

# Stock Volatility Modelling with Augmented GARCH Model with Jumps

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**Abstract**—Knowing the characteristics of news in numerical indices one can use them in mathematical and statistical models and automated trading systems. Currently, the tools of the news analytics have been increasingly used by traders in the U.S. and Europe. The interest in news analytics is related to the ability to predict changes of prices, volatility and trading volume on the stock market.

The emphasis of the paper is on assessing the added value of using news analytics data in improving the explanatory power of the GARCH-Jump model. Based on empirical evidences for some of FTSE100 companies, the paper examines two GARCH models with jumps. First we consider the well-known GARCH model with autoregressive conditional jump intensity proposed in [1]. Then we introduce the GARCH-Jumps model augmented with news intensity and obtain some empirical results. The main assumption of the model is that jump intensity might change over time and that jump intensity depends linearly on the number of news (the news intensity). The comparison of the values of log likelihood supports the hypothesis of impact of news on the jump intensity of volatility.

**Index Terms**—stock volatility modelling, GARCH models, news analytics.

## I. INTRODUCTION

Empirical studies based on the log return time series data of some stocks showed that serial dependence is present in the data; volatility changes over time; distribution of the data is heavy-tailed, asymmetric and therefore not Gaussian. These facts show that a random walk with Gaussian increments is not a very realistic model for financial time series. The ARCH (Autoregressive Conditionally Heteroscedastic) model was introduced by Engle in 1982 [2]. In the model it is supposed that the conditional variance (squared volatility) is not constant over time and shows autoregressive structure. This model is a convenient way of modeling time-dependent conditional variance. Some years later, Bollerslev [3] generalized this model as the GARCH model (Generalized Autoregressive Conditional Heteroscedasticity). A distinctive feature of the modern financial series is the presence of jump dynamics of asset prices. Some of the models describing this behavior is GARCH model with jumps was proposed in [4], [1].

Recent studies on the volatility of stock returns have been dominated by time series models of conditional heteroscedasticity and have found strong support for ARCH-GARCH-type effects. However, ARCH-GARCH-type models do not provide a theoretical explanation of volatility or what, if any, the exact contributions of information flows are in the volatility-generating process.

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Different measures of information arrivals were employed in variety of empirical studies in order to test the impact of the rate of information on the market volatility:

- macroeconomic news, in the paper [5];
- the number of daily newspaper headlines and earnings announcements, in the paper [6];
- the number of specific stock market announcements, in the paper [7].

In the papers [8], [9] volatility of log returns depends on the intensity of news flow on market directly. It is worth to be mentioned the works [10] and [11]. In the paper of [10] firm-specific announcements were used as a proxy for information flows. It was shown that there exists a positive and significant impact of the arrival rate of the selected news variable on the conditional variance of stock returns on the Australian Stock Exchange in a GARCH framework. They split all their press releases into different categories according to their subject. In the second of the papers the author examines impact of news releases on *index* volatility, while in our work we analyze the impact on *stock* volatility following study of [10].

In the papers [12] and [13] authors analyze the impact of extraneous sources of information (viz. news and trade volume) on stock volatility by considering some augmented Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models. Following the study of [14], it was supposed that trading volume can be considered as a proportional proxy for information arrivals to the market. Also it was considered the daily number of press releases on a stock (news intensity) as an alternative explanatory variable in the basic equation of GARCH model. The papers [12] and [13] restrict the choice by some of the FTSE100 companies, while [10] considered some French companies. It was shown that the GARCH(1,1) model augmented with volume does remove GARCH and ARCH effects for the most of the companies, while the GARCH(1,1) model augmented with news intensity has difficulties in removing the impact of log return on volatility.

Based on empirical evidences for some of FTSE100 companies, this paper examines two GARCH models with jumps to evaluate the impact of news flow intensity on stock volatility. First it will be considered the well-known GARCH model with jumps proposed in [1]. Then we will introduce the GARCH-Jumps model augmented with news intensity and obtain some empirical results. The main assumption of the model is that jump intensity might change over time and that jump intensity depends linearly on the number of news. It is not clear whether news adds any value to a jump-GARCH model. However, the comparison of the values of log likelihood shows that the GARCH-Jumps model augmented with news intensity performs slightly better than "pure" GARCH or the GARCH model with Jumps. We

restrict our choice by some of the FTSE100 companies. Our emphasis is on assessing the added value of using news intensity in improving the explanatory power of the GARCH–Jump model.

The paper extends the ideas of the work [15] in the three directions:

- the present work examines the GARCH–Jump models with autoregressive conditional jump intensity proposed in [1], while the paper [15] uses the GARCH–Jump model with constant jump intensity proposed in [4];
- the empirical results of the work [15] presented for 10 of FTSE100 companies; in this paper we will present results for 12 companies;
- in this paper we extend the time period of our empirical analysis’s data from 3 to 6 years.

## II. MODELS DESCRIPTION

Let  $X_t$  be the log return of a particular stock or the market portfolio from time  $t - 1$  to time  $t$ . Let  $I_{t-1}$  denote the past information set containing the realized values of all relevant variables up to time  $t - 1$ . Suppose investors know the information  $I_{t-1}$  when they make their investment decision at time  $t - 1$ . Then the relevant expected return  $\mu_t$  to the investors is the conditional expected value of  $X_t$ , given  $I_{t-1}$ , i.e.

$$\mu_t = E(X_t|I_{t-1}).$$

The relevant expected volatility  $\sigma_t^2$  to the investors is conditional variance of  $X_t$ , given  $I_{t-1}$ , i.e.

$$\sigma_t^2 = Var(X_t|I_{t-1}).$$

Then

$$\epsilon_t = X_t - \mu_t$$

is the unexpected return at time  $t$ .

### A. GARCH model

We recall [3] that a process  $(\epsilon_t)$  is said to be the generalized autoregressive conditionally heteroscedastic or GARCH(1,1) process if  $\epsilon_t = \sigma_t u_t, t \in \mathbb{Z}$ , where  $(\sigma_t)$  is a nonnegative process such that

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \quad (1)$$

and  $(u_t)$  is a sequence of i.i.d. random variables such that  $u_t \sim N(0, 1)$ .

