Likelihood and Bayesian Estimation in Stress Strength Model from Generalized Exponential Distribution Containing Outliers

Chunping Li, Huibing Hao

Abstract—**This paper studies the estimation of $R = P(Y < X)$ when X and Y are two independent generalized exponential distributions containing one outlier. The maximum likelihood estimator (MLE) and Bayesian estimator of R are obtained under exchangeable and identifiable models, respectively. Monte Carlo simulation is used to compare and verify the proposed model and approaches. The simulation results show that the performance of MLE is more satisfactory than Bayes estimator.**

Index Terms—**generalized exponential distribution, outlier, maximum likelihood estimator, bayes estimator.**

I. Introduction

In reliability contexts, inferences about stress strength model $R=P(Y<X)$ is an interest subject. For example, in mechanical reliability of a system, if X is the strength of a component which is subject to the stress Y, then we know that R is a measure of system performance. The system fails, if at any time the applied stress Y is greater than its strength X.

The problem of estimating $R=P(Y<X)$, where X and Y belong to a certain family of probability distributions, has been widely studied in the literature, such as burr distributions (Mokhlis, 2005), Weibull distribution (Kundu et al., 2006), Gompertz distribution (Saraçoglu et al., 2007), exponential distribution (Jiang et al., 2008), the generalized gamma distribution (Pham et al., 1995) and the generalized pareto distribution (Rezaei et al., 2010), et al.. Recently, in the stress strength model literature, Kundu and Gupta (2005) and Raqab et al. (2008) has considered the ordinary samples from the generalized exponential distribution (GED), and Baklizi (2008) has considered the record data from the GED. All these papers assume that the sample observations are independently and identically distributed. In fact, the sample data may contain outliers in many cases, because outliers are usually caused by measurement error or erroneous procedures (seen in Barnett et al., 1984).

Kim and Chung (2006) and Jeevanand and Nair (1994) have considered the outlier from the burr-X distribution and exponential distribution, but they only considered the Bayes estimation of R. It is well known that the prior function plays an important role in Bayes method, thus the other estimation method should be considered.

In this paper, we focus on estimation of $R=P(Y<X)$, where X and Y follow the GED(θ) and GED(β). The maximum likelihood estimator (MLE) and the Bayes estimation of R are obtained from the samples containing outliers, which has not been studied before.

The rest of the paper is organized as follows: In the next Section, the generalized exponential distribution and the stress strength model are introduced. In Section 3, we introduce the joint distribution of (X_1, X_2, … , X_n) with one outlier. In Sections 4 and 5, the MLE and the Bayes estimator of R under exchangeable and identifiable model are obtained. In Section 6, we present some numerical results, and compare the Bias and the mean squares errors (MSE). Section 7 concludes the paper.

II. Generalized Exponential Distribution and Stress Strength Model

The generalized exponential distribution (GED) has firstly been introduced by Gupta and Kundu (1999). Due to the convenient structure of distribution function, the GED can be used to analyze various lifetime data. In recent years, many scholars have studied about this distribution, such as Raqab and Ahsanullah(2001), Zheng(2002), Gupta and Kundu (2003).

The probability density function with one parameter GED(θ) is given by

$$f(x)=\theta e^{-x}(1-e^{-x})^{\theta-1},\ x>0,\theta>0 \tag{1}$$

And the corresponding cumulative distribution function is

$$F(x)=(1-e^{-x})^{\theta},\ x>0,\theta>0 \tag{2}$$

Suppose that X~GED(θ) and Y~GED(β) are two independent random variables, then the reliability of stress strength model can be obtained as

$$R=P(Y<X)$$
$$=\int_0^{\infty}P(Y<X\mid X=x)P(X=x)dx=\frac{\theta}{\theta+\beta} \tag{3}$$

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C. P. Li is with the Department of Mathematics, Hubei Engineering University, Hubei, 432000, China. e-mail: lichunping315@163.com.

H. B. Hao is the corresponding author with the Department of Mathematics, Hubei Engineering University, Hubei, 432000, China. e-mail: haohuibing1979@163.com.

