The Oscillating Flows and Heat Transfer of a Generalized Oldroyd-B Fluid in Magnetic Field

Yaqing Liu, Liancun Zheng, Xinxin Zhang, Fenglei Zong

Abstract—**This paper presents an analysis for magnetohydrodynamic (MHD) flow of an incompressible generalized Oldroyd-B fluid with fractional derivative. The effect of radiation on the heat transfer is considered and the fractional calculus approach is used to establish the constitutive relationship model of a viscoelastic fluid. Exact solutions are obtained for the velocity field and temperature field in integral and series form in terms of G function by means of Fourier sine transfer and Laplace transform technique for the fractional calculus. Moreover, the figures are plotted to show the effects of different parameters on velocity field and temperature field.**

Index Terms—**Oldroyd-B fluid, oscillation, Fourier sine transfer, Laplace transform, G function.**

I. INTRODUCTION

The interest for motion problems of non-Newtonian fluids has considerably grown because of the wide range of their applications. These fluids have been modeled in a number of diverse manners with their constitutive equations varying greatly in complexity. Among them the Oldroyd-B fluid as a special viscolesatic non-Newtonian fluid has had some success in describing polymeric liquids, it being more amenable to analysis and more importantly experimental.

Recently, the fractional derivatives [1] are found to be quite flexible for describing the behaviors of viscoelastic fluids. Many researchers have studied different problems related to such fluids. In their works, the constitutive equations for generalized non-Newtonian fluids are modified from the well known fluid models by replacing the time derivative of an integer order by the so-called Riemann-Liouville fractional calculus operators. Qi and Xu [2] investigated the Stokes' problem for a viscoelastic fluid with a generalized Oldroyd-B model. Khan and Hyder *et al.* [3-4] considered some fluid with generalized Oldroyd-B model. Hyder [5] discussed the flows of generalized Oldroyd-B fluid between two side walls perpendicular to the plate. Fetecau *et al.* [6-9] investigated some accelerated flows of a generalized Oldroyd-B fluid. Hayat *et al.* [10-11] studied the flow of a Maxwell fluid between two side walls. Moreover, MHD flows have wide converged on the development of energy generation and in astrophysical and geophysical fluid dynamics. Recently, the theory of MHD has received much attention, see [12-16] and reference therein. The effect of radiation on the heat and fluid over an unsteady stretching surface is analyzed [17-18]. However, there are no attempts to consider the viscoelastic fluids under the effect of thermal radiation.

In this paper, we consider the MHD flow of an incompressible generalized Oldroyd-B fluid. Exact solutions for the velocity field and temperature field are obtained by using the Fourier sine transform and Laplace transform technique for the fractional calculus. The magnetic field and thermal radiation and their influence on the flow are considered. A parametric study of some physical parameters involved is performed to illustrate the influence of these parameters.

II. GOVERNING EQUATIONS

The constitutive equation of an incompressible and unsteady Oldroyd-B fluid is written in the form [2]:

$$\mathbf{T}=-p\mathbf{I}+\mathbf{S}\,,\ \mathbf{S}+\lambda\frac{\mathrm{D}^{\alpha}\mathbf{S}}{\mathrm{D}t^{\alpha}}=\mu(1+\lambda_r\frac{\mathrm{D}^{\beta}}{\mathrm{D}t^{\beta}})\mathbf{A}\,. \quad (1)$$

where $\mathbf{T}$ is the Cauchy stress tensor, $-p\mathbf{I}$ denotes the indeterminate spherical stress, $\mathbf{S}$ is the extra-stress tensor, $\mathbf{A}=\mathbf{L}+\mathbf{L}^{\mathrm{T}}$ is the first Rivlin-Ericksen tensor, $\mathbf{L}$ is the velocity gradient, μ,λ,λ_r are material constants, known as the viscosity coefficient, the relaxation and retardation times, respectively , and

$$\frac{\mathrm{D}^{\alpha}\mathbf{S}}{\mathrm{D}t^{\alpha}}=\mathrm{D}_t^{\alpha}\mathbf{S}+\mathbf{V}\cdot\nabla\mathbf{S}-\mathbf{L}\mathbf{S}-\mathbf{S}\mathbf{L}^{\mathrm{T}},$$

$$\frac{\mathrm{D}^{\beta}\mathbf{A}}{\mathbf{D}t^{\beta}}=\mathrm{D}_t^{\beta}\mathbf{A}+\mathbf{V}\cdot\nabla\mathbf{A}-\mathbf{L}\mathbf{A}-\mathbf{A}\mathbf{L}^{\mathrm{T}}. \quad (2)$$

