Nonexistence of Global Solutions to a Nonlinear Fractional Reaction-Diffusion System

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Abstract—**In this paper we consider a Cauchy problem presented as a parabolic fractional reaction diffusion system with nonlinear terms of the form $|u|^{p_i}|v|^{q_i}$, $i = 1, 2$ where $p_1, q_2 \geq 0$ and $p_2, q_1 > 1$ are constants. We prove a nonexistence result which is more general than the interesting result obtained by Kirane *et al.* [8] which concerns the case $p_1 = q_2 = 0$. We also show that this result extends the works of Yamauchi [15] and Zheng [16] done in the classical case.**

Index Terms—**Fractional derivatives, nonlinear reaction diffusion system, test-function, critical exponent.**

I. Introduction

THE fractional calculus is a field of mathematics which is as old as the differential calculus, the concept is born from a question asked by l'Hospital in 1695 to Leibniz about the meaning of $d^n y/dx^n = D^n y$ if $n = 1/2$ and what happens for D^n if n is not an integer number; Leibniz replied "That is a paradox from which one day useful consequences will be drawn". In the last few years non integer derivatives and integrals have been used in different fields of science to describe many phenomena and they are nowadays playing a great role in modelling engineering problems including fluid flow, rheology, diffusive transport, networks, probability and hereditary properties of various materials. Fractional and boundary value problems for nonlinear fractional differential equations have received much attention and it becomes natural that many authors try to solve the fractional derivatives, fractional integrals and fractional differential equations, see , e.g., [1], [6], [8], [10], [11], [12], [13], [14] and references therein.

The present paper is concerned with the following Cauchy problem for the nonlinear reaction-diffusion system

$$\begin{cases} \mathbf{D}^{\alpha_1}_{0|t}u + (-\Delta)^{\beta_1/2}u = |u|^{p_1}|v|^{q_1}, \\ \mathbf{D}^{\alpha_2}_{0|t}v + (-\Delta)^{\beta_2/2}v = |u|^{p_2}|v|^{q_2}, \\ u(0,x) = u_0(x) \geq 0, \not\equiv 0, \\ v(0,x) = v_0(x) \geq 0, \not\equiv 0, \end{cases} \tag{1}$$

where $(t, x) \in R^+ \times R^N$, $p_1 \geq 0$, $q_2 \geq 0$, $p_2 > 1$, $q_1 > 1$, $0 < \alpha_i < 1$, $1 \leq \beta_i \leq 2$, $i = 1, 2$ are constants. $\mathbf{D}^{\alpha_i}_{0|t}$ denotes the derivatives of order α_i in the sense of Caputo (see, e.g., [13]) and $(-\Delta)^{\beta_i/2}$ is the fractional power of the Laplacian $(-\Delta)$ defined by

$$(-\Delta)^{\beta_i/2}u(t,x) = \mathcal{F}^{-1}(|\xi|^{\beta_i}\mathcal{F}(u)(\xi))(t,x),$$

where $\mathcal{F}$ is the Fourier transform and $\mathcal{F}^{-1}$ its inverse.

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In the case where $\alpha_i = 1$, $\beta_i = 2$, $i = 1, 2$, the problem (1) was treated by many authors in several contexts, see for example [2], [3], [4], [15], [16]. Escobedo and Herrero [2] proved that if $p_2 q_1 > 1$, $p_1 = q_2 = 0$ and $(\gamma + 1)/(p_2 q_1 - 1) \geq N/2$ with $\gamma = \max(p_2, q_1)$, then the only solution of the problem (1) is the trivial one, i.e., $u \equiv v \equiv 0$. Later in [3] Escobedo and Levine showed that if $p_1 \geq 1$ and $p_2 + q_2 \geq p_1 + q_1 > 0$, then the problem (1) behaves like the Cauchy problem for the single equation $u_t - \Delta u = u^{p_1+q_1}$, with respect to Fujita-type blowup theorems, see [5]. In [15], Yamauchi considired the problem

$$\begin{cases} u_t - \Delta u = |x|^{\sigma_1}|u|^{p_1}|v|^{q_1}, \\ v_t - \Delta v = |x|^{\sigma_2}|u|^{p_2}|v|^{q_2}, \\ u(0,x) = u_0(x) \geq 0, \not\equiv 0, \\ v(0,x) = v_0(x) \geq 0, \not\equiv 0, \end{cases}$$

where p_i, $q_i \geq 0$, $\sigma_i \geq \max(-2, -N)$, $i = 1, 2$. He proved a nonexistence results under some conditions concerning relation between exponents p_i, q_i, σ_i and initial data.