In the model,  $\alpha$  reflects the influence of random deviations in the previous period on  $\sigma_t$ , whereas  $\beta$  measures the part of the realized variance in the previous period that is carried over into the current period. The sizes of the parameters  $\alpha$  and  $\beta$  determine the short-run dynamics of the resulting volatility time series, i.e. the sum  $\alpha + \beta$  of these parameters reflects the degree of persistence. Large ARCH error coefficients  $\alpha$  mean that volatility reacts intensely to market movements, while large GARCH lag coefficients  $\beta$  indicate that shocks to volatility persist over time.

### B. GARCH–Jump model with autoregressive conditional jump intensity

For the first time the GARCH–Jumps model was proposed and studied in [4]. This paper proposes a model of conditional variance of returns implied by the impact of different type of news. The development of GARCH–Jumps model of [4] can be found in the papers [16] and [1], where it is assumed that the conditional jump intensity, i.e. the expected number of jumps occurring between time  $t - 1$  and  $t$  conditional on information  $I_{t-1}$ , is autoregressive and related both to the last period’s conditional jump intensity and to an intensity residual.

In GARCH–Jumps model it is supposed that news process have two separate components (normal and unusual news), which cause two types of innovation (smooth and jump-like innovations):

$$\epsilon_t = \epsilon_{1,t} + \epsilon_{2,t}. \quad (2)$$

These two news innovations have a different impact on return volatility. It is assumed that the first component  $\epsilon_{1,t}$  reflects the impact of unobservable normal news innovations, while the second one  $\epsilon_{2,t}$  is caused by unusual news events.

The first term in (2) reflects the impact of normal news to volatility:

$$\epsilon_{1,t} = \sigma_t u_t, t \in \mathbb{Z}, \quad (3)$$

where  $(u_n)$  be a sequence of i.i.d. random variables such that  $u_t \sim N(0, 1)$ ,  $(\sigma_t)$  is a nonnegative GARCH(1,1) process such that

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (4)$$

and  $\alpha_0, \alpha_1, \beta_1 > 0$ . Note that  $\mathbb{E}(\epsilon_{1,t}|I_{t-1}) = 0$ .

The second term in (2) is a jump innovation with  $\mathbb{E}(\epsilon_{2,t}|I_{t-1}) = 0$ . The component  $\epsilon_{2,t}$  is a result of unexpected events and is responsible for jumps in volatility.

The distribution of jumps is assumed to be Poisson distribution. Let  $\lambda_t$  be intensity parameter of Poisson distribution. Denote  $n_t$  a number of jumps occurring between time  $t - 1$  and  $t$ .

The model supposes that the intensity parameter  $\lambda_t$  conditionally varies over time. It is assumed that the conditional jump intensity  $\lambda_t = \mathbb{E}(n_t|I_{t-1})$ , i.e. the expected number of jumps occurring between time  $t - 1$  and  $t$  conditional on information  $I_{t-1}$ , has dynamics

$$\lambda_t = a + b\lambda_{t-1} + c\zeta_{t-1}. \quad (5)$$

The process (5) is called an autoregressive conditional jump intensity and was proposed in the paper [16]. The model based on the assumption that the conditional jump intensity is autoregressive and related both to the last period’s conditional jump intensity and to an intensity residual  $\zeta_{t-1}$ . The intensity residual  $\zeta_{t-1}$  is defined as

$$\begin{aligned} \zeta_{t-1} &= \mathbb{E}(n_{t-1}|I_{t-1}) - \lambda_{t-1} = \\ &= \sum_{j=0}^{\infty} jP(n_{t-1} = j|I_{t-1}) - \lambda_{t-1}. \end{aligned}$$

Here  $\mathbb{E}(n_{t-1}|I_{t-1})$  is the expected number of jumps occurring from  $t - 2$  to  $t - 1$ , and  $\lambda_{t-1}$  is the conditional expectation of numbers of jumps  $n_{t-1}$  given the information  $I_{t-2}$  available at the moment  $t - 2$ . Thus

$$\zeta_{t-1} = \mathbb{E}(n_{t-1}|I_{t-1}) - \mathbb{E}(n_{t-1}|I_{t-2})$$

i.e.  $\zeta_{t-1}$  represents the change in the econometrician's conditional forecast of  $n_{t-1}$  as the information set is updated from  $t-2$  to  $t-1$ . It is easy to see that  $\mathbb{E}(\zeta_t|I_{t-1}) = 0$ , i.e.  $\zeta_t$  is a martingale difference sequence with respect to  $I_{t-1}$ , and therefore  $\mathbb{E}(\zeta_t) = 0$ ,  $\text{Cov}(\zeta_t, \zeta_{t-i}) = 0$  for all  $i > 0$ .

Denote  $Y_{t,k}$  the size of  $k$ -th jump that occur from time  $t-1$  to  $t$ ,  $1 \leq k \leq n_t$ . The model supposes that the jump size  $Y_{t,k}$  is realization of normal distributed random:

$$Y_{t,k} \sim \mathcal{N}(\theta, \delta^2).$$

Then the cumulative jump size  $J_t$  from  $t-1$  to  $t$  is equal to the sum of all jumps occurring from time  $t-1$  to  $t$ :

$$J_t = \sum_{k=1}^{n_t} Y_{t,k}.$$

The jump innovation  $\epsilon_{2,t}$  defined by

$$\epsilon_{2,t} = J_t - \mathbb{E}(J_t|I_{t-1}).$$

It follows from

$$\mathbb{E}(J_t|I_{t-1}) = \theta\lambda_t$$

that

$$\epsilon_{2,t} = \sum_{k=1}^{n_t} Y_{t,k} - \theta\lambda_t.$$

Therefore we have

$$\mathbb{E}(\epsilon_{2,t}|I_{t-1}) = 0.$$

GARCH-Jump model with constant jump intensity studied in [4] supposes that the intensity parameter  $\lambda$  is constant over time.

