Pricing Dual Spread Options by the Lie-Trotter Operator Splitting Method

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Abstract — **In this paper, based upon the Lie-Trotter operator splitting method proposed by Lo (2014), we present a simple closed-form approximation for pricing the (three-asset) dual spread options. Illustrative numerical examples show that the proposed approximation is not only extremely fast and robust, but also it is very accurate for typical volatilities and maturities up to two years. Moreover, for the case of a vanishing strike the proposed approximation becomes exact.**

Keywords: Spread options; Kirk's approximation; Lie-Trotter operator splitting method

I. INTRODUCTION

A spread option is an option whose payoff is linked to the price difference of two underlying assets and forms the simplest type of multi-asset options. Spread options are very popular in interest rate markets, currency and foreign exchange markets, commodity markets, and energy markets nowadays.[1] Unlike pricing single-asset options, pricing spread options is a very challenging task. The major difficulty stems from the lack of knowledge about the distribution of the spread of two correlated lognormal random variables. The simplest approach is to evaluate the expectation of the final payoff over the joint probability distribution of the two correlated lognormal underlyings by means of numerical integration. However, practitioners often prefer to use analytical approximations rather than numerical methods because of their computational ease. Among various analytical approximations, Kirk's approximation seems to be the most popular and is the current market standard, especially in the energy markets.[2] It is well known that Kirk's approximation extends from Margrabe's exchange option formula with no rigorous derivation.[3] Recently, Lo (2013) applied the idea of WKB method to provide a derivation of Kirk's approximation and discuss its validity.[4] Nevertheless, it is not straightforward to apply Lo's approach either to provide a generalization of Kirk's approximation for the multi-asset case or to improve Kirk's approximation while retaining its favourable features.

In order to overcome these shortcomings, Lo (2014) subsequently presented a simple unified approach,[5,6] namely the Lie-Trotter operator splitting method,[7-12] not only to rigorously derive Kirk's approximation but also to obtain a generalization for the case of multi-asset spread option in a straightforward manner. The derived price formula for the multi-asset spread option bears a striking resemblance to Kirk's approximation in the two-asset case. Illustrative numerical examples have demonstrated that the multi-asset generalization retains all the favourable features of Kirk's approximation. More importantly, the proposed approach is able to provide a new perspective on Kirk's approximation and the generalization; that is, they are simply equivalent to the Lie-Trotter operator splitting approximation to the Black-Scholes equation.

It is the aim of this communication to apply the Lie-Trotter operator splitting method proposed by Lo (2014) to derive a closed-form approximate price formula for the (three-asset) *dual spread options*,[5,6] whose final payoff has the form $\max(S_1 - S_3 - K, S_2 - S_3 - K, 0)$ with S_i being the price of the asset i and K being the strike price. The final payoff is a generalisation of the case of a standard three-asset spread option and closely resembles the payoff of a European "best of two" option.[13] The derived approximate price formula bears a great resemblance to that of a European "best of two" option as well. Illustrative numerical examples are also

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(Advance online publication: 28 November 2014)

shown to demonstrate both the accuracy and efficiency of the proposed approximation. Furthermore, it should be noted that for $K = 0$ the proposed approximation becomes exact.

II. PRICING DUAL SPREAD OPTIONS BY LIE-TROTTER SPLITTING APPROXIMATION

To price a European three-asset dual call spread option, we need to solve the three-dimensional Black-Scholes equation

$$\sum_{i,j=1}^{3} \frac{1}{2}\rho_{ij}\sigma_i\sigma_j F_i F_j \frac{\partial^2 P(F_1,F_2,F_3,\tau)}{\partial F_i \partial F_j} = \frac{\partial P(F_1,F_2,F_3,\tau)}{\partial \tau} \quad (1)$$

subject to the final payoff condition

$$P(F_1,F_2,F_3,0) = \max(F_1 - F_3 - K, F_2 - F_3 - K, 0) \;, \quad (2)$$

where F_i is the future price of the underlying asset i with the volatility σ_i, ρ_{ij} is the correlation between the assets i and j, K is the strike price, and τ is the time-to-maturity.

