Multiobjective Fuzzy Random Linear Programming Problems Based on Coefficients of Variation

Hitoshi Yano, Kota Matsui and Mikiya Furuhashi

***Abstract*—In this paper, an interactive decision making method for multiobjective fuzzy random linear programming problems based on coefficients of variation is proposed. In the proposed method, it is assumed that the decision maker intends to not only maximize the expected degrees of possibilities that the original objective functions attain the corresponding fuzzy goals, but also minimize coefficients of variation for such possibilities, and such fuzzy goals are quantified by eliciting the corresponding membership functions. Using the fuzzy decision, both the expected degrees of possibilities and the membership functions of coefficients of variation are integrated. In the integrated membership space, a satisfactory solution is obtained from among an CV-Pareto optimal solution set through the interaction with the decision maker. In order to show the efficiency of the proposed method, the interactive processes for the numerical example under the hypothetical decision maker are demonstrated, and the proposed method is compared with the V-model based method.**

***Index Terms*—multiobjective programming, fuzzy random variables, expectations, coefficients of variation, fuzzy decision, interactive method.**

I. Introduction

In the real world decision making situations, we often have to make a decision under uncertainty. In order to deal with decision problems involving uncertainty, stochastic programming approaches [1], [2], [3], [6] and fuzzy programming approaches [12], [14], [25] have been developed. Recently, mathematical programming problems with fuzzy random variables [11] have been formulated [13], [15], [17], whose concept includes both probabilistic uncertainty and fuzzy one simultaneously. Extensions to multiobjective fuzzy random linear programming problems (MOFRLP) have been done and interactive methods to obtain a satisfactory solution for the decision maker have been proposed [7], [9], [15]. In their methods, it is required in advance for the decision maker to specify permissible possibility levels in a probability maximization model or permissible probability levels in a fractile optimization model [16]. However, it seems to be very difficult for the decision maker to specify such permissible levels appropriately. From such a point of view, a fuzzy approach to MOFRLP has been proposed [21], in which the decision maker specifies not the values of permissible levels but the membership functions for the fuzzy goals of permissible levels. In the proposed method, it is assumed that the decision maker adopts the fuzzy decision [5], [14] to integrate the membership functions. As a natural extension of such methods, interactive fuzzy decision making methods for MOFRLP to obtain a satisfactory solution from among an extended Pareto optimal solution set have been proposed [19], [20], [23], [24]

On the other hand, some decision maker may prefer to adopt an expectation model (E-model) or a variance model (V-model) rather than a probability maximization model or a fractile optimization model to deal with MOFRLP, because the expectation value or the variance is a well-known statistical quantity. From such a point of view, Katagiri et al. [8], [10] proposed interactive decision making methods for MOFRLP to obtain a satisfactory solution of the decision maker, using E-model and V-model [18]. However, when adopting E-model, the effects for the variance of the random variable coefficients of fuzzy random variables is ignored. Similarly, when adopting V-model, although the effects for the variance of the random variable coefficients of fuzzy random variables is considered in the formulation processes of MOFRLP, the decision maker must specify in advance a permissible expectation level for each objective function of MOFRLP subjectively. In general, the minimization of a permissible expectation level in a minimization problem conflicts with the minimization of the variance. Therefore, it seems to be difficult for the decision maker to specify appropriately a permissible expectation level for each objective function of MOFRLP.

In this paper, it is assumed that the decision maker intends to not only maximize the expected degrees of possibilities [5] that the original objective functions involving fuzzy random variable coefficients attain the corresponding fuzzy goals, but also minimize coefficients of variation for such possibilities in MOFRLP [8], [10]. In order to deal with such decision making situations in MOFRLP, we introduce an CV-Pareto optimal solution concept, in which both the expected degrees of possibilities and the corresponding coefficients of variation for such possibilities are integrated through the fuzzy decision [5], [14]. To obtain an CV-Pareto optimal solution, minmax problem is formulated. An interactive algorithm is proposed to obtain a satisfactory solution from among an CV-Pareto optimal solution set by solving the minmax problem on the basis of convex programming technique [22]. In order to illustrate the proposed method, a three-objective fuzzy random linear programming problem is formulated, and the interactive processes under the hypothetical decision maker are demonstrated, and the proposed method is compared with the V-model based method.

H. Yano is with Department of Social Sciences, Graduate School of Humanities and Social Sciences, Nagoya City University, Nagoya, 467-8501, Japan, e-mail: yano@hum.nagoya-cu.ac.jp

K. Matsui is with Department of Computer Science and Mathematical Informatics, Graduate School of Information Science, Nagoya University, Nagoya, 464-8601, Japan, e-mail: matsui@math.cm.is.nagoya-u.ac.jp

M. Furuhashi is with LiNCREA Corp., Minato-ku, Tokyo, 108-9975, Japan, e-mail: m.furuhashi@lincrea.co.jp

II. MULTIOBJECTIVE FUZZY RANDOM LINEAR PROGRAMMING PROBLEMS

In this section, we focus on multiobjective programming problems involving fuzzy random variable coefficients in objective functions called multiobjective fuzzy random linear programming problem (MOFRLP).
[MOFRLP]

$$\min_{\boldsymbol{x}\in X} \tilde{\bar{C}}\boldsymbol{x} = (\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}, \cdots, \tilde{\bar{\boldsymbol{c}}}_k\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{x} = (x_1, \cdots, x_n)^T$ is an n dimensional decision variable column vector. X is a linear constraint set with respect to $\boldsymbol{x}$. $\tilde{\bar{\boldsymbol{c}}}_i = (\tilde{\bar{c}}_{i1}, \cdots, \tilde{\bar{c}}_{in}), i = 1, \cdots, k$ are coefficient vectors of objective function $\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}$, whose elements are fuzzy random variables (The symbols "–" and "~" mean randomness and fuzziness respectively).

In this paper, we assume that under the occurrence of each scenario $\ell_i \in \{1, \cdots, L_i\}$, $\tilde{c}_{ij\ell_i}$ is a realization of a fuzzy random variable $\tilde{\bar{c}}_{ij}$ which is a fuzzy number whose membership function is defined as follows [15].

$$\mu_{\tilde{c}_{ij\ell_i}}(t) = \begin{cases} \max\left\{1 - \frac{d_{ij\ell_i} - t}{\alpha_{ij}}, 0\right\}, & t \le d_{ij\ell_i} \\ \max\left\{1 - \frac{t - d_{ij\ell_i}}{\beta_{ij}}, 0\right\}, & t > d_{ij\ell_i} \end{cases} \tag{2}$$

where the parameters $\alpha_{ij} > 0, \beta_{ij} > 0$ are constants and $d_{ij\ell_i}$ varies depending on which a scenario ℓ_i occurs. Moreover, we assume that a scenario ℓ_i occurs with a probability $p_{i\ell_i}$, where $\sum_{\ell_i=1}^{L_i} p_{i\ell_i} = 1$ for $i = 1, \cdots, k$.