### C. Augmented GARCH-Jump model with constant jump intensity

Many investment companies in the U.S. and Europe have been using news analytics to improve the quality of its business [17]. Interest in news analytics is related to the ability to predict changes of prices, volatility and trading volume on the stock market [18]. News analytics uses some methods and technics of data mining [19] and relies on methods of computer science, artificial intelligence (including algorithms for natural language processing), financial engineering, mathematical statistics and mathematical modeling. News analytics software signalize traders about the most important events or send their output data directly to automated trading algorithms, which take into account this signals automatically during the trade.

Unlike [4] we consider the model (2), (3), (4), (??), where  $N_t$  is a Poisson random variable with conditional jump intensity

$$\lambda_t = \lambda + \rho n_{t-1}, \tag{6}$$

where  $n_{t-1}$  is the number of news from  $t-2$  to  $t-1$  respectively. Therefore we directly take into account the qualitative data of news intensity (source: RavenPack News Scores).

### D. Augmented GARCH-Jump model with autoregressive conditional jump intensity

We are going to analyze the impact of news process intensity on stock volatility by extending GARCH-Jumps model proposed and studied in [1]. The main assumption of the model is that jump intensity might change over time and that jump intensity depends linearly on the news intensity (the number of company news per day).

Following [1] we suppose that news process have two separate components: normal and unusual news,

$$\epsilon_t = \epsilon_{1,t} + \epsilon_{2,t}. \tag{7}$$

The first term in (7) reflects the impact of normal news to volatility and follows the standard GARCH process (3), (4).

The second term in (7) reflects the result of unexpected events and describe jumps in volatility:

$$\epsilon_{2,t} = \sum_{k=1}^{N_t} Y_{t,k} - \theta\lambda_t, \tag{8}$$

where  $Y_{t,k} \sim \mathcal{N}(\theta, \delta^2)$ ,  $N_t$  is a Poisson random variable with conditional jump intensity

$$\lambda_t = a + b\lambda_{t-1} + c\zeta_{t-1} + \rho n_{t-1}, \tag{9}$$

where  $\zeta_{t-1} = \mathbb{E}(N_{t-1}|I_{t-1}) - \theta\lambda_{t-1}$ , and  $n_{t-1}$  is the number of news from  $t-2$  to  $t-1$  respectively. Therefore we directly take into account the qualitative data of news intensity (source: RavenPack News Scores).

### E. Maximum Likelihood Estimation of GARCH Model with Jumps

Note that GARCH-Jump model can be calibrated either with generalized method of moments or with quasi-maximum likelihood approach. We have chosen to apply the latter approach here. The subsection describes quasi-maximum likelihood estimation (QML) of GARCH model with Jumps. The vector of model parameters is

$$\Theta = (\alpha_0, \alpha_1, \beta_1, \delta, \theta, a, b, c)^T.$$

We will assume that  $\theta$  belongs to the set

$$S := \{(\alpha_0, \alpha_1, \beta_1, \delta, \theta, a, b, c)^T : \alpha_0 \geq 0, \alpha_1 > 0, \beta_1 > 0\}.$$

Denote

$$\Theta^* = (\alpha_0^*, \alpha_1^*, \beta_1^*, \delta^*, \theta^*, a^*, b^*, c^*)^T$$

the vector of the true values of parameters. The aim is to find  $\Theta^*$  that maximize a QML function given an observation sequence

$$\epsilon_0, \dots, \epsilon_n$$

of length  $n$ .

Define the sequence  $(\tilde{\sigma}_1, \dots, \tilde{\sigma}_n)$  by recursion:

$$\tilde{\sigma}_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \beta_1\tilde{\sigma}_{t-1}^2.$$

If we assume that the likelihood function is Gaussian, then the log-likelihood function can be written as (see e.g. [16]):

$$F_n(\Theta) := \sum_{t=1}^n \log f(\epsilon_t|I_{t-1}, \Theta),$$

where

$$f(\epsilon_t|I_{t-1}, \Theta) = \sum_{j=0}^{\infty} \frac{\exp(-\tilde{\lambda}_t)\tilde{\lambda}_t^j}{j!} f(\epsilon_t|n_t = j, I_{t-1}, \Theta) \tag{10}$$

and

$$f(\epsilon_t|n_t = j, I_{t-1}, \Theta) = \frac{1}{\sqrt{2\pi(\tilde{\sigma}_t^2 + j\delta^2)}} \exp\left(-\frac{(\epsilon_t + \theta\lambda_t - \theta j)^2}{2(\tilde{\sigma}_t^2 + j\delta^2)}\right). \tag{11}$$

The sequence of  $\tilde{\lambda}_t$  is defined by recursion:

$$\tilde{\lambda}_t = a + b\tilde{\lambda}_{t-1} + c\zeta_{t-1},$$

where

$$\zeta_{t-1} = \mathbb{E}(n_{t-1}|I_{t-1}) - \tilde{\lambda}_{t-1},$$

and

$$\begin{aligned} \mathbb{E}(n_{t-1}|I_{t-1}) &= \sum_{j=0}^{\infty} jP(n_{t-1} = j|I_{t-1}) = \\ &= \sum_{j=0}^{\infty} j \frac{f(\epsilon_t|n_{t-1} = j, I_{t-2}, \Theta)P(n_{t-1} = j|I_{t-2})}{f(\epsilon_t|I_{t-2}, \Theta)} = \\ &= \frac{\sum_{j=1}^{\infty} \frac{\exp(-\tilde{\lambda}_{t-1})\tilde{\lambda}_{t-1}^j}{j!\sqrt{2\pi(\tilde{\sigma}_{t-1}^2 + j\delta^2)}} \exp\left(-\frac{(\epsilon_{t-1} + \theta\lambda_{t-1} - \theta j)^2}{2(\tilde{\sigma}_{t-1}^2 + j\delta^2)}\right)}{f(\epsilon_{t-1}|I_{t-2}, \Theta)} \end{aligned} \tag{12}$$

The maximum likelihood estimator of  $\Theta$  is defined by

$$\Theta^* = \arg \max_{\Theta \in S} F_n(\Theta).$$

Since the densities (11) has an infinite sum, it is impossible to use them for parameters' estimation. There are two ways of using equation (11):

- taking a finite Taylor expansions of (11);
- truncation of the sum (11), i.e. limitation of the number of terms in the sum.

The problem of calibration of GARCH–Jumps models is difficult due to its non convexity and noisiness. We have use different solvers for global optimization in MatLab.

