On Accelerated MHD Flows of Second Grade Fluid in a Porous Medium and Rotating Frame

Faisal Salah, Zainal Abdul Aziz, and Dennis Ling Chuan Ching

Abstract—**The rotating magnetohydrodynamics (MHD) flow of a second grade fluid due to an accelerated plate is examined. Modified Darcy's law is used to formulate the physical problem in a porous space. Two cases of interest namely constant and variable accelerated MHD flows are addressed. Fourier sine transform technique is adopted to solve analytically the resulting problems. Finally, from the exact solutions the implied effects of emerging flow parameters on the velocity field are displayed and discussed.**

Index Terms—**Accelerated MHD flows, second grade fluid, porous medium, rotating frame, Fourier sine transform, exact solution**

I. INTRODUCTION

THE studies of non-Newtonian fluids have received considerable attention because of numerous applications in industry, geophysics and engineering. Some investigations are notably important in industries related to paper, food stuff, personal care products, textile coating and suspension solutions [1]. A large class of real fluids does not exhibit the linear relationship between stress and rate of strain as Newtonian fluid does. Due to the non-linear dependence, the analysis of the behavior of fluid motion of non-Newtonian fluids tends to be much more complicated and subtle in comparison with that of Newtonian fluids.

Unlike the Newtonian fluids, one cannot recommend a single constitutive relationship between stress and shear rate which exhibits all the characteristics of non-Newtonian fluids. Hence many empirical or semi-empirical expressions of non - Newtonian fluids are presented. The non - Newtonian fluids have been mainly classified under the differential, rate and integrals types. The second grade fluids are the subclass of non - Newtonian fluids and are the simplest subclass of differential type fluids which can show the normal stress effects [2]. It was employed to study various problems due to their relatively simple structure. Moreover, one can reasonably hope to obtain exact solutions from this type of second grade fluid. This motivates us to choose the second grade model in this study. The exact solutions are important as these provide standard for checking the accuracies of many approximate solutions which can be numerical or empirical. They can also be used as tests for verifying numerical schemes that are developed for studying more complex flow problems [3]–[5].

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In general, the governing equation of unidirectional flow of second grade fluid is third order which is more than the second order Navier - Stokes equation. Further, the order of equation in second grade fluid is reduced when higher order non-linearities are ignored, which is not possible in the Navier - Stokes equations. In view of such interesting features, several researchers have discussed the flows of second grade fluid in different configurations (for instance see studies [6]–[11]). There are available few attempts [12]–[20] in which the flows of non - Newtonian fluids have been investigated in a rotating frame.

To the best of our knowledge no literature exists in which accelerated MHD flows of non - Newtonian fluids in rotating frame and porous space have been studied. Therefore the objective of this work is to investigate the rotating and accelerated magnetohydrodynamic (MHD) flows in a second grade fluid filling up the porous space. The analysis has been performed for the two cases (a) constant accelerated MHD flow and (b) variable accelerated MHD flow. In addition the graphical results are plotted and discussed.

II. PROBLEM FORMULATION

Let us consider a Cartesian coordinate system (x, y, z). We consider a fluid saturated porous half space bounded by an infinite accelerated plate at $z = 0$ ($z-$ axis is taken normal to the plate). The whole system is rotating uniformly with a constant angular velocity Ω about the $z-$ axis, (refer to Figure 1.0). The porous space is described by the modified Darcy's law. A constant magnetic field $B_\circ$ acts in the $z-$ direction. The applied and induced magnetic fields are chosen to be zero.

The unsteady flow in a porous medium is governed by the following equations of motion and continuity:

$$\rho\left[\frac{\partial\vec{V}}{\partial t}+(\vec{V}.\nabla)\vec{V}+2\Omega\times\vec{V}+\Omega\times(\Omega\times r)\right]$$
$$=-\nabla p+div\vec{S}+\vec{J}\times\vec{B}+\vec{R}, \quad (1)$$
$$div\vec{V}=0. \quad (2)$$

In the above equations $\vec{V}=(u,v,w)$ is the velocity filed, $\vec{J}$ is the current density, $\vec{B}=B_\circ+b$ is the total magnetic field, $B_\circ$ and b are the applied and induced magnetic fields respectively, ρ is the fluid density and $\vec{R}$ is the Darcy resistance.

For small magnetic Reynolds number, the Lorentz force $\vec{J}\times\vec{B}$ becomes $\sigma\left(\vec{V}\times\vec{B}\right)\times\vec{B}$ when imposed and induced electric fields are negligible and only the applied magnetic field $\vec{B_0}$ contributes to the current density $\vec{J}=\sigma\left(\vec{V}\times\vec{B}\right)$; σ is the electrical conductivity of the fluid. In view of these considerations the Lorentz force due to magnetic field becomes $\vec{J}\times\vec{B}=\sigma\vec{B}_0^2\vec{V}$.