III. JOINT DISTRIBUTION WITH OUTLIER

Let X~GED(θ), $X_1, X_2, \cdots, X_n$ be a random sample from X, and (n-1) of them have the same probability density function as follow

$$f(x)=\theta e^{-x}(1-e^{-x})^{\theta-1},\ x>0,\theta>0 \tag{4}$$

the remaining one's probability density function is given as

$$g(x)=b\theta e^{-x}(1-e^{-x})^{b\theta-1},\ x>0,b>1,\theta>0 \tag{5}$$

Therefore, the joint probability density function of (X_1, $X_2,\cdots, X_n$) can be obtained as (see Dixit and Nooghabi, 2011)

$$f(x_1,x_2,\cdots,x_n)=\frac{(n-1)!}{n!}\prod_{i=1}^{n} f(x_i)\sum_{j=1}^{n}\frac{g(x_j)}{f(x_j)}$$

where $f(x)$ and $g(x)$ are given in (4) and (5). Then

$$f(x_1,x_2,\cdots,x_n)=\frac{1}{n}b\theta^n\exp(-\sum_{i=1}^{n}x_i)$$

$$\cdot\prod_{i=1}^{n}[1-\exp(-x_i)]^{\theta-1}\sum_{j=1}^{n}\frac{(1-x_j)^{b\theta-1}}{(1-x_j)^{\theta-1}}$$

$$=\frac{1}{n}b\theta^n\exp(-\sum_{i=1}^{n}x_i)\sum_{i=1}^{n}\exp\big(-\theta[T_1(x)$$

$$+(b-1)T(x_i)]+T_1(x)\big) \tag{6}$$

where

$$T_1(x)=-\sum_{i=1}^{n}\log[1-\exp(-x_i)],$$

$$T(x_i)=-\log[1-\exp(-x_i)].$$

Similarly, let Y~ GED(β) and Y_1, $Y_2,\cdots$, Y_n be a random sample from Y, and (n-1) of them have the same probability density function as

$$f(y)=\beta e^{-y}(1-e^{-y})^{\beta-1},\ y>0,\beta>0$$

the remaining one's probability density function is given as

$$g(y)=c\beta e^{-y}(1-e^{-y})^{c\beta-1},\ y>0,c>1,\beta>0$$

Then, we can get the joint distribution of Y_1, $Y_2,\cdots$, Y_n as

$$f(y_1,y_2,\cdots,y_n)=\frac{1}{n}c\beta^n\exp(-\sum_{j=1}^{n}y_j)$$

$$\cdot\sum_{j=1}^{n}\exp\big(-\beta[T_1(y)+(c-1)T(y_j)]+T_1(y)\big) \tag{7}$$

where

$$T_1(y)=-\sum_{j=1}^{n}\log[1-\exp(-y_j)],$$

$$T(y_j)=-\log[1-\exp(-y_j)]$$

IV. MLE AND BAYES ESTIMATION OF R UNDER EXCHANGEABLE MODEL

The exchangeable model assumes that outliers are not identifiable and any observation in sample is as likely to be discordant as any other. In this section, we will obtain the MLE and the Bayes estimation of R under this condition.

A. The MLE of R

Suppose that X=(X_1, X_2, $\cdots$, X_n) follows GED(θ) and contains one outlier, according to the expressions (6), the log likelihood function is given by

$$\ln f=-\ln n+n\ln\theta+\ln b-\sum_{i=1}^{n}x_i$$

$$+\ln\{\sum_{i=1}^{n}\exp[-\theta(T_1(x)+(b-1)T(x_i))+T_1(x)]\}$$

Differentiating ln$f$ with respect to θ and b, respectively, we can get

$$\frac{\partial\ln f}{\partial\theta}=\frac{n}{\theta}$$

$$\frac{\sum_{i=1}^{n}\exp[-\theta(T_1(x)+(b-1)T(x_i))+T_1(x)][T_1(x)+(b-1)T(x_i))]}{\sum_{i=1}^{n}\exp[-\theta(T_1(x)+(b-1)T(x_i))+T_1(x)]} \tag{8}$$

$$\frac{\partial\ln f}{\partial b}=\frac{1}{b}$$

$$-\frac{\sum_{i=1}^{n}\exp[-\theta(T_1(x)+(b-1)T(x_i))+T_1(x)][\theta T(x_i))]}{\sum_{i=1}^{n}\exp[-\theta(T_1(x)+(b-1)T(x_i))+T_1(x)]} \tag{9}$$