In the above relations $\mathbf{V}$ is the velocity, ∇ is the gradient operator, D_t^{α} and D_t^{β} are based on Riemann- Liouville's definition is defined as [1]:

$$\mathrm{D}_t^{p}f(t)=\frac{1}{\Gamma(1-p)}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t\frac{f(\tau)}{(t-\tau)^p}\mathrm{d}\tau\,,0\le p<1\,, \quad (3)$$

where $\Gamma(\cdot)$ is the Gamma function.

Assuming the velocity field and stress of the form

$$\mathbf{V}=u(y,t)\mathbf{i}\,,\ \mathbf{S}=S(y,t)\,. \quad (4)$$

Where u is the velocity and $\mathbf{i}$ is the unit vectors in the x-direction. Substituting Eq.(4) into Eq.(1) and taking account of the initial condition

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$$S(y,0)=0\,,\,y>0\,, \tag{5}$$

the fluid being at rest up to the time $t=0$, we get

$$(1+\lambda \mathrm{D}_t^{\alpha})S_{xy}=\mu(1+\lambda_r \mathrm{D}_t^{\beta})\partial_y u(y,t)\,, \tag{6}$$

$S_{yy}=S_{zz}=S_{xz}=S_{yz}=0$, $S_{xy}=S_{yx}$. We consider a generalized Oldroyd-B fluid. The fluid is permeated by an imposed magnetic field B_0 which acts in the positive y -coordinate. In the low- magnetic Reynolds number approximation, the magnetic body force is represented $\sigma B_0^2 u$. Then, in the absence of a pressure gradient in the x-direction, the equation of motion yields the following scalar equation:

$$\rho\frac{\partial u}{\partial t}=\frac{\partial}{\partial y}S_{xy}-\sigma B_0^2 u\,, \tag{7}$$

where ρ is the constant density of the fluid. Eliminating S_{xy} between Eq.(6) and Eq.(7), we arrive at the following fractional differential equation

$$(1+\lambda \mathrm{D}_t^{\alpha})\frac{\partial u(y,t)}{\partial t}=\nu(1+\lambda_r \mathrm{D}_t^{\beta})\frac{\partial^2 u(y,t)}{\partial y^2}-M(1+\lambda \mathrm{D}_t^{\alpha})u(y,t) \tag{8}$$

where $\nu=\mu/\rho$ is the kinematic viscosity and $M=\sigma B_0^2/\rho$.

The fluid is considered to be a gray, absorbing-emitting radiation but non-scattering medium. When the Fourier's law of heat conduction is considered, the energy equation may be written in the form:

$$\frac{\partial\theta}{\partial t}=\frac{k_T}{\rho C_p}\frac{\partial^2\theta}{\partial y^2}+\frac{\nu}{C_p}\left[\frac{\partial u}{\partial y}\right]^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{9}$$

where k_T is the thermal conductivity, ρ is the density, C_p is the specific heat of a fluid at constant pressure and q_r is the radiative heat flux.