By Zadeh's extension principle, the realization $\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}$ becomes a fuzzy number which characterized by the following membership function.

$$\mu_{\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}}(y) = \begin{cases} \max\left\{1 - \frac{\boldsymbol{d}_{i\ell_i}\boldsymbol{x} - y}{\boldsymbol{\alpha}_i\boldsymbol{x}}, 0\right\}, & y \le \boldsymbol{d}_{i\ell_i}\boldsymbol{x} \\ \max\left\{1 - \frac{y - \boldsymbol{d}_{i\ell_i}\boldsymbol{x}}{\boldsymbol{\beta}_i\boldsymbol{x}}, 0\right\}, & y > \boldsymbol{d}_{i\ell_i}\boldsymbol{x} \end{cases} \tag{3}$$

where $\boldsymbol{d}_{i\ell_i} = (d_{i1\ell_i}, \cdots, d_{in\ell_i})$, $\boldsymbol{\alpha}_i = (\alpha_{i1}, \cdots, \alpha_{in}) \ge \boldsymbol{0}$, $\boldsymbol{\beta}_i = (\beta_{i1}, \cdots, \beta_{in}) \ge \boldsymbol{0}$.

Considering the imprecise nature of the decision maker's judgment, it is natural to assume that the decision maker has a fuzzy goal for each objective function in MOFRLP. In this paper, it is assumed that such a fuzzy goal $\tilde{G}_i$ can be quantified by eliciting the corresponding membership function defined as follows.

$$\mu_{\tilde{G}_i}(y_i) = \begin{cases} 1 & y_i < z_i^1 \\ \dfrac{y_i - z_i^0}{z_i^1 - z_i^0} & z_i^1 \le y_i \le z_i^0 \\ 0 & y_i > z_i^0 \end{cases} \tag{4}$$

where z_i^0 represents the minimum value of an unacceptable level of the objective function, and z_i^1 represents the maximum value of a sufficiently satisfactory level of the objective function. By using a concept of possibility measure [5], the degree of possibility that the objective function value $\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}$ satisfies the fuzzy goal $\tilde{G}_i$ is expressed as follows [9].

$$\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i) \stackrel{\text{def}}{=} \sup_y \min\{\mu_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(y), \mu_{\tilde{G}_i}(y)\} \tag{5}$$

It should be noted here that if a scenario ℓ_i occurs with probability $p_{i\ell_i}$ then the value of possibility measure can be represent as

$$\Pi_{\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}}(\tilde{G}_i) \stackrel{\text{def}}{=} \sup_y \min\{\mu_{\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}}(y), \mu_{\tilde{G}_i}(y)\}. \tag{6}$$

Using the above possibility measure, MOFRLP can be transformed into the following multiobjective stochastic programming problem (MOSP).
[MOSP]

$$\max_{\boldsymbol{x}\in X}(\Pi_{\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}}(\tilde{G}_1), \cdots, \Pi_{\tilde{\bar{\boldsymbol{c}}}_k\boldsymbol{x}}(\tilde{G}_k)) \tag{7}$$

III. AN EXPECTATION MODEL AND A VARIANCE MODEL FOR MOFRLP

Katagiri et al.[8], [10] formulated MOFRLP as the multiobjective programming problems through expectation model (E-model) and variance model (V-model) respectively. At First, we explain E-model for MOFRLP formulated as follows.
[MOP-E1]

$$\max_{\boldsymbol{x}\in X}(E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}}(\tilde{G}_1)], \cdots, E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_k\boldsymbol{x}}(\tilde{G}_k)]) \tag{8}$$

where $E[\cdot]$ denotes the expectation operator. In order to deal with MOP-E1, we introduce an E-Pareto optimal solution concept.

Definition 1: $\boldsymbol{x}^* \in X$ is said to be an E-Pareto optimal solution to MOP-E1, if and only if there does not exist another $\boldsymbol{x} \in X$ such that $E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)] \ge E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}^*}(\tilde{G}_i)]$, $i = 1, \cdots, k$ with strict inequality holding for at least one i.

It should be noted here that (6) can be represented as follows [15].

$$\Pi_{\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}}(\tilde{G}_i) = \frac{\sum_{j=1}^n(\alpha_{ij} - d_{ij\ell_i})x_j + z_i^0}{\sum_{j=1}^n \alpha_{ij}x_j - z_i^1 + z_i^0} \tag{9}$$

Since the probability that a scenario ℓ_i occurs is $p_{i\ell_i}$, $E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)]$ can be computed as follows.

$$\begin{aligned} & E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)] \\ = & \sum_{\ell_i=1}^{L_i} p_{i\ell_i}\Pi_{\tilde{\boldsymbol{c}}_{i\ell_i}\boldsymbol{x}}(\tilde{G}_i) \\ = & \frac{\sum_{j=1}^n(\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i}d_{ij\ell_i})x_j + z_i^0}{\sum_{j=1}^n \alpha_{ij}x_j - z_i^1 + z_i^0} \\ \stackrel{\text{def}}{=} & Z_i^E(\boldsymbol{x}) \end{aligned} \tag{10}$$

Then, MOP-E1 can be transformed into MOP-E2.
[MOP-E2]

$$\max_{\boldsymbol{x}\in X}(Z_1^E(\boldsymbol{x}), \cdots, Z_k^E(\boldsymbol{x})) \tag{11}$$

Next, consider V-model for MOFRLP. The multiobjective programming problem based on V-model can be formulated as follows.
[MOP-V1]

$$\min_{\boldsymbol{x}\in X}(V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}}(\tilde{G}_1)], \cdots, V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_k\boldsymbol{x}}(\tilde{G}_k)]) \tag{12}$$

subject to

$$E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)] \ge \xi_i, \quad i = 1, \cdots, k \tag{13}$$

where $V[\cdot]$ denotes the variance operator, and ξ_i represents a permissible expectation level for $E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)]$. Now, we denote the feasible set of MOP-V1 as

$$X(\boldsymbol{\xi}) \stackrel{\text{def}}{=} \{\boldsymbol{x} \in X | E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i\boldsymbol{x}}(\tilde{G}_i)] \ge \xi_i, i = 1, \cdots, k\}. \tag{14}$$

Similar to E-model, in order to deal with MOP-V1, a V-Pareto optimal solution concept is defined.

Definition 2: $\boldsymbol{x}^* \in X(\boldsymbol{\xi})$ is said to be a V-Pareto optimal solution to MOP-V1, if and only if there does not exist another $\boldsymbol{x} \in X(\boldsymbol{\xi})$ such that $V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)] \leq V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}^*}(\tilde{G}_i)]$, $i = 1, \cdots, k$ with strict inequality holding for at least one i.

