Distance-Based Indices for Some Families of Dendrimer Nanostars

Wei Gao, Li Shi, and Mohammad Reza Farahani*

Abstract—**In theoretical chemistry, chemical compounds and drugs are modeled as graphs in which each vertex represents an atom of molecule and covalent bounds between atoms are represented by edges between the corresponding vertices. Such a graph constructed according to a chemical compounds is often called its molecular graph and the indices defined over this molecular graph have been shown to be strongly correlated to various chemical properties of the compounds. In this paper, in terms of analyzing the structure of chemical molecular graph and the techniques of edge dividing, we report several distance-based indices for four families of dendrimer Nanostars, including second ABC index, second GA related indices, PI related indices and polynomials and Szeged related indices and polynomials. Furthermore, we discuss the distance-based indices of three kinds of widely appeared molecular structures: $TC_4C_8(R)$ Nanotorus, H-Naphtalenic nanotubes and 1,3-adamantane array. The formulations of second ABC index, second GA related indices, vertex PI and Szeged polynomials are manifested as additional conclusions.**

Index Terms—**molecular graph, second atom bond connectivity index, second geometric-arithmetic index, vertex-edge Szeged index, edge-vertex Szeged index**

I. INTRODUCTION

ONE of the most important applications of chemical graph theory is to measure chemical, physical and pharmaceutical properties of molecules which are called alkanes. Several indices relied on the graphical structure of the alkanes are defined and employed to model both the melting point and boiling point of the molecules. Molecular graph is a topological representation of a molecule in which each vertex represents an atom of molecule, and covalent bounds between atoms are represented by edges between the corresponding vertices.

Specifically, let G be a molecular graph, then a topological index can be regarded as a score function f: $G \rightarrow \mathbb{R}^+$, with this property that $f(G_1) = f(G_2)$ if G_1 and G_2 are isomorphic. As numerical descriptors of the molecular structure obtained from the corresponding molecular graph, topological indices have found several applications in theoretical chemistry, especially in QSPR/QSAR study. For instance, Wiener index, Zagreb index, harmonic index and sum connectivity index are introduced to reflect certain structural features of organic molecules. Several papers contributed to determining these distance-based indices of special molecular graph (See Yan et al., [1-2], Farahani and Gao [3], Gao et al., [4-5], Gao and Shi [6], Xi and Gao [7], Gao and Wang [8-9], and Farahani [10] for more detail). Other contributions on this field can refer to [11-14]. The notation and terminology used but undefined in this paper can be found in [15].

Let $e=uv$ be an edge of the molecular graph G. The number of vertices of G whose distance to the vertex u is smaller than that to the distance to the vertex v is denoted by $n_u(e)$. Analogously, $n_v(e)$ is the number of vertices of G whose distance to the vertex v is smaller than the distance to the vertex u. Graovac and Ghorbani [16] defined a new version of the atom-bond connectivity index, i.e., the second atom bond connectivity index:

$$ABC_2(G) = \sum_{uv\in E(G)} \sqrt{\frac{n_u(e)+n_v(e)-2}{n_u(e)n_v(e)}}.$$

Rostami et. al., [17] calculated some upper bounds for the second atom-bond connectivity index. Gao and Wang [8] determined the second atom-bond connectivity index of unilateral polyomino chain and unilateral hexagonal chain. Also, they deduced the second ABC indices of V-phenylenic nanotubes and Nanotori.

Recently, Fath-Tabar et al., [18] defined a new version of the geometric-arithmetic index, i.e., the second geometric-arithmetic index:

$$GA_2(G) = \sum_{uv\in E(G)} \frac{2\sqrt{n_u(e)n_v(e)}}{n_u(e)+n_v(e)}.$$

In Zhan and Qiao., [19], the maximum and the minimum second geometric-arithmetic index of the star-like tree were studied in view of an increasing or decreasing transformation of the second geometric arithmetic index of trees. Furthermore, they determined the corresponding extremal trees.

Gao and Wang [20] yielded the computational formulas for calculating the second geometric-arithmetic index and introduced the general second geometric-arithmetic index as follows:

$$GA_2^{\gamma}(G) = \sum_{uv\in E(G)} \left(\frac{2\sqrt{n_u(e)n_v(e)}}{n_u(e)+n_v(e)}\right)^{\gamma},$$

where γ is a real number.

