Analytical Solution of Time-Fractional Drinfeld-Sokolov-Wilson System Using Residual Power Series Method

H.M. Jaradat, Safwan Al-Shar'a, Qamar J.A. Khan, Marwan Alquran, Kamel Al-Khaled

***Abstract*—The aim of this paper is to introduce a novel study of obtaining an approximate solution to a generalized time-fractional Drinfeld-Sokolov-Wilson system. An iterative technique based on the generalized Taylor series residual power series (RPS) is extended to handle such a system. Description of the method is given and the obtained results reveal that RPS is a new significant method for exploring nonlinear fractional models.**

***Index Terms*—Generalized Taylor series, Residual power series, Time-fractional Drinfeld-Sokolov-Wilson system.**

I. Introduction

FUNDAMENTAL works and developments on the fractional derivative and fractional differential equations have been done over the past four decades. Oldham and Spanier-1974 [1], Miller and Ross-1993 [2], Samko et al-1993 [3], Podlubny-1999 [4], Kilbas et al-2006 [5] and others [6], [7] are the pioneer in this field; their works form an introduction to the theory of fractional differential equations and provide a systematic understanding of the fractional calculus such as the existence and the uniqueness of solutions. In [8], Hernandez et al-2010 published a paper on recent developments in the theory of abstract differential equations with fractional derivative. Application of financial risk assessment model for predicting market behavior has been used in [9] that uses a solution to a non-stationary fractional diffusion equation. An application of telescoping decomposition method is developed in [10] for ordinary differential equations and is extended to derive approximate analytical solutions of fractional differential equations. The authors in [11], [12] discussed the nonexistence of weak solutions and blow up solutions to nonlinear fractional wave equations. Finally, interested various applications of fractional calculus in the field of interdisciplinary sciences such as image processing and control theory have been studied by Magin et al [13] and Mainardi [14].

In general, there exists no method that produce an exact solution for nonlinear fractional differential equations defined in the Caputo fractional derivative and Riemann-Liouville. Only approximate solutions can be derived using linearization or successive or perturbation methods. Such methods are: Variational iteration method and multivariate Pade approximations [15], Iterative Laplace transform method [16], decomposition method [17], [18], [19], [20], Homotopy analysis method [21], [22] and Sumudu transform method [23]. In 2007, a very restricted modified Riemann-Liouville derivative is defined by Jumarie [24]. He suggested a simple transformation that converts fractional nonlinear partial differential equations (PDEs) into classical PDEs. Based on Jumarie definition, authors could use different solitary wave methods to construct exact solutions. For example, the well known (G'/G)-expansion method is used in [25], [26], [27] to seek solitary wave solutions for space-time fractional nonlinear PDEs.

The main objective of this paper is to present a new generalization of Drinfeld-Sokolov-Wilson (DSW) system by replacing the first order time derivative by a fractional derivative of order α, and takes the form

$$\begin{aligned} D_t^{\alpha} u &= -a v v_x, \\ D_t^{\alpha} v &= -b v_{xxx} - \gamma u v_x - \epsilon v u_x, \end{aligned} \qquad (1)$$

where $u = u(x,t),\ v = v(x,t)$ and $a,\ b,\ \gamma,\ \epsilon$ are nonzero parameters and α is the fractional derivative with $0 \le \alpha \le 1$. Note that for $\alpha = 1$, Equation (1) is reduced to the standard Drinfeld-Sokolov-Wilson system which was first proposed by Drinfeld and Sokolov [28], [29] and Wilson [30] when $a = 1,\ b = \gamma = 2,\ \epsilon = 1$. To best of our knowledge the system given in (1) is new and to be explored in this study where the residual power series method is adopted [31], [32], [33], [34].

The pattern of the current paper is as follows: In section 2, some definitions and theorems regards Caputo's derivative and fractional power series are given. Detailed derivation of the RPS solution of the fractional DSW system has been discussed in section 3. Finally, the performance of the RPS method has been tested in section 4 by considering a specific example of the fractional DSW system.

