha} + C)(| y^1 - y^2 |^2 + | z^1 - z^2 |^2)^{\frac{1}{2}} \\ \le\ & \sqrt{2}(\frac{2}{\alpha} + C)(| y^1 - y^2 | + | z^1 - z^2 |). \quad (3) \end{aligned}$$

Let $w^1 = (y, z^1)$ and $w^2 = (y, z^2)$, then by Lemma 2.5, we have

$$(F^\alpha(t,w^1) - F^\alpha(t,w^2), w^1 - w^2) \le -C | w^1 - w^2 |^2$$

so $(f^\alpha(t,y,z^1) - f^\alpha(t,y,z^2), y - y) + (g^\alpha(t,y,z^2) - g^\alpha(t,y,z^1), z^1 - z^2) \le -C | z^1 - z^2 |^2$

thus

$$| g^\alpha(t,y,z^1) - g^\alpha(t,y,z^2) | \cdot | z^1 - z^2 | \ge C | z^1 - z^2 |^2$$

$$| g^\alpha(t,y,z^1) - g^\alpha(t,y,z^2) | \ge C | z^1 - z^2 | \quad (4)$$

By the above inequalities (3) and (4), f^α and g^α satisfy the assumptions in Theorem 2.3. Hence, there exists a unique adapted solution $u^\alpha = (y^\alpha, z^\alpha)$ for Eq.(2) in $M^2(0, T; R^n \times R^{n\times d})$.

Step 2. There exists a constant L, such that $E\int_0^T | u^\alpha |^2 dt \leq L$.

Applying the Itô formula to $| y_t^\alpha |^2$ and taking the expectation, we get

$$E\xi^2 = E | y_0^\alpha |^2 - E\int_0^T 2(y_t^\alpha, f^\alpha(t, y_t^\alpha, z_t^\alpha))dt$$
$$+E\int_0^T | g^\alpha(t, y_t^\alpha, z_t^\alpha) |^2 dt$$

so

$$E\int_0^T 2(y_t^\alpha, f^\alpha(t, y_t^\alpha, z_t^\alpha) - f^\alpha(t,0,0))dt$$
$$-E\int_0^T 2((z_t^\alpha, g^\alpha(t, y_t^\alpha, z_t^\alpha) - g^\alpha(t,0,0)))dt + E\xi^2$$
$$= -E\int_0^T 2(y_t^\alpha, f^\alpha(t,0,0))dt + E\int_0^T | g^\alpha(t, y_t^\alpha, z_t^\alpha) |^2 dt$$
$$+E | y_0^\alpha |^2 - E\int_0^T 2((z_t^\alpha, g^\alpha(t, y_t^\alpha, z_t^\alpha) - g^\alpha(t,0,0)))dt$$

hence

$$\begin{aligned} & E\xi^2 - E\int_0^T 2C | u^\alpha |^2 dt \\ \geq\ & E | y_0^\alpha |^2 + E\int_0^T | g^\alpha(t, y_t^\alpha, z_t^\alpha) |^2 dt \\ & -E\int_0^T 2(y_t^\alpha, f^\alpha(t,0,0))dt \\ & -E\int_0^T 2((z_t^\alpha, g^\alpha(t, y_t^\alpha, z_t^\alpha) - g^\alpha(t,0,0)))dt \end{aligned}$$

then

$$\begin{aligned} & E | y_0^\alpha |^2 + E\int_0^T 2C | u^\alpha |^2 dt \\ \leq\ & E\int_0^T 2(y_t^\alpha, f^\alpha(t,0,0))dt \\ & +E\int_0^T 2((z_t^\alpha, g^\alpha(t, y_t^\alpha, z_t^\alpha) - g^\alpha(t,0,0)))dt \\ & +E\xi^2 - E\int_0^T | g^\alpha(t, y_t^\alpha, z_t^\alpha) |^2 dt \end{aligned}$$