Similarly, the number of edges of G whose distance to the vertex u is smaller than the distance to the vertex v is denoted by $m_u(e)$, and $m_v(e)$ is the number of edges of G whose

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distance to the vertex v is smaller than the distance to the vertex u. Then, the third atom bond connectivity index and third GA index were introduced as

$$ABC_3(G)=\sum_{uv\in E(G)}\sqrt{\frac{m_u(e)+m_v(e)-2}{m_u(e)m_v(e)}},$$

$$GA_3(G)=\sum_{uv\in E(G)}\frac{2\sqrt{m_u(e)m_v(e)}}{m_u(e)+m_v(e)}.$$

Furthermore, the general version of the third GA index is stated as

$$GA_3^{\gamma}(G)=\sum_{uv\in E(G)}\left(\frac{2\sqrt{m_u(e)m_v(e)}}{m_u(e)+m_v(e)}\right)^{\gamma}.$$

Several related distance-based indices were also introduced to measure the properties of chemical compounds and medicine. The PI index of molecular graph G is denoted by

$$PI(G)=\sum_{uv\in E(G)}(m_u(e)+m_v(e)).$$

The vertex PI index of molecular graph G is defined as

$$PI_v(G)=\sum_{uv\in E(G)}(n_u(e)+n_v(e)).$$

As the extension of PI index and vertex PI index, the PI polynomial and vertex PI polynomial are introduced by Ashrafi et. al., [21] stated as

$$PI(G,x)=\sum_{uv\in E(G)}x^{m_u(e)+m_v(e)},$$

and

$$PI_v(G)=\sum_{uv\in E(G)}x^{n_u(e)+n_v(e)}.$$

The Szeged index and edge Szeged index of molecular graph G is denoted by

$$Sz_v(G)=\sum_{uv\in E(G)}(n_u(e)n_v(e)).$$

and

$$Sz_e(G)=\sum_{uv\in E(G)}(m_u(e)m_v(e)).$$

The corresponding Szeged polynomial and edge Szeged polynomial are defined as

$$Sz_v(G,x)=\sum_{uv\in E(G)}x^{n_u(e)n_v(e)}.$$

and

$$Sz_e(G,x)=\sum_{uv\in E(G)}x^{m_u(e)m_v(e)}.$$

Also, the vertex-edge Szeged index and edge-vertex Szeged index of molecular graph G were introduced as

$$Sz_{ve}(G)=\frac{1}{2}\sum_{uv\in E(G)}(m_u(e)n_u(e)+m_v(e)n_v(e)),$$

and

$$Sz_{ev}(G)=\frac{1}{2}\sum_{uv\in E(G)}(m_u(e)n_v(e)+m_v(e)n_u(e)).$$

For any edge $e=uv$, let $n(e)$ and $m(e)$ be the number of vertices and edges which have equal distance to u and v, respectively. Then, the modified version of Szeged index and edge Szeged index for a molecular graph G were defined as

$$Sz_v^*(G)=\sum_{uv\in E(G)}((n_u(e)+\frac{n(e)}{2})(n_v(e)+\frac{n(e)}{2})).$$

and

$$Sz_e^*(G)=\sum_{uv\in E(G)}((m_u(e)+\frac{m(e)}{2})(m_v(e)+\frac{m(e)}{2})).$$

Furthermore, the normalized revised Szeged index was defined as

$$Szs^*(G)=\sqrt{\frac{Sz_v^*(G)}{|E(G)|}}.$$

At last, the total Szeged index of a molecular graph G was introduced as

$$Sz_T(G)=\sum_{uv\in E(G)}((m_u(e)+n_u(e))(m_v(e)+n_v(e))).$$

Nanostar dendrimers are common structures in chemical compounds and drugs and there are different kinds of definitions for nanostar dendrimers. In this paper, we mainly focus on the four families of nanostar dendrimers which have appeared in former articles (see Ashrafi and Mirzargar [22-23] and Ashrafi and Karbasioun [24]). In these three families of nanostar dendrimers, the first family is NSC_5C_6, and other three families are denoted by $NS_1[n]$, $NS_2[n]$ and $NS_3[n]$, respectively. Here, n is the number of stratums.

Although there have been several advances in PI index, Szeged index and edge Szeged index for some families of nanostar dendrimers, the study of edge-vertex Szeged index, vertex-edge Szeged index, total Szeged index, second atom bond connectivity index, third atom bond connectivity index, second geometric-arithmetic index and third geometric-arithmetic index of special families of nanostar dendrimers have been largely limited. In addition, as widespread and critical chemical structures, large families of nanostar dendrimers are widely used in medical science and pharmaceutical field. For these reasons, we have attracted tremendous interests from academic and industrial fields to research into the more distance-based indices of commonly used nanostar dendrimer families from a mathematical point of view.

The main contributions of our paper are four-fold. First, we compute some distance-based indices of unilateral polyomino chain. Second, we discuss some relevant indices of $NS_1[n]$. Then, larger amount of distance-based indices of $NS_2[n]$ are calculated. At last, we present vertex distance version indices for $NS_3[n]$. As supplemental contributions, we determine some vertex version of distance-based indices and polynomials of three classes of chemical molecular structures. All the theoretical results yielded in our paper show the characteristics of the related compounds and drugs, make up the shortage of chemical experiments and provide a theoretical basis for the synthesis of new compounds and manufacture of new drugs.