Equating (8) and (9) to zero, we have

$$\hat{\theta}=\frac{n\sum_{i=1}^{n}\exp[-\hat{\theta}(T_1(x)+(\hat{b}-1)T(x_i))+T_1(x)]}{\sum_{i=1}^{n}\exp[-\hat{\theta}(T_1(x)+(\hat{b}-1)T(x_i))+T_1(x)][T_1(x)+(\hat{b}-1)T(x_i))]} \tag{10}$$

$$\hat{b}=\frac{\sum_{i=1}^{n}\exp[-\hat{\theta}(T_1(x)+(\hat{b}-1)T(x_i))+T_1(x)]}{\sum_{i=1}^{n}\exp[-\hat{\theta}(T_1(x)+(\hat{b}-1)T(x_i))+T_1(x)][\hat{\theta}T(x_i))]} \tag{11}$$

In the same way, we can obtain the MLE of β and c as

$$\hat{\beta}=\frac{n\sum_{j=1}^{n}\exp[-\hat{\beta}(T_1(y)+(\hat{c}-1)T(y_j))+T_1(y)]}{\sum_{j=1}^{n}\exp[-\hat{\beta}(T_1(y)+(\hat{c}-1)T(y_j))+T_1(y)][T_1(y)+(\hat{c}-1)T(y_j))]} \tag{12}$$

$$\hat{c}=\frac{\sum_{j=1}^{n}\exp[-\hat{\beta}(T_1(y)+(\hat{c}-1)T(y_j))+T_1(y)]}{\sum_{j=1}^{n}\exp[-\hat{\beta}(T_1(y)+(\hat{c}-1)T(y_j))+T_1(y)][\hat{\beta}T(y_j))]} \tag{13}$$

Then, it is easy to obtain the MLE of R under exchangeable model as

$$\hat{R}=\frac{\hat{\theta}}{\hat{\theta}+\hat{\beta}} \tag{14}$$

B. The Bayes estimation of R

In this subsection, we consider the Bayes estimation of R under the squared error loss function. Let $X=(X_1, X_2,\cdots, X_n)$ and $Y=(Y_1, Y_2,\cdots, Y_n)$ be two independent random samples from GED with parameters (θ, b) and (β, c), respectively. Then, the likelihood functions are proportion to

$$l(X\mid\theta,b)\propto b\theta^n\sum_{i=1}^{n}\exp[-\theta(T_1(x)+(b-1)T(x_i))+T_1(x)]$$

and

$$l(Y\mid\beta,c)\propto c\beta^n\sum_{j=1}^{n}\exp[-\beta(T_1(y)+(c-1)T(y_j))+T_1(y)] \tag{15}$$

In Bayesian framework, we assume that the parameters θ and b are taken to be independently distributed to a gamma distribution and a non informative prior distribution. Then the joint distribution of (θ, b) is proportion to

$$\pi(\theta,b)\propto\theta^{p-1}\exp(-u\theta)$$

In the similar manner, the joint distribution of (β, c) is proportion to

$$\pi(\beta,c)\propto\beta^{q-1}\exp(-v\beta)$$

(16)

where p, q, u and v are known.

Based on the above assumption, we can derive the joint probability density function of X and Y as follows

$$l(X,Y\mid\theta,\beta,b,c)=l(X\mid\theta,b)l(Y\mid\beta,c)\pi(\theta,b)\pi(\beta,c)$$

the posterior density function of (θ, β, b, c) is obtained

$$\pi(\theta,\beta,b,c\mid X,Y)=\frac{l(X,Y\mid\theta,b,\beta,c)}{\int_0^\infty\int_0^\infty\int_1^\infty\int_1^\infty l(X,Y\mid\theta,b,\beta,c)d\theta d\beta dbdc}$$

Then

$$\pi(\theta,\beta\mid X,Y)=\int_1^\infty\int_1^\infty\pi(\theta,\beta,b,c\mid X,Y)dbdc$$