Main result:

The price of the dual call spread option can be approximated by

$$\begin{aligned} & P(F_1,F_2,F_3,\tau) \\ \approx\ & e^{-r\tau}\left[F_1\left\{N(\Lambda_1) - N_2(\Lambda_1,\Lambda_2,\Gamma_1)\right\} - \right. \\ & (F_3+K)\,N_2\left(\Lambda_1 - \tilde{\sigma}_1\sqrt{\tau}, \Lambda_3, -\Gamma_1\right) + \\ & F_2\left\{N(\Lambda_4) - N_2\left(\Lambda_4, -\Lambda_2 - \tilde{\sigma}_-\sqrt{\tau}, \Gamma_2\right)\right\} - \\ & \left.(F_3+K)\,N_2\left(\Lambda_4 - \tilde{\sigma}_2\sqrt{\tau}, -\Lambda_3, -\Gamma_2\right)\right] \;. \quad (3) \end{aligned}$$

where

$$\begin{aligned} \Lambda_1 &= \frac{\ln(F_1/[F_3+K]) + \frac{1}{2}\tilde{\sigma}_1^2\tau}{\tilde{\sigma}_1\sqrt{\tau}} \\ \Lambda_2 &= \frac{\ln(F_2/F_1) - \frac{1}{2}\tilde{\sigma}_-^2\tau}{\tilde{\sigma}_-\sqrt{\tau}} \\ \Lambda_3 &= \frac{\ln(F_1/F_2) - \frac{1}{2}\left(\tilde{\sigma}_1^2 - \tilde{\sigma}_2^2\right)\tau}{\tilde{\sigma}_-\sqrt{\tau}} \\ \Lambda_4 &= \frac{\ln(F_2/[F_3+K]) + \frac{1}{2}\tilde{\sigma}_2^2\tau}{\tilde{\sigma}_2\sqrt{\tau}} \\ \Gamma_1 &= \frac{\tilde{\rho}_{12}\tilde{\sigma}_2 - \tilde{\sigma}_1}{\tilde{\sigma}_-} \\ \Gamma_2 &= \frac{\tilde{\rho}_{12}\tilde{\sigma}_1 - \tilde{\sigma}_2}{\tilde{\sigma}_-} \\ \tilde{\sigma}_- &= \sqrt{\tilde{\sigma}_1^2 - 2\tilde{\rho}_{12}\tilde{\sigma}_1\tilde{\sigma}_2 + \tilde{\sigma}_2^2} \\ \tilde{\sigma}_1 &= \sqrt{\sigma_1^2 - 2\rho_{13}\sigma_1\tilde{\sigma}_3 + \tilde{\sigma}_3^2} \\ \tilde{\sigma}_2 &= \sqrt{\sigma_2^2 - 2\rho_{23}\sigma_2\tilde{\sigma}_3 + \tilde{\sigma}_3^2} \\ \tilde{\sigma}_3 &= \sigma_3\left(\frac{F_3}{F_3+K}\right) \\ \tilde{\rho}_{12} &= \frac{\rho_{12}\sigma_1\sigma_2 - (\rho_{13}\sigma_1 + \rho_{23}\sigma_2 - \tilde{\sigma}_3)\,\tilde{\sigma}_3}{\tilde{\sigma}_1\tilde{\sigma}_2} \quad (4) \end{aligned}$$

and $N_2(\cdot)$ is the cumulative bivariate normal distribution function.

Derivation:

In terms of the new variables

$$R_1 = \frac{F_1}{F_3+K} \;,\; R_2 = \frac{F_2}{F_3+K} \;,\; R_3 = F_3 + K \quad (5)$$

we can express Eq.(1) as

$$\left\{\hat{L}_0 + \hat{L}_1 + \hat{L}_2 - r\right\} P(R_1,R_2,R_3,\tau) = \frac{\partial P(R_1,R_2,R_3,\tau)}{\partial \tau} \;, \quad (6)$$

where

$$\begin{aligned} \hat{L}_0 &= \frac{1}{2}\tilde{\sigma}_1^2 R_1^2 \frac{\partial^2}{\partial R_1^2} + \frac{1}{2}\tilde{\sigma}_2^2 R_2^2 \frac{\partial^2}{\partial R_2^2} + \\ & \quad \tilde{\rho}_{12}\tilde{\sigma}_1\tilde{\sigma}_2 R_1 R_2 \frac{\partial^2}{\partial R_1 \partial R_2} \\ \hat{L}_1 &= (\rho_{13}\sigma_1 - \tilde{\sigma}_3)\,\tilde{\sigma}_3 R_1 R_3 \frac{\partial^2}{\partial R_1 \partial R_3} \\ & \quad - (\rho_{13}\sigma_1 - \tilde{\sigma}_3)\,\tilde{\sigma}_3 R_1 \frac{\partial}{\partial R_1} \\ & \quad + \frac{1}{2}\tilde{\sigma}_3^2 R_3^2 \frac{\partial^2}{\partial R_3^2} \\ \hat{L}_2 &= (\rho_{23}\sigma_2 - \tilde{\sigma}_3)\,\tilde{\sigma}_3 R_2 R_3 \frac{\partial^2}{\partial R_2 \partial R_3} \\ & \quad - (\rho_{23}\sigma_2 - \tilde{\sigma}_3)\,\tilde{\sigma}_3 R_2 \frac{\partial}{\partial R_2} \\ & \quad + \frac{1}{2}\tilde{\sigma}_3^2 R_3^2 \frac{\partial^2}{\partial R_3^2} \;. \quad (7) \end{aligned}$$