II. DISTANCE-BASED INDICES OF NANOSTAR DENDRIMER NSC_5C_6

Let $G(n)$ be a nanostar dendrimer NSC_5C_6 with n stratum. Manuel et al., [25] studied this structure and manifested the total Szeged index of NSC_5C_6. In this section, we continue on their work and present more distance-based indices for this family of nanostar dendrimers.

Any two diametrically opposite vertices in a hexagon are denoted by apex vertex and base vertex, respectively. Any vertex in a pentagon can be regarded as an apex vertex, and vertices *x*, *y* are denoted as left and right base vertices of pentagon if $d(v,x) = d(v,y)$ =2 (here, *d* is distance function between two vertices). An edge is called apex edge if it is incident to an apex vertex, and an edge is called base edge if it is incident to left and right base vertices. There is one hexagon h_1^1 and two pentagons p_1^1 and p_1^2 in the first stratum. For $2 \le i \le n$, there are 2^i hexagons h_i^k (here $1 \le k \le 2^i$) and 2^i pentagons p_i^k (here $1 \le k \le 2^i$) in the *i*-th stratum. Let x_1^1 and x_1^2 be two edges which joint to apex and base vertices of h_1^1. For $1 \le k \le 2$, set y_1^k as an edge join to apex vertex of p_1^k which satisfies $d''(x_1^k, y_1^k)$ =0, and set z_1^k as a pendant edge with the property $d''(x_1^k, z_1^k) = d''(y_1^k, z_1^k)$ =0. Here, let *e*=*uv*, *f*=*xy* be two edges, *w* be a vertex, d' be the distance function between vertex and edge, and d'' be the distance function between two edges which are defined by

$$d'(w,e) = \min\{d(w,u), d(w,v)\}$$

and

$$d''(e,f) = \min\{d'(e,x), d'(e,y)\},$$

respectively.

For $2 \le i \le n$ and $1 \le k \le 2^i$, let x_i^k be an edge joint to the base vertex of h_i^k, y_i^k be an edge jointed apex vertex of p_i^k meet $d''(x_i^k, y_i^k)$ =0, z_i^k be a pendant edge satisfies $d''(x_i^k, z_i^k) = d''(y_i^k, z_i^k)$ =0. For $1 \le i \le n-1$ and $1 \le k \le 2^{i+1}$, if $k \equiv 1$ (mod 2), set a_i^k as an edge jointed the left base vertex of $p_i^{\lfloor \frac{k}{2} \rfloor}$; if $k \equiv 0$ (mod 2), set a_i^k as an edge jointed the right base vertex of $p_i^{\frac{k}{2}}$. The other end of a_i^k connected to an edge b_i^k, then the other end of b_i^k connected to an edge c_i^k, and at last the other end of c_i^k connected to the apex vertex of h_{i+1}^k. Let d_i^k be a pendant edge satisfies $d''(b_i^k, d_i^k) = d''(c_i^k, d_i^k)$ =0. For $1 \le k \le 2^n$, there are two pendant edges joined to each base vertices of p_n^k which can be expressed as $e_n^{k,l}$, where $1 \le l \le 4$. It is easy to check that $|V(G(n))|$ = $9 \cdot 2^{n+2} - 44$ and $|E(G(n))| = 10 \cdot 2^{n+2} - 50$. The Figure 1 shows the basic structure of $G(2)$.

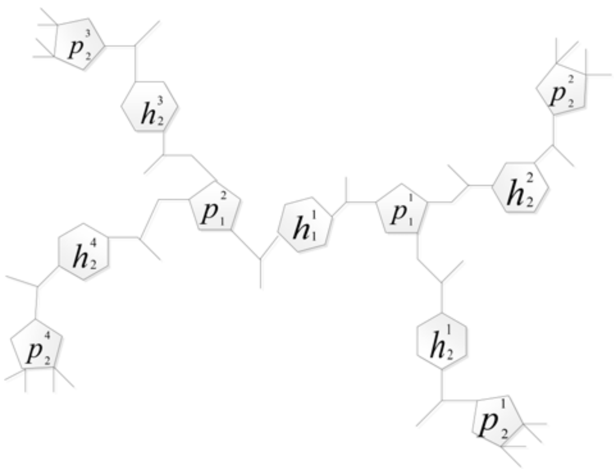

Fig. 1. The basic structure of $G(2)$.

By analyzing the structure of $G(n)$, we check the following fact which will be used to determine our main results in this section.