In the case of real order $0 < \alpha_i < 1$ and $1 \leq \beta_i \leq 2$, Kirane *et al.* [8] considered the following Cauchy problem

$$\begin{cases} \mathbf{D}^{\alpha_1}_{0|t}u + (-\Delta)^{\beta_1/2}u = |v|^{q_1}, \\ \mathbf{D}^{\alpha_2}_{0|t}v + (-\Delta)^{\beta_2/2}v = |u|^{p_2}, \\ u(0,x) = u_0(x) \geq 0, \\ v(0,x) = v_0(x) \geq 0, \end{cases} \tag{2}$$

and they proved that if

$$q_1 > 1,\ p_2 > 1,\ q_1 q_1' = q_1 + q_1',\ p_2 p_2' = p_2 + p_2'$$

and

$$N \leq \max\left\{ \frac{\frac{\alpha_2}{p_2} + \alpha_1 - (1 - \frac{1}{p_2 q_1})}{\frac{\alpha_2}{\beta_2 p_2 q_1'} + \frac{\alpha_1}{\beta_1 p_2'}}, \frac{\frac{\alpha_1}{q_1} + \alpha_2 - (1 - \frac{1}{p_2 q_1})}{\frac{\alpha_1}{\beta_1 p_2' q_1} + \frac{\alpha_2}{\beta_2 q_1'}} \right\},$$

then the problem (2) does not admit nontrivial global weak nonnegative solutions.

The aim of this paper is to prove a nonexistence result which is more general than the above interesting result cited in [8] who concerns the case $p_1 = q_2 = 0$.

II. Preliminaries

In this section, we present some definitions and results concerning fractional derivatives that will be used hereafter.

Definition 1: Let $0 < \alpha < 1$ and $\phi \in L^1(0, T)$. The left-sided and right-sided Riemann-Liouville derivatives of order α for ϕ are defined, respectively, by:

$$D^{\alpha}_{0|t}\phi(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{\phi(\sigma)}{(t-\sigma)^{\alpha}}d\sigma,$$

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and

$$D^{\alpha}_{t|T}\phi(t) = -\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_t^T \frac{\phi(\sigma)}{(\sigma-t)^{\alpha}}d\sigma,$$

where Γ denotes the gamma function.

Definition 2: Let $0 < \alpha < 1$ and $\phi' \in L^1(0,T)$. The left-sided and respectively right-sided Caputo derivatives of order α for ϕ' are defined as:

$$\mathbf{D}^{\alpha}_{0|t}\phi(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{\phi'(\sigma)}{(t-\sigma)^{\alpha}}d\sigma,$$

and

$$\mathbf{D}^{\alpha}_{t|T}\phi(t) = -\frac{1}{\Gamma(1-\alpha)}\int_t^T \frac{\phi'(\sigma)}{(\sigma-t)^{\alpha}}d\sigma,$$

where Γ denotes, as usual, the gamma function.
The relation between Caputo and Riemann-Liouville derivatives is written as

$$\mathbf{D}^{\alpha}_{0|t}\phi(t) = D^{\alpha}_{0|t}[\phi(t)-\phi(0)].$$

Finally, taking into account the following integration by parts formula:

$$\int_0^T f(t)(D^{\alpha}_{0|t}g)(t)dt = \int_0^T (D^{\alpha}_{t|T}f)(t)g(t)dt,$$

we adopt the following definition concerning the weak formulation for the problem (1).