The final payoff condition now becomes

$$P(R_1,R_2,R_3,0) = R_3 \max(R_1 - 1, R_2 - 1, 0) \;. \quad (8)$$

It is obvious that Eq.(6) has the formal solution

$$\begin{aligned} & P(R_1,R_2,R_3,\tau) \\ =\ & e^{-r\tau}\exp\left\{\tau\left(\hat{L}_0 + \hat{L}_1 + \hat{L}_2\right)\right\} \\ & \times R_3 \max(R_1 - 1, R_2 - 1, 0) \;. \quad (9) \end{aligned}$$

Then, applying the Lie-Trotter operator splitting method to Eq.(9) yields an approximate solution,[7,8] namely (see the Appendix)

$$\begin{aligned} & P^{LT}(R_1,R_2,R_3,\tau) \\ = & e^{-r\tau}\exp\left\{\tau\hat{L}_0\right\}\exp\left\{\tau\left(\hat{L}_1+\hat{L}_2\right)\right\} \\ & \times R_3\max(R_1-1,R_2-1,0) \\ = & e^{-r\tau}R_3\exp\left\{\tau\hat{L}_0\right\}\max(R_1-1,R_2-1,0) \\ \equiv & e^{-r\tau}R_3 C(R_1,R_2,\tau)\ . \end{aligned} \tag{10}$$

Here $C(R_1,R_2,\tau)$ satisfies the partial differential equation

$$\begin{aligned} & \frac{\partial C(R_1,R_2,\tau)}{\partial\tau} \\ = & \left\{\frac{1}{2}\tilde{\sigma}_1^2R_1^2\frac{\partial^2}{\partial R_1^2}+\tilde{\rho}_{12}\tilde{\sigma}_1\tilde{\sigma}_2R_1R_2\frac{\partial^2}{\partial R_1\partial R_2}+\right. \\ & \left.\frac{1}{2}\tilde{\sigma}_2^2R_2^2\frac{\partial^2}{\partial R_2^2}-r\right\}C(R_1,R_2,\tau) \end{aligned} \tag{11}$$

with the initial condition: $C(R_1,R_2,\tau=0) = \max(R_1-1,R_2-1,0)$. Since $\tilde{\sigma}_1$, $\tilde{\sigma}_2$ and $\tilde{\rho}_{12}$ are independent of R_1 and R_2, we have a problem of pricing a European "best-of-two" option.[13] The desired solution of Eq.(11) is simply given by

$$\begin{aligned} & C(R_1,R_2,\tau) \\ = & e^{-r\tau}\int_{-\infty}^{\infty}dx_{10}\int_{-\infty}^{\infty}dx_{20}f(x_1,x_2,\tau;x_{10},x_{20}) \\ & \times\max\left(e^{x_{10}}-1,e^{x_{20}}-1,0\right) \\ = & e^{-r\tau}\int_0^{\infty}dx_{10}\int_{-\infty}^{x_{10}}dx_{20}f(x_1,x_2,\tau;x_{10},x_{20}) \\ & \times\left(e^{x_{10}}-1\right)+ \\ & e^{-r\tau}\int_0^{\infty}dx_{20}\int_{-\infty}^{x_{20}}dx_{10}f(x_1,x_2,\tau;x_{10},x_{20}) \\ & \times\left(e^{x_{20}}-1\right) \end{aligned} \tag{12}$$

where $x_{10}=\ln R_{10}$, $x_{20}=\ln R_{20}$, $x_1=\ln R_1$, $x_2=\ln R_2$, and