II. Preliminaries

Many definitions and studies of fractional calculus have been proposed in the literature. These definitions include: Grunwald-Letnikov, Riemann-Liouville, Weyl, Riesz and Caputo sense. In the Caputo case, the derivative of a constant is zero and one can define, properly, the initial conditions for the fractional differential equations which can be handled by using an analogy with the classical integer

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case. For these reasons, researchers prefer to use the Caputo fractional derivative [35] which is defined as

Definition 1. For m to be the smallest integer that exceeds α, the Caputo fractional derivatives of order $\alpha > 0$ is defined as

$$D_t^\alpha u(x,t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} \frac{\partial^m u(x,\tau)}{\partial \tau^m} d\tau, \\ m-1 < \alpha < m \\ \frac{\partial^m u(x,t)}{\partial t^m}, \quad \alpha = m \in N \end{cases} \tag{2}$$

Now, we survey some needed definitions and theorems regards the fractional power series, where there is much theory to be found in [36], [37].

Definition 2. A power series expansion of the form

$\sum_{m=0}^{\infty} c_m (t-t_0)^{m\alpha} = c_0 + c_1(t-t_0)^\alpha + c_2(t-t_0)^{2\alpha} + \ldots \quad : 0 \le n-1 < \alpha \le n,\ t \le t_0$

is called fractional power series PS about $t = t_0$

Theorem 1. Suppose that f has a fractional PS representation at $t = t_0$ of the form

$f(t) = \sum_{m=0}^{\infty} c_m (t-t_0)^{m\alpha}, \quad t_0 \le t < t_0 + R.$

If $D^{m\alpha} f(t),\ \ m = 0, 1, 2, ..$ are continuous on $(t_0, t_0 + R)$, then $c_m = \frac{D^{m\alpha} f(t_0)}{\Gamma(1+m\alpha)}$.

Definition 3. A power series expansion of the form

$\sum_{m=0}^{\infty} f_m(x)(t-t_0)^{m\alpha}$

is called multiple fractional power series PS about $t = t_0$

Theorem 2. Suppose that $u(x,t)$ has a multiple fractional PS representation at $t = t_0$ of the form

$u(x,t) = \sum_{m=0}^{\infty} f_m(x)(t-t_0)^{m\alpha}, \quad x \in I,\ t_0 \le t < t_0 + R.$

If $D_t^{m\alpha} u(x,t), \quad m = 0, 1, 2, ..$ are continuous on $I \times (t_0, t_0 + R)$, then $f_m(x) = \frac{D_t^{m\alpha} u(x,t_0)}{\Gamma(1+m\alpha)}$.

From the last theorem, it is clear that if $n+1$-dimensional function has a multiple fractional PS representation at $t = t_0$, then it can be derived in the same manner. i.e.

Corollary 1. Suppose that $u(x,y,t)$ has a multiple fractional PS representation at $t = t_0$ of the form

$u(x,y,t) = \sum_{m=0}^{\infty} g_m(x,y)(t-t_0)^{m\alpha}, \quad (x,y) \in I_1 \times I_2, \quad t_0 \le t < t_0 + R.$

If $D_t^{m\alpha} u(x,y,t), \quad m = 0, 1, 2, ..$ are continuous on $I_1 \times I_2 \times (t_0, t_0 + R)$, then $g_m(x,y) = \frac{D_t^{m\alpha} u(x,y,t_0)}{\Gamma(1+m\alpha)}$.

Next, we present in details the derivation of the residual power series solution to the generalized fractional DSW system.