III. Statement of the Problem

Supposed that a generalized Oldroyd-B fluid occupying the space above a flat plate. Initially the fluid as well as the plate is at rest, and at time $t=0^+$ the plate oscillate in its plane with the velocity $V\cos(\omega t)$ or $V\sin(\omega t)$ (V is a constant). Let θ_w denotes temperature of the plate for $t\ge 0$, and suppose the temperature of the fluid at the moment $t=0$ is θ_∞ . Due to the shear, the fluid is moved gradually. Accordingly, the initial and boundary conditions of velocity field are:

Initial condition: $u(y,0)=\dfrac{\partial u(y,0)}{\partial t}=0\,,\ \ y>0\,.$ (10)

Boundary conditions: $u(0,t)=\mathrm{V}\sin(\omega \mathrm{t})$ or

$$u(0,t)=\mathrm{V}\cos(\omega \mathrm{t})\,,\ t>0\,. \tag{11}$$

$$u(y,t)\,,\ \frac{\partial u(y,t)}{\partial y}\to 0 \text{ as } y\to\infty\,,\ t>0\,. \tag{12}$$

where u is velocity in the x-coordinate direction.

The corresponding initial and boundary conditions of energy equation are:

Initial condition: $\theta(y,0)=0\,,$ for $y>0\,.$ (13)

Boundary conditions: $\theta(0,t)=\theta_w\,,$ for $t\ge 0\,.$ (14)

$$\theta(y,t)\to\theta_\infty\,,\ \frac{\partial\theta(y,t)}{\partial y}\to 0\,,\quad \text{for } y\to\infty\,. \tag{15}$$

IV. Velocity Field

Employing the non-dimensional quantities

$$u^*=\frac{u}{\mathrm{V}},\ y^*=\frac{y\mathrm{V}}{\nu},\ t^*=\frac{t\mathrm{V}^2}{\nu},$$

$$\lambda^*=\lambda\left(\frac{\mathrm{V}^2}{\nu}\right)^{\alpha},\lambda_r^*=\lambda_r\left(\frac{\mathrm{V}^2}{\nu}\right)^{\beta},M^*=\frac{M\nu}{A^2}. \tag{16}$$

We obtain the dimensionless motion equation as follows (for brevity the dimensionless mark "*" is omitted here)

$$(1+\lambda \mathrm{D}_t^{\alpha})\frac{\partial u(y,t)}{\partial t}=(1+\lambda_r \mathrm{D}_t^{\beta})\frac{\partial^2 u(y,t)}{\partial y^2}-M(1+\lambda \mathrm{D}_t^{\alpha})u(y,t) \tag{17}$$

Initial condition: $u(y,0)=\dfrac{\partial u(y,0)}{\partial t}=0\,,\ \ y>0\,.$ (18)

Boundary conditions: $u(0,t)=\cos(\omega \mathrm{t})$ or

$$u(0,t)=\sin(\omega \mathrm{t})\,,\ t>0\,. \tag{19}$$

$$u(y,t)\,,\ \frac{\partial u(y,t)}{\partial y}\to 0 \text{ as } y\to\infty\,,\ t>0\,. \tag{20}$$

In order to solve the above problem, we use Fourier sine transform [19] and Laplace transform for fractional derivative. Firstly, multiplying both sides of Eq.(17) by $\sqrt{2/\pi}\sin(\xi y)$, integrating then with respect to y from 0 to ∞ and take account corresponding initial and boundary conditions (18)-(20), we obtain

$$(1+\lambda \mathrm{D}_t^{\alpha})\frac{\partial u_s(\xi,t)}{\partial t}=(1+\lambda_r \mathrm{D}_t^{\beta})[\sqrt{\frac{2}{\pi}}\xi\cos(\omega t)-\xi^2 u_s(\xi,t)]-M(1+\lambda \mathrm{D}_t^{\alpha})u_s(\xi,t)\,. \tag{21}$$

and

$$(1+\lambda \mathrm{D}_t^{\alpha})\frac{\partial u_s(\xi,t)}{\partial t}=(1+\lambda_r \mathrm{D}_t^{\beta})[\sqrt{\frac{2}{\pi}}\xi\sin(\omega t)-\xi^2 u_s(\xi,t)]-M(1+\lambda \mathrm{D}_t^{\alpha})u_s(\xi,t)\,. \tag{22}$$

where the Fourier sine transform $u_s(\xi,t)$ of $u(y,t)$ has to satisfy the conditions