$$\begin{aligned} & f(x_1,x_2,\tau;x_{10},x_{20}) \\ = & \frac{1}{2\pi\tilde{\sigma}_1\tilde{\sigma}_2\tau\sqrt{1-\tilde{\rho}_{12}^2}}\times \\ & \exp\left\{-\frac{1}{2\tilde{\sigma}_1^2\tau\left(1-\tilde{\rho}_{12}^2\right)}\left(x_{10}-x_1+\frac{1}{2}\tilde{\sigma}_1^2\tau\right)^2\right. \\ & +\frac{\tilde{\rho}_{12}}{\tilde{\sigma}_1\tilde{\sigma}_2\tau\left(1-\tilde{\rho}_{12}^2\right)}\left(x_{10}-x_1+\frac{1}{2}\tilde{\sigma}_1^2\tau\right) \\ & \times\left(x_{20}-x_2+\frac{1}{2}\tilde{\sigma}_2^2\tau\right)- \\ & \left.\frac{1}{2\tilde{\sigma}_2^2\tau\left(1-\tilde{\rho}_{12}^2\right)}\left(x_{20}-x_2+\frac{1}{2}\tilde{\sigma}_2^2\tau\right)^2\right\}\ . \end{aligned} \tag{13}$$

After carrying out the integrals in Eq.(12), the solution can be determined in closed form as follows:

$$\begin{aligned} & C(R_1,R_2,\tau) \\ = & e^{-r\tau}\left[R_1\left\{N(\Lambda_1)-N_2(\Lambda_1,\Lambda_2,\Gamma_1)\right\}\right. \\ & -N_2\left(\Lambda_1-\tilde{\sigma}_1\sqrt{\tau},\Lambda_3,-\Gamma_1\right)+ \\ & R_2\left\{N(\Lambda_4)-N_2\left(\Lambda_4,-\Lambda_2-\tilde{\sigma}_-\sqrt{\tau},\Gamma_2\right)\right\} \\ & \left.-N_2\left(\Lambda_4-\tilde{\sigma}_2\sqrt{\tau},-\Lambda_3,-\Gamma_2\right)\right]\ . \end{aligned} \tag{14}$$

As a result, the approximate solution $P^{LT}(R_1,R_2,R_3,\tau) = R_3C(R_1,R_2,\tau)$ yields the approximate price formula given in Eq.(3).

In terms of the spot asset prices, namely $S_i \equiv F_i\exp(-r\tau)$, the price formula given in Eq.(3) has the form

$$\begin{aligned} & P_{Kirk}(S_1,S_2,S_3,\tau) \\ = & S_1\left\{N(d_1)-N_2(d_1,q,\Gamma_1)\right\}- \\ & \left(S_3+Ke^{-r\tau}\right)N_2(d_2,p,-\Gamma_1)+ \\ & S_2\left\{N(h_1)-N_2\left(h_1,-q-\tilde{\sigma}_-\sqrt{\tau},\Gamma_2\right)\right\} \\ & -\left(S_3+Ke^{-r\tau}\right)N_2(h_2,-p,-\Gamma_2) \end{aligned} \tag{15}$$

where

$$\begin{aligned} d_1 &= \frac{\ln\left(S_1/\left[S_3+Ke^{-r\tau}\right]\right)+\frac{1}{2}\tilde{\sigma}_1^2\tau}{\tilde{\sigma}_1\sqrt{\tau}} \\ d_2 &= d_1-\tilde{\sigma}_1\sqrt{\tau} \\ h_1 &= \frac{\ln\left(S_2/\left[S_3+Ke^{-r\tau}\right]\right)+\frac{1}{2}\tilde{\sigma}_2^2\tau}{\tilde{\sigma}_2\sqrt{\tau}} \\ h_2 &= h_1-\tilde{\sigma}_2\sqrt{\tau} \\ p &= \frac{\ln(S_1/S_2)-\frac{1}{2}\left(\tilde{\sigma}_1^2-\tilde{\sigma}_2^2\right)\tau}{\tilde{\sigma}_-\sqrt{\tau}} \\ q &= \frac{\ln(S_2/S_1)-\frac{1}{2}\tilde{\sigma}_-^2\tau}{\tilde{\sigma}_-\sqrt{\tau}}\ . \end{aligned} \tag{16}$$

It should be noted that the Lie-Trotter operator splitting approximation is particularly applicable for those dual spread options with short maturities, *i.e.* $\tilde{\sigma}_1^2\tau \ll 1$ and $\tilde{\sigma}_2^2\tau \ll 1$. Furthermore, for $K=0$, the operators $\hat{L}_0$, $\hat{L}_1$ and $\hat{L}_2$ commute so that the Lie-Trotter splitting approximation becomes exact.