It should be noted here that $V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)]$ can be represented as follows [15].

$$\begin{aligned} & V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)] \\ = & \frac{1}{(\sum_{j=1}^n \alpha_{ij} x_j - z_i^1 + z_i^0)^2} V\left[\sum_{j=1}^n \bar{d}_{ij} x_j\right] \\ = & \frac{1}{(\sum_{j=1}^n \alpha_{ij} x_j - z_i^1 + z_i^0)^2} \boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x} \qquad (15) \\ \stackrel{\text{def}}{=} & Z_i^V(\boldsymbol{x}) \end{aligned}$$

where $\boldsymbol{V}_i$ is the variance-covariance matrix of $\bar{\boldsymbol{d}}_i$ expressed by

$$\boldsymbol{V}_i = \begin{pmatrix} v_{11}^i & v_{12}^i & \cdots & v_{1n}^i \\ v_{21}^i & v_{22}^i & \cdots & v_{2n}^i \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1}^i & v_{n2}^i & \cdots & v_{nn}^i \end{pmatrix}, i = 1, \cdots, k, \qquad (16)$$

and

$$\begin{aligned} v_{jj}^i &= V[\bar{d}_{jj}] \\ &= \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}^2 - \left(\sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}\right)^2, \\ & \quad j = 1, \cdots, n, \qquad (17) \\ v_{jr}^i &= \text{Cov}[\bar{d}_{ij}, \bar{d}_{ir}] \\ &= E[\bar{d}_{ij} \cdot \bar{d}_{ir}] - E[\bar{d}_{ij}] E[\bar{d}_{ir}] \\ &= \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} d_{ir\ell_i} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ir\ell_i}, \\ & \quad j, r = 1, \cdots, n, j \neq r \qquad (18) \end{aligned}$$

Furthermore, the inequalities (13) can be expressed by the following forms.

$$\begin{aligned} & \sum_{j=1}^n \left(\sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} - (1 - \xi_i) \alpha_{ij}\right) x_j \\ & \leq z_i^0 - \xi_i (z_i^0 - z_i^1), i = 1, \cdots, k \qquad (19) \end{aligned}$$

Then, MOP-V1 can be transformed into MOP-V2.

[MOP-V2]

$$\min_{\boldsymbol{x} \in X} (Z_1^V(\boldsymbol{x}), \cdots, Z_k^V(\boldsymbol{x})) \qquad (20)$$

subject to

$$\sum_{j=1}^n \left(\sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} - (1 - \xi_i) \alpha_{ij}\right) x_j \leq z_i^0 - \xi_i (z_i^0 - z_i^1),$$

$$i = 1, \cdots, k.$$

From the fact that $\sum_{j=1}^n \alpha_{ij} x_j - z_i^1 + z_i^0 > 0$, $\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x} > 0$, due to the positive-semidefinite property of V_i, MOP-V2 can be equivalently transformed to MOP-V3

[MOP-V3]

$$\min_{\boldsymbol{x} \in X} (Z_1^{SD}(\boldsymbol{x}), \cdots, Z_k^{SD}(\boldsymbol{x})) \qquad (21)$$

subject to

$$\sum_{j=1}^n \left(\sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} - (1 - \xi_i) \alpha_{ij}\right) x_j \leq z_i^0 - \xi_i (z_i^0 - z_i^1),$$

$$i = 1, \cdots, k,$$

where

$$Z_i^{SD}(\boldsymbol{x}) \stackrel{\text{def}}{=} \frac{\sqrt{\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x}}}{\sum_{j=1}^n \alpha_{ij} x_j - z_i^1 + z_i^0}.$$

It should be noted here that $Z_i^E(\boldsymbol{x})$ and $Z_i^{SD}(\boldsymbol{x})$ are the statical values for the same random function $\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)$. When solving MOFRLP, it is natural for the decision maker to consider both $Z_i^E(\boldsymbol{x})$ and $Z_i^{SD}(\boldsymbol{x})$ for each objective function $\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)$ of MOSP simultaneously, rather than considering either of them. Moreover, it seems be difficult for the decision maker to express his/her preference for the standard deviations $Z_i^{SD}(\boldsymbol{x})$, $i = 1, \cdots, k$. From such a point of view, in the following sections, we propose the hybrid model for MOFRLP, in which E-model and V-model are incorporated simultaneously, and define an CV-Pareto optimality concept. In order to derive a satisfactory solution of the decision maker from among an CV-Pareto optimal solution set, the interactive algorithm is developed.

IV. CV-Model for MOFRLP

In this section, we consider the following hybrid model for MOFRLP, where both E-model and V-model are considered simultaneously.

[MOP-EV1]

$$\begin{aligned} \max_{\boldsymbol{x} \in X} \quad & \big(Z_1^E(\boldsymbol{x}), \cdots, Z_k^E(\boldsymbol{x}), \\ & -Z_1^{SD}(\boldsymbol{x}), \cdots, -Z_k^{SD}(\boldsymbol{x})\big) \qquad (22) \end{aligned}$$

In MOP-EV1, $Z_i^E(\boldsymbol{x})$ and $Z_i^{SD}(\boldsymbol{x})$ means the expected value and the standard deviation of the objective function $\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)$ in MOSP. It should be noted here that $Z_i^E(\boldsymbol{x})$ can be interpreted as an expected value of the satisfactory degree for $\Pi_{\tilde{\bar{\boldsymbol{c}}}_i \boldsymbol{x}}(\tilde{G}_i)$, but $Z_i^{SD}(\boldsymbol{x})$ does not mean the satisfactory degree itself. Here, instead of $Z_i^{SD}(\boldsymbol{x})$, let us consider the coefficient of variation defined as follows.

$$\begin{aligned} Z_i^{CV}(\boldsymbol{x}) & \stackrel{\text{def}}{=} \frac{Z_i^{SD}(\boldsymbol{x})}{Z_i^E(\boldsymbol{x})} \\ & = \frac{\sqrt{\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x}}}{\sum_{j=1}^n (\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}) x_j + z_i^0} \end{aligned} \qquad (23)$$

By using the coefficient of variation $Z_i^{CV}(\boldsymbol{x})$, we can transform MOP-EV1 into MOP-EV2.