$$u_s(\xi,0)=\frac{\partial u_s(\xi,0)}{\partial t}=0\,,\ \xi>0\,. \tag{23}$$

Applying Laplace transform for sequential fractional derivative to Eqs.(21)-(22) and using the initial condition Eq.(23), we get

$$\overline{u}_s(\xi,s)=\sqrt{\frac{2}{\pi}}\frac{1}{\xi}\frac{s}{s^2+\omega^2}\frac{\xi^2(1+\lambda_r s^{\beta})}{[(s+M)(1+\lambda s^{\alpha})+\xi^2(1+\lambda_r s^{\beta})]} \tag{24}$$

and

$$\overline{u}_s(\xi,s)=\sqrt{\frac{2}{\pi}}\frac{1}{\xi}\frac{\omega}{s^2+\omega^2}\frac{\xi^2(1+\lambda_r s^\beta)}{[(s+M)(1+\lambda s^\alpha)+\xi^2(1+\lambda_r s^\beta)]} \tag{25}$$

Where $\overline{u}_s(\xi,s)$ is the Laplace transform of $u_s(\xi,t)$ with respect to t. In order to avoid the lengthy procedure of residues and contour integrals, we rewrite Eqs.(24)-(25) into the series form

$$\overline{u}_s(\xi,s)=\sqrt{\frac{2}{\pi}}\frac{1}{\xi}\frac{s}{s^2+\omega^2}-\sqrt{\frac{2}{\pi}}\frac{1}{\xi}\frac{s}{s^2+\omega^2}\sum_{k=0}^{\infty}(-1)^k \times\frac{1}{\xi^{2(k+1)}\lambda_r^{k+1}}\sum_{m,l\geq 0}^{m+l=k+1}\frac{(k+1)!}{m!l!}M^m\sum_{n,w\geq 0}^{n+w=k}\frac{k!}{n!w!}\lambda^n \times(1+\lambda s^\alpha)\frac{s^{\alpha n+l}}{(\lambda_r^{-1}+s^\beta)^{k+1}}. \tag{26}$$

and

$$\overline{u}_s(\xi,s)=\sqrt{\frac{2}{\pi}}\frac{1}{\xi}\frac{\omega}{s^2+\omega^2}-\sqrt{\frac{2}{\pi}}\frac{1}{\xi}\frac{\omega}{s^2+\omega^2}\sum_{k=0}^{\infty}(-1)^k \times\frac{1}{\xi^{2(k+1)}\lambda_r^{k+1}}\sum_{m,l\geq 0}^{m+l=k+1}\frac{(k+1)!}{m!l!}M^m\sum_{n,w\geq 0}^{n+w=k}\frac{k!}{n!w!}\lambda^n \times(1+\lambda s^\alpha)\frac{s^{\alpha n+l}}{(\lambda_r^{-1}+s^\beta)^{k+1}}. \tag{27}$$

Taking the discrete inverse Laplace transform, we obtain

$$u_s(\xi,t)=\sqrt{\frac{2}{\pi}}\frac{1}{\xi}\cos(\omega t)-\sqrt{\frac{2}{\pi}}\frac{1}{\xi}\sum_{k=0}^{\infty}(-1)^k\frac{1}{\xi^{2(k+1)}\lambda_r^{k+1}} \times\sum_{m,l\geq 0}^{m+l=k+1}\frac{(k+1)!}{m!l!}M^m\sum_{n,w\geq 0}^{n+w=k}\frac{k!}{n!w!}\lambda^n\int_0^t\cos(\omega(t-\tau)) \times\{G_{\beta,\alpha n+l,k+1}(-\lambda_r^{-1},\tau)+\lambda G_{\beta,\alpha n+l+\alpha,k+1}(-\lambda_r^{-1},\tau)\}d\tau \tag{28}$$