### III. INTRODUCTION TO NEWS ANALYTICS

This section is a short review of the tools, methods and providers of news analytics. It also presents preliminary analysis of news analytics data.

News analytics can be described as a measurement of the following quantitative and qualitative characteristics of news:

- 1) **The nature of news** (it determines the impact of news (positive or negative), i.e. how news affects stock prices change; it is believed that positive news about the company leads to a growth in the stock prices of its shares, and negative, on the contrary, can leads to decreasing);
- 2) **The impact of news** (it is characterized by the influence of news on the scale of the changes caused by the news);
- 3) **The relevance** (describes how the events, described in a news report, are connected with the trader's interest security);
- 4) **The novelty** (shows how much news is informative, usually it is inversely correlated with the number of

references to events that are written in this news report, with other news).

News analysis is a relatively new tool designed to improve the trading strategies of investors. It is closely connected with the theory of behavioral finance and in some sense, is contrary to the classical economic theory.

Indeed, the famous "efficient markets hypothesis" states [20] that any available information is already reflected in share prices. This condition makes it impossible to attempt to outperform the market in a long period of time through the use of information available on the market. On the other hand, in the modern world, the intensity level of various news agencies is so high (for example, Thomson Reuters has more than 4000 messages per day) that the trader is unable on its own to handle this information flow. Events that are potentially change the situation on the stock exchange, may be lost or omitted in a huge stream of news. In this context, it is unlikely that at any one time all traders will be equally informed of all events affecting the price of certain stocks. That is why the news analytics is an effective tool to gain advantage over other market participants.

Knowing the characteristics of news in numerical indices one can use them in mathematical and statistical models and automated trading systems. Currently, the tools of the news analytics have been increasingly used by traders in the U.S. and Europe.

The process of news analysis in information systems is automated and usually includes the following steps:

- 1) collecting news from different sources;
- 2) preliminary analysis of news;
- 3) analysis of news-related expectations (sentiments), taking into account the current market situation;
- 4) designing and using of quantitative models.

The process of news analytics is described in more details in the following sections.

It worth be noted that managers of investment funds rarely use tools of news analytics, since they usually create investment portfolios for a long period of time, and in this case portfolio management does not suggest a frequent resale of securities.

#### A. Data Sources

News data can be obtained from various sources:

- **News sources of news agencies.** Until recently, the news had been spread by printed sources, radio, television and it was quite difficult to obtain an overall picture of the news flow. The Internet has changed the process of news analysis; the using of tagging and indexing has made possible their automatic processing.
- **Pre-news** is a raw information material which is used in the preparation of news by reporters. It can be obtained from different primary sources, for example, SEC reports, court documents, reports of various government agencies, business resources, company reports, announcements, industrial, and macroeconomic statistics.
- **Social media** (blogs, social networks, etc.). The quality of news from this type of sources can be vary highly, and this information is often useless. However, you can

keep track (evaluate) the mood of a large number of these messages and apply results in trade strategies.

In addition, the financial news can be classified in terms of their expectations. Expected news come out at a scheduled time and often their contents can be predicted on the basis of pre-news. They have a structured format and generally include numeric data, which is convenient for automated analysis (e.g., usually all companies publish annual or quarterly financial reports in the same time). Macroeconomic reports have a strong influence on liquid markets (foreign exchange, futures, government bonds) and are widely used in the automatic trading. Speed and accuracy of processing of such information are important technological requirements. Reports of incomes and losses affect directly the change in stock prices and are widely used in trading strategies.

The main difficulties of the processing of financial information are associated with unexpected news, since the time of their appearance is unknown and, often they have a unstructured text format and do not contain numeric data. They are difficult to process quickly and efficiently, but they may contain information about the causes and consequences of the event. To analyze unexpected news one can use the artificial intelligence systems based on methods of natural language processing.

News analytics evaluates the relevance, nature, novelty and the importance of news. The results of processing of news information are used to create signals for investors and traders. These signals can be combined with forecasts from other primary or processed sources.

### B. Providers of News Analytics

In the world there are more than 50 providers of economic news. Bloomberg, Dow Jones and Thomson Reuters are the three largest of them. About 200 agencies are involved in providing of financial analytics.

The most well-known providers of news analytics and data are:

- **RavenPack** (<http://www.ravenpack.com/>) is one of the leading providers of real-time news analysis services. The company specializes in linguistic analysis of large volumes of news in real time from news providers. RavenPack News Scores measures the news sentiment and news flow of the global equity market based on all major investable equity securities. News scores include analytics on more than 27,000 companies in 83 countries and covers over 98% of the investable global market. All relevant news items about companies are classified and quantified according to their sentiment, relevance, topic, novelty, and market impact; the result is a data product that can be segmented into many distinct benchmarks and used in various applications. RavenPack is working with news feeds from the company Dow Jones.
- **Media Sentiment** ([www.mediasentiment.com/](http://www.mediasentiment.com/)) has a resource library of nearly 2,000,000 articles and it regularly searches and analyzes output from 6,000+ sources in near-real time to bring investors updated news media sentiment about publicly traded companies, both quickly and effortlessly.
- **Thomson Reuters News Analytics** (<http://thomsonreuters.com>) automatically analyzes

news providing improved buy/hold/sell signals within milliseconds. The system can scan and analyze stories on thousands of companies in real-time and feed the results into your quantitative strategies. With its ability to track news sentiment over time, Thomson Reuters News Analytics provides a more comprehensive understanding of a company's news coverage, helping to guide trading and investment decisions. It delivers unparalleled insight into a company's market reputation, giving money managers a unique advantage. *Reuters NewsScope* and *Sentiment Analysis* are new software products, which provide financial news (interest rates, consumer price indices, etc.). These programs are designed for use in automated trading.

### C. Preliminary Analysis of News Analytics Data

Our sample covers a period ranging from January 4, 2005 to January 28, 2011 (i.e. approx 1500 trading days). Our sample is composed of the 12 UK stocks that were part of the FTSE100 index in the beginning of 2005 and which survived through the period of 6 years.