- If *e* is an edge of h_i^k, where $2 \le i \le n$, $1 \le k \le 2^i$, or $(i,k) = (1,1)$. Then, for all six edges, we infer $n_u(e) = 9 \cdot 2^{n-i+2} - 22$, $n_v(e) = 9 \cdot 2^{n+2} - 9 \cdot 2^{n-i+2} - 22$, $m_u(e) = 10 \cdot 2^{n-i+2} - 26$, $m_v(e) = 10 \cdot 2^{n+2} - 10 \cdot 2^{n-i+2} - 26$.
- If *e* is an apex edge of p_i^k, where $1 \le i \le n$, $1 \le k \le 2^i$. Then, we deduce $n_u(e) = 9 \cdot 2^{n-i+1} - 14$, $n_v(e) = 9 \cdot 2^{n+2} - 9 \cdot 2^{n-i+2} - 15$, $m_u(e) = 10 \cdot 2^{n-i+1} - 16$, $m_v(e) = 10 \cdot 2^{n+2} - 10 \cdot 2^{n-i+2} - 17$.
- If *e* is an edge of p_i^k which is adjacent to both apex edge and base edge where $1 \le i \le n$, $1 \le k \le 2^i$. Then, we yield $n_u(e) = 9 \cdot 2^{n-i+2} - 30$, $n_v(e) = 9 \cdot 2^{n+2} - 9 \cdot 2^{n-i+2} - 15$, $m_u(e) = 10 \cdot 2^{n-i+2} - 34$, $m_v(e) = 10 \cdot 2^{n+2} - 10 \cdot 2^{n-i+2} - 17$.
- If *e* is a base edge of p_i^k where $1 \le i \le n$ and $1 \le k \le 2^i$. Then, we obtain $n_u(e) = 9 \cdot 2^{n-i+1} - 14$, $n_v(e) = 9 \cdot 2^{n-i+1} - 14$, $m_u(e) = 10 \cdot 2^{n-i+1} - 16$, $m_v(e) = 10 \cdot 2^{n-i+1} - 16$.
- For $1 \le k \le 2^n$ and $1 \le l \le 4$, we have $n_u(e_n^{k,l})$ =1, $n_v(e_n^{k,l}) = 9 \cdot 2^{n+2} - 45$, $m_u(e_n^{k,l})$ =0, $m_v(e_n^{k,l}) = 10 \cdot 2^{n+2} - 51$.
- For $1 \le i \le n$, $1 \le k \le 2^i$, we get $n_u(x_i^k) = 9 \cdot 2^{n-i+2} - 25$, $n_v(x_i^k) = 9 \cdot 2^{n+2} - 9 \cdot 2^{n-i+2} - 19$, $m_u(x_i^k) = 10 \cdot 2^{n-i+2} - 29$, $m_v(x_i^k) = 10 \cdot 2^{n+2} - 10 \cdot 2^{n-i+2} - 22$.
- For $1 \le i \le n$ and $1 \le k \le 2^i$, we check that $n_u(y_i^k) = 9 \cdot 2^{n-i+2} - 27$, $n_v(y_i^k) = 9 \cdot 2^{n+2} - 9 \cdot 2^{n-i+2} - 17$, $m_u(y_i^k) = 10 \cdot 2^{n-i+2} - 31$,

$m_v(y_i^k)=10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-20$.

- For $1\le i\le n$ and $1\le k\le 2^i$, we infer $n_u(z_i^k)=1$, $n_v(z_i^k)=9\cdot 2^{n+2}-45$, $m_u(z_i^k)=0$, $m_v(z_i^k)=10\cdot 2^{n+2}-51$.
- For $1\le i\le n-1$ and $1\le k\le 2^{i+1}$, we yield $n_u(a_i^k)=9\cdot 2^{n-i+1}-16$, $n_v(a_i^k)=9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-28$, $m_u(a_i^k)=10\cdot 2^{n-i+1}-19$, $m_v(a_i^k)=10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-32$.
- For $1\le i\le n-1$ and $1\le k\le 2^{i+1}$, we deduce $n_u(b_i^k)=9\cdot 2^{n-i+1}-17$, $n_v(b_i^k)=9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-27$, $m_u(b_i^k)=10\cdot 2^{n-i+1}-20$, $m_v(b_i^k)=10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-31$.
- For $1\le i\le n-1$ and $1\le k\le 2^{i+1}$, we obtain $n_u(c_i^k)=9\cdot 2^{n-i+1}-19$, $n_v(c_i^k)=9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-25$, $m_u(c_i^k)=10\cdot 2^{n-i+1}-22$, $m_v(c_i^k)=10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-29$.
- For $1\le i\le n-1$ and $1\le k\le 2^{i+1}$, we get $n_u(d_i^k)=1$, $n_v(d_i^k)=9\cdot 2^{n+2}-45$, $m_u(d_i^k)=0$, $m_v(d_i^k)=10\cdot 2^{n+2}-51$.