III. Residual power series (RPS) for solving time-fractional DSW system

Consider the time-fractional DSW system

$$\begin{aligned} D_t^\alpha u &= -avv_x, \\ D_t^\alpha v &= -bv_{xxx} - \gamma u v_x - \epsilon v u_x, \end{aligned} \tag{3}$$

subject to the initial conditions:

$$\begin{aligned} u(x,0) &= f(x), \\ v(x,0) &= g(x). \end{aligned} \tag{4}$$

The RPS method propose the solution for Eqs. (3-4) as a fractional PS about the initial point $t = 0$

$$\begin{aligned} u(x,t) &= \sum_{n=0}^{\infty} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \\ v(x,t) &= \sum_{n=0}^{\infty} g_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \end{aligned} \tag{5}$$

where $0 < \alpha \le 1,\ x \in I,\ 0 \le t < R$. Next, we let $u_k(x,t),\ v_k(x,t)$ to denote the k-th truncated series of $u(x,t),\ v(x,t)$, respectively, i.e.

$$\begin{aligned} u_k(x,t) &= \sum_{n=0}^{k} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \\ v_k(x,t) &= \sum_{n=0}^{k} g_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}. \end{aligned} \tag{6}$$

It is clear that by condition (4) the 0-th RPS approximate solutions of $u(x,t)$ and $v(x,t)$ are

$$\begin{aligned} u_0(x,t) &= f_0(x) = u(x,0) = f(x) \\ v_0(x,t) &= g_0(x) = v(x,0) = g(x). \end{aligned} \tag{7}$$

Also, Eqs. (6) can be written as

$$\begin{aligned} u_k(x,t) &= f(x) + \sum_{n=1}^{k} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \\ v_k(x,t) &= g(x) + \sum_{n=1}^{k} g_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \end{aligned} \tag{8}$$

where $0 < \alpha \le 1,\ x \in I,\ 0 \le t < R,\ k = 1, 2, 3, \ldots$. Now, we define the residual functions, $Res_u,\ Res_v$, for Eqs. (3)

$$\begin{aligned} Res_u &= D_t^\alpha u + avv_x, \\ Res_v &= D_t^\alpha v + bv_{xxx} + \gamma u v_x + \epsilon u_x v, \end{aligned} \tag{9}$$

and therefore, the k-th residual functions, $Res_{u,k},\ Res_{v,k}$, are

$$\begin{aligned} Res_{u,k} &= D_t^\alpha u_k + av_k \frac{\partial v_k}{\partial x}, \\ Res_{v,k} &= D_t^\alpha v_k + b\frac{\partial^3 v_k}{\partial x^3} + \gamma u_k \frac{\partial v_k}{\partial x} + \epsilon \frac{\partial u_k}{\partial x} v_k. \end{aligned} \tag{10}$$

Authors in [38], [39] showed that $Res(x,t) = 0$ and $\lim_{k\to\infty} Res_k(x,t) = Res(x,t)$ for all $x \in I$ and $t \ge 0$. Therefore, $D_t^{r\alpha} Res(x,t) = 0$ since the fractional derivative of a constant in the Caputo's sense is 0. Also, the fractional derivative $D_t^{r\alpha}$ of $Res(x,t)$ and $Res_k(x,t)$ are matching at

$t=0$ for each $r=0,1,2,...,k$.

To clarify the RPS technique, we substitute the k-th truncated series of $u(x,t)$, $v(x,t)$ into Eqs. (10), find the fractional derivative formula $D_t^{(k-1)\alpha}$ of both $Res_{u,k}(x,t)$, $Res_{v,k}$, $k=1,2,3,...$, and then, we solve the obtained algebraic system

$$\begin{aligned} D_t^{(k-1)\alpha}Res_{u,k}(x,0) &= 0,\\ D_t^{(k-1)\alpha}Res_{v,k}(x,0) &= 0, \end{aligned} \tag{11}$$

to get the required coefficients $f_n(x)$, $g_n(x)$, $n=1,2,3,...,k$ in Eqs. (8). Now, we follow the following steps.