III. ILLUSTRATIVE NUMERICAL EXAMPLES

In this section illustrative numerical examples are presented to demonstrate the accuracy of the proposed approximation for the dual spread options. We examine a simple dual spread option with the final payoff $\max(S_1-S_3-K,S_2-S_3-K,0)$. Table I tabulates the approximate option prices for different values of the strike price K and time-to-maturity

T. Other input model parameters are set as follows: $r = 0.05$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.3$, $\rho_{12} = 0.2$, $\rho_{23} = 0.4$, $\rho_{13} = 0.8$, $S_1 = 150$, $S_2 = 60$ and $S_3 = 50$. Monte Carlo estimates and the corresponding standard deviations are also presented for comparison. It is observed that the computed errors of the approximate option prices are capped at 0.2% (in magnitude). In fact, most of them are less than 0.1%. Then, in Table II the effect of increasing the three volatilities (from 0.3 to 0.6) upon the approximate estimation of the option prices is investigated. Obviously only a small increase occurs in the computed errors, and these errors are still less than 0.7% (in magnitude).

Table I: Prices of a European dual call spread option. Other input parameters are: $r = 0.05$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.3$, $\rho_{12} = 0.2$, $\rho_{23} = 0.4$, $\rho_{13} = 0.8$, $S_1 = 150$, $S_2 = 60$ and $S_3 = 50$. Here "LT" refers to the proposed approximation based upon the Lie-Trotter operator splitting method while "MC" denotes the Monte Carlo estimates with 900,000,000 replications. The relative errors of the "LT" option prices with respect to the "MC" estimates are also presented.

$K \backslash T$	0.25	0.5	1	2	
30	70.3727	70.7428	71.5561	73.7670	LT
	0.0%	0.0%	0.0%	0.0%	error
	70.3732	70.7422	71.5578	73.7724	MC
	0.0104	0.0168	0.0227	0.0333	σ_{MC}
35	65.4348	65.8662	66.8025	69.2787	LT
	0.0%	0.0%	0.0%	0.0%	error
	65.4351	65.8669	66.8025	69.2917	MC
	0.0119	0.0168	0.0215	0.0334	σ_{MC}
40	60.4969	60.9899	62.0562	64.8309	LT
	0.0%	0.0%	0.0%	−0.1%	error
	60.4971	60.9899	62.0581	64.8664	MC
	0.0107	0.0160	0.0220	0.0324	σ_{MC}
45	55.5590	56.1146	57.3291	60.4500	LT
	0.0%	0.0%	0.0%	−0.1%	error
	55.5585	56.1165	57.3421	60.5071	MC
	0.0110	0.0147	0.0228	0.0301	σ_{MC}
50	50.6212	51.2440	52.6426	56.1672	LT
	0.0%	0.0%	−0.1%	−0.2%	error
	50.6213	51.2469	52.6677	56.2557	MC
	0.0108	0.0140	0.0200	0.0303	σ_{MC}

Finally, we study a case in which all the three volatilities are different, namely $\sigma_1 = 0.3$, $\sigma_2 = 0.4$ and $\sigma_3 = 0.5$, while the other parameters remain the same. According to Table III, the computed errors are generally reduced a little bit in this case and they do not exceed 0.4% (in magnitude). Moreover, since the approximate option price formula is given in closed form, its evaluation is instantaneous. As a result, it can be concluded that the proposed closed-form approximation for the dual spread options are found to be very accurate and efficient.

IV. CONCLUSION

In this paper, based upon the Lie-Trotter operator splitting method proposed by Lo (2014),[5,6] we have presented a simple closed-form approximation for pricing the (three-asset) dual spread options. The derived price formula bears a close resemblance to that of a European "best of two" option. As demonstrated by illustrative numerical examples for the dual spread options, the proposed approximation is not only extremely fast and robust, but also it is very accurate for typical volatilities and maturities of up to two years. Moreover, for the case of a vanishing strike, *i.e.* $K = 0$, the proposed approximation becomes exact.

Table II: Prices of a European dual call spread option. Other input parameters are: $r = 0.05$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.6$, $\rho_{12} = 0.2$, $\rho_{23} = 0.4$, $\rho_{13} = 0.8$, $S_1 = 150$, $S_2 = 60$ and $S_3 = 50$. Here "LT" refers to the proposed approximation based upon the Lie-Trotter operator splitting method while "MC" denotes the Monte Carlo estimates with 900,000,000 replications. The relative errors of the "LT" option prices with respect to the "MC" estimates are also presented.