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For the second grade fluid we have the extra stress tensor $\vec{S}$ given in [4] as

$$\vec{S} = \mu\vec{A_1} + \alpha_1\vec{A_2} + \alpha_2\vec{A}_1^2 \tag{3}$$

where μ is the dynamic viscosity and $\alpha_i\ (i = 1, 2)$ are material constants satisfying $\alpha_1 \geq 0, \alpha_1 + \alpha_2 = 0$, $\vec{A_1}$ and $\vec{A_2}$ are the first two Rivlin - Eriksen tensors and the velocity field is assumed in the form

$$\vec{V} = (u(z,t), \upsilon(z,t), w(z,t)) \tag{4}$$

On the basis of Oldroyd constitutive equation, the following law describing both relaxation and retardation phenomena in an unbounded porous medium has been suggested by Tan and Masuoka [9]:

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \nabla p = -\frac{\mu}{k}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \vec{V_D} \tag{5}$$

In the above equation (5), λ_1 and λ_2 are the relaxation and retardation times respectively, k is the permeability, $\vec{V_D}\left(= \phi\vec{V}\right)$ is the Darcian velocity and ϕ is the porosity. For the second grade fluid $\lambda_1 = 0$, then equation (5) takes the form

$$\nabla p = -\frac{\mu\phi}{k}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \vec{V} \tag{6}$$

Note that equation (5) reduces into the classical Darcy's law for $\lambda_1 = 0$ and $\lambda_2 = 0$. Since the pressure gradient in equation (6) can also be interpreted as a measure of the resistance to flow in the bulk of the porous medium and R is a measure of the flow resistance offered by the solid matrix. Therefore R can be inferred from equation (6) to satisfy the following equation:

$$\vec{R} = -\frac{\mu\phi}{k}\left(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t}\right) \vec{V} \tag{7}$$

Equation (7) is a measure of flow resistance for the second grade fluid. In view of equations (2) and (4), it turns out that $w = 0$ for the rigid plate. Upon making use of this result in conjunction with equation (4) and assuming the modified pressure gradient equal to zero, equations (1), (3) and (7) lead to equations in [4],

$$\begin{aligned}\rho\left(\frac{\partial u}{\partial t} - 2\Omega\upsilon\right) &= \mu\frac{\partial^2 u}{\partial z^2} + \alpha_1\frac{\partial^3 u}{\partial z^2 \partial t} \\ &\quad -\sigma B_\circ^2 u - \frac{\mu\phi}{k}\left(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t}\right) u\end{aligned} \tag{8}$$

$$\begin{aligned}\rho\left(\frac{\partial \upsilon}{\partial t} + 2\Omega u\right) &= \mu\frac{\partial^2 \upsilon}{\partial z^2} + \alpha_1\frac{\partial^3 \upsilon}{\partial z^2 \partial t} \\ &\quad -\sigma B_\circ^2 \upsilon - \frac{\mu\phi}{k}\left(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t}\right) \upsilon\end{aligned} \tag{9}$$

The initial and boundary conditions are subjected to constantly accelerated MHD flow and are given as follows

$$u = \upsilon = 0\ , \text{when}\ t = 0,\ \ z > 0, \tag{10}$$

$$u(0,t) = At,\ \ \upsilon(0,t) = 0\ \text{ for }\ t > 0, \tag{11}$$

$$u,\ \frac{\partial u}{\partial z},\ \upsilon,\ \frac{\partial \upsilon}{\partial z} \to 0,\ \text{ as }\ z \to \infty,\ \ t > 0 \tag{12}$$

where A has dimension of $\frac{L}{T^2}$.

III. Exact Solution for Constant Accelerated MHD Flow

Making use of the following expression

$$F = u + i\upsilon, \tag{13}$$

and defining

$$\xi = Z\left(\frac{A}{\nu^2}\right)^{\frac{1}{3}},\ \ \tau = t\left(\frac{A^2}{\nu}\right)^{\frac{1}{3}},\ \ G = \frac{F}{(\nu A)^{\frac{1}{3}}},$$

$$P = 1 + \frac{\alpha_1\phi\nu}{\mu k},\ \ M = \frac{\sigma B_o^2}{\rho}\left(\frac{\nu}{A^2}\right)^{\frac{1}{3}},\ \ \omega = \left(\Omega\frac{\nu^{\frac{1}{3}}}{A^{\frac{2}{3}}}\right),$$

$$\frac{1}{B} = \frac{\phi}{k}\left(\frac{\nu^2}{A}\right)^{\frac{2}{3}},\ \ c = 2i\omega + M + \frac{1}{B},\ \ \alpha = \frac{\alpha_1}{\rho}\left(\frac{\nu}{A}\right)^{\frac{2}{3}}, \tag{14}$$

we arrive at

$$P\frac{\partial G(\xi,\tau)}{\partial\tau} + cG(\xi,\tau) = \left[1 + \alpha\frac{\partial}{\partial\tau}\right]\frac{\partial^2 G(\xi,\tau)}{\partial\xi^2} \tag{15}$$

$$G(0,\tau) = \tau,\ \ \tau > 0, \tag{16}$$

$$G(\xi,0) = 0,\ \ \xi > 0, \tag{17}$$

$$G(\xi,\tau),\ \ \frac{\partial G(\xi,\tau)}{\partial\xi} \to 0\ \text{ as }\ \ \xi \to \infty;\ \ \tau > 0. \tag{18}$$