For each news wire, Raven Pack generate the following fields for sources of news analytics data: time stamp, company name, company id, relevance of the news, event category, event sentiment, novelty of the news, novelty id, composite sentiment score of the news, word/phrase level score, projections by company, editorials & commentary, reports corp actions, news impact projection, story ID. Company, relevance score, composite sentiment score are the main fields of interest. One piece of news can of course concern several companies, industries and subjects. To avoid any redundancy and duplicate announcements that do not bring any additional information value, some researchers restrict the sample to news released with high relevance score (more or equal to 90). Some of researchers also do not eliminate all news releases with the same headlines and lead paragraphs, since it is supposed that the number of the same news published by different news agencies reflects the importance of the news.

For example, there was more than 20000 financial HSBC news releases with relevance  $\geq 90$  over the whole sample period. Figure 1 presents the histogram of news intensity for HSBC Holdings (January 4, 2005 – January 28, 2011).

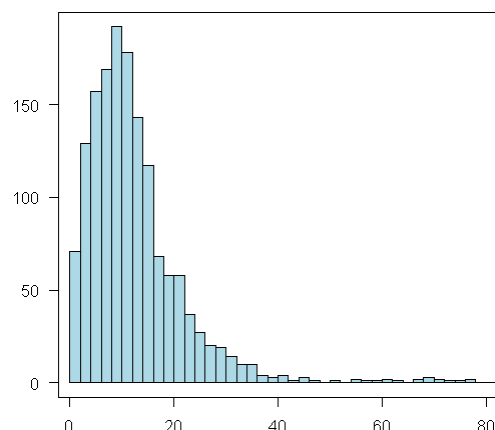


Fig. 1. The histogram of news intensity for HSBC Holdings

There is no clear trend of the total daily number of news wires for FTSE100 companies. It could indicate that the news time-series is stationary and reduce the risk of dummy results due to a possible simultaneous increase over time of the stock volatility. Some periods have rate of news intensity below the average (e.g. holidays, Christmas time). On the other hand, one can witness the increase of the rate at the periods of the quarterly reports and releases of the intermediate figures and earnings of companies.

There is a clear presence of weekly seasonality in the data (the average number of a company’s news announcements released during the week-end is much lower than the one of the other weekdays). The same picture is held for all FTSE100 companies indeed. Since that we exclude all weekend news from our analysis.

In the world there are more than 50 providers of economic news. Bloomberg, Dow Jones and Thomson Reuters are the three largest of them. About 200 agencies are involved in providing of financial analytics. In our research we use the Raven Pack data, one of the most well-known providers of news analytics data.

For every new instance a company is reported in the news, RavenPack produces a company level record. Each record contains 16 fields including a time stamp, company identifiers, scores for relevance, novelty and sentiment, and a unique identifier for each news story analyzed. In the historical data files, each row in the file represents a company-level record. Empirical properties of news analytics data for 12 companies can be found in Table I.

TABLE I  
EMPIRICAL PROPERTIES OF DAILY NEWS INTENSITY (THE NUMBER OF NEWS PER STOCK) IN THE SAMPLE

Company	mean	min	max	S	K
AstraZeneca	4.95	0	86	4.63	30.72
Barclays	8.45	0	108	4.08	26.46
BP	14.59	0	384	6.10	66.59
British Sky Broadcasting	2.46	0	81	5.74	43.26
HSBC Holdings	13.44	0	155	4.67	38.39
Int Consolidated Airlines	5.19	0	73	3.33	16.71
Johnson Matthey	4.02	0	54	7.31	69.74
London Stock Exch Group	3.48	0	79	4.58	33.59
National Grid	2.40	0	52	5.28	45.12
RBS Group	6.31	0	111	4.80	36.34
Shire plc	3.09	0	223	11.59	220.96
Tesco	2.77	0	76	5.61	40.50

We restrict the sample to news released with high relevance score (more or equal to 90). We do not eliminate all news releases with the same headlines and lead paragraphs, since we suppose that the number of the same news published by different news agencies reflects the importance of the news.

#### IV. EMPIRICAL RESULTS

Our sample covers a period ranging from January 4, 2005 to January 28, 2011 (i.e. approximately 1500 trading days). Our sample is composed of the 12 UK stocks that were part of the FTSE100 index in the beginning of 2005 and which survived through the period of 6 years (see Table II). For

TABLE III  
MLE OF THE GARCH(1,1) MODEL

Company	$\alpha$	$\beta$	$\alpha + \beta$	$LLF_1$
AstraZeneca	0.12	0.81	0.93	4451.29
Barclays	0.21	0.79	0.99	3867.86
BP	0.13	0.83	0.96	4564.14
British Sky Broadcasting	0.17	0.83	0.99	4358.39
HSBC Holdings	0.12	0.88	0.99	4883.65
Int Consolidated Airlines	0.04	0.96	0.99	3835.74
Johnson Matthey	0.06	0.92	0.98	4256.86
London Stock Exch Group	0.11	0.89	0.99	3712.82
National Grid	0.20	0.71	0.92	4551.73
RBS Group	0.49	0.51	0.99	3607.11
Shire	0.15	0.00	0.15	3332.29
Tesco	0.69	0.31	0.99	4302.32

our analysis we chose the companies with the high level of news intensity.

Daily stock closing prices (the last daily transaction price of the security), as well as daily transactions volume (number of shares traded during the day) are obtained from Yahoo Finance database. Table II presents

- the list of stocks,
- the Kiefer-Salmon skewness test statistic (S)
- the Kiefer-Salmon kurtosis statistic (K)
- p-value of the Shapiro-Wilk statistic (marginal significance level)
- the Box-Ljung  $Q$ -statistic, constructed for maximum lag of 20.

It is well-known that  $S$  and  $K$  are asymptotically  $\chi^2(1)$ -distributed, and  $K + S$  is  $\chi^2(2)$ -distributed.

Based on the results presented in Table II we can conclude that the null hypothesis of normality is rejected for all stocks. The values of skewness is more than 3 for all companies.

The Box-Ljung  $Q$ -statistic shows that there is no autocorrelation of log returns. Using this fact, we do not include autoregressive and moving average terms in mean equation. We will assume  $\mu = \mathbb{E}(r_t)$ .