$$\begin{aligned} \leq\ & E\xi^2 + E\int_0^T | y_t^\alpha |^2 dt \\ & +E\int_0^T | f^\alpha(t,0,0) |^2 dt - E\int_0^T | g^\alpha(t, y_t^\alpha, z_t^\alpha) |^2 dt \\ & +E\int_0^T 2((z_t^\alpha, g^\alpha(t, y_t^\alpha, z_t^\alpha)))dt - E\int_0^T | z_t^\alpha |^2 dt \\ & -E\int_0^T 2((z_t^\alpha, g^\alpha(t,0,0)))dt + E\int_0^T | z_t^\alpha |^2 dt \\ \leq\ & E\int_0^T | f^\alpha(t,0,0) |^2 dt - E\int_0^T 2((z_t^\alpha, g^\alpha(t,0,0)))dt \\ & +E\int_0^T | z_t^\alpha |^2 dt + E\xi^2 + E\int_0^T | y_t^\alpha |^2 dt \\ =\ & E\int_0^T | f^\alpha(t,0,0) |^2 dt - E\int_0^T 2((z_t^\alpha, g^\alpha(t,0,0)))dt \\ & -2E\int_0^T | g^\alpha(t,0,0) |^2 dt - \frac{1}{2}E\int_0^T | z_t^\alpha |^2 dt \\ & +2E\int_0^T | g^\alpha(t,0,0) |^2 dt + \frac{1}{2}E\int_0^T | z_t^\alpha |^2 dt \\ & +E\int_0^T | z_t^\alpha |^2 dt + E\xi^2 + E\int_0^T | y_t^\alpha |^2 dt \end{aligned}$$

By $| F^\alpha(x) | \leq | F(x) | + 2C | x |$, we have $| F^\alpha(0) |^2 \leq | F(0) |^2$. Thus $| f^\alpha(t,0,0) |^2 + | g^\alpha(t,0,0) |^2 \leq | f(t,0,0) |^2 + | g(t,0,0) |^2 = | f(t,0,0) |^2$. Therefore we get

$$\begin{aligned} & E | y_0^\alpha |^2 + E\int_0^T 2C | u^\alpha |^2 dt \\ \leq\ & E\xi^2 + E\int_0^T | y_t^\alpha |^2 dt + 2E\int_0^T | f(t,0,0) |^2 dt \\ & +\frac{3}{2}E\int_0^T | z_t^\alpha |^2 dt \\ \leq\ & E\xi^2 + \frac{3}{2}E\int_0^T | u^\alpha |^2 dt + 2E\int_0^T | f(t,0,0) |^2 dt \end{aligned}$$

so

$$\begin{aligned} & E | y_0^\alpha |^2 + E\int_0^T (2C - \frac{3}{2}) | u^\alpha |^2 dt \\ \leq\ & E\xi^2 + 2E\int_0^T | f(t,0,0) |^2 dt = M \end{aligned}$$

Because $\frac{3}{4} \leq C \leq 1$, then let $L = \frac{2M}{4C-3}$, we have $E\int_0^T | u^\alpha |^2 dt \leq L$.

Step 3. $u^\alpha = (y^\alpha, z^\alpha)$ converges in $M^2(0,T; R^n \times R^{n\times d})$.

Let $\alpha > 0$ and $\beta > 0$, applying the Itô formula to $| y_t^\alpha - y_t^\beta |^2$ and taking the expectation, we get

$$0 = E\int_0^T 2(y_t^\alpha - y_t^\beta, f^\beta(t, y_t^\beta, z_t^\beta) - f^\alpha(t, y_t^\alpha, z_t^\alpha))dt$$
$$+E\int_0^T | g^\alpha(t, y_t^\alpha, z_t^\alpha) - g^\beta(t, y_t^\beta, z_t^\beta) |^2 dt$$
$$+E | y_0^\alpha - y_0^\beta |^2$$

hence

$$E\int_0^T | g^\alpha(t, y_t^\alpha, z_t^\alpha) - g^\beta(t, y_t^\beta, z_t^\beta) |^2 dt$$

$$
\begin{aligned}
& -E\int_0^T 2((z_t^\alpha - z_t^\beta, g^\alpha(t,y_t^\alpha,z_t^\alpha) - g^\beta(t,y_t^\beta,z_t^\beta)))dt \\
& +E \mid y_0^\alpha - y_0^\beta \mid^2 \\
= \; & E\int_0^T 2(y_t^\alpha - y_t^\beta, f^\alpha(t,y_t^\alpha,z_t^\alpha) - f^\beta(t,y_t^\beta,z_t^\beta))dt \\
& -E\int_0^T 2((z_t^\alpha - z_t^\beta, g^\alpha(t,y_t^\alpha,z_t^\alpha) - g^\beta(t,y_t^\beta,z_t^\beta)))dt \\
= \; & E\int_0^T 2[F^\alpha(t,u^\alpha) - F^\beta(t,u^\beta), u^\alpha - u^\beta]dt \\
\leq \; & -2CE\int_0^T \mid u^\alpha - u^\beta \mid^2 dt +
\end{aligned}
$$