and

$$u_s(\xi,t)=\sqrt{\frac{2}{\pi}}\frac{1}{\xi}\sin(\omega t)-\sqrt{\frac{2}{\pi}}\frac{1}{\xi}\sum_{k=0}^{\infty}(-1)^k\frac{1}{\xi^{2(k+1)}\lambda_r^{k+1}} \times\sum_{m,l\geq 0}^{m+l=k+1}\frac{(k+1)!}{m!l!}M^m\sum_{n,w\geq 0}^{n+w=k}\frac{k!}{n!w!}\lambda^n\int_0^t\sin(\omega(t-\tau)) \times\{G_{\beta,\alpha n+l,k+1}(-\lambda_r^{-1},\tau)+\lambda G_{\beta,\alpha n+l+\alpha,k+1}(-\lambda_r^{-1},\tau)\}d\tau \tag{29}$$

where [20]

$$G_{\alpha,a,b}(d,t)=\sum_{l=0}^{\infty}\frac{(b)_l t^{\alpha(l+b)-a-1}}{\Gamma(l+1)\Gamma((l+b)\alpha-a)}(d)^l. \tag{30}$$

$(b)_l=b(b+1)\cdots(b+l-1)$ is the Pochhammer polynomial.

Then, in terms of the inverse Fourier sine transform, we find the following expression for the velocity field

$$u(y,t)=\cos(\omega t)-\frac{2}{\pi}\int_0^\infty\frac{\sin(\xi y)}{\xi}\sum_{k=0}^{\infty}(-1)^k\frac{1}{\xi^{2(k+1)}\lambda_r^{k+1}} \times\sum_{m,l\geq 0}^{m+l=k+1}\frac{(k+1)!}{m!l!}M^m\sum_{n,w\geq 0}^{n+w=k}\frac{k!}{n!w!}\lambda^n\int_0^t\cos(\omega(t-\tau)) \times\{G_{\beta,\alpha n+l,k+1}(-\lambda_r^{-1},\tau)+\lambda G_{\beta,\alpha n+l+\alpha,k+1}(-\lambda_r^{-1},\tau)\}d\tau d\xi \tag{31}$$

and

$$u(y,t)=\sin(\omega t)-\frac{2}{\pi}\int_0^\infty\frac{\sin(\xi y)}{\xi}\sum_{k=0}^{\infty}(-1)^k\frac{1}{\xi^{2(k+1)}\lambda_r^{k+1}} \times\sum_{m,l\geq 0}^{m+l=k+1}\frac{(k+1)!}{m!l!}M^m\sum_{n,w\geq 0}^{n+w=k}\frac{k!}{n!w!}\lambda^n\int_0^t\sin(\omega(t-\tau)) \times\{G_{\beta,\alpha n+l,k+1}(-\lambda_r^{-1},\tau)+\lambda G_{\beta,\alpha n+l+\alpha,k+1}(-\lambda_r^{-1},\tau)\}d\tau d\xi \tag{32}$$

V. Temperature Field

Using the Rosseland approximation for radiation, the radiative heat flux is simplified as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial\theta^4}{\partial y}. \tag{33}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow such as that the term θ^4 may be expressed as a linear function of temperature. Hence, expanding θ^4 in a Taylor series about a free stream temperature θ_∞ and neglecting higher-order terms we get

$$\theta^4\cong 4\theta_\infty^3\theta-3\theta_\infty^4. \tag{34}$$

It should be noted that the above radiative transfer pertains to an optically thick model.

In view of Eqs.(33)and (34), Eq.(9) reduces to

$$\frac{\partial\theta}{\partial t}=\frac{k_T}{\rho C_p}\left[\frac{3N_R+4}{3N_R}\right]\frac{\partial^2\theta}{\partial y^2}+\frac{\nu}{C_p}\left[\frac{\partial u}{\partial y}\right]^2. \tag{35}$$

where $N_R=\dfrac{k^*k_T}{4\sigma^*T_\infty^3}$.