[MOP-EV2]

$$\begin{aligned} \max_{\boldsymbol{x} \in X} \quad & \big(Z_1^E(\boldsymbol{x}), \cdots, Z_k^E(\boldsymbol{x}), \\ & -Z_1^{CV}(\boldsymbol{x}), \cdots, -Z_k^{CV}(\boldsymbol{x})\big) \qquad (24) \end{aligned}$$

In MOP-EV2, we assume that the decision maker has fuzzy goals for $Z_i^{CV}(\boldsymbol{x}), i = 1, \cdots, k$, and the corresponding linear membership functions are defined as $\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})), i =$

$1, \cdots, k$.

$$\mu_i^{CV}(s_i) = \begin{cases} 1 & s_i < q_i^1 \\ \dfrac{s_i - q_i^0}{q_i^1 - q_i^0} & q_i^1 \leq s_i \leq q_i^0 \\ 0 & s_i > q_i^0 \end{cases} \tag{25}$$

where q_i^0 represents the minimum value of an unacceptable level of the coefficient of variation $Z_i^{CV}(\boldsymbol{x})$, and q_i^1 represents the maximum value of a sufficiently satisfactory level of the coefficient of variation $Z_i^{CV}(\boldsymbol{x})$.

In order to elicit the linear membership function $\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))$ appropriately, we can compute a range of $\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))$ as follows.

$$CV_{i\min} \stackrel{\text{def}}{=} \min_{\boldsymbol{x}\in X} \frac{\sqrt{\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x}}}{\sum_{j=1}^n (\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}) x_j + z_i^0} \tag{26}$$

$$CV_{i\max} \stackrel{\text{def}}{=} \max_{\boldsymbol{x}\in X} \frac{\sqrt{\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x}}}{\sum_{j=1}^n (\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}) x_j + z_i^0} \tag{27}$$

The problem (26) for $CV_{i\min}$ is easily solved by applying a Dinkelbach-type algorithm [4], or a hybrid method of the bisection method and convex programming technique. Unfortunately, the problem (27) for $CV_{i\max}$ becomes a non-convex optimization problem. On the interval $[CV_{i\min}, CV_{i\max}]$, the decision maker sets his/her membership function $\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))$, which is strictly decreasing and continuous.

From the point of view that both $Z_i^E(\boldsymbol{x})$ and $\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))$ means the satisfactory degree for $\Pi_{\tilde{\boldsymbol{C}}_i \boldsymbol{x}}(\tilde{G}_i)$, we introduce the integrated membership function in which the both satisfactory levels $Z_i^E(\boldsymbol{x})$ and $\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))$ are incorporated simultaneously through the fuzzy decision [5], [14].

$$\mu_{D_i}(\boldsymbol{x}) \stackrel{\text{def}}{=} \min\{Z_i^E(\boldsymbol{x}), \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))\} \tag{28}$$

Then, MOP-EV2 can be transformed into the following multiobjective programming problem.

[MOP-EV3]

$$\max_{\boldsymbol{x}\in X} \quad (\mu_{D_1}(\boldsymbol{x}), \cdots, \mu_{D_k}(\boldsymbol{x})) \tag{29}$$

$\mu_{D_i}(\boldsymbol{x})$ can be interpreted as an overall satisfactory degree for the fuzzy goal $\tilde{G}_i$. For MOP-EV3, we introduce an CV-Pareto optimal solution concept defined as follows.

Definition 3: $\boldsymbol{x}^* \in X$ is an CV-Pareto optimal solution to MOP-EV3, if and only if there does not exist another $\boldsymbol{x} \in X$ such that $\mu_{D_i}(\boldsymbol{x}) \geq \mu_{D_i}(\boldsymbol{x}^*)$, $i = 1, \cdots, k$ with strict inequality holding for at least one i.

In order to generate a candidate of a satisfactory solution from among an CV-Pareto optimal solution set, the decision maker is asked to specify the reference membership values [14]. For the reference membership values $\hat{\boldsymbol{\mu}} = (\hat{\mu}_1, \cdots, \hat{\mu}_k)$, the corresponding CV-Pareto optimal solution is obtained by solving the following minmax problem.

[MINMAX($\hat{\boldsymbol{\mu}}$)]

$$\min_{\boldsymbol{x}\in X, \lambda\in\Lambda} \lambda \tag{30}$$

subject to

$$\hat{\mu}_i - Z_i^E(\boldsymbol{x}) \leq \lambda, i = 1, \cdots, k \tag{31}$$

$$\hat{\mu}_i - \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \leq \lambda, i = 1, \cdots, k \tag{32}$$

where

$$\Lambda \stackrel{\text{def}}{=} \left[\max_{i=1,\cdots,k} \hat{\mu}_i - 1, \max_{i=1,\cdots,k} \hat{\mu}_i\right] = [\lambda_{\min}, \lambda_{\max}] \tag{33}$$

From the definition of $Z_i^E(\boldsymbol{x})$ and $\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))$, the constraints (31) and (32) can be equivalently transformed into the following forms respectively.

$$\begin{aligned} &\sum_{j=1}^n (\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}) x_j + z_i^0 \\ &\geq \left(\sum_{j=1}^n \alpha_{ij} x_j - z_i^1 + z_i^0\right) \cdot (\hat{\mu}_i - \lambda), i = 1, \cdots, k \end{aligned} \tag{34}$$

$$\begin{aligned} &\left(\sum_{j=1}^n \left(\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}\right) x_j + z_i^0\right) \\ &\cdot \mu_i^{CV^{-1}}(\hat{\mu}_i - \lambda) \geq \sqrt{\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x}}, i = 1, \cdots, k \end{aligned} \tag{35}$$

The relationship between the optimal solution $(\boldsymbol{x}^*, \lambda^*)$ of MINMAX($\hat{\boldsymbol{\mu}}$) and CV-Pareto optimal solutions of MOP-EV3 can be characterized by the following theorems.

Theorem 1: If $\boldsymbol{x}^* \in X$, $\lambda^* \in \Lambda$ is an unique optimal solution of MINMAX($\hat{\boldsymbol{\mu}}$) then $\boldsymbol{x}^*$ is an CV-Pareto optimal solution of MOP-EV3.

(Proof)

Let us assume that $\boldsymbol{x}^* \in X$ is not an CV-Pareto optimal solution of MOP-EV3. Then, there exists $\boldsymbol{x} \in X$ such that $\mu_{D_i}(\boldsymbol{x}) \geq \mu_{D_i}(\boldsymbol{x}^*), i = 1, \cdots, k$, with strict inequality holding for at least one i. This implies that

$$\begin{aligned} & && \mu_{D_i}(\boldsymbol{x}) \geq \mu_{D_i}(\boldsymbol{x}^*) \\ &\Leftrightarrow && \hat{\mu}_i - \min\{Z_i^E(\boldsymbol{x}), \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))\} \\ & &\leq\ & \hat{\mu}_i - \min\{Z_i^E(\boldsymbol{x}^*), \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*))\} \\ &\Leftrightarrow && \max\{\hat{\mu}_i - Z_i^E(\boldsymbol{x}), \hat{\mu}_i - \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))\} \\ & &\leq\ & \max\{\hat{\mu}_i - Z_i^E(\boldsymbol{x}^*), \hat{\mu}_i - \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*))\} \\ & &\leq\ & \lambda^*, i = 1, \cdots, k. \end{aligned}$$