Consistent with the findings in [14], we find that the  $p$ -values of Shapiro-Wilk statistic of log returns for all companies are close to zero. We may conclude that all series are non-normal.

Let  $r_t$  and  $r_t^*$  denote log return of the stock and log return of FTSE100 index on interval  $t$  respectively. We will consider a process  $(\epsilon_t) = r_t - (\theta_1 + \theta_2 r_t^*)$ , where  $\theta_1$  and  $\theta_2$  are parameters of models.

The GARCH model of [3] provides a flexible and parsimonious approximation to conditional variance dynamics. Maximum likelihood estimates (MLE) of the GARCH(1,1) model defined by (1) for log returns of closing daily prices are presented in Table III. Using GARCH estimates, Table III shows that volatility persistence, i.e.  $\alpha + \beta$ , is more than 0.9 for almost all companies except Shire plc. It provides clear evidence of GARCH effect. The coefficients of the model are significant with levels of 5%.

Table IV shows the maximum likelihood estimates of GARCH(1,1)-Jumps model with autoregressive jump intensity for log returns of the closing daily prices of the 12 companies for 6 years (January 4, 2005 – January 28, 2011). Values in parenthesis are standard deviations.

TABLE II  
EMPIRICAL PROPERTIES OF DAILY LOG RETURNS IN THE SAMPLE, DAILY LOG RETURNS ARE IN %

Company	S	K	SW(p)	Q(20)	mean	min	max	st.dev.
AstraZeneca	-0.17	8.57	0.94	44.40 (0,001)	0.03	-11.47	9.63	1.61
Barclays	1.39	38.68	0.77	63.24 (0,000)	-0.05	-29.82	56.41	4.01
BP	-0.18	10.33	0.91	51.22 (0,000)	0.00	-14.04	10.58	1.96
British Sky Broadcasting	0.30	16.31	0.86	56.82 (0,000)	0.02	-15.51	15.33	1.86
HSBC Holdings	-0.84	22.12	0.82	63.22 (0,000)	-0.02	-20.80	14.42	2.07
Int Consolidated Airlines	-0.17	5.39	0.97	22.05 (0,338)	0.01	-13.25	11.74	2.84
Johnson Matthey	0.01	11.83	0.91	43.96 (0,002)	0.04	-17.59	17.25	2.22
London Stock Exch Group	0.65	13.44	0.87	52.56 (0,000)	0.02	-15.13	26.67	2.82
National Grid	-0.40	18.38	0.86	77.30 (0,000)	0.01	-14.10	15.33	1.63
RBS Group	-7.66	163.76	0.59	129.93 (0,000)	-0.20	-109.57	30.50	5.01
Shire	-0.11	14.23	0.90	23.82 (0,250)	0.08	-15.99	13.99	1.90
Tesco	0.24	69.93	0.77	101.57 (0,000)	0.01	-28.12	28.61	1.90

TABLE IV  
MLE OF GARCH-JUMP MODEL WITH AUTOREGRESSIVE JUMP INTENSITY

Company	$\alpha$	$\beta$	$\delta$	$\theta$	$a$	$b$	$c$	LLF
AstraZeneca	0.01 (0.00)	0.98 (0.00)	2.27 (0.89)	0.15 (0.21)	0.00 (0.00)	0.89 (0.39)	0.07 (0.06)	4576.56
Barclays	0.19 (0.03)	0.78 (0.04)	8.77 (1.75)	2.04 (1.00)	0.00 (0.00)	0.69 (0.24)	0.03 (0.05)	3956.26
BP	0.06 (0.02)	0.89 (0.02)	3.79 (1.00)	0.89 (0.89)	0.02 (0.01)	0.20 (0.28)	0.04 (0.03)	4654.34
British Sky Broadcasting	0.18 (0.03)	0.66 (0.07)	2.60 (1.09)	0.66 (0.31)	0.00 (0.00)	0.99 (0.04)	0.01 (0.01)	4543.52
HSBC Holdings	0.07 (0.03)	0.90 (0.03)	2.83 (0.87)	-0.21 (0.32)	0.00 (0.00)	0.63 (0.26)	0.07 (0.05)	4933.97
Int Consolidated Airlines	0.04 (0.01)	0.95 (0.01)	2.11 (0.95)	0.82 (0.52)	0.10 (0.11)	0.44 (0.37)	0.00 (0.00)	3867.82
Johnson Matthey	0.10 (0.02)	0.84 (0.04)	3.89 (0.88)	1.65 (0.78)	0.00 (0.00)	0.63 (0.29)	0.04 (0.05)	4485.08
London Stock Exch Group	0.07 (0.03)	0.83 (0.05)	4.61 (0.46)	0.75 (0.35)	0.00 (0.00)	0.73 (0.15)	0.14 (0.09)	4007.43
National Grid	0.13 (0.03)	0.72 (0.07)	3.49 (1.08)	-1.17 (0.73)	0.00 (0.00)	0.98 (0.02)	0.00 (0.00)	4639.80
RBS Group	0.15 (0.04)	0.81 (0.04)	15.41 (1.35)	-0.19 (1.02)	0.00 (0.00)	0.52 (0.14)	0.05 (0.04)	3923.51
Shire	0.15 (0.02)	0.00 (0.00)	5.50 (2.52)	1.67 (2.08)	0.03 (0.00)	0.15 (0.09)	0.03 (0.01)	3598.71
Tesco	0.10 (0.02)	0.81 (0.04)	4.85 (3.27)	0.00 (0.71)	0.00 (0.02)	0.93 (0.04)	0.01 (0.00)	4523.99

It can be seen that the coefficients  $\alpha, \beta$  of the model are highly significant. Table IV shows that volatility persistence, i.e.  $\alpha + \beta$ , is more than 0.9. It provides clear evidence of GARCH effect.

Note that jumps are mainly related with negative movements in the price, because the estimates of parameter  $\theta$  are either negative or insignificant. The size of jumps (standard deviation of jumps,  $\delta$ ) is the highest for the Royal Bank of Scotland Group ( $\delta = 15.41$ ) and is the lowest for International Consolidated Airlines ( $\delta = 2.11$ ).

Despite the fact that many of parameters are non-significant, the Box-Ljung statistics reject the model only for the company Intl. Cons. Air Grp.