Taking the Fourier sine transform, the above problem takes the form

$$\begin{aligned}&\left(P + \alpha\eta^2\right)\frac{\partial G_s(\eta,\tau)}{\partial\tau} + \left[c + \eta^2\right] G_s(\eta,\tau) \\ &\qquad = \sqrt{\frac{2}{\pi}}\eta\left[\tau + \alpha\right];\ \ \eta,\tau > 0,\end{aligned} \tag{19}$$

$$G_s(\eta,0) = 0,\ \ \eta > 0, \tag{20}$$

in which $G_s(\eta,\tau)$ indicates the Fourier sine transform of $G(\xi,\tau)$.

The above system is satisfied by the following relation

$$\begin{aligned}G_s(\eta,\tau) &= \sqrt{\frac{2}{\pi}}\frac{\eta}{(c+\eta^2)^2} * \\ &\left[\tau\left(c+\eta^2\right) + (\alpha c - P) + (P - \alpha c)\, e^{\frac{-\left(c+\eta^2\right)\tau}{(P+\alpha\eta^2)}}\right]\end{aligned} \tag{21}$$

Fourier inversion of equation (21) yields

$$\begin{aligned}G(\xi,\tau) &= \tau e^{-\xi\sqrt{c}} - \frac{2(P - c\alpha)}{\pi} * \\ &\int_0^\infty \left[\frac{1 - e^{\frac{-\left(c+\eta^2\right)\tau}{(P+\alpha\eta^2)}}}{\left(c+\eta^2\right)^2}\right]\eta\sin(\xi\eta)d\eta.\end{aligned} \tag{22}$$

Equation (22) represents the exact solution for the constant accelerated MHD flow of second grade fluid in a porous medium and rotating frame.

For hydrodynamic second grade fluid in a non - porous medium and non-rotational, one has , $c = 0$ and therefore equation (22) reduces to

$$G(\xi,\tau) = \tau - \frac{2P}{\pi}\int_0^\infty\left[\frac{1 - e^{\frac{-\eta^2\tau}{(P+\alpha\eta^2)}}}{\eta^3}\right]\sin(\xi\eta)d\eta. \tag{23}$$

The velocity field $G(\xi,\tau)$, as given by equation (23), is equivalently of the form to that obtained by Fetecau et al. (2007, Eq. (23)) For a magnetohydrodynamic Newtonian fluid (i.e.$\alpha = 0$) in a porous space, equation (22) reduces to

$$G(\xi,\tau) = \tau - \frac{2}{\pi}\int_0^\infty \left[\frac{1-e^{-\eta^2\tau}}{\eta^3}\right]\sin(\xi\eta)d\eta. = 4\tau i^2 Erfc\left(\frac{\xi}{2\sqrt{\tau}}\right) \quad (24)$$

In order to address the influence of several pertinent parameters on the dimensionless velocity field components, graphical illustrations in the Figures 2 to 5 are presented for the constant accelerated MHD flows generated by a plate. The panels (a) and (b) in each plot represent the real and imaginary parts of the derived velocity field respectively.

IV. Exact Solution for Variable Accelerated MHD Flow

The problem here is described by equations (8 -10), (12) and the boundary conditions

$$u = v = 0 \text{ at } t = 0,\ z > 0,$$
$$u(0,t) = \Re t^2,\ v(0,t) = 0;\ t > 0. \quad (25)$$

where $\Re$ has dimension of $\frac{L}{T^3}$.

We now introduce the following quantities

$$\zeta = z\left(\frac{\Re}{\nu^3}\right)^{\frac{1}{5}},\ \delta = t\left(\frac{\Re^2}{\nu}\right)^{\frac{1}{5}},\ S = \frac{F}{(\nu^2\Re)^{\frac{1}{5}}},$$
$$P = 1 + \frac{\alpha_1\phi\nu}{\mu k},\ \in = \frac{\alpha_1}{\rho}\left(\frac{\nu}{\Re^3}\right)^{\frac{2}{5}},\ H = \frac{\sigma B_o^2}{\rho}\left(\frac{\nu}{\Re}\right)^{\frac{1}{5}},$$
$$\psi = \left(\Omega\frac{\nu^{\frac{1}{5}}}{\Re^{\frac{2}{5}}}\right),\ \frac{1}{L} = \frac{\phi}{k}\left(\frac{\nu}{\Re^2}\right)^{\frac{2}{5}},\ D = 2i\psi + H + \frac{1}{L}. \quad (26)$$