This contradicts the assumption that $\boldsymbol{x}^* \in X$, $\lambda^* \in \Lambda$ is an unique optimal solution of MINMAX($\hat{\mu}$). □

Theorem 2: If $\boldsymbol{x}^* \in X$ is an CV-Pareto optimal solution of MOP-EV3, then there exists a reference membership values $\hat{\boldsymbol{\mu}} = (\hat{\mu}_1, \cdots, \hat{\mu}_k)$ such that $\boldsymbol{x}^* \in X$, $\lambda^* = \hat{\mu}_i - \mu_{D_i}(\boldsymbol{x}^*), i = 1, \cdots, k$ is an optimal solution of MINMAX($\hat{\boldsymbol{\mu}}$)

(Proof)

Let us assume that $\boldsymbol{x}^* \in X$, $\lambda^* = \hat{\mu}_i - \mu_{D_i}(\boldsymbol{x}^*) = \max\{\hat{\mu}_i - Z_i^E(\boldsymbol{x}^*), \hat{\mu}_i - \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*))\}$, $i = 1, \cdots, k$, is not an optimal solution of MINMAX($\hat{\boldsymbol{\mu}}$). Then, there exists $\boldsymbol{x} \in X$ and $\lambda < \lambda^*$ such that

$$\begin{aligned} & && \begin{cases} \hat{\mu}_i - Z_i^E(\boldsymbol{x}) \leq \lambda < \lambda^* \\ \hat{\mu}_i - \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \leq \lambda < \lambda^* \end{cases} \\ &\Leftrightarrow && \hat{\mu}_i - \mu_{D_i}(\boldsymbol{x}) \leq \lambda < \lambda^* \\ &\Leftrightarrow && \hat{\mu}_i - \mu_{D_i}(\boldsymbol{x}) < \hat{\mu}_i - \mu_{D_i}(\boldsymbol{x}^*) \\ &\Leftrightarrow && \mu_{D_i}(\boldsymbol{x}) > \mu_{D_i}(\boldsymbol{x}^*) \end{aligned}$$

for all $i = 1, \cdots, k$. This contradicts the fact that $\boldsymbol{x}^* \in X$ is an CV-Pareto optimal solution of MOP-EV3. □

Unfortunately, since MINMAX($\hat{\boldsymbol{\mu}}$) is a nonlinear programming problem, it seems to be difficult to solve it directly. To overcome such difficulties, we consider the following function with respect to the constraints (34) and (35).

$$\begin{aligned} g_i(\boldsymbol{x}, \lambda) &\stackrel{\text{def}}{=} \sqrt{\boldsymbol{x}^T \boldsymbol{V}_i \boldsymbol{x}} - {\mu_i^{CV}}^{-1}(\hat{\mu}_i - \lambda) \\ &\quad \cdot \left(\sum_{j=1}^{n} \left(\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i} \right) x_j + z_i^0 \right), \\ &\quad i = 1, \cdots, k \end{aligned} \tag{36}$$

$$\begin{aligned} h_i(\boldsymbol{x}, \lambda) &\stackrel{\text{def}}{=} \left(\sum_{j=1}^{n} \alpha_{ij} x_j - z_i^1 + z_i^0 \right) \cdot (\hat{\mu}_i - \lambda) \\ &\quad - \sum_{j=1}^{n} (\alpha_{ij} - \sum_{\ell_i=1}^{L_i} p_{i\ell_i} d_{ij\ell_i}) x_j - z_i^0, \\ &\quad i = 1, \cdots, k \end{aligned} \tag{37}$$

It should be noted here that $g_i(\boldsymbol{x}, \lambda)$, $h_i(\boldsymbol{x}, \lambda)$, $i = 1, \cdots, k$ are convex with respect to $\boldsymbol{x} \in X$ for any fixed $\lambda \in \Lambda$. Let us define the following feasible set $X(\lambda)$ for some fixed $\lambda \in \Lambda$.

$$X(\lambda) \stackrel{\text{def}}{=} \{\boldsymbol{x} \in X \mid g_i(\boldsymbol{x}, \lambda) \leq 0, h_i(\boldsymbol{x}, \lambda) \leq 0, i = 1, \cdots, k\} \tag{38}$$

Then, it is clear that $X(\lambda)$ is a convex set and satisfies the following property.

Property 1: If $\lambda_1, \lambda_2 \in \Lambda, \lambda_1 \leq \lambda_2$, then it holds that $X(\lambda_1) \subset X(\lambda_2)$.

In the following, it is assumed that $X(\lambda_{\min}) = \phi$, $X(\lambda_{\max}) \neq \phi$. From Property 1, we can obtain the optimal solution $(\boldsymbol{x}^*, \lambda^*)$ of MINMAX($\hat{\boldsymbol{\mu}}$) using the following simple algorithm which is based on the bisection method and the convex programming technique.

[Algorithm 1]

Step 1: Set $\lambda_0 \leftarrow \lambda_{\min}$, $\lambda_1 \leftarrow \lambda_{\max}$, $\lambda \leftarrow (\lambda_0 + \lambda_1)/2$.

Step 2: Solve the convex programming problem for the fixed $\lambda \in \Lambda$,

$$\min_{\boldsymbol{x} \in X} h_j(\boldsymbol{x}, \lambda)$$

subject to

$$\begin{aligned} g_i(\boldsymbol{x}, \lambda) &\leq 0, i = 1, \cdots, k, \\ h_i(\boldsymbol{x}, \lambda) &\leq 0, i = 1, \cdots, k, \end{aligned}$$

where the index j is one of $\{1, 2, \cdots, k\}$, and denote the optimal solution as $\boldsymbol{x}(\lambda)$.

Step 3: If $|\lambda_0 - \lambda_1| < \delta$, go to Step 4, where δ is a sufficiently small positive number. If $g_i(\boldsymbol{x}(\lambda), \lambda) \leq 0$ and $h_i(\boldsymbol{x}(\lambda), \lambda) \leq 0$, for any $i = 1, \cdots, k$, set $\lambda_1 \leftarrow \lambda$, $\lambda \leftarrow (\lambda_0 + \lambda_1)/2$. Otherwise, set $\lambda_0 \leftarrow \lambda$, $\lambda \leftarrow (\lambda_0 + \lambda_1)/2$. And return to Step 2.

Step 4: Adopt $\boldsymbol{x}^* \leftarrow \boldsymbol{x}(\lambda)$, $\lambda^* \leftarrow \lambda$ as an optimal solution of MINMAX($\hat{\boldsymbol{\mu}}$).