The average jump intensity is different for different companies. For example, the average of the jump intensity for HSBC Holdings is equal to  $E(\lambda_t) = 0,053$ , i.e. jumps are occurred every 19 days in average. As the jump size become larger, the intensity of the jumps diminishes.

Parameter  $b$  gives the important persistence in  $\lambda_t$  implying a smooth evolution of the  $\lambda_t$  through time. The comparison of log-likelihoods of the 'pure' GARCH model and the GARCH-Jump model shows a statistically significant change.

The expectation of standard deviation for HSBC Holdings is equal to  $E(\sigma_t) = 0,0148$ .

The part of standard deviation explained by jumps for HSBC Holdings is equal to 0,128, or 12,8%.

We can estimate the temporal evolution of the probability

that a jump took place over a given day,  $\Pr(n_t \geq 1|I_t) = 1 - \Pr(n_t = 0|I_t)$ . Figure 2 presents the time evolution of the jump probability  $\Pr(n_t \geq 1|I_t)$  for HSBC Holdings (January 4, 2005 – January 28, 2011). Relatively high value of the parameter  $b$  gives the smoothness of  $\lambda_t$  in some periods of time.

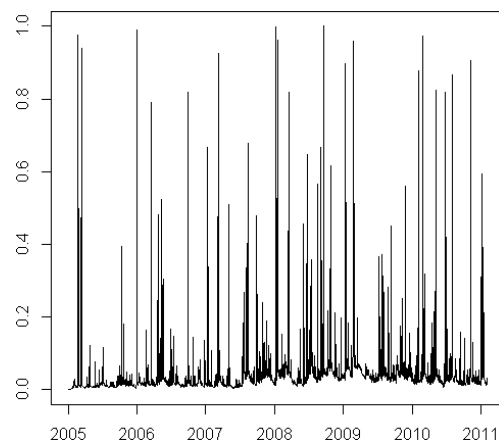


Fig. 2. Estimation of the jump probability  $\Pr(n_t \geq 1|I_t)$  for HSBC Holdings

Moreover, we can consider the temporal evolution implied by the model for skewness and kurtosis.

Figure 3 presents the time evolution for estimation of the conditional skewness for HSBC Holdings (January 4, 2008

– January 28, 2011).

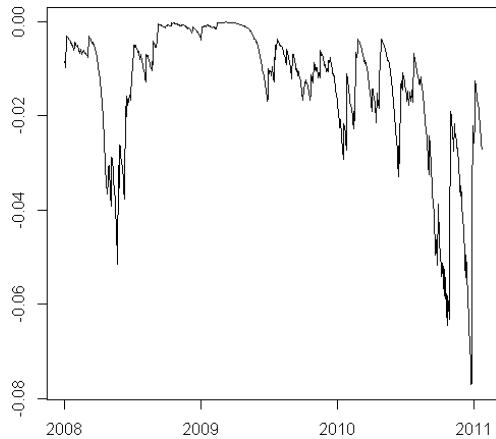


Fig. 3. Evolution of the conditional skewness for HSBC Holdings

Figure 4 presents the time evolution for estimation of the conditional kurtosis for HSBC Holdings (January 4, 2008 – January 28, 2011).

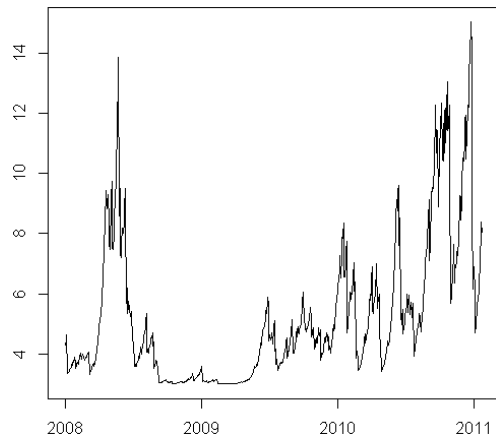


Fig. 4. Evolution of the conditional kurtosis for HSBC Holdings

These figures show that both skewness and kurtosis of the data evolve over time.

Table V shows the maximum likelihood estimates of GARCH–Jump model with autoregressive jump intensity augmented with news analytics data for log returns of the closing daily prices of the 12 companies for 6 years (January 4, 2005 – January 28, 2011). The jump size  $\theta$  is negative or insignificant reflecting the stylized fact that returns are negatively skewed.

Despite the fact that many of parameters are non-significant, the Box-Ljung statistics reject the model only for the company Intl. Cons. Air Grp. After adding news intensity in the GARCH–Jump model, the jumps autoregressive parameters ( $a, b, c$ ) became insignificant for almost all companies.

Note that the GARCH model with jumps (the null model) is a special case of the augmented GARCH–Jumps model (the alternative model). Therefore, to compare the fit of two models it can be used a likelihood ratio test (see e.g. [21]). It is the most common approach to testing problem. This test has been discussed in the papers [22] and [23]. We use this approach to test the augmented GARCH–Jumps model against 'pure' GARCH model with jumps.

Let  $H_0$  denote the 'pure' GARCH–Jumps model with autoregressive jump intensity and  $H_1$  denote the augmented GARCH–Jumps model with autoregressive jump intensity. Let  $\epsilon_t$  be a random variable that has a mean and a variance conditionally on the information set  $I_{t-1}$ .

Denote the corresponding log likelihood functions by  $LLF_{H_0}(\epsilon; \theta_0)$  and  $LLF_{H_1}(\epsilon; \theta_1)$ , respectively.

We will consider the test statistic defined by

$$LR = 2(LLF_{H_1}(\epsilon; \tilde{\theta}_1) - LLF_{H_0}(\epsilon; \tilde{\theta}_0)). \quad (13)$$

While the asymptotic null distribution of (13) is unknown, it can be approximated by Monte Carlo simulation.

We can assume that the augmented GARCH–Jumps model is the alternative model and that  $\theta_1$  is the true parameter. Using Monte Carlo approach we will generate  $N$  realizations of  $T$  observations  $\epsilon^{(i)} = (\epsilon_t^{(i)})_{t=1}^T$ ,  $i = 1, \dots, N$ , from this model. Then we will estimate both models and calculates the value of (13) using each realization  $\epsilon^{(i)}$ .