Employing the non-dimensional quantities:

$$\theta^*=\frac{\theta-\theta_\infty}{\theta_w-\theta_\infty},\ u^*=\frac{u}{u_0(0)},\ y^*=\frac{u_0(0)y}{\nu}, \quad t^*=\frac{u_0^2(0)t}{\nu},\ \eta^*=\frac{u_0^2(0)}{C_p(\theta_w-\theta_\infty)},\ Pr^*=\frac{C_p\mu}{k_T}. \tag{36}$$

Eqs.(35), (13)-(15)can reduce to dimensionless equations as follows (for brevity the dimensionless mark "*"are omitted here):

$$\frac{\partial\theta(y,t)}{\partial t}=\frac{1}{Pr}\left[\frac{3N_R+4}{3N_R}\right]\frac{\partial^2\theta(y,t)}{\partial y^2}+\eta\left[\frac{\partial u(y,t)}{\partial y}\right]^2 \tag{37}$$

Letting $g(y,t)=\left[\dfrac{\partial u(y,t)}{\partial y}\right]^2$, $k_0=\dfrac{3N_R}{3N_R+4}$, Eq.(37) can be rewritten as

$$\frac{\partial\theta(y,t)}{\partial t}=\frac{1}{k_0 Pr}\frac{\partial^2\theta(y,t)}{\partial y^2}+\eta g(y,t). \tag{38}$$

The corresponding initial and boundary conditions become:

$$\theta(y,0)=0,\ \text{for } y>0, \tag{39}$$

$$\theta(0,t)=1,\ \text{for } t\geq 0, \tag{40}$$

$$\theta(y,t)\to 0\,,\ \frac{\partial\theta(y,t)}{\partial y}\to 0\,,\ \text{ for } y\to\infty\,. \qquad (41)$$

Applying Fourier sine transform to Eqs.(38)-(39), we obtain

$$\frac{d\theta_s(\xi,t)}{dt}+\frac{1}{k_0 Pr}\xi^2\theta_s(\xi,t)=\sqrt{\frac{2}{\pi}}\frac{\xi}{k_0 Pr}+\eta g_s(\xi,t) \qquad (42)$$

$$\theta_s(\xi,0)=0\,. \qquad (43)$$

Where $\theta_s(\xi,t)$ and $g_s(\xi,t)$ denote the Fourier sine transform of $\theta(y,t)$ and $g(y,t)$ with respect to y, respectively. The solution of the ordinary differential equation Eq.(42) subject to the initial condition (43) is given by

$$\theta_s(\xi,t)=e^{-\xi^2 t/(k_0 Pr)}\int_0^t\left[\sqrt{\frac{2}{\pi}}\frac{\xi}{k_0 Pr}+\eta g_s(\xi,\tau)\right]e^{\xi^2\tau/(k_0 Pr)}d\tau \qquad (44)$$

Inverting Eq.(44) by means of Fourier sine transform, we get

$$\theta(y,t)=\sqrt{\frac{2}{\pi}}\int_0^\infty \sin(\xi y)e^{-\xi^2 t/(k_0 Pr)}\int_0^t[\sqrt{\frac{2}{\pi}}\frac{\xi}{k_0 Pr}+\eta g_s(\xi,\tau)]e^{\xi^2\tau/(k_0 Pr)}d\tau d\xi\,. \qquad (45)$$

VI. RESULTS AND DISCUSSION

In this paper, we have presented some oscillating flow of a generalized Oldroyd-B fluid. The effect of radiation on the heat transfer is considered. Exact analytic solutions are obtained for the velocity and temperature fields by means of Fourier sine transform coupled with Laplace transform. In there, we analyze the characteristics of velocity field and temperature field by using the analytical solutions obtained in sections 4-5.

The motion of the fluid was due to the oscillation of the plate parallel x direction with angular frequency ω. The velocity profiles are displayed for different time $\omega t = k\pi/4$ $(k=1,2,3,\ 4,5,6,7,8)$ with $\omega=1.5$ in Fig.1. Fig.2 shows the velocity in the case of magnetohydrodynamic fluid is