From what we have discussed above, we have the following conclusions by means of definitions of indices.

$$ABC_2(G(n))=\frac{3}{9\cdot 2^n-11}\sqrt{9\cdot 2^{n+2}-46}$$
$$+\sum_{i=2}^{n}2^i\cdot 6\sqrt{\frac{9\cdot 2^{n+2}-46}{(9\cdot 2^{n-i+2}-22)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-22)}}$$
$$+\sum_{i=1}^{n}2^i\cdot 2\sqrt{\frac{9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-31}{(9\cdot 2^{n-i+1}-14)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-15)}}$$
$$+\sum_{i=1}^{n}2^i\cdot 2\sqrt{\frac{9\cdot 2^{n+2}-47}{(9\cdot 2^{n-i+2}-30)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-15)}}$$
$$+\sum_{i=1}^{n}2^i\sqrt{\frac{9\cdot 2^{n-i+2}-30}{(9\cdot 2^{n-i+1}-14)(9\cdot 2^{n-i+1}-14)}}$$
$$+(2^{n+3}-6)\sqrt{\frac{9\cdot 2^{n+2}-46}{9\cdot 2^{n+2}-45}}$$
$$+\sum_{i=1}^{n}2^i\sqrt{\frac{9\cdot 2^{n+2}-46}{(9\cdot 2^{n-i+2}-25)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-19)}}$$
$$+\sum_{i=1}^{n}2^i\sqrt{\frac{9\cdot 2^{n+2}-46}{(9\cdot 2^{n-i+2}-27)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-17)}}$$
$$+\sum_{i=1}^{n-1}2^{i+1}\sqrt{\frac{9\cdot 2^{n+2}-46}{(9\cdot 2^{n-i+1}-16)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-28)}}$$
$$+\sum_{i=1}^{n-1}2^{i+1}\sqrt{\frac{9\cdot 2^{n+2}-46}{(9\cdot 2^{n-i+1}-17)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-27)}}$$
$$+\sum_{i=1}^{n-1}2^{i+1}\sqrt{\frac{9\cdot 2^{n+2}-46}{(9\cdot 2^{n-i+1}-19)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-25)}}.$$

$$ABC_3(G(n))=\frac{3}{5\cdot 2^{n+1}-13}\sqrt{10\cdot 2^{n+2}-54}$$
$$+\sum_{i=2}^{n}2^i\cdot 6\sqrt{\frac{10\cdot 2^{n+2}-54}{(10\cdot 2^{n-i+2}-26)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-26)}}$$
$$+\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-35)}{(10\cdot 2^{n-i+1}-16)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-17)}}$$
$$+\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{10\cdot 2^{n+2}-53}{(10\cdot 2^{n-i+2}-34)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-17)}}$$
$$+\sum_{i=1}^{n}\frac{2^{i-1}}{5\cdot 2^{n-i+1}-8}\sqrt{10\cdot 2^{n-i+2}-34}+$$
$$\sum_{i=1}^{n}2^i\sqrt{\frac{10\cdot 2^{n+2}-53}{(10\cdot 2^{n-i+2}-29)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-22)}}$$
$$+\sum_{i=1}^{n}2^i\sqrt{\frac{10\cdot 2^{n+2}-53}{(10\cdot 2^{n-i+2}-31)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-20)}}$$
$$+\sum_{i=1}^{n-1}2^{i+1}\sqrt{\frac{10\cdot 2^{n+2}-53}{(10\cdot 2^{n-i+1}-19)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-32)}}$$
$$+\sum_{i=1}^{n-1}2^{i+1}\sqrt{\frac{10\cdot 2^{n+2}-53}{(10\cdot 2^{n-i+1}-20)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-31)}}$$
$$+\sum_{i=1}^{n-1}2^{i+1}\sqrt{\frac{10\cdot 2^{n+2}-53}{(10\cdot 2^{n-i+1}-22)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-29)}}.$$

$$GA_2^{\gamma}(G(n))=2^{n+1}+4+$$
$$\sum_{i=2}^{n}2^i\cdot 6\left(\frac{\sqrt{(9\cdot 2^{n-i+2}-22)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-22)}}{9\cdot 2^{n+1}-22}\right)^{\gamma}$$
$$+\sum_{i=1}^{n}2^{i+1}\left(\frac{2\sqrt{(9\cdot 2^{n-i+1}-14)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-15)}}{9\cdot 2^{n-i+1}+9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-29}\right)^{\gamma}$$
$$+\sum_{i=1}^{n}2^{i+1}\left(\frac{2\sqrt{(9\cdot 2^{n-i+2}-30)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-15)}}{9\cdot 2^{n+2}-45}\right)^{\gamma}$$
$$+(2^{n+3}-6)\left(\frac{\sqrt{9\cdot 2^{n+2}-45}}{9\cdot 2^{n+1}-22}\right)^{\gamma}+$$
$$\sum_{i=1}^{n}2^i\left(\frac{\sqrt{(9\cdot 2^{n-i+2}-25)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-19)}}{9\cdot 2^{n+1}-22}\right)^{\gamma}$$
$$+\sum_{i=1}^{n}2^i\left(\frac{\sqrt{(9\cdot 2^{n-i+2}-27)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-17)}}{9\cdot 2^{n+1}-22}\right)^{\gamma}$$
$$+\sum_{i=1}^{n-1}2^{i+1}\left(\frac{\sqrt{(9\cdot 2^{n-i+1}-16)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-28)}}{9\cdot 2^{n+1}-22}\right)^{\gamma}$$
$$+\sum_{i=1}^{n-1}2^{i+1}\left(\frac{\sqrt{(9\cdot 2^{n-i+1}-17)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-27)}}{9\cdot 2^{n+1}-22}\right)^{\gamma}$$