V. An interactive algorithm

In Theorem 1, if the optimal solution $(\boldsymbol{x}^*, \lambda^*)$ of MINMAX($\hat{\boldsymbol{\mu}}$) is not unique, the CV-Pareto optimality can not be guaranteed. In order to guarantee the CV-Pareto optimality for $(\boldsymbol{x}^*, \lambda^*)$, we formulate the CV-Pareto optimality test problem. Before formulating such a test problem, without loss of generality, we assume that the following inequalities hold at the optimal solution $\boldsymbol{x}^* \in X, \lambda^* \in \Lambda$.

$$Z_i^E(\boldsymbol{x}^*) \leq \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)), i \in I_1 \tag{39}$$

$$Z_i^E(\boldsymbol{x}^*) > \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)), i \in I_2 \tag{40}$$

$$I_1 \cup I_2 = \{1, \cdots, k\}, I_1 \cap I_2 \neq \phi \tag{41}$$

Under the above conditions, we formulate the following CV-Pareto optimality test problem.

[CV-Pareto optimality test problem]

$$\max_{\boldsymbol{x} \in X, \epsilon_i \geq 0, i=1,\cdots,k} \sum_{i=1}^{k} \epsilon_i$$

subject to

$$\begin{aligned} &Z_i^E(\boldsymbol{x}) \geq Z_i^E(\boldsymbol{x}^*) + \epsilon_i, i \in I_1 \\ &\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \geq Z_i^E(\boldsymbol{x}^*) + \epsilon_i, i \in I_1 \\ &Z_i^E(\boldsymbol{x}) \geq \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)) + \epsilon_i, i \in I_2 \\ &\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \geq \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)) + \epsilon_i, \in I_2 \end{aligned}$$

The following theorem shows the relationships between the optimal solution of CV-Pareto optimality test problem and the CV-Pareto optimal solution for MOP-EV3.

Theorem 3: Let $\check{\boldsymbol{x}} \in X$, $\check{\epsilon}_i \geq 0$, $i = 1, \cdots, k$ be an optimal solution of the CV-Pareto optimality test problem for $(\boldsymbol{x}^*, \lambda^*)$. If $\sum_{i=1}^{k} \check{\epsilon}_i = 0$, then $\boldsymbol{x}^* \in X$ is an CV-Pareto optimal solution.

(Proof)

Assume that $\check{\epsilon}_i = 0$, $i = 1, \cdots, k$. If $\boldsymbol{x}^* \in X$ is not an CV-Pareto optimal solution, there exists some $\boldsymbol{x} \in X$ such that $\mu_{D_i}(\boldsymbol{x}) \geq \mu_{D_i}(\boldsymbol{x}^*)$,$i = 1, \cdots, k$, with strict inequality holding for at least one i. From the inequalities (39) and (40), this is equivalent to the following relations.

$$\begin{aligned} &\min\{Z_i^E(\boldsymbol{x}), \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))\} \\ \geq\ &\min\{Z_i^E(\boldsymbol{x}^*), \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*))\} \\ =\ &\begin{cases} Z_i^E(\boldsymbol{x}^*), & i \in I_1 \\ \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)), & i \in I_2 \end{cases} \end{aligned}$$

As a result, the following inequalities holds.

$$\begin{cases} Z_i^E(\boldsymbol{x}) \geq Z_i^E(\boldsymbol{x}^*), & i \in I_1 \\ \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \geq Z_i^E(\boldsymbol{x}^*), & i \in I_1 \\ Z_i^E(\boldsymbol{x}) \geq \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)), & i \in I_2 \\ \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x})) \geq \mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}^*)), & i \in I_2 \end{cases} \tag{42}$$

with strict inequality holding for at least one $i \in I_1 \cup I_2$. Hence, there must exist at least one i such that $\check{\epsilon}_i > 0$. This contradicts the assumption that $\check{\epsilon}_i = 0$, $i = 1, \cdots, k$. □

Now, following the above discussions, we can construct the interactive algorithm in order to derive a satisfactory solution from among an CV-Pareto optimal solution set.

[An interactive algorithm]

Step 1: The decision maker sets the membership function $\mu_{\tilde{G}_i}(y)$, $i = 1, \cdots, k$ for the fuzzy goals of the objective functions in MOFRLP.

Step 2: Considering the interval $CV_{i\min}, CV_{i\max}$, the decision maker sets the membership function $\mu_i^{CV}(Z_i^{CV}(\boldsymbol{x}))$, $i = 1, \cdots, k$.

TABLE I
PARAMETERS OF OBJECTIVE FUNCTIONS

	$\ell_1=1$	$\ell_1=2$	$\ell_1=3$	α_{1j},β_{1j}
$d_{11\ell_1}$	-2.5	-2.0	-1.5	0.4
$d_{12\ell_1}$	-3.5	-3.0	-2.5	0.5
$d_{13\ell_1}$	-2.25	-2.0	-1.75	0.4
p_1	0.25	0.4	0.35	
	$\ell_2=1$	$\ell_2=2$	$\ell_2=3$	α_{2j},β_{2j}
$d_{21\ell_2}$	-2.5	-2.0	-1.5	0.3
$d_{22\ell_2}$	-0.75	-0.5	-0.25	0.4
$d_{23\ell_2}$	-2.5	-2.25	-2.0	0.3
p_2	0.3	0.5	0.2	
	$\ell_3=1$	$\ell_3=2$	$\ell_3=3$	α_{3j},β_{3j}
$d_{31\ell_3}$	3.0	3.25	3.5	0.4
$d_{32\ell_3}$	2.5	2.75	3.0	0.5
$d_{33\ell_3}$	4.5	4.75	5.0	0.4
p_3	0.2	0.45	0.35	

TABLE II
PARAMETERS OF MEMBERSHIP FUNCTIONS

	z_i^0	z_i^1	q_i^0	q_i^1
$i=1$	-91.667	-126.25	5.779	4.244
$i=2$	-9.1666	-77.5	1.773	0.841
$i=3$	185	91.666	2.258	0.438

Step 3: Set the initial reference membership values as $\hat{\mu}_i=1,\ i=1,\cdots,k$.
Step 4: Solve MINMAX($\hat{\boldsymbol{\mu}}$) by applying Algorithm 1, and obtain the optimal solution $\boldsymbol{x}^*\in X,\lambda^*\in\Lambda$. In order to guarantee CV-Pareto optimality, solve the CV-Pareto optimality test problem for $\boldsymbol{x}^*\in X$.
Step 5: If the decision maker is satisfied with the current value of the CV-Pareto optimal solution $\boldsymbol{x}^*\in X$, then stop. Otherwise, the decision maker updates his/her reference membership values $\hat{\mu}_i,\ i=1,\cdots,k$ and return to Step 4.