Step 1. To determine $f_1(x)$, $g_1(x)$, we consider $(k=1)$ in (10)

$$\begin{aligned} Res_{u,1}(x,t) &= D_t^\alpha u_1 + a v_1\frac{\partial v_1}{\partial x},\\ Res_{v,1}(x,t) &= D_t^\alpha v_1 + b\frac{\partial^3 v_1}{\partial x^3} + \gamma u_1\frac{\partial v_1}{\partial x}\\ &+ \epsilon\frac{\partial u_1}{\partial x}v_1. \end{aligned} \tag{12}$$

But, $u_1(x,t)=f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}$ and $v_1(x,t)=g(x)+g_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}$. Therefore,

$$\begin{aligned} Res_{u,1}(x,t) &= f_1(x) + a\left(g(x)+g_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\ &\times \left(g'(x)+g_1'(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right),\\ Res_{v,1}(x,t) &= g_1(x) + b\left(g'''(x)+g_1'''(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\ &+ \gamma\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\ &\times \left(g'(x)+g_1'(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\ &+ \epsilon\left(f'(x)+f_1'(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\ &\times \left(g(x)+g_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right). \end{aligned} \tag{13}$$

From Eqs. (11) we deduce that $Res_{u,1}(x,0)=0$, $Res_{v,1}(x,0)=0$ and thus,

$$\begin{aligned} f_1(x) &= -ag(x)g'(x),\\ g_1(x) &= -bg'''(x)-\gamma f(x)g'(x)-\epsilon f'(x)g(x). \end{aligned} \tag{14}$$

Therefore, the 1-st RPS approximate solutions are

$$\begin{aligned} u_1(x,t) &= f(x)-ag(x)g'(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\\ v_1(x,t) &= g(x)-(bg'''(x)+\gamma f(x)g'(x)+\epsilon f'(x)g(x))\\ &\times \frac{t^\alpha}{\Gamma(1+\alpha)}. \end{aligned} \tag{15}$$

Step 2. To obtain $f_2(x)$, $g_2(x)$, we substitute the 2-nd truncated series $u_2(x,t)=f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}$ and $v_2(x,t)=g(x)+g_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+g_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}$ into the 2-nd residual function $Res_{u,2}(x,t)$ and $Res_{v,2}(x,t)$, i.e.

$$\begin{aligned} Res_{u,2}(x,t) &= D_t^\alpha u_2 + a v_2\frac{\partial v_2}{\partial x}\\ &= f_1(x)+\frac{f_2(x)t^\alpha}{\Gamma(1+\alpha)}\\ &+ a\left(g(x)+...+\frac{g_2(x)t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &\times \left(g'(x)+...+\frac{g_2'(x)t^{2\alpha}}{\Gamma(1+2\alpha)}\right) \end{aligned} \tag{16}$$

and

$$\begin{aligned} Res_{v,2}(x,t) &= D_t^\alpha v_2 + b\frac{\partial^3 v_2}{\partial x^3} + \gamma u_2\frac{\partial v_2}{\partial x} + \epsilon\frac{\partial u_2}{\partial x}v_2\\ &= g_1(x)+g_2(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\\ &+ b\left(g'''(x)+...+g_2'''(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &+ \gamma\left(f(x)+...+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &\times \left(g'(x)+...+g_2'(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &+ \epsilon\left(f'(x)+...+f_2'(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &\times \left(g(x)+...+g_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right). \end{aligned} \tag{17}$$