The dimensionless flow problem then takes the following form,

$$P\frac{\partial S(\zeta,\delta)}{\partial\delta} + DS(\zeta,\delta) = \left[1 + \in\frac{\partial}{\partial\delta}\right]\frac{\partial^2 S(\zeta,\delta)}{\partial\zeta^2};\ \zeta,\ \delta > 0, \quad (27)$$
$$S(0,\delta) = \delta^2,\ \delta > 0, \quad (28)$$
$$S(\zeta,0) = 0,\ \xi > 0 \quad (29)$$
$$S(\zeta,\delta),\ \frac{\partial S(\zeta,\delta)}{\partial\zeta} \to 0,\ as\ \zeta \to \infty; \delta > 0. \quad (30)$$

Adopting the solution procedures of the previous section we directly write the exact solution the variable accelerated MHD flow of second grade fluid in a porous medium and rotating frame as

$$\begin{aligned} S(\zeta,\delta) = {} & \frac{2\delta^2}{\pi}\int_0^\infty \frac{\eta\sin(\zeta\eta)}{D+\eta^2}d\eta \\ & + \frac{4(\in D - P)\delta}{\pi}\int_0^\infty \frac{\eta sin(\zeta\eta)d\eta}{(D+\eta^2)^2} \\ & - \frac{4(\in D - P)}{\pi}\int_0^\infty (P + \in\eta^2) \\ & \times\left(1 - e^{-\left(\frac{D+\eta^2}{P+\in\eta^2}\right)\delta}\right)\eta\frac{sin(\zeta\eta)d\eta}{(D+\eta^2)^3} \end{aligned} \quad (31)$$

or on the another form the solution (31) becomes

$$\begin{aligned} S(\zeta,\delta) = {} & \delta^2 e^{-\zeta\sqrt{D}} + \left(\frac{(\in D - P)\delta}{\sqrt{D}}\zeta\right)e^{-\zeta\sqrt{D}} - \frac{4(P - \in D)}{\pi} \\ & \times\int_0^\infty \left[\frac{(P + \in\eta^2)\left(1 - e^{\frac{-(D+\eta^2)\delta}{(P+\in\eta^2)}}\right)}{(D+\eta^2)^3}\eta\sin(\zeta\eta)d\eta\right] \end{aligned} \quad (32)$$

For hydrodynamic second grade fluid in a non - porous medium and non- rotational one has $D = 0$, and therefore equation (31) reduces to

$$S(\zeta,\delta) = \delta^2 - \frac{4P\delta}{\pi}\int_0^\infty \frac{sin(\zeta\eta)d\eta}{\eta^3}$$
$$+\frac{4P}{\pi}\int_0^\infty (P + \in\eta^2)\left(1 - e^{-\left(\frac{\eta^2}{P+\in\eta^2}\right)\delta}\right)\frac{sin(\zeta\eta)d\eta}{\eta^5} \quad (33)$$

For Newtonian magnetohydrodynamic fluid (i.e.$\in = 0$, $P = 1$) in a porous space, equation (32) reduces then to

$$S(\zeta,\delta) = \delta^2 e^{-(\sqrt{D})\zeta} - \frac{4\delta}{\pi}\int_0^\infty \frac{\eta\sin(\zeta\eta)}{(D+\eta^2)^2}d\eta + \frac{4}{\pi}\int_0^\infty \left[1 - e^{(D+\eta^2)\delta}\right]\frac{\eta\sin(\zeta\eta)}{(D+\eta^2)^3}d\eta \quad (34)$$

For hydrodynamic viscous fluid in a non - porous medium and non- rotational one has, $D = 0$ and therefore equation (34) reduces to

$$\begin{aligned} S(\zeta,\delta) = {} & \delta^2 - \frac{4\delta}{\pi}\int_0^\infty \frac{\sin(\zeta\eta)}{\eta^3}d\eta + \frac{4}{\pi}\int_0^\infty \left[1 - e^{\eta^2\delta}\right]\frac{\sin(\zeta\eta)}{\eta^5}d\eta \\ = {} & 32\delta^2 i^4 Erfc\left(\frac{\zeta}{2\sqrt{\delta}}\right) \end{aligned} \quad (35)$$

In order to see the effects of several relevant parameters on the dimensionless velocity field components, some graphical illustrations in the Figures 6 to 9 are presented for the variable accelerated MHD flows generated by a plate. The panels (a) and (b) in each plot represent the real and imaginary parts of the derived velocity field respectively.

V. Discussion of the Results

The aim of this section is to address the influence of several pertinent parameters on the dimensionless velocity field components. Graphical illustrations are presented for the two situations namely constant and variable accelerated MHD flows generated by a plate. The panels (a) and (b) in each plot represent the real and imaginary parts of the derived velocity profile respectively.

In order to analyze the whole situation, we have drawn the Figures 2 to 9. It should be pointed out that the result of velocity profile for constant accelerated plate has been shown in the Figures 2 to 5. Furthermore, the flow induced by variable accelerated plate has been sketched in the Figures 6 to 9.