Let $r=\theta/(\theta+\beta)$ and $\rho=\theta+\beta$, $\rho>0$, $0<r<1$. We can get

$$\pi(r,\rho\mid X,Y)=\rho\int_1^\infty\int_1^\infty\pi(r\rho,(1-r)\rho,b,c\mid X,Y)dbdc$$

Therefore, we obtain the marginal posterior density function of R as

$$\pi(r\mid X,Y)=\int_0^\infty\pi(r,\rho\mid X,Y)d\rho$$

$$=C_1\sum_{i=1}^{n}\sum_{j=1}^{n}(T(x_i)T(y_j))^{-2}r^{m-3}(1-r)^{s-3}(Ar+1)^{-(m+s-2)}$$

$$\cdot[(Ar+1)^2D^2+(m+s-4)(Ar+1)(rT(x_i)$$

$$+(1-r)T(y_j))Q_2+(m+s-3)(m+s-4)(T(x_i)T(y_j))r(1-r)]$$

where

$m=n+p, s=n+q, Q_1=T_1(x)+u$, $A=\frac{Q_1}{Q_2}-1$,

$Q_2=T_1(y)+v$, $B(m-2,s-2)=\frac{\Gamma(m-2)\Gamma(s-2)}{\Gamma(m+s-4)}$,

$$C_1^{-1}=\sum_{i=1}^{n}\sum_{j=1}^{n}(T(x_i)T(y_j))^{-2}[(m-2)T(x_i)+Q_1]$$

$$\cdot[(s-2)T(y_j)+Q_2]B(m-2,s-2)\left(\frac{Q_2}{Q_1}\right)^{m-1} \tag{17}$$

Hence, under the squared error loss function, the Bayes estimator of R is

$$\hat{R}=E(r\mid date)=\int_0^1 r\pi(r\mid date)dr$$

$$=C_1\sum_{i=1}^{n}\sum_{j=1}^{n}(T(x_i)T(y_j))^{-2}\{Q_2^2H(0,0,0)$$

$$+(m+s-4)T(x_i)Q_2H(1,1,0)+(m+s-4)T(y_j)Q_2H(1,0,1)$$

$$+(m+s-3)(m+s-4)T(x_i)T(y_j)H(2,1,1)\} \tag{18}$$

where

$$H(a,b,c)=F_{2,1}(m+s-4-a,m+b-1,m+s+a-3,-A)$$

$$\cdot B(m+b-1,s+c-2)$$

$B(m+b-1,s+c-2)=\frac{\Gamma(m+b-1)\Gamma(s+c-2)}{\Gamma(m+s+b+c-3)}$,

$F_{2,1}(a,b;c;z)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1\frac{t^{b-1}(1-t)^{c-b-1}}{(1-tz)^a}dt$.

V. MLE AND BAYES ESTIMATION OF R UNDER IDENTIFIABLE MODEL

The identifiable model assumes that outliers are identifiable. We treat the largest observation in the sample as an outlier because the largest order statistics in the sample has the largest posterior probability (see Kale and Kale, 1992). We will obtain the MLE and Bayes estimations of R under this model.

A. The MLE of R

Suppose that $X=(X_1, X_2, \cdots, X_n)$ follows GED(θ) and contains one outlier, according to Kale and Kale (1992), we know that the largest order statistics in the sample is the outlier when $b>1$, and we treat $X_{(n)}$ as outlier. Then we can get the likelihood function and the log likelihood function as

$$L(\theta,b\mid X)=C_2\prod_{i=1}^{n-1}f(x_i)f(x_{(n)})$$

$$=C_2b\theta^n\exp(-\sum_{i=1}^{n}x_i)\exp[-\theta(T_1(x)+(b-1)T(x_{(n)}))+T_1(x)]$$

and

$$\ln L=\ln C_2+n\ln\theta+\ln b$$

$$-\sum_{i=1}^{n}x_i-\theta(T_1(x)+(b-1)T(x_{(n)}))+T_1(x)$$

where

C_2 is a constant, $T_1(x) = -\sum_{i=1}^{n} \log[1-\exp(-x_i)]$,

$T(x_{(n)}) = -\log[1-\exp(-x_{(n)})]$.