VI. NUMERICAL EXAMPLE

In order to demonstrate the proposed method and the interactive processes, we consider the following three-objective linear programming problem with fuzzy random variable coefficients.
[MOFRLP]

$$\min_{\boldsymbol{x}\in X} z_1(\boldsymbol{x})=\tilde{\bar{c}}_{11}x_1+\tilde{\bar{c}}_{12}x_2+\tilde{\bar{c}}_{13}x_3$$
$$\min_{\boldsymbol{x}\in X} z_2(\boldsymbol{x})=\tilde{\bar{c}}_{21}x_1+\tilde{\bar{c}}_{22}x_2+\tilde{\bar{c}}_{23}x_3$$
$$\min_{\boldsymbol{x}\in X} z_3(\boldsymbol{x})=\tilde{\bar{c}}_{31}x_1+\tilde{\bar{c}}_{32}x_2+\tilde{\bar{c}}_{33}x_3$$

where

$$\begin{aligned} X &= \{(x_1,x_2,x_3)\in \mathrm{R}^3_+ \mid 3x_1+2x_2+x_3\le 85,\\ &\quad 2x_1+x_2+2x_3\le 115, 3x_1+4x_2+3x_3\le 155,\\ &\quad x_1+3x_2+2x_3\ge 110\}\end{aligned}$$

and it is assumed that a realization $\tilde{c}_{ij\ell_i}$ of a fuzzy random variable $\tilde{\bar{c}}_{ij}$ is an triangular-type fuzzy number whose membership function is defined as (2) where the parameters $d_{ij\ell_i}$, α_{ij}, β_{ij} are given in Table I. According to (17) and (18), the variance-covariance matrices $V_i, i=1,2,3$ are computed as follows.

$$V_1=\begin{pmatrix}0.1475 & 0.1475 & 0.07375\\ 0.1475 & 0.1475 & 0.07375\\ 0.07375 & 0.07375 & 0.036875\end{pmatrix}$$

$$V_2=\begin{pmatrix}0.1225 & 0.06125 & 0.06125\\ 0.06125 & 0.030625 & 0.030625\\ 0.06125 & 0.030625 & 0.030625\end{pmatrix}$$

$$V_3=\begin{pmatrix}0.032969 & 0.032969 & 0.032969\\ 0.032969 & 0.032969 & 0.032969\\ 0.032969 & 0.032969 & 0.032969\end{pmatrix}$$

Let us assume that the hypothetical decision maker sets the membership functions $\mu_{\tilde{G}_i}(\cdot)$, $\mu_i^{CV}(\cdot)$, $i=1,2,3$ as follows.

$$\begin{aligned}\mu_{\tilde{G}_i}(y) &= \frac{y-z_i^0}{z_i^1-z_i^0},\quad z_i^1\le y\le z_i^0, i=1,2,3\\ \mu_i^{CV}(s) &= \frac{s-q_i^0}{q_i^1-q_i^0},\quad q_i^1\le s\le q_i^0, i=1,2,3\end{aligned}$$

where the parameters z_i^0, z_i^1, q_i^0, q_i^1 are given in Table II. Then, after the decision maker specifies the reference membership values $\hat{\boldsymbol{\mu}}=(\hat{\mu}_1,\cdots,\hat{\mu}_k)$ in his/her subjective manner, the corresponding minmax problem is formulated as follows.
[MINMAX($\hat{\boldsymbol{\mu}}$)]

$$\min_{\boldsymbol{x}\in X,\lambda\in\Lambda}\lambda$$

subject to

$$\begin{aligned}&\sum_{j=1}^n(\alpha_{ij}-\sum_{\ell_i=1}^{L_i}p_{i\ell_i}d_{ij\ell_i})x_j+z_i^0\\ &\ge\left(\sum_{j=1}^n\alpha_{ij}x_j-z_i^1+z_i^0\right)\cdot(\hat{\mu}_i-\lambda), i=1,\cdots,k\\ &\left(\sum_{j=1}^n\left(\alpha_{ij}-\sum_{\ell_i=1}^{L_i}p_{i\ell_i}d_{ij\ell_i}\right)x_j+z_i^0\right)\\ &\cdot\mu_i^{CV^{-1}}(\hat{\mu}_i-\lambda)\ge\sqrt{\boldsymbol{x}^T\boldsymbol{V}_i\boldsymbol{x}}, i=1,\cdots,k\end{aligned}$$

At Step 3, set the initial reference membership values as $(\hat{\mu}_1,\hat{\mu}_2,\hat{\mu}_3)=(1,1,1)$. Solve MINMAX($\hat{\boldsymbol{\mu}}$) by applying Algorithm 1, and obtain the optimal solution $(\mu_{D_1}(\boldsymbol{x}^*),\mu_{D_2}(\boldsymbol{x}^*),\mu_{D_3}(\boldsymbol{x}^*))=(0.5831,0.5831,0.5831)$. Since the hypothetical decision maker is not satisfied with the current value, the decision maker updates his/her reference membership values as $(\hat{\mu}_1,\hat{\mu}_2,\hat{\mu}_3)=(1,0.9,1)$. The interactive processes under the hypothetical decision maker are summarized in Table III. Let us compare the proposal method based on CV-model with V-model (MOP-V1) proposed by Katagiri et al.[10]. According to V-model (MOP-V1), we set ξ_i in (13) heuristically as $(\xi_1,\xi_2,\xi_3)=(0.8,0.7,0.7)$.
[MOP-V1]

$$\min_{\boldsymbol{x}\in X}(V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}}(\tilde{G}_1)],V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_2\boldsymbol{x}}(\tilde{G}_2)],V[\Pi_{\tilde{\bar{\boldsymbol{c}}}_3\boldsymbol{x}}(\tilde{G}_3)])$$

subject to

$$E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_1\boldsymbol{x}}(\tilde{G}_1)]\ge 0.8$$
$$E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_2\boldsymbol{x}}(\tilde{G}_2)]\ge 0.7$$
$$E[\Pi_{\tilde{\bar{\boldsymbol{c}}}_3\boldsymbol{x}}(\tilde{G}_3)]\ge 0.7$$

TABLE III
INTERACTIVE PROCESSES

Iteration	1	2	3
$\hat{\mu}_1$	1	1	1
$\hat{\mu}_2$	1	0.9	0.9
$\hat{\mu}_3$	1	1	0.9
x_1^*	0.0014	0	0
x_2^*	29.029	31.005	29.522
x_3^*	12.054	10.325	12.303
$Z_1^E(\boldsymbol{x}^*)$	0.6883	0.7345	0.7261
$Z_2^E(\boldsymbol{x}^*)$	0.5831	0.5499	0.5943
$Z_3^E(\boldsymbol{x}^*)$	0.5831	0.6087	0.5617
$\mu_1^{CV}(Z_1^{CV}(\boldsymbol{x}^*))$	0.5831	0.6087	0.6617
$\mu_2^{CV}(Z_2^{CV}(\boldsymbol{x}^*))$	0.7645	0.6853	0.7491
$\mu_3^{CV}(Z_3^{CV}(\boldsymbol{x}^*))$	0.7754	0.7908	0.7416
$\mu_{D_1}(\boldsymbol{x}^*)$	0.5831	0.6087	0.6617
$\mu_{D_2}(\boldsymbol{x}^*)$	0.5831	0.5499	0.5943
$\mu_{D_3}(\boldsymbol{x}^*)$	0.5831	0.6087	0.5617