The velocity field components versus ξ have been plotted in Figure 2 when different values of α are accounted for. It is concluded that the real part of velocity profile increases when there is an increase in α (for example in the physical process involving stress thickening). Further, the magnitude of imaginary part of velocity profile decreases when α increases. Plots in figure 3 indicate the behaviour of M (physical process relating to action of the Lorentz force) on the velocity field components versus ξ . It is noted that the real part of velocity profile is a decreasing function of M whiles the magnitude of the imaginary part increases with large values of M . A comparative study of Figures 3 and 4 yields an opposite features associated with M and B on the real and imaginary parts of the velocity profile. Plots in Figure 5 characterize the influence of rotational parameter ω on the real and imaginary parts of velocity profile. Panel (a) in this Figures shows that Re $(G(\xi,\tau)$) decreases when large values of ω are taken into account. However, the imaginary part of velocity profile in panel (b) first decreases and then increases. Moreover, it is found that variations of all these parameters in variable accelerated MHD flow i.e. in Figures 6 to 9 are similar to that of constant accelerated MHD flow presented in the Figures 2 to 5 in a qualitative manner.

VI. CONCLUSIONS

In this article, the unsteady rotating flow engendered by an accelerated plate has been studied in the presence of an applied magnetic field. From the presented analysis, the main observations are described as below:

- The variation of α on the magnitude of real and imaginary parts of velocity profile is different.
- Effect of M on the magnitudes of real and imaginary components of velocity profile is quite the opposite.
- The flow characteristics subject to M and B on the velocity profile components leads to an opposite effects.
- The results corresponding to viscous fluid can be obtained by choosing $\alpha = 0$.

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APPENDIX

In order to obtain Equation (32) from Equation (31), we use the result in this work (Grandshteyn and Ryzhik, 1994):

- $\int_0^\infty \frac{\sin(ax)}{x} dx = \frac{\pi}{2}$
- $\int_0^\infty \frac{x\sin(ax)}{\beta^2+x^2} dx = \frac{\pi}{2} e^{-a\beta}$, $(a > 0, \mathrm{Re}\beta > 0)$
- $\int_0^\infty \frac{x\sin(ax)}{(\beta^2+x^2)^2} dx = \frac{\pi}{4\beta} a e^{-a\beta}$, $(a > 0, \beta > 0)$

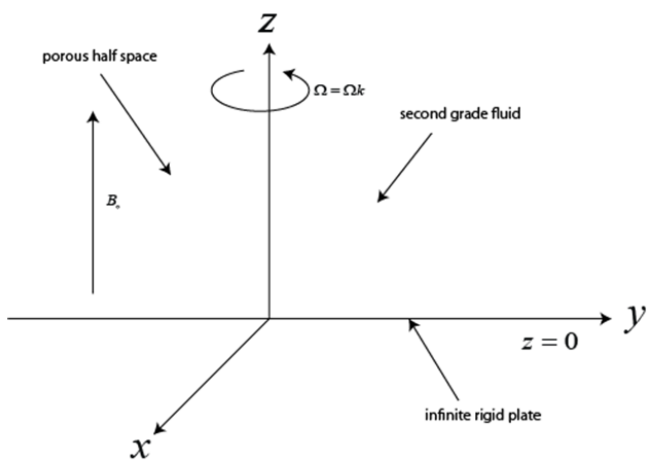
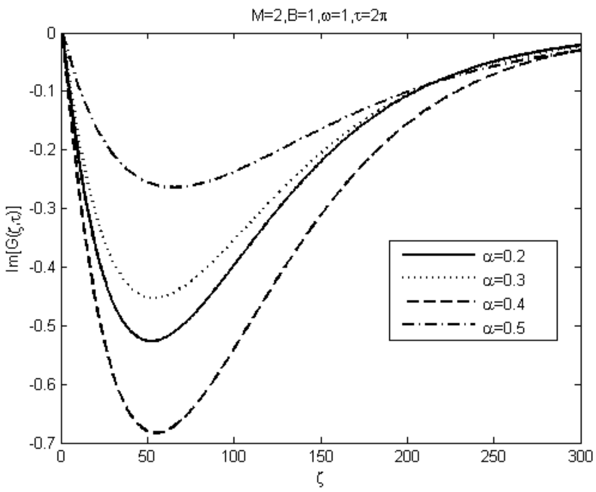
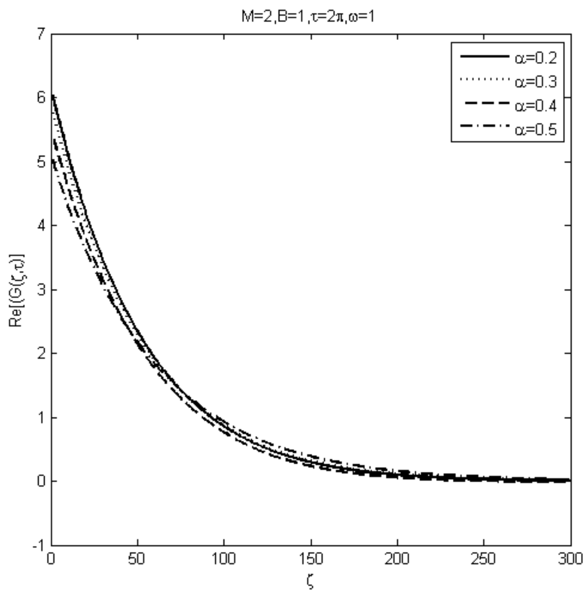


Fig. 1. Graphical representation of the physical problem for second grade fluid in a rotating frame.