Applying D_t^α on both sides of Eqs. (16) and (17) gives

$$\begin{aligned} D_t^\alpha Res_{u,2}(x,t) &= f_2(x) + a\left(g_1(x)+\frac{g_2(x)t^\alpha}{\Gamma(1+\alpha)}\right)\\ &\times \left(g'(x)+...+\frac{g_2'(x)t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &+ a\left(g(x)+...+\frac{g_2(x)t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &\times \left(g_1'(x)+\frac{g_2'(x)t^\alpha}{\Gamma(1+\alpha)}\right) \end{aligned} \tag{18}$$

and

$$\begin{aligned} D_t^\alpha Res_{v,2}(x,t) &= g_2(x) + b\left(g_1'''(x)+g_2'''(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\ &+ \gamma\left(f_1(x)+f_2(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\ &\times \left(g'(x)+...+g_2'(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &+ \gamma\left(f(x)+...+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &\times \left(g_1'(x)+g_2'(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\ &+ \epsilon\left(f_1'(x)+f_2'(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\ &\times \left(g(x)+...+g_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &+ \epsilon\left(f'(x)+...+f_2'(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\ &\times \left(g_1(x)+g_2(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right). \end{aligned} \tag{19}$$

By the fact that $D_t^{\alpha} Res_{u,2}(x,0) = 0 = D_t^{\alpha} Res_{v,2}(x,0)$ and solving the resulting system in (18) and (19) for the unknown coefficient functions $f_2(x), g_2(x)$, we get

$$f_2(x) = -a(g(x)g_1'(x) + g_1(x)g'(x)),$$

$$\begin{aligned} g_2(x) &= -bg_1'''(x) - \gamma(f_1(x)g'(x) + f(x)g_1'(x)) \\ &- \epsilon(f'(x)g_1(x) + f_1'(x)g(x)). \end{aligned} \tag{20}$$

Step 3. We can obtain the other coefficients $f_i(x),\ g_i(x)$ by expanding and differentiating once in accordance of having $D_t^{(i-1)\alpha} Res_{u,i}(x,0) = D_t^{(i-1)\alpha} Res_{v,i}(x,0) = 0$. Therefore,

$$f_3(x) = -a\left(g_2(x)g'(x) + 2g_1(x)g_1'(x) + g(x)g_2'(x)\right),$$

$$\begin{aligned} g_3(x) &= -bg_2'''(x) \\ &- \gamma\left(f_2(x)g'(x) + 2f_1(x)g_1'(x) + f(x)g_2'(x)\right) \\ &- \epsilon\left(f'(x)g_2(x) + 2f_1'(x)g_1(x) + f'(x)g_2(x)\right). \end{aligned} \tag{21}$$

Step 4. Considering the results obtained in (21) and follow the same process, then we reach at

$$\begin{aligned} f_4(x) &= -a\{g_3(x)g'(x) + 3g_2(x)g_1'(x) + 3g_1(x)g_2'(x) \\ &+ g(x)g_3'(x)\}, \end{aligned}$$

$$\begin{aligned} g_4(x) &= -bg_3'''(x) - \gamma\{f_3(x)g'(x) + 3f_2(x)g_1'(x) \\ &+ 3f_1(x)g_2'(x) + f(x)g_3'(x)\} \\ &- \epsilon\{f'(x)g_3(x) + 3f_1'(x)g_2(x) \\ &+ 3f_2'(x)g_1(x) + f_3'(x)g(x)\}. \end{aligned} \tag{22}$$

Finally, we derive the equations given in (22) in a way that obey the fact $D_t^{4\alpha} Res_{u,5}(x,0) = D_t^{4\alpha} Res_{v,5}(x,0) = 0$. Thus

$$\begin{aligned} f_5(x) &= -a\{g_4(x)g'(x) + 4g_3(x)g_1'(x) + 6g_2(x)g_2'(x) \\ &+ 4g_1(x)g_3'(x) + g(x)g_4'(x)\}, \end{aligned}$$

$$\begin{aligned} g_5(x) &= -bg_4'''(x) - \gamma\{f_4(x)g'(x) + 4f_3(x)g_1'(x) \\ &+ 6f_2(x)g_2'(x) + 4f_1(x)g_3'(x) + f(x)g_4'(x)\} \\ &- \epsilon\{f'(x)g_4(x) + 4f_1'(x)g_3(x) \\ &+ 6f_2'(x)g_2(x) + 4f_3'(x)g_1(x) + f_4'(x)g(x)\}. \end{aligned} \tag{23}$$

It is to be noted here that the RPS method is a numerical technique based on the generalized Taylor series formula which constructs an analytical solution in the form of a convergent series. Therefore, one can achieve a good approximation with the exact solution by using few terms only and thus, the overall errors can be made smaller by adding more new terms of the RPS approximations. So, by the above obtained 5-th RPS approximate solution, we present some graphical results regards the time-fractional DSW system.