$K \backslash T$	0.25	0.5	1	2	
30	70.4659	71.6660	75.5696	84.3876	LT
	0.0%	0.0%	−0.1%	−0.3%	error
	70.4663	71.6745	75.6231	84.6051	MC
	0.0223	0.0306	0.0473	0.0749	σ_{MC}
35	65.5319	66.8473	71.0669	80.4493	LT
	0.0%	0.0%	−0.2%	−0.4%	error
	65.5335	66.8692	71.1709	80.7580	MC
	0.0215	0.0341	0.0476	0.0731	σ_{MC}
40	60.6083	62.0915	66.7001	76.6848	LT
	0.0%	−0.1%	−0.2%	−0.5%	error
	60.6151	62.1381	66.8578	77.0725	MC
	0.0216	0.0331	0.0466	0.0800	σ_{MC}
45	55.7114	57.4357	62.5004	73.1061	LT
	0.0%	−0.1%	−0.3%	−0.6%	error
	55.7275	57.5148	62.7155	73.5664	MC
	0.0221	0.0320	0.0391	0.0658	σ_{MC}
50	50.8689	52.9202	58.4926	69.7183	LT
	−0.1%	−0.2%	−0.5%	−0.7%	error
	50.8995	53.0358	58.7584	70.2351	MC
	0.0218	0.0292	0.0528	0.0727	σ_{MC}

Table III: Prices of a European dual call spread option. Other input parameters are: $r = 0.05$, $\sigma_1 = 0.3$, $\sigma_2 = 0.4$, $\sigma_3 = 0.5$, $\rho_{12} = 0.2$, $\rho_{23} = 0.4$, $\rho_{13} = 0.8$, $S_1 = 150$, $S_2 = 60$ and $S_3 = 50$. Here "LT" refers to the proposed approximation based upon the Lie-Trotter operator splitting method while "MC" denotes the Monte Carlo estimates with 900,000,000 replications. The relative errors of the "LT" option prices with respect to the "MC" estimates are also presented.

$K\backslash T$	0.25	0.5	1	2	
30	70.3728	70.7578	71.7849	74.8637	LT
	0.0%	0.0%	0.0%	−0.2%	error
	70.3726	70.7577	71.7923	74.9890	MC
	0.0093	0.0135	0.0210	0.0254	σ_{MC}
35	65.4351	65.8813	67.0308	70.3651	LT
	0.0%	0.0%	0.0%	−0.2%	error
	65.4351	65.8810	67.0426	70.5045	MC
	0.0105	0.0143	0.0213	0.0298	σ_{MC}
40	60.4970	61.0048	62.2798	65.8854	LT
	0.0%	0.0%	0.0%	−0.2%	error
	60.4972	61.0050	62.2946	66.0457	MC
	0.0092	0.0138	0.0196	0.0288	σ_{MC}
45	55.5591	56.1286	57.5358	61.4321	LT
	0.0%	0.0%	0.0%	−0.3%	error
	55.5597	56.1302	57.5556	61.6195	MC
	0.0093	0.0146	0.0191	0.0293	σ_{MC}
50	50.6212	51.2535	52.8076	57.0343	LT
	0.0%	0.0%	−0.1%	−0.4%	error
	50.6216	51.2539	52.8346	57.2514	MC
	0.0100	0.0126	0.0216	0.0286	σ_{MC}

APPENDIX:

Suppose that one needs to exponentiate an operator $\hat{C}$ which can be split into two different parts, namely $\hat{A}$ and $\hat{B}$. For simplicity, let us assume that $\hat{C} = \hat{A} + \hat{B}$, where the exponential operator $\exp\left(\hat{C}\right)$ is difficult to evaluate but $\exp\left(\hat{A}\right)$ and $\exp\left(\hat{B}\right)$ are either solvable or easy to deal with. Under such circumstances the exponential operator $\exp\left(\varepsilon\hat{C}\right)$, with ε being a small parameter, can be approximated by the Lie-Trotter operator splitting formula:[7-12]

$$\exp\left(\varepsilon\hat{C}\right) = \exp\left(\varepsilon\hat{A}\right)\exp\left(\varepsilon\hat{B}\right) + \mathcal{O}\left(\varepsilon^2\right) \quad . \quad (A.1)$$

The Lie-Trotter splitting approximation is particularly useful for studying the short-time behaviour of the solutions of evolutionary partial differential equations of parabolic type because for this class of problems it is sensible to split the spatial differential operator into several parts each of which corresponds to a different physical contribution (e.g., reaction and diffusion).

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