(a)



(b)

Fig. 2. Velocity profiles for different values of α

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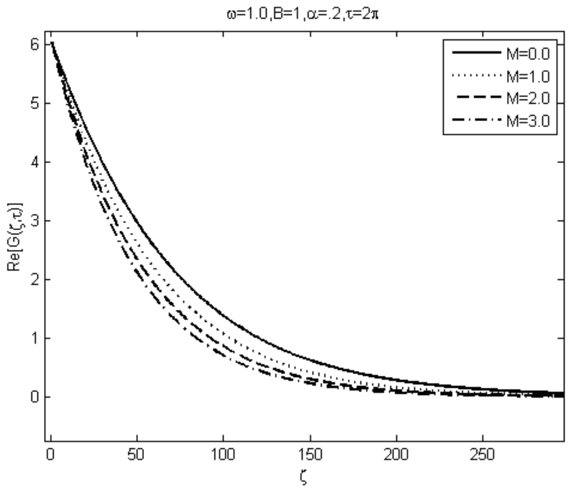

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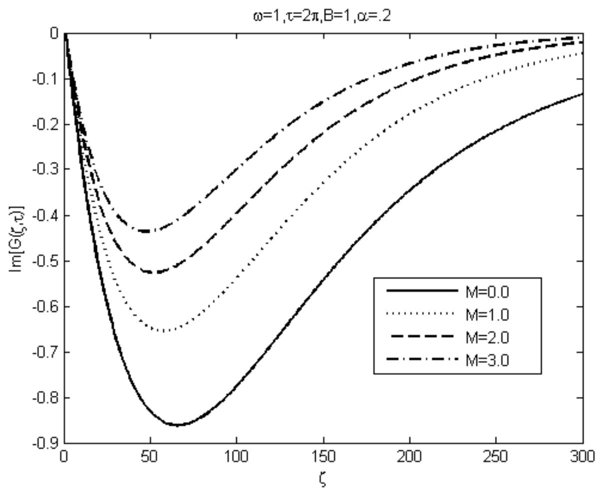

(b)

Fig. 3. Velocity profiles for different values of M

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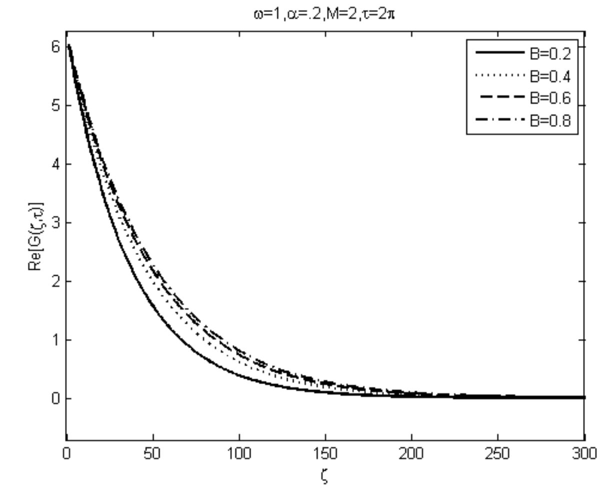
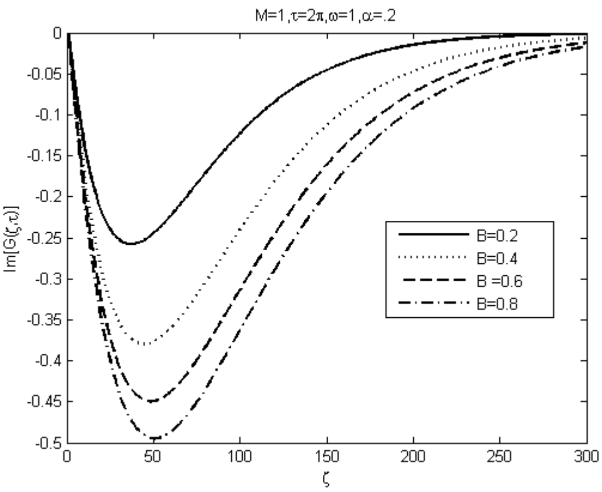