IV. Numerical example

The purpose of this section is to test the derivation of residual power series solutions of DSW system.

Example If we set $a = 3,\ b = 2,\ \gamma = 2,\ \epsilon = 1$, the DSW is reduced to [40]

$$\begin{aligned} D_t^{\alpha} u &= -3vv_x, \\ D_t^{\alpha} v &= -2v_{xxx} - 2uv_x - vu_x), \end{aligned} \tag{24}$$

subject to the initial conditions:

$$\begin{aligned} u(x,0) &= f(x) = -\frac{3c}{2}\text{sech}^2\left(\sqrt{\frac{c}{2}}(x-ct)\right), \\ v(x,0) &= g(x) = c\ \text{sech}\left(\sqrt{\frac{c}{2}}(x-ct)\right). \end{aligned} \tag{25}$$

Figure 1, represents the 5-th RPS approximate solution of the function $u(x,t)$ for different values of the fractional derivative α. Figure 2, represents the corresponding 5-th RPS approximate solutions of the function $v(x,t)$. It is clear from these figures that when α is decreasing toward 0, the solutions bifurcate and provide wave-like pattern. But, when α is close to 1, there is no pattern. Also, as the fractional order α being increasing the subfigures are nearly coinciding and similar in their behavior. While on the other hand, for special case $\alpha = 1$ subfigures Figure 1-(d), (e) and Figure 2-(4), (5) are nearly identical and in excellent agreement to each other in terms of the accuracy. Meanwhile, it is easy to conclude that the wave solution is getting smooth around the zero zone as α tends to 1.

V. Conclusions

In this paper, a new analytical iterative technique based on the residual power series RPS is proposed to obtain an approximate solution to a nonlinear time-fractional DSW system. It has been found that the construction of this recent RPS method possesses in general a very rapid convergent series due to the embedded generalized Taylor series. The RPS method is promising technique based on its simplicity and accuracy and it is considered to be an additive tool for the field of fractional theory and computations. As future work, we will extend the RPS method to handle $(2+1)$-dimensional linear and nonlinear space and time-fractional physical models.

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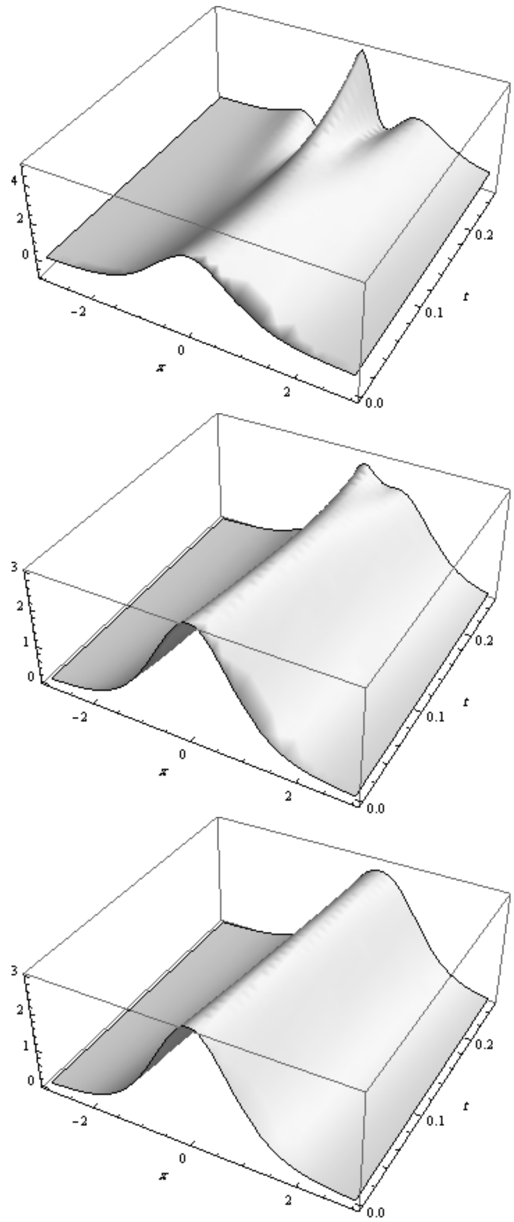