Definition 3: Let $Q_T = (0,T)\times R^N, 0 < T < +\infty$. We say that $(u,v) \in (L^1_{loc}(Q_T))^2$is a local weak solution to problem (1) on Q_T, if $u^{p_i}v^{q_i} \in L^1_{loc}(Q_T), i = 1,2$, and it is such that

$$\begin{aligned}&\int_{Q_T} u_0 D^{\alpha_1}_{t|T}\varphi_1 dtdx + \int_{Q_T} |u|^{p_1}|v|^{q_1}\varphi_1 dtdx \\ &= \int_{Q_T} u D^{\alpha_1}_{t|T}\varphi_1 dtdx + \int_{Q_T} u(-\Delta)^{\frac{\beta_1}{2}}\varphi_1 dtdx, \qquad (3)\end{aligned}$$

and

$$\begin{aligned}&\int_{Q_T} v_0 D^{\alpha_2}_{t|T}\varphi_2 dtdx + \int_{Q_T} |u|^{p_2}|v|^{q_2}\varphi_2 dtdx + \\ &= \int_{Q_T} v D^{\alpha_2}_{t|T}\varphi_2 dtdx + \int_{Q_T} v(-\Delta)^{\frac{\beta_2}{2}}\varphi_2 dtdx. \qquad (4)\end{aligned}$$

for all test functions $\varphi_i \in C^{1,2}_{t,x}(Q_T)$ such that $\varphi_i(T,x) = 0$, $i = 1,2$.

III. Main results

We now state our main result as follows.

Theorem 1: Let $p_1 \geq 0, q_2 \geq 0, p_2 > 1, q_1 > 1$, $p_2p_2' = p_2 + p_2'$, $q_1q_1' = q_1 + q_1'$. Let u_0, v_0 in $L^{\infty}(R^N)$ suth that $u_0, v_0 \geq 0$ and $u_0, v_0 \not\equiv 0$. If

$$N \leq \max\left\{\frac{\frac{\alpha_2}{p_2}+\alpha_1-(1-\frac{1}{p_2q_1})}{\frac{\alpha_2}{\beta_2p_2q_1'}+\frac{\alpha_1}{\beta_1p_2'}}, \frac{\frac{\alpha_1}{q_1}+\alpha_2-(1-\frac{1}{p_2q_1})}{\frac{\alpha_1}{\beta_1p_2'q_1}+\frac{\alpha_2}{\beta_2q_1'}}\right\},$$

then the problem (1) does not admit global weak solutions.

Proof: The proof is by contradiction. Suppose that (u,v) is a global weak solution to problem (1). Since $u_0, v_0 \geq 0$ and $u_0, v_0 \not\equiv 0$, then $u(t), v(t) > 0$ for all $t \in (0,T^*)$ for any arbitrary $T^* > 0$.
Let T and θ be two real numbers such that

$$0 < T < T^* \text{ and } \theta = \min\left\{\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}\right\}.$$

Let $\Phi \in C^2_0(R_+)$, $0 \leq \Phi(r) \leq 1$ for all $r \geq 0$, a smooth nonnegative nonincreasing function such that

$$\Phi(r) = \begin{cases} 1 & \text{if} \quad 0 \leq r \leq 1, \\ 0 & \text{if} \quad r \geq 2, \end{cases}$$

and

$$\Psi(s) = \begin{cases} (1-s)^{\gamma} & \text{if} \quad 0 \leq s \leq 1, \\ 0 & \text{if} \quad s \geq 1, \end{cases}$$

where $\gamma \geq \max\{\alpha_1q_1', \alpha_2p_2'\}$.
We choose

$$\varphi_i(t,x) = \phi^l(x)\psi(t), \quad i = 1,2.$$

with

$$\phi(x) = \Phi(\frac{|x|}{T^{\theta}}),\ \psi(t) = \Psi(\frac{t}{T}) \text{ and } l \geq \max\{q_1', p_2'\}.$$