Differentiating ln$L$ with respect to θ and b, respectively. We can obtain the following solutions

$$\frac{\partial \ln L}{\partial \theta} = \frac{n}{\theta} - [T_1(x) + (b-1)T(x_{(n)})] = 0 \tag{19}$$

$$\frac{\partial \ln L}{\partial b} = \frac{1}{b} - \theta[T(x_{(n)})] = 0 \tag{20}$$

From the above two equations, we derive

$$\hat{\theta} = \frac{n-1}{T_1(x) - T(x_{(n)})} \tag{21}$$

Similarly, suppose that $Y=(Y_1,Y_2,\cdots,Y_n)$ follows GED(β) and contains one outlier, the log likelihood function from the sample is given by

$$\ln L = \ln C_2 + n\ln\beta + \ln c$$
$$- \sum_{j=1}^{n} y_j - \beta(T_1(y) + (c-1)T(y_{(n)})) + T_1(y)$$

where

$T_1(y) = -\sum_{i=1}^{n} \log[1-\exp(-y_i)]$,

$T(y_{(n)}) = -\log[1-\exp(-y_{(n)})]$.

Similar to (19) and (20), we can get

$$\hat{\beta} = \frac{n-1}{T_1(y) - T(y_{(n)})} \tag{22}$$

Therefore, from (21) and (22), the MLE of R under the exchangeable model is

$$\hat{R} = \frac{\hat{\theta}}{\hat{\theta} + \hat{\beta}} \tag{23}$$

B. The Bayes estimation of R

In this subsection, we consider the Bayes estimation of R under the squared error loss function. Let $X=(X_1, X_2,\cdots, X_n)$ and $Y=(Y_1, Y_2, \cdots, Y_n)$ be the two independent random samples from GED with parameters (θ, b) and (β, c), respectively. Then, the likelihood functions are proportion to

$$l_1(X\,|\,\theta,b) \propto b\theta^n \exp[-\theta(T_1(x)+(b-1)T(x_{(n)}))+T_1(x)]$$

and

$$l_2(Y\,|\,\beta,c) \propto c\beta^n \exp[-\beta(T_1(y)+(c-1)T(y_{(n)}))+T_1(y)]$$

Assuming the joint prior distribution of (θ, b) and (β, c) are the same as given (15), we can get

$$l(X,Y\,|\,\theta,\beta,b,c) = l_1(X\,|\,\theta,b)\,l_2(Y\,|\,\beta,c)\,\pi(\theta,b)\,\pi(\beta,c)$$

Using similar way, we can get the marginal posterior distribution of R as follow

$$\pi(r\,|\,X,Y) = \int_0^\infty \pi(r,\rho\,|\,X,Y)d\rho$$

$$= C_3(T(x_{(n)})T(y_{(n)}))^{-2} r^{m-3}(1-r)^{s-3}(Ar+1)^{-(m+s-2)}$$
$$\cdot[(Ar+1)^2 D^2 + (m+s-4)(Ar+1)(rT(x_{(n)})+(1-r)T(y_{(n)}))Q_2$$
$$+(m+s-3)(m+s-4)(T(x_{(n)})T(y_{(n)}))r(1-r)]$$

where

$$C_3^{-1} = [(m-2)T(x_i)+Q_1][(s-2)T(y_j)+Q_2]B(m-2,s-2)\left(\frac{Q_2}{Q_1}\right)^{m-1}$$

Hence, under the mean squared error loss function, the Bayes estimator of R is

$$\hat{R} = E(r\,|\,date) = \int_0^1 r\pi(r\,|\,date)dr$$
$$= C_3\{Q_2^2 H(0,0,0) + (m+s-4)T(x_{(n)})Q_2 H(1,1,0)$$
$$+(m+s-4)T(y_{(n)})Q_2 H(1,0,1)$$
$$+(m+s-3)(m+s-4)T(x_{(n)})T(y_{(n)})H(2,1,1)\} \tag{24}$$