Fig. 1. The 5-th RPS approximate solution of the function $u(x,t)$ when $\alpha = 0.7$, $\alpha = 0.8$, and $\alpha = 0.9$, respectively, for, $c = 2$ and $-3 < x < 3$

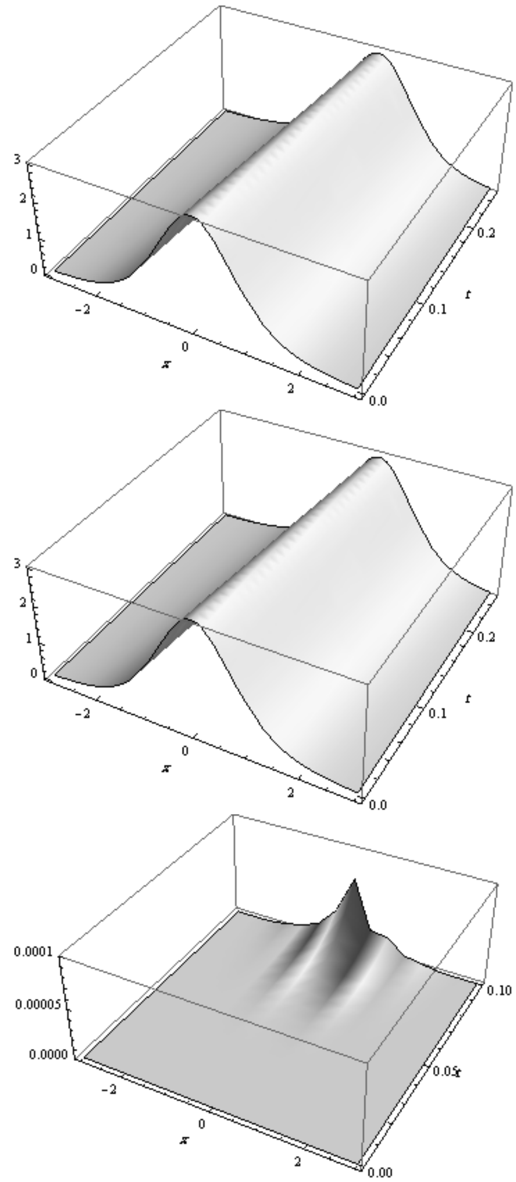

Fig. 2. The 5-th RPS solution $u_5(x,t,\alpha = 1)$, the exact solution $u(x,t,\alpha = 1)$ and the absolute error for, $c = 2$ and $-3 < x < 3$