Applying Hölder inequality to the right hand sides of the weak formulations (3) and (4) on Σ, where

$$\Sigma = (0,T)\times\{x \in R^N : |x| \leq 2T^{\theta}\},$$

we obtain

$$\int_{\Sigma} u|D^{\alpha}_{t|T}\varphi_1| \leq \left(\int_{\Sigma}|u|^{p_2}|v|^{q_2}\varphi_2\right)^{\frac{1}{p_2}}\cdot A,$$

and

$$\int_{\Sigma} u|(-\Delta)^{\frac{\beta_1}{2}}\varphi_1| \leq \left(\int_{\Sigma}|u|^{p_2}v^{q_2}\varphi_2\right)^{\frac{1}{p_2}}\cdot B,$$

where

$$A = \left(\int_{\Sigma}|D^{\alpha_1}_{t|T}\varphi_1|^{p_2'}|v|^{-\frac{p_2'q_2}{p_2}}\varphi_2^{-\frac{p_2'}{p_2}}\right)^{\frac{1}{p_2'}},$$

and

$$B = \left(\int_{\Sigma}|(-\Delta)^{\frac{\beta_1}{2}}\varphi_1|^{p_2'}|v|^{-\frac{p_2'q_2}{p_2}}\varphi_2^{-\frac{p_2'}{p_2}}\right)^{\frac{1}{p_2'}}.$$

Consequently

$$\int_{\Sigma}|u|^{p_1}|v|^{q_1}\varphi_1 \leq \left(\int_{\Sigma}|u|^{p_2}|v|^{q_2}\varphi_2\right)^{\frac{1}{p_2}}\cdot A_1, \qquad (5)$$

where

$$A_1 = A + B.$$

The same way give us the next estimate

$$\int_{\Sigma}|u|^{p_2}|v|^{q_2}\varphi_2 \leq \left(\int_{\Sigma}|u|^{p_1}|v|^{q_1}\varphi_1\right)^{\frac{1}{q_1}}\cdot A_2, \qquad (6)$$

where

$$A_2 = \left(\int_{\Sigma}|D^{\alpha_2}_{t|T}\varphi_2|^{q_1'}|u|^{-\frac{p_1q_1'}{q_1}}\varphi_1^{-\frac{q_1'}{q_1}}\right)^{\frac{1}{q_1'}} + \left(\int_{\Sigma}|(-\Delta)^{\frac{\beta_2}{2}}\varphi_2|^{q_1'}|u|^{-\frac{p_1q_1'}{q_1}}\varphi_1^{-\frac{q_1'}{q_1}}\right)^{\frac{1}{q_1'}}.$$

Using (5) and (6) one can write

$$\left(\int_{\Sigma}|u|^{p_1}|v|^{q_1}\varphi_1\right)^{1-\frac{1}{p_2q_1}} \leq A_1A_2^{\frac{1}{p_2}}, \qquad (7)$$

and

$$\left(\int_{\Sigma}|u|^{p_2}|v|^{q_2}\varphi_2\right)^{1-\frac{1}{p_2q_1}} \leq A_1^{\frac{1}{q_1}}A_2. \qquad (8)$$

Therefore, as $u>0$ and $v>0$, then using Ju's inequality $(-\Delta)^{\frac{\beta_i}{2}}\phi^l \leq l\phi^{l-1}(-\Delta)^{\frac{\beta_i}{2}}\phi$ (see [7]), and introducing the change of variables $t=T\tau,\ \ x=T^{\frac{\alpha_i}{\beta_i}}\xi$ in A_i, we obtain

$$\left(\int_{\Sigma}|u|^{p_1}|v|^{q_1}\varphi_1\right)^{1-\frac{1}{p_2q_1}} \leq CT^{-\left(\frac{\gamma_1}{p_2'}+\frac{\gamma_2}{p_2q_1'}\right)}, \qquad (9)$$

and

$$\left(\int_{\Sigma}|u|^{p_2}|v|^{q_2}\varphi_2\right)^{1-\frac{1}{p_2q_1}} \leq CT^{-\left(\frac{\gamma_1}{p_2'q_1}+\frac{\gamma_2}{q_1'}\right)}, \qquad (10)$$

where

$$\gamma_1 = \alpha_1p_2'-\frac{\alpha_1}{\beta_1}N-1,$$
$$\gamma_2 = \alpha_2q_1'-\frac{\alpha_2}{\beta_2}N-1.$$