$2(\alpha+\beta)E\int_0^T(\mid F(t,u^\alpha) \mid + \mid F(t,u^\beta) \mid + C(\mid u^\alpha \mid + \mid u^\beta \mid))^2 dt$

therefore

$$E \mid y_0^\alpha - y_0^\beta \mid^2 + 2CE\int_0^T \mid u^\alpha - u^\beta \mid^2 dt$$

$\leq 2(\alpha+\beta)E\int_0^T(\mid F(t,u^\alpha) \mid + \mid F(t,u^\beta) \mid + C \mid u^\alpha \mid + C \mid u^\beta \mid)^2 dt$

$$
\begin{aligned}
& +E\int_0^T 2((z_t^\alpha - z_t^\beta, g^\alpha(t,y_t^\alpha,z_t^\alpha) - g^\beta(t,y_t^\beta,z_t^\beta)))dt \\
& -E\int_0^T \mid z_t^\alpha - z_t^\beta \mid^2 dt + E\int_0^T \mid z_t^\alpha - z_t^\beta \mid^2 dt \\
& -E\int_0^T \mid g^\alpha(t,y_t^\alpha,z_t^\alpha) - g^\beta(t,y_t^\beta,z_t^\beta) \mid^2 dt
\end{aligned}
$$

$$
\begin{aligned}
& \leq 8(\alpha+\beta)E\int_0^T(\mid F(t,u^\alpha) \mid^2 + \mid F(t,u^\beta) \mid^2 \\
& \qquad + C^2 \mid u^\alpha \mid^2 + C^2 \mid u^\beta \mid^2)dt \\
& +E\int_0^T \mid z_t^\alpha - z_t^\beta \mid^2 dt + E\int_0^T \mid y_t^\alpha - y_t^\beta \mid^2 dt
\end{aligned}
$$

then

$$
\begin{aligned}
& E \mid y_0^\alpha - y_0^\beta \mid^2 + (2C-1)E\int_0^T \mid u^\alpha - u^\beta \mid^2 dt \\
\leq \; & 8(\alpha+\beta)E\int_0^T(\mid F(t,u^\alpha) \mid^2 + \mid F(t,u^\beta) \mid^2 \\
& + C^2 \mid u^\alpha \mid^2 + C^2 \mid u^\beta \mid^2)dt
\end{aligned}
$$

Because $\mid F(t,u) \mid^2 = \mid f(t,u) \mid^2 + \mid z \mid^2$, $\mid f(t,u) \mid \leq \mid f(t,0) \mid + C_1 \mid u \mid$ and $E\int_0^T \mid u \mid^2 dt \leq L$, we deduce that there exists a constant $k > 0$, such that

$$E \mid y_0^\alpha - y_0^\beta \mid^2 + (2C-1)E\int_0^T \mid u^\alpha - u^\beta \mid^2 dt \leq k(\alpha+\beta)$$

Because $\frac{3}{4} \leq C \leq 1$, then $\{u^\alpha, \alpha > 0\}$ is a Cauchy sequence in $M^2(0,T;R^n \times R^{n\times d})$. We denote its limit by $u = (y,z) \in M^2(0,T;R^n \times R^{n\times d})$.

Step 4. Taking weak limits in the approximating equations (2).