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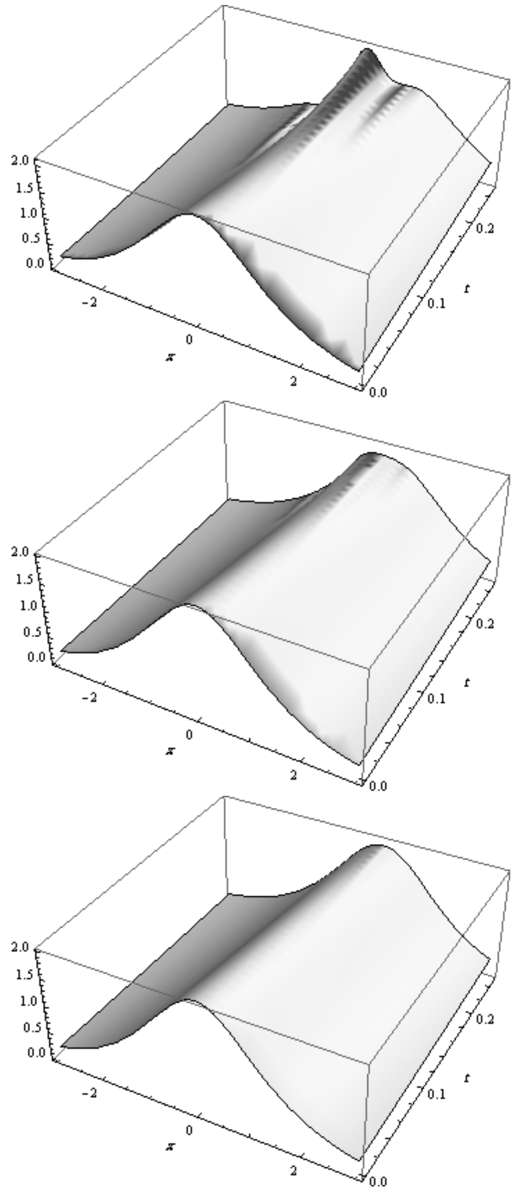

Fig. 3. The 5-th RPS approximate solution of the function $v(x,t)$ when $\alpha = 0.7$, $\alpha = 0.8$, and $\alpha = 0.9$, respectively, for, $c = 2$ and $-3 < x < 3$

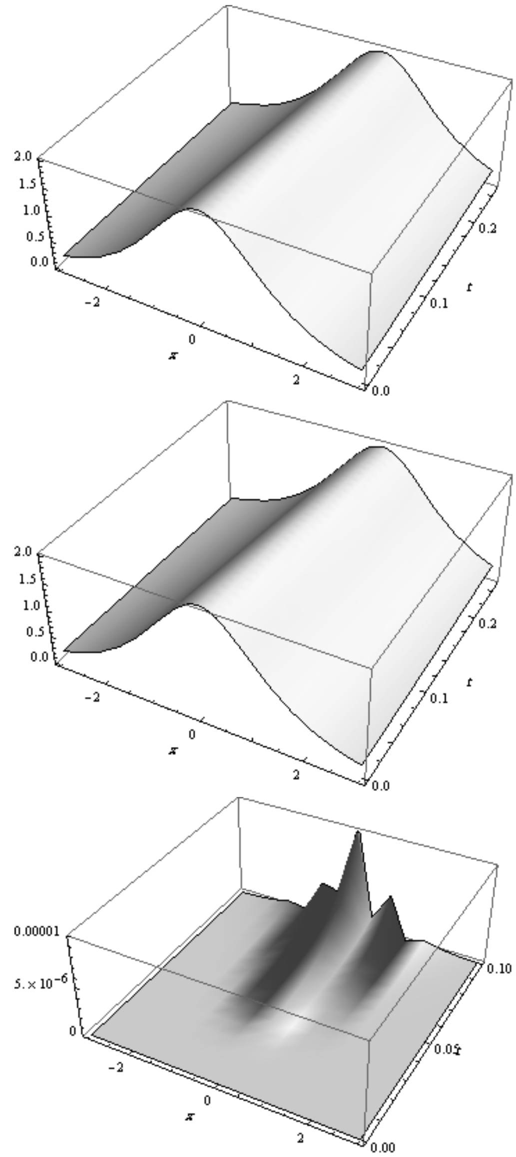

Fig. 4. The 5-th RPS solution $v_5(x,t,\alpha=1)$, the exact solution $v(x,t,\alpha=1)$ and the absolute error for, $c = 2$ and $-3 < x < 3$

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