VI. MONTE CARLO SIMULATION STUDY

In this section, Monte Carlo simulation is used to compare performance of the proposed models and methods. Without loss of generality, let θ = 19, β = 1 and b = c = 10. We consider sample size to be (n, m) = (5, 5), (10, 10), (15, 15), (20, 20), (25, 25), (30, 30). For a given generated sample, compute the MLE and Bayes estimators of R and replicate the process 3000 times. For the different prior parameters: $p = q = u = v = 0$, $p = q = u = v = 1$, and $p = q = u = v = 2$, we obtain the Bayes estimators for R as Bayes-1, Bayes-2, and Bayes-3, respectively. The Bias and MSE of the MLE estimator and Bayes estimator are computed by

$$Bias(\hat{R}) = \frac{1}{3000}\sum_{i=1}^{3000}\left|R - \hat{R}_i\right|,$$

and

$$MSE(\hat{R}) = \frac{1}{3000}\sum_{i=1}^{3000}(R - \hat{R}_i)^2 \tag{25}$$

Table I and Table II show the simulation results for the Bias and MSE of MLE and Bayes estimators for R under different sample sizes and different prior parameter values.

From the simulation results, it is quite clear that the performances of both the MLE and the Bayes estimators under the two models are quite satisfactory even for very small sample sizes. It is noted that the Bias and MSE of the MLE are smaller than the Bayes estimators under the two models when the sample contains outliers.

In addition, it is observed that the Bias and MSE decrease for all the estimators under the two models when the sample size increases. Meanwhile, the Biases and MSE of the Bayes estimators decrease when the parameters of prior distributions, p, q, u and v increase.

Moreover, from the simulation results, it is observed that the Biases and MSE of the MLE, Bayes-1, Bayes-2 and Bayes-3 under the identifiable model are smaller than the corresponding values under the exchangeable model.

Based on all those knowledge, when the sample contains one outlier, we recommend use the MLE whenever the model is the exchangeable or identifiable.

VII. Conclusion

This paper deals with the estimation of $R=P(Y<X)$ when X and Y are two independent generalized exponential distributed random variables. We assume that the sample from each population contains one outlier. The MLE and Bayes estimator of R are obtained under the exchangeable and identifiable models. A simulation study is presented to compare the two estimation methods under the different model. Based on the simulation results, the performances of the MLE are more satisfactory than Bayes estimator even for very small sample sizes. When there is more than one outlier, the problem becomes quite more complicated. The corresponding estimation methods need to be explored in the future. It may be mentioned that although it has been assumed that the samples are from generalized exponential distributions containing one outlier, it may be extended to some other distributions also, for example, the Weibull or gamma distribution containing outliers. Work is in progress, and it will be reported later.

TABLE I
Bias and MSE under the exchangeable model

(n, m)		(5,5)	(10, 10)	(15, 15)	(20, 20)	(25, 25)	(30, 30)
MLE	bias	0.0423	0.0222	0.0172	0.0142	0.0125	0.0109
	mse	0.0039	0.0011	0.0007	0.0005	0.0004	0.0003
Bsyes-1	bias	0.0729	0.0636	0.0573	0.0518	0.0487	0.0449
	mse	0.0081	0.0063	0.0051	0.0042	0.0037	0.0032
Bsyes-2	bias	0.0602	0.0507	0.0458	0.0423	0.0396	0.0368
	mse	0.0053	0.0040	0.0033	0.0028	0.0025	0.0021
Bsyes-3	bias	0.0526	0.0454	0.0419	0.0400	0.0379	0.0355
	mse	0.0040	0.0032	0.0027	0.0025	0.0023	0.0020

TABLE II
Bias and MSE under the identifiable model

(n, m)		(5,5)	(10, 10)	(15, 15)	(20, 20)	(25, 25)	(30, 30)
MLE	bias	0.0333	0.0180	0.0158	0.0136	0.0110	0.0103
	mse	0.0021	0.0007	0.0005	0.0004	0.0003	0.0002
Bsyes-1	bias	0.0584	0.0425	0.0364	0.0316	0.0287	0.0254
	mse	0.0051	0.0028	0.0021	0.0015	0.0013	0.0010
Bsyes-2	bias	0.0539	0.0398	0.0319	0.0291	0.0253	0.0229
	mse	0.0043	0.0403	0.0320	0.0281	0.0233	0.0221
Bsyes-3	bias	0.0477	0.0403	0.0320	0.0281	0.0233	0.0221
	mse	0.0035	0.0025	0.0016	0.0012	0.0009	0.0006

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