From Lemma 2.5 and Assumption (H1), there exist constants l and m such that

$$
\begin{aligned}
\mid F^\alpha(t,u^\alpha) \mid^2 & \leq (\mid F(t,u^\alpha) \mid + 2C \mid u^\alpha \mid)^2 \\
& \leq l \mid f(t,0) \mid^2 + m \mid u^\alpha \mid^2
\end{aligned}
$$

So, there exists a constant $C_2 > 0$ such that $E\int_0^T \mid F^\alpha(t,u^\alpha) \mid^2 dt \leq C_2$. Therefore there exists a subsequences of $\{F^\alpha(.,u^\alpha), \alpha > 0\}$ converge weakly to limits $G = (H,-B)$ in the space $M^2(0,T;R^n \times R^{n\times d})$. Taking weak limits in the approximating equations (2) yields

$$y_t = \xi + \int_t^T H(s)ds - \int_t^T B(s)dw_s$$

Similar to the proof of Hu[3], we can prove that $G = (H,-B) = F(t,y_t,z_t) = (f(t,y_t,z_t), -g(t,y_t,z_t))$.

Therefore

$$y_t = \xi + \int_t^T f(s,y_s,z_s)ds - \int_t^T z_s dW_s$$

We deduce that (y,z) is an adapted solution of Eq.(1). The existence is proved.

Next, we prove the uniqueness of solution of Eq.(1).

Let $u^1 = (y_t^1, z_t^1)$ and $u^2 = (y_t^2, z_t^2)$ be two solutions of Eq.(1). $\hat{y}_t = y_t^1 - y_t^2$ and $\hat{z}_t = z_t^1 - z_t^2$, then we have

$$d\hat{y}_t = (f(t,y_t^2,z_t^2) - f(t,y_t^1,z_t^1))dt + (z_t^1 - z_t^2)dw_t$$

Applying the Itô formula to $\mid \hat{y}_t \mid^2$ and taking the expectation, we get

$$
\begin{aligned}
0 = E \mid \hat{y}_0 \mid^2 & + E\int_0^T 2(\hat{y}_t, f(t,y_t^2,z_t^2) - f(t,y_t^1,z_t^1))dt \\
& + E\int_0^T \mid z_t^1 - z_t^2 \mid^2 dt
\end{aligned}
$$

Hence by Assumption 2.4, we get

$$
\begin{aligned}
& E \mid \hat{y}_0 \mid^2 - E\int_0^T \mid z_t^1 - z_t^2 \mid^2 dt \\
= \; & E\int_0^T 2(\hat{y}_t, f(t,y_t^1,z_t^1) - f(t,y_t^2,z_t^2))dt \\
& -2E\int_0^T \mid z_t^1 - z_t^2 \mid^2 dt \\
\leq \; & -2CE\int_0^T (\mid \hat{z}_t \mid^2 + \mid \hat{y}_t \mid^2)dt
\end{aligned}
$$

Then $2CE\int_0^T(|\ \hat{z}_t\ |^2+|\ \hat{y}_t\ |^2)dt-E\int_0^T|\ \hat{z}_t\ |^2\ dt\le 0$. Because $\frac{3}{4}\le C\le 1$, then we have $E\int_0^T|\ \hat{y}_t\ |^2\ dt=0$ and $E\int_0^T|\ \hat{z}_t\ |^2\ dt=0$. So $u^1=u^2$.

Thus there exists a unique adapted solution $(y(.),z(.))$ for Eq.(1) in $M^2(0,T;\quad R^n\times R^{n\times d})$. The proof is completed.

References

[1] G. Da Prato and J. Zabczyk, *Stochastic equations in infinite dimensions*, Cambridge University Press, Cambridge, 1992.

[2] G. Da Prato and J. Zabczyk, *Ergodicity for infinite dimensional systems*, Cambridge University Press, Cambridge, 1996.

[3] Hu,Y., "On the solution of forward-backward SDEs with monotone and continuous coefficients," *Nonlinear analysis*, v42,pp.1-12,2000

[4] Lepeltier,J.P., and Martin,J.S. , "Backward stochastic differential equations with continuous coefficient," *Statistics Probability Letters*,v32, pp.425-430,1997

[5] Mao,X. ," Adapted solutions of backward stochastic differential equations with non-Lipschitz coefficient," *Stochastic Processes and their Applications*, v58, pp.281-292,1995

[6] Pardoux,E. and Peng,S., "Adapted solution of a backward stochastic differential equation," *Systems control Letter*,v14, pp.51-61,1990