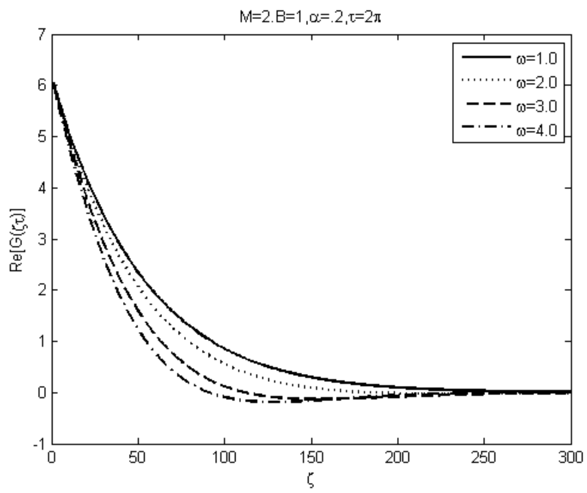
(a)



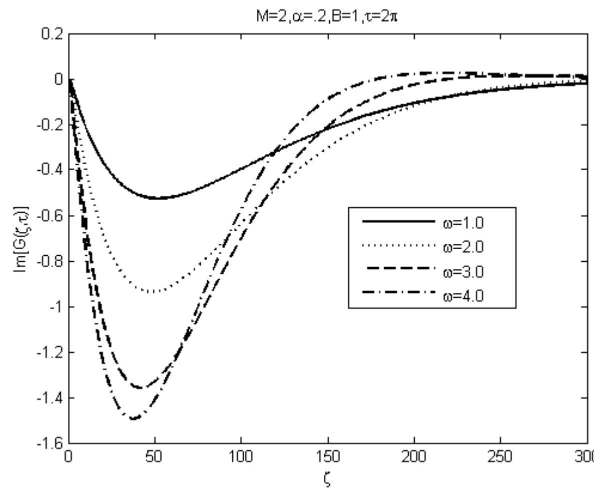
(b)

Fig. 4. Velocity profiles for different values of B

.



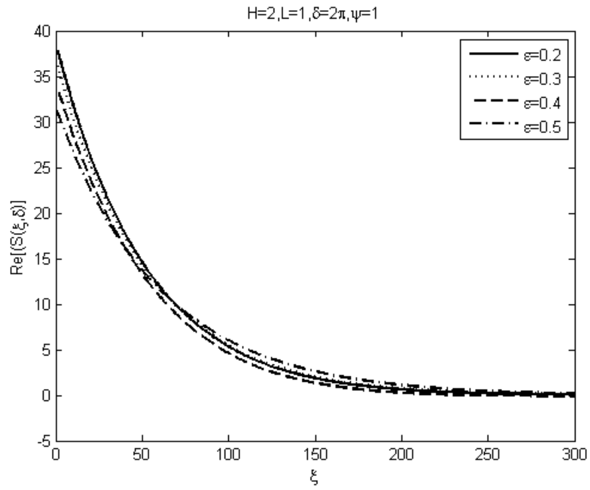
(a)



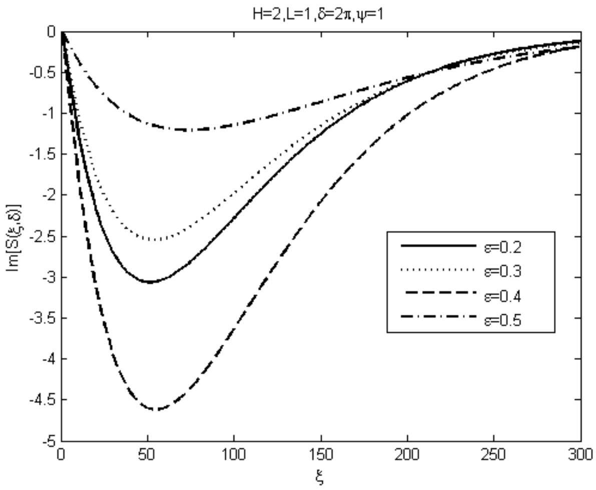
(b)

Fig. 5. Velocity profiles for different values of ω

.



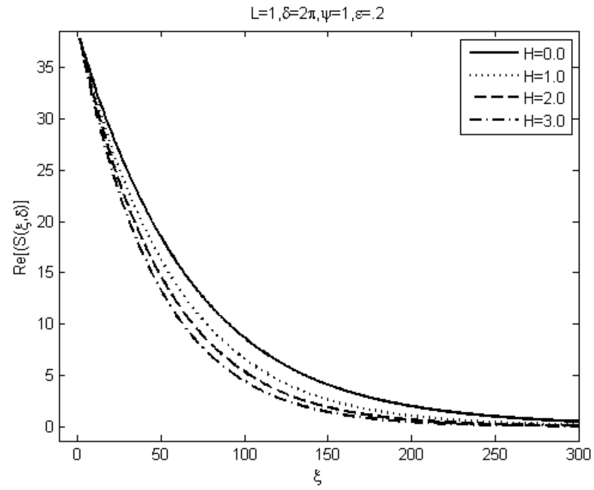
(a)