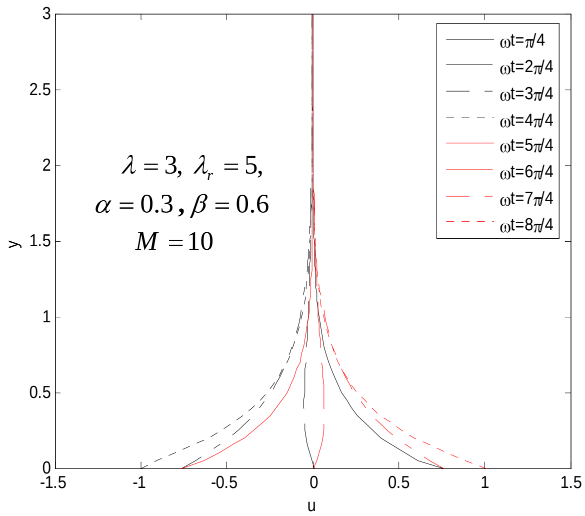

Fig.1 Profiles of the velocity field at different times for $u(0,t)=\cos(\omega t)$.

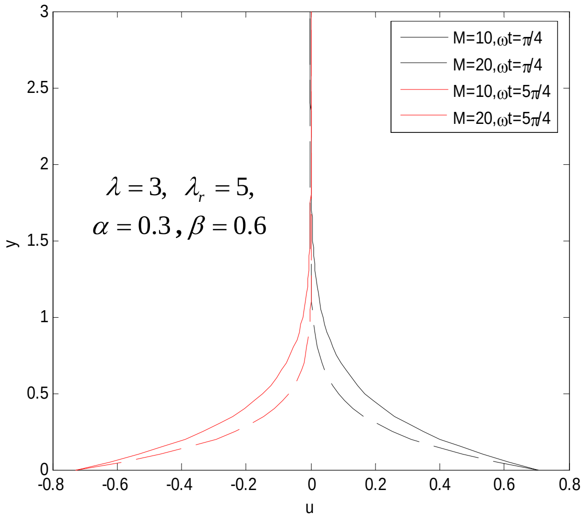

Fig.2 Profiles of the velocity field at different times and M for $u(0,t)=\cos(\omega t)$.

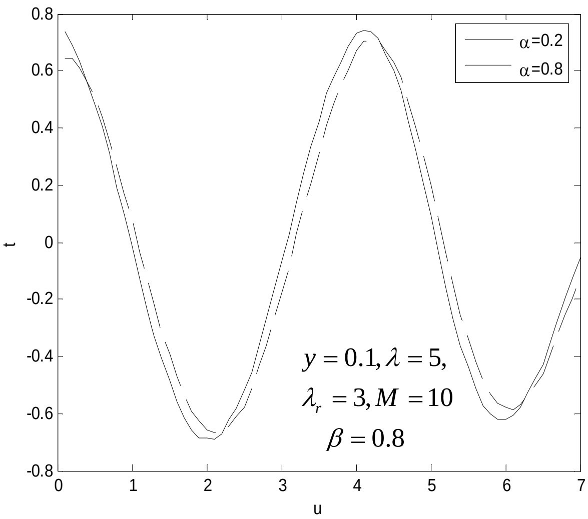

Fig.3 Profiles of the velocity field with different values of α for $u(0,t)=\cos(\omega t)$.

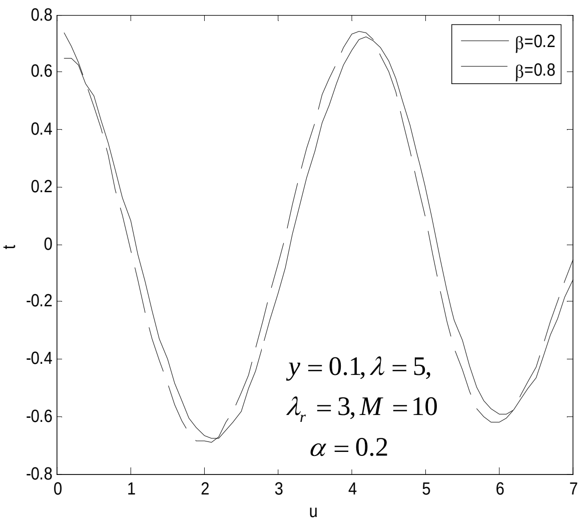

Fig.4 Profiles of the velocity field with different values of β for $u(0,t)=\cos(\omega t)$.