Now, if we choose $N<N^*$ and pass to the limit in (9) and (10), as T goes to infinity, we get

$$\int_{R^+\times R^N}|u|^{p_1}|v|^{q_1}\varphi_1=0,$$

or

$$\int_{R^+\times R^N}|u|^{p_2}|v|^{q_2}\varphi_2=0.$$

Using the dominated convergence theorem and the continuity in time and space of u and v, we infer that

$$\int_{R^+\times R^N}|u|^{p_1}|v|^{q_1}=0,$$

or

$$\int_{R^+\times R^N}|u|^{p_2}|v|^{q_2}=0.$$

This implies that $u\equiv 0$ or $v\equiv 0$, which is a contradiction.

In the case $N=N^*$, we modify the previous function ϕ by introducing a new number R, $0<R<T$, such that

$$\phi(x)=\Phi\left(\frac{|x|}{(T/R)^{\theta}}\right),$$

and we set

$$\Sigma_R=(0,T)\times\left\{x\in R^N:|x|\leq 2(T/R)^{\theta}\right\},$$

$$\Delta_R=(0,T)\times\left\{x\in R^N:(T/R)^{\theta}\leq|x|\leq 2(T/R)^{\theta}\right\}.$$

Since, from (9) and (10), we find that

$$\left(\int_{R^+\times R^N}|u|^{p_1}|v|^{q_1}\varphi_1\right)^{1-\frac{1}{p_2q_1}} \leq C,$$

or

$$\left(\int_{R^+\times R^N}|u|^{p_2}|v|^{q_2}\varphi_2\right)^{1-\frac{1}{p_2q_1}} \leq C,$$

Then we have

$$\lim_{T\longrightarrow+\infty}\int_{\Delta_R}|u|^{p_1}|v|^{q_1}\varphi_1dtdx=0, \qquad (11)$$

or

$$\lim_{T\longrightarrow+\infty}\int_{\Delta_R}|u|^{p_2}|v|^{q_2}\varphi_2dtdx=0. \qquad (12)$$

Applying Hölder inequality to the right hand sides of the weak formulations (3) and (4) on Σ_R, we obtain

$$\int_{\Sigma_R}u|D^{\alpha}_{t|T}\varphi_1| \leq \left(\int_{\Sigma_R}|u|^{p_2}|v|^{q_2}\varphi_2\right)^{\frac{1}{p_2}}\cdot B_1,$$

and

$$\int_{\Sigma_R}u|(-\Delta)^{\frac{\beta_1}{2}}\varphi_1| \leq \left(\int_{\Sigma_R}|u|^{p_2}|v|^{q_2}\varphi_2\right)^{\frac{1}{p_2}}\cdot C_1,$$

where

$$B_1=\left(\int_{\Sigma_R}|D^{\alpha_1}_{t|T}\varphi_1|^{p_2'}|v|^{-\frac{p_2'q_2}{p_2}}\varphi_2^{-\frac{p_2'}{p_2}}\right)^{\frac{1}{p_2'}},$$

and

$$C_1=\left(\int_{\Delta_R}|(-\Delta)^{\frac{\beta_1}{2}}\varphi_1|^{p_2'}|v|^{-\frac{p_2'q_2}{p_2}}\varphi_2^{-\frac{p_2'}{p_2}}\right)^{\frac{1}{p_2'}}.$$

Consequently

$$\begin{aligned}\textstyle\int_{\Sigma_R}|u|^{p_1}|v|^{q_1}\varphi_1 &\leq \left(\textstyle\int_{\Sigma_R}|u|^{p_2}|v|^{q_2}\varphi_2\right)^{\frac{1}{p_2}}\cdot B_1\\ &+\left(\textstyle\int_{\Delta_R}|u|^{p_2}|v|^{q_2}\varphi_2\right)^{\frac{1}{p_2}}\cdot C_1,\end{aligned} \qquad (13)$$