$$+\sum_{i=1}^{n-1}2^{i+1}(\frac{\sqrt{(9\cdot 2^{n-i+1}-19)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-25)}}{9\cdot 2^{n+1}-22})^{\gamma}.$$

$$GA_3^{\gamma}(G(n))=2^{n+1}+4$$

$$+\sum_{i=2}^{n}2^{i}\cdot 6(\frac{\sqrt{(10\cdot 2^{n-i+2}-26)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-26)}}{10\cdot 2^{n+1}-26})^{\gamma}$$

$$+\sum_{i=1}^{n}2^{i+1}(\frac{2\sqrt{(10\cdot 2^{n-i+1}-16)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-17)}}{10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-33})^{\gamma}$$

$$+\sum_{i=1}^{n}2^{i+1}(\frac{2\sqrt{(10\cdot 2^{n-i+2}-34)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-17)}}{10\cdot 2^{n+2}-51})^{\gamma}$$

$$+\sum_{i=1}^{n}2^{i}(\frac{2\sqrt{(10\cdot 2^{n-i+2}-29)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-22)}}{10\cdot 2^{n+2}-51})^{\gamma}$$

$$+\sum_{i=1}^{n}2^{i}(\frac{2\sqrt{(10\cdot 2^{n-i+2}-31)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-20)}}{10\cdot 2^{n+2}-51})^{\gamma}$$

$$+\sum_{i=1}^{n-1}2^{i+1}(\frac{2\sqrt{(10\cdot 2^{n-i+1}-19)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-32)}}{10\cdot 2^{n+2}-51})^{\gamma}$$

$$+\sum_{i=1}^{n-1}2^{i+1}(\frac{2\sqrt{(10\cdot 2^{n-i+1}-20)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-31)}}{10\cdot 2^{n+2}-51})^{\gamma}$$

$$+\sum_{i=1}^{n-1}2^{i+1}(\frac{2\sqrt{(10\cdot 2^{n-i+1}-22)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-29)}}{10\cdot 2^{n+2}-51})^{\gamma}.$$

$$GA_2(G(n))=2^{n+1}+4+$$

$$\sum_{i=2}^{n}2^{i}\cdot 3\frac{\sqrt{(9\cdot 2^{n-i+2}-22)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-22)}}{9\cdot 2^{n}-11}$$

$$+\sum_{i=1}^{n}2^{i+2}\frac{\sqrt{(9\cdot 2^{n-i+1}-14)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-15)}}{9\cdot 2^{n-i+1}+9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-29}$$

$$+\sum_{i=1}^{n}2^{i+2}\frac{\sqrt{(9\cdot 2^{n-i+2}-30)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-15)}}{9\cdot 2^{n+2}-45}$$

$$+(2^{n+2}-3)\frac{\sqrt{9\cdot 2^{n+2}-45}}{9\cdot 2^{n}-11}+$$

$$\sum_{i=1}^{n}2^{i-1}\frac{\sqrt{(9\cdot 2^{n-i+2}-25)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-19)}}{9\cdot 2^{n}-11}$$

$$+\sum_{i=1}^{n}2^{i-1}\frac{\sqrt{(9\cdot 2^{n-i+2}-27)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-17)}}{9\cdot 2^{n}-11}$$

$$+\sum_{i=1}^{n-1}2^{i}\frac{\sqrt{(9\cdot 2^{n-i+1}-16)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-28)}}{9\cdot 2^{n}-11}$$

$$+\sum_{i=1}^{n-1}2^{i}\frac{\sqrt{(9\cdot 2^{n-i+1}-17)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-27)}}{9\cdot 2^{n}-11}$$

$$+\sum_{i=1}^{n-1}2^{i}\frac{\sqrt{(9\cdot 2^{n-i+1}-19)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-25)}}{9\cdot 2^{n}-11}.$$

$$GA_3(G(n))=2^{n+1}+4$$

$$+\sum_{i=2}^{n}2^{i}\cdot 3\frac{\sqrt{(10\cdot 2^{n-i+2}-26)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-26)}}{10\cdot 2^{n}-13}$$

$$+\sum_{i=1}^{n}2^{i+2}\frac{\sqrt{(10\cdot 2^{n-i+1}-16)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-17)}}{10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-33}$$

$$+\sum_{i=1}^{n}2^{i+2}\frac{\sqrt{(10\cdot 2^{n-i+2}-34)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-17)}}{10\cdot 2^{n+2}-51}$$

$$+\sum_{i=1}^{n}2^{i+1}\frac{\sqrt{(10\cdot 2^{n-i+2}-29)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-22)}}{10\cdot 2^{n+2}-51}$$

$$+\sum_{i=1}^{n}2^{i+1}\frac{\sqrt{(10\cdot 2^{n-i+2}-31)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-20)}}{10\cdot 2^{n+2}-51}$$

$$+\sum_{i=1}^{n-1}2^{i+2}\frac{\sqrt{(10\cdot 2^{n-i+1}-19)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-32)}}{10\cdot 2^{n+2}-51}$$

$$+\sum_{i=1}^{n-1}2^{i+2}\frac{\sqrt{(10\cdot 2^{n-i+1}-20)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-31)}}{10\cdot 2^{n+2}-51}$$

$$+\sum_{i=1}^{n-1}2^{i+2}\frac{\sqrt{(10\cdot 2^{n-i+1}-22)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-29)}}{10\cdot 2^{n+2}-51}.$$

$$Sz_{ve}(G(n))=\frac{1}{2}\{6\cdot((9\cdot 2^{n+1}-22)(10\cdot 2^{n+1}-26)$$

$$+(9\cdot 2^{n+1}-22)(10\cdot 2^{n+1}-26))+$$

$$\sum_{i=2}^{n}2^{i}\cdot 6\cdot((9\cdot 2^{n-i+2}-22)(10\cdot 2^{n-i+2}-26)$$

$$+(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-22)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-26))$$

$$+\sum_{i=1}^{n}2^{i}\cdot 2\cdot((9\cdot 2^{n-i+1}-14)(10\cdot 2^{n-i+1}-16)$$

$$+(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-15)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-17))$$

$$+\sum_{i=1}^{n}2^{i}\cdot 2\cdot((9\cdot 2^{n-i+2}-30)(10\cdot 2^{n-i+2}-34)$$

$$+(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-15)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-17))$$

$$+\sum_{i=1}^{n}2^{i}\cdot((9\cdot 2^{n-i+1}-14)(10\cdot 2^{n-i+1}-16)$$

$$+(9\cdot 2^{n-i+1}-14)(10\cdot 2^{n-i+1}-16))$$

$$+4\cdot 2^{n}(9\cdot 2^{n+2}-45)(10\cdot 2^{n-i+1}-51)+$$

$$\sum_{i=1}^{n}2^{i}\cdot((9\cdot 2^{n-i+2}-25)(10\cdot 2^{n-i+2}-29)$$

$$+(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-19)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-22))$$

$$+\sum_{i=1}^{n}2^{i}\cdot((9\cdot 2^{n-i+2}-27)(10\cdot 2^{n-i+2}-31)$$

$$+(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-17)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-20))$$

$$+\sum_{i=1}^{n}2^{i}\cdot(9\cdot 2^{n+2}-45)(10\cdot 2^{n+2}-51)+$$

$$\sum_{i=1}^{n-1}2^{i+1}\cdot((9\cdot 2^{n-i+1}-16)(10\cdot 2^{n-i+1}-19)$$

$$+(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-28)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-32))$$
$$+\sum_{i=1}^{n-1}2^{i+1}\cdot((9\cdot 2^{n-i+1}-17)(10\cdot 2^{n-i+1}-20)$$
$$+(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-27)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-31))$$
$$+\sum_{i=1}^{n-1}2^{i+1}\cdot((9\cdot 2^{n-i+1}-19)(10\cdot 2^{n-i+1}-22)$$
$$+(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-25)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-29))$$
$$+\sum_{i=1}^{n-1}2^{i+1}\cdot(9\cdot 2^{n+2}-45)(10\cdot 2^{n+2}-51)\}$$
$$=66048\cdot 2^{n}-30497+3928\cdot 2^{n}\cdot n-69020\cdot 4^{n}$$
$$+32220\cdot 8^{n}+720\cdot 16^{n}-25272\cdot 4^{n}\cdot n.$$

$$Sz_{v}(G(n),x)=6x^{(9\cdot 2^{n+1}-22)(9\cdot 2^{n+1}-22)}+$$
$$\sum_{i=2}^{n}2^{i}6x^{(9\cdot 2^{n-i+2}-22)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-22)}$$
$$+\sum_{i=1}^{n}2^{i}2x^{(9\cdot 2^{n-i+1}-14)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-15)}$$
$$+\sum_{i=1}^{n}2^{i+1}x^{(9\cdot 2^{n-i+2}-30)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-15)}$$
$$+\sum_{i=1}^{n}2^{i}x^{(9\cdot 2^{n-i+1}-14)(9\cdot 2^{n-i+1}-14)}+(10\cdot 2^{n}-6)x^{9\cdot 2^{n+2}-45}$$
$$+\sum_{i=1}^{n}2^{i}x^{(9\cdot 2^{n-i+2}-25)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-19)}$$
$$+\sum_{i=1}^{n}2^{i}x^{(9\cdot 2^{n-i+2}-27)(9\cdot 2^{n+2}-9\cdot 2^{n-i+2}-17)}$$
$$+\sum_{i=1}^{n-1}2^{i+1}x^{(9\cdot 2^{n-i+1}-16)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-28)}$$
$$+\sum_{i=1}^{n-1}2^{i+1}x^{(9\cdot 2^{n-i+1}-17)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-27)}$$
$$+\sum_{i=1}^{n-1}2^{i+1}x^{(9\cdot 2^{n-i+1}-19)(9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-25)}.$$

$$Sz_{e}(G(n),x)=6x^{(10\cdot 2^{n+1}-26)(10\cdot 2^{n+1}-26)}$$
$$+\sum_{i=2}^{n}2^{i}6x^{(10\cdot 2^{n-i+2}-26)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-26)}$$
$$+\sum_{i=1}^{n}2^{i+1}x^{(10\cdot 2^{n-i+1}-16)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-17)}$$
$$+\sum_{i=1}^{n}2^{i+1}x^{(10\cdot 2^{n-i+2}-34)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-17)}$$
$$+\sum_{i=1}^{n}2^{i}x^{(10\cdot 2^{n-i+1}-16)(10\cdot 2^{n-i+1}-16)}$$
$$+\sum_{i=1}^{n}2^{i}x^{(10\cdot 2^{n-i+2}-29)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-22)}$$
$$+\sum_{i=1}^{n}2^{i}x^{(10\cdot 2^{n-i+2}-31)(10\cdot 2^{n+2}-10\cdot 2^{n-i+2}-20)}$$
$$+\sum_{i=1}^{n-1}2^{i+1}x^{(10\cdot 2^{n-i+1}-19)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-32)}$$
$$+\sum_{i=1}^{n-1}2^{i+1}x^{(10\cdot 2^{n-i+1}-20)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-31)}$$
$$+\sum_{i=1}^{n-1}2^{i+1}x^{(10\cdot 2^{n-i+1}-22)(10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-29)}.$$

$$PI(G(n),x)=(2\cdot 2^{n}+2)x^{10\cdot 2^{n+2}-52}+$$
$$\sum_{i=1}^{n}2^{i+1}x^{10\cdot 2^{n+2}-10\cdot 2^{n-i+1}-33}+\sum_{i=1}^{n}2^{i}x^{10\cdot 2^{n-i+2}-32}+$$
$$(14\cdot 2^{n}-20)x^{10\cdot 2^{n+2}-51}.$$

$$PI_{v}(G(n),x)=(30\cdot 2^{n}-40)x^{9\cdot 2^{n+2}-44}$$
$$+\sum_{i=1}^{n}2^{i+1}x^{9\cdot 2^{n+2}-9\cdot 2^{n-i+1}-29}+(4\cdot 2^{n}-4)x^{9\cdot 2^{n+2}-45}$$
$$+\sum_{i=1}^{n}2^{i}x^{9\cdot 2^{n-i+2}-28}.$$

$$PI(G(n))=800\cdot 4^{n}-1894\cdot 2^{n}+984.$$
$$PI_{v}(G(n))=1368\cdot 4^{n}-3400\cdot 2^{n}+2112.$$

III. DISTANCE-BASED INDICES OF FIRST FAMILY OF NANOSTAR DENDRIMERS

In this section, we focus on the first family of nanostar dendrimers $NS_1[n]$. The structure of this kind of nanostar dendrimers can be referred to Figure 2. By analyzing of graph structure, we check that $|V(NS_1[n])|=120\cdot 2^{n}-108$ and $|E(NS_1[n])|=140\cdot 2^{n}-127$. The techniques in this section mainly follow the ways given by Ashrafi and Mirzargar [22], but some insight tricks are given.

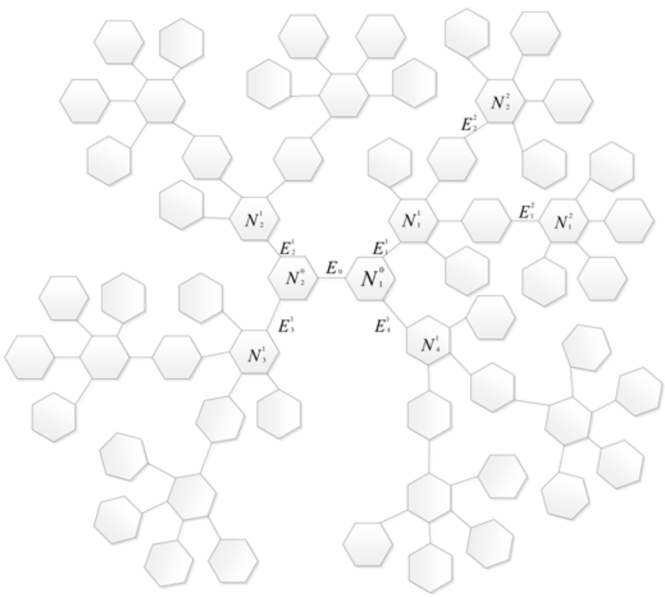

Fig. 2. The basic structure of $NS_1[2]$.