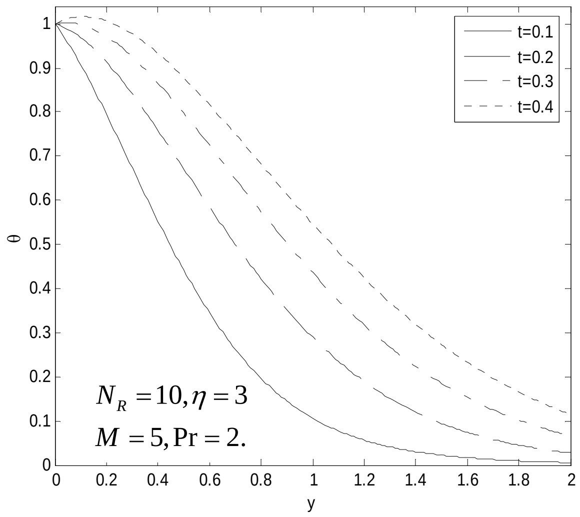

Fig.5 Profiles of the temperature field with different values of t for $u(0,t) = \cos(\omega t)$.

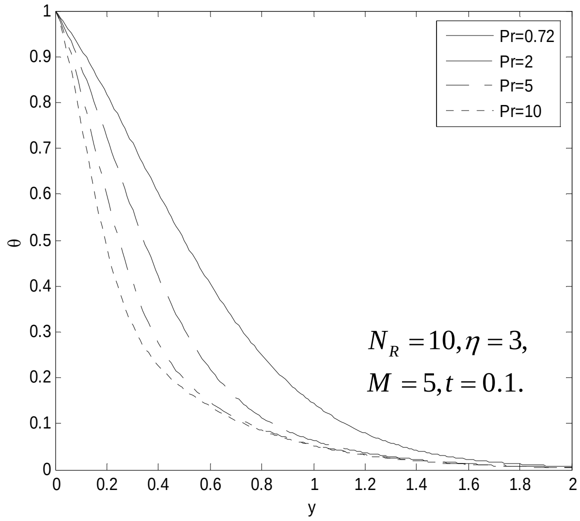

Fig.6 Profiles of the temperature field with different values of Pr for $u(0,t) = \cos(\omega t)$.

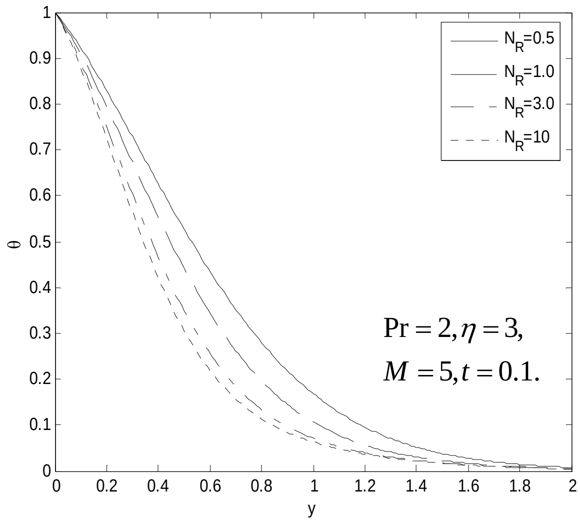

Fig.7 Profiles of the temperature field with different values of N_R for $u(0,t) = \cos(\omega t)$.

more steady than hydrodynamic. And the magnetic body force is favorable to decay of the velocity. Fig.3-4 demonstrate the velocity changes with the fractional parameters α and β . We can see that their effects on both motions are opposite. The non-Newtonian effects are stronger at large values of α . The smaller the values of α , the more steady of the velocity field.

Fig.5 displays the influence of time on temperature field. As it was to be expected, it clearly results that the non-Newtonian effects are stronger at lager values of t . The greater the value of t , the higher the temperature. Fig.6 is the graph for temperature distribution θ for different values of Pr , it is clear that there is a fall in temperature with increasing the Prandtl number. Fig.7 depicts the effect of varying N_R for temperature field. The results show marked decrease in the temperature distributions with increase in N_R .

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