The same way give us the next estimate

$$\begin{aligned}\textstyle\int_{\Sigma_R}|u|^{p_2}|v|^{q_2}\varphi_2 &\leq \left(\textstyle\int_{\Sigma_R}|u|^{p_1}|v|^{q_1}\varphi_1\right)^{\frac{1}{q_1}}\cdot B_2\\ &+\left(\textstyle\int_{\Delta_R}|u|^{p_1}|v|^{q_1}\varphi_1\right)^{\frac{1}{q_1}}\cdot C_2,\end{aligned} \qquad (14)$$

where

$$B_2=\left(\int_{\Sigma}|D^{\alpha_2}_{t|T}\varphi_2|^{q_1'}|u|^{-\frac{p_1q_1'}{q_1}}\varphi_1^{-\frac{q_1'}{q_1}}\right)^{\frac{1}{q_1'}},$$

and

$$C_2=\left(\int_{\Delta_R}|(-\Delta)^{\frac{\beta_2}{2}}\varphi_2|^{q_1'}|u|^{-\frac{p_1q_1'}{q_1}}\varphi_1^{-\frac{q_1'}{q_1}}\right)^{\frac{1}{q_1'}}.$$

If we introduce the change of variables

$$t=T\tau,\ \ x=(T/R)^{\frac{\alpha_i}{\beta_i}}\xi$$

in B_i and C_i, and using (13) and (14), we obtain via (11) and (12), after passing the limit as T goes to infinity

$$\left(\int_{R^+\times R^N}|u|^{p_1}|v|^{q_1}\varphi_1\right)^{1-\frac{1}{p_2q_1}} \leq CR^{-\left(\frac{\gamma_1'}{p_2'}+\frac{\gamma_2'}{p_2q_1'}\right)}, \qquad (15)$$

or

$$\left(\int_{R^+\times R^N}|u|^{p_2}|v|^{q_2}\varphi_2\right)^{1-\frac{1}{p_2q_1}} \leq CR^{-\left(\frac{\gamma_1'}{p_2'q_1}+\frac{\gamma_2'}{q_1'}\right)}, \qquad (16)$$

where

$$\gamma_i' = \frac{\alpha_i}{\beta_i} N, \ i = 1, 2.$$

Then, taking the limit when R goes to infinity, we obtain $u \equiv 0$ or $v \equiv 0$, contradiction.

Remark 1: When $p_1 = q_2 = 0$, we recover the case studied by Kirane *et al.* [8], however we have to impose the constraints $u_0 \geq 0, \not\equiv 0$ and $v_0 \geq 0, \not\equiv 0$, while Kirane *et al.* require only the conditions $u_0 \geq 0$ and $v_0 \geq 0$.

Remark 2: We can extend our result to the more general system

$$\begin{cases} \mathbf{D}^{\alpha_1}_{0|t} u + (-\Delta)^{\beta_1/2} \left(|u|^{m-1} u \right) = h|u|^{p_1}|v|^{q_1} + g|u|^{r_1}|v|^{s_1}, \\ \\ \mathbf{D}^{\alpha_2}_{0|t} v + (-\Delta)^{\beta_2/2} \left(|v|^{m-1} v \right) = k|u|^{p_2}|v|^{q_2} + l|u|^{r_2}|v|^{s_2}, \end{cases}$$

under some suitable conditions on $h(t,x)$, $g(t,x)$, $k(t,x)$ and $l(t,x)$.

IV. Conclusion

We have established some new results for a class of nonlinear fractional differential systems with nonlinear terms of the form $|u|^{p_i}|v|^{q_i}$, $i = 1, 2$ where $p_1, q_2 \geq 0$ and $p_2, q_1 > 1$ are constants. We note that these results can also be applied to study the problem (1) for a high order $\alpha_i > 1$. Finally, we wish that the present manuscript will open wide avenues for further research in the field of fractional calculus and other domains.

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