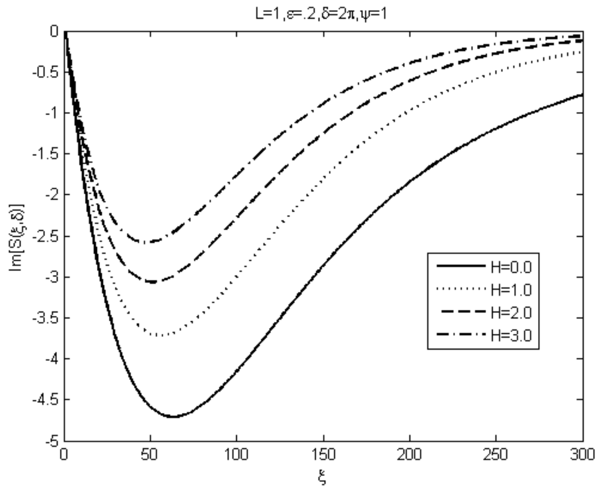
(b)

Fig. 6. Velocity profiles for different values of ϵ

.



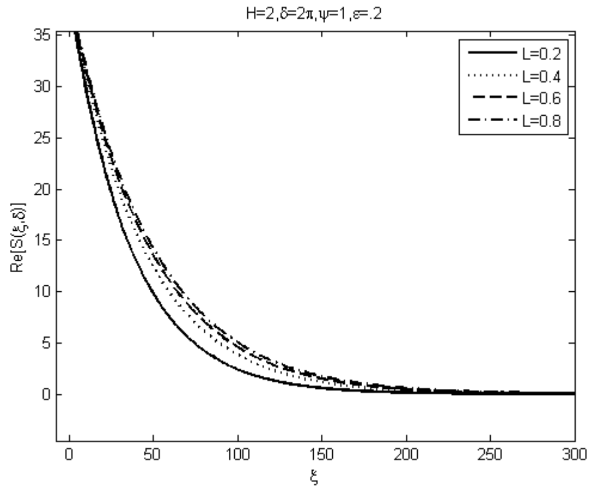
(a)



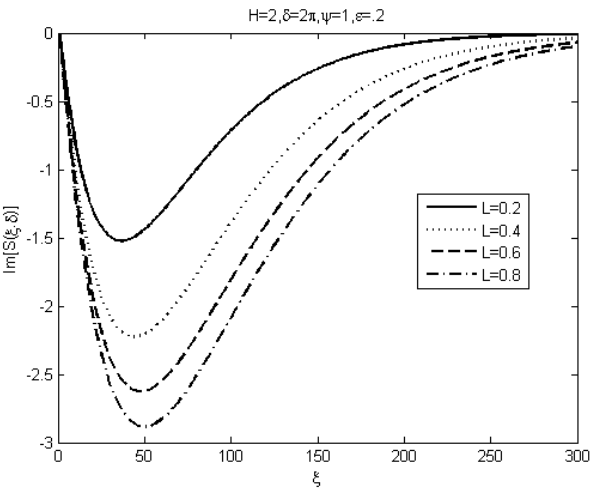
(b)

Fig. 7. Velocity profiles for different values of H

.



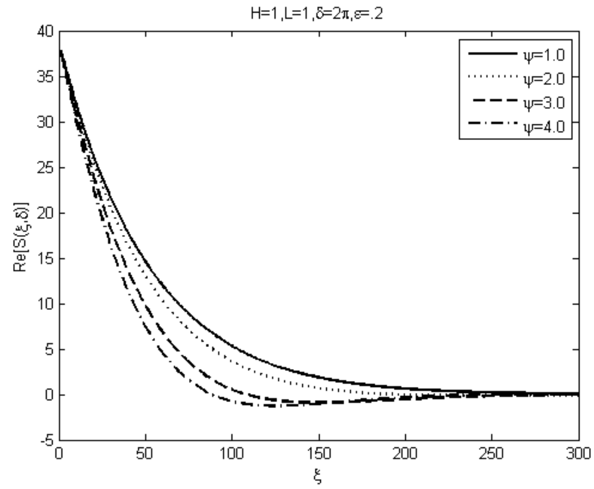
(a)



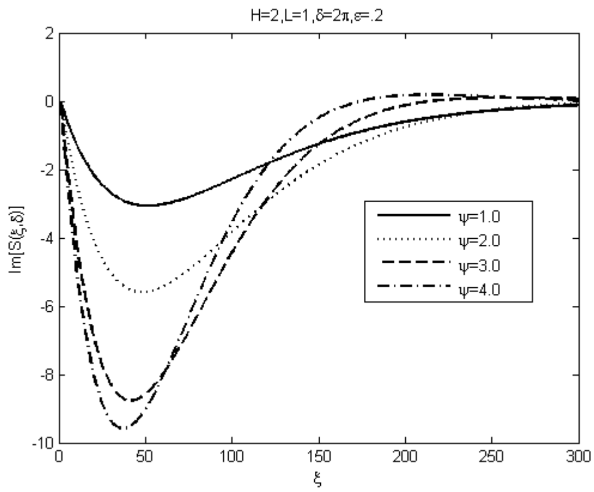
(b)

Fig. 8. Velocity profiles for different values of L

.



(a)



(b)

Fig. 9. Velocity profiles for different values of ψ

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