TABLE IV
COMPARISON BETWEEN CV-MODEL AND V-MODEL

Model	CV-model	V-model
$\hat{\mu}_1$	1	1
$\hat{\mu}_2$	1	1
$\hat{\mu}_3$	1	1
x_1^*	0.0014	3.0840
x_2^*	29.029	29.252
x_3^*	12.054	9.5795
$Z_1^E(\boldsymbol{x}^*)$	0.6883	0.8
$Z_2^E(\boldsymbol{x}^*)$	0.5831	0.7
$Z_3^E(\boldsymbol{x}^*)$	0.5831	0.7
$\mu_1^{CV}(Z_1^{CV}(\boldsymbol{x}^*))$	0.5831	0.3908
$\mu_2^{CV}(Z_2^{CV}(\boldsymbol{x}^*))$	0.7645	0.5670
$\mu_3^{CV}(Z_3^{CV}(\boldsymbol{x}^*))$	0.7754	0.7668
$\mu_{D_1}(\boldsymbol{x}^*)$	0.5831	0.3908
$\mu_{D_2}(\boldsymbol{x}^*)$	0.5831	0.5670
$\mu_{D_3}(\boldsymbol{x}^*)$	0.5831	0.7

The results are summarized in Table IV. It is shown that the proper balance between the membership functions $\mu_{D_i}(\boldsymbol{x}), i = 1, 2, 3$ is attained in the proposed method.

VII. CONCLUSION

In this paper, under the assumption that the decision maker intends to not only maximize the expected degrees of possibilities that the original objective functions attain the corresponding fuzzy goals, but also minimize coefficients of variation for such possibilities, an interactive decision making method for MOFRLP is proposed. In the proposed method, a satisfactory solution is obtained from among an CV-Pareto optimal solution set through the interaction with the decision maker.

REFERENCES

[1] J.R. Birge, F. Louveaux, *Introduction to Stochastic Programming*, Springer, London, 1997.

[2] A. Charnes, W.W. Cooper, Chance constrained programming, *Management Science*, 6 (1959) 73–79.

[3] G.B. Danzig, Linear programming under uncertainty, *Management Science*, 1 (1955) 197–206.

[4] W. Dinkelbach, On nonlinear fractional programming, *Management Science*, 13 (1967) 492–498.

[5] D. Dubois, H. Prade, *Fuzzy Sets and Systems*, Academic Press, New York, 1980.

[6] P. Kall, J. Mayer, *Stochastic Linear Programming Models, Theory, and Computation*, Springer, New York, 2005.

[7] H. Katagiri, M. Sakawa, Interactive multiobjective fuzzy random linear programming through the level set-based probability model, *Information Sciences*, 181 (2011) 1641–1650.

[8] H. Katagiri, M. Sakawa, H. Ishii, Multiobjective fuzzy random linear programming using E-model and possibility measure, *Joint 9-th IFSA World Congress and 20-th NAFIPS International Conference, Vancouver*, (2001) 2295–2300.

[9] H. Katagiri, M. Sakawa, K. Kato, I. Nishizaki, Interactive multiobjective fuzzy random linear programming: maximization of possibility and probability, *European Journal of Operational Research*, 188 (2008) 530–539.

[10] H. Katagiri, M. Sakawa, S. Osaki, An interactive satisficing method through the variance minimization model for fuzzy random linear programming problems, *Multi-Objective Programming and Goal-Programming: Theory and Applications (Advances in Soft Computing)*, Springer-Verlag, (2003) 171–176.

[11] H. Kwakernaak, Fuzzy random variable-1, *Information Sciences*, 15 (1978) 1–29.

[12] V.J. Lai, C.L. Hwang, *Fuzzy Mathematical Programming*, Springer, Berlin, 1992.

[13] M.K. Luhandjula, M.M. Gupta, On fuzzy stochastic optimization, *Fuzzy Sets and Systems*, 81 (1996) 47–55.

[14] M. Sakawa, *Fuzzy Sets and Interactive Multiobjective Optimization*, Plenum Press, New York, 1993.

[15] M. Sakawa, I. Nishizaki, H. Katagiri, *Fuzzy Stochastic Multiobjective Programming*, Springer, 2011.

[16] M. Sakawa, H. Yano, and I. Nishizaki, *Linear and Multiobjective Programming with Fuzzy Stochastic Extensions* , Springer, 2013.

[17] G.-Y. Wang, Z. Qiao, Linear programming with fuzzy random variable coefficients, *Fuzzy Sets and Systems*, 57 (1993) 295–311.

[18] D.J. White, *Optimality and Efficiency*, John Wiley and Sons, 1982.

[19] H. Yano, Interactive Decision Making for Fuzzy Random Multiobjective Linear Programming Problems with Variance-Covariance Matrices Through Probability Maximization, *Proceedings of The 6th International Conference on Soft Computing and Intelligent Systems and 13th International Symposium on Advanced Intelligent Systems* , (2012) 965–970.

[20] H. Yano, Fuzzy decision making for fuzzy random multiobjective linear programming problems with variance covariance matrices, *Information Sciences*, 272 (2014), 111–125.

[21] H. Yano, K. Matsui, Fuzzy approaches for multiobjective fuzzy random linear programming problems through a probability maximization model, *Lecture Notes in Engineering and Computer Science: Proceedings of The International MultiConference of Engineers and Computer Scientists 2011*, (2011), 1349–1354.

[22] H. Yano, K. Matsui and M. Furuhashi, Interactive Decision Making for Multiobjective Fuzzy Random Linear Programming Problems Using Expectations and Coefficients of Variation, Lecture Notes in Engineering and Computer Science: Proceedings of The International MultiConference of Engineers and Computer Scientists 2014, IMECS 2014, 12-14 March, 2014, Hong Kong, 1251–1256.

[23] H. Yano, M. Sakawa, Interactive multiobjective fuzzy random linear programming through fractile criteria, *Advances in Fuzzy Systems*, 2012 (2012) 1–9.

[24] H. Yano, M. Sakawa, Interactive fuzzy programming for multiobjective fuzzy random linear programming problems through possibility-based probability maximization, *Operational Research : An International Journal*, 2013/07 (2013) 1–19.

[25] H.-J. Zimmermann, *Fuzzy Sets, Decision-Making and Expert Systems*, Kluwer Academic Publishers, Boston, 1987.