Ranking the  $N$  values gives an empirical distribution with which one compares the original value of (13). The true value of  $\theta_1$  is unknown, but the approximation error due to the use of  $\tilde{\theta}_1$  as a replacement vanishes asymptotically as  $T \rightarrow \infty$ .

If the value of (13) is more or equal to the  $100(1 - \alpha)\%$  quantile of the empirical distribution, the null model is rejected at significance level  $\alpha$ . As it was mentioned in [22] the models under comparison need not have the same number of parameters, and the value of the statistic can also be negative. Reversing the roles of the models, it can be possible to test GARCH–Jumps model with constant jump intensity against the augmented GARCH–Jumps model.

The data-generating model is defined by equations (7)–(9) given before. Notice that the error term in the mean equation is drawn from a normal distribution with mean zero and variance that changes over time according to equations (7)–(9).

Finally, we have set the number of trials  $N$  in each Monte Carlo experiment to 1000.

Table VI presents the results of the Monte Carlo simulation for the likelihood ratio statistic to compare GARCH–Jumps model with autoregressive jump intensity and the augmented GARCH–Jumps model with autoregressive jump intensity (Null Hypothesis) on the finite sample performance of the MLE estimator. In particular, we study the significance of the MLE estimators of the parameters of the variance equation. For all companies the alternative models is preferable with confidence level of 1%.

## V. CONCLUSION

We have studied GARCH model augmented with news analytics data to examine the impact of news intensity on stock volatility. Likelihood ratio test has shown that the GARCH–Jump model augmented with the news intensity performs efficiently than the 'pure' GARCH–Jump model. To calibrate the models we have used the Quasi Maximum Likelihood Estimation (QMLE) methods. We have used RavenPack news analytics data. We may conclude that

- the likelihood ratio test supports the hypothesis of impact of news on jump intensity of volatility;
- GARCH–Jump model augmented with the news intensity does not remove GARCH and ARCH effects for all companies.



TABLE V  
MLE OF GARCH–JUMP MODEL WITH AUTOREGRESSIVE JUMP INTENSITY AUGMENTED WITH NEWS ANALYTICS DATA

Company	$\alpha$	$\beta$	$\delta$	$\theta$	$a$	$b$	$c$	$100\rho$	$LLF$
AstraZeneca	0.02 (0.01)	0.97 (0.01)	1.79 (0.43)	0.40 (0.07)	0.00 (0.00)	0.86 (0.29)	0.03 (0.08)	2.98 (1.03)	4617.95
Barclays	0.17 (0.05)	0.78 (0.05)	6.83 (1.48)	1.93 (0.94)	0.00 (0.00)	0.05 (0.08)	0.01 (0.01)	0.51 (0.30)	3974.35
BP	0.11 (0.03)	0.77 (0.07)	2.55 (0.90)	0.30 (0.38)	0.00 (0.00)	0.15 (0.25)	0.00 (0.00)	0.64 (0.33)	4675.72
British Sky Broadcasting	0.16 (0.04)	0.66 (0.06)	2.01 (0.23)	0.23 (0.11)	0.00 (0.00)	0.97 (0.00)	0.01 (0.00)	4.11 (0.47)	4572.09
HSBC Holdings	0.07 (0.01)	0.90 (0.01)	2.00 (0.87)	0.12 (0.13)	0.00 (0.00)	0.37 (0.35)	0.00 (0.00)	0.60 (0.38)	4950.70
Int Consolidated Airlines	0.04 (0.01)	0.95 (0.01)	2.05 (0.83)	0.74 (0.75)	0.02 (0.03)	0.21 (0.27)	0.00 (0.00)	3.11 (1.57)	3898.02
Johnson Matthey	0.10 (0.02)	0.84 (0.03)	4.14 (0.93)	2.05 (0.72)	0.00 (0.00)	0.00 (0.00)	0.01 (0.01)	0.72 (0.43)	4492.86
London Stock Exch Group	0.07 (0.02)	0.84 (0.03)	3.98 (1.01)	0.06 (0.09)	0.00 (0.00)	0.48 (0.24)	0.17 (0.09)	2.23 (0.90)	4032.99
National Grid	0.14 (0.02)	0.71 (0.04)	3.16 (0.49)	-0.91 (0.39)	0.00 (0.00)	0.98 (0.02)	0.00 (0.00)	0.83 (0.88)	4644.74
RBS Group	0.15 (0.03)	0.81 (0.03)	11.75 (1.59)	-0.08 (0.17)	0.00 (0.00)	0.28 (0.36)	0.02 (0.02)	0.53 (0.21)	3938.06
Shire	0.15 (0.04)	0.00 (0.00)	4.78 (0.58)	-0.21 (0.41)	0.01 (0.00)	0.00 (0.00)	0.00 (0.00)	1.58 (0.70)	3640.42
Tesco	0.11 (0.02)	0.80 (0.04)	3.77 (1.76)	1.49 (0.64)	0.00 (0.01)	0.65 (0.21)	0.02 (0.00)	1.00 (0.41)	4543.16

TABLE VI  
RESULTS OF THE LIKELIHOOD RATIO TEST FOR THE GARCH MODEL WITH JUMPS AND THE AUGMENTED GARCH-JUMPS MODEL

Company	Null Hypothesis
AstraZeneca	rejected
Barclays	rejected
BP	rejected
British Sky Broadcasting	rejected
HSBC Holdings	rejected
Int Consolidated Airlines	rejected
Johnson Matthey	rejected
London Stock Exch Group	rejected
National Grid	rejected
RBS Group	rejected
Shire	rejected
Tesco	rejected

Based on the research it can be suggested some directions of future work.

- The first problem is to develop a GARCH-type model with news analytics data for prediction VaR with better performance than the "pure" GARCH model.
- It is worth considering the problem of mutual dependence of volatility and news intensity.
- Future work may be also associated with the study of
  - Markov – Switching GARCH models.
  - HMM – GARCH Model.

There are some evidences (see e.g. [17]) that effect of news on prices is short-term, therefore it is more likely that we need tick by tick data to examine impact of news on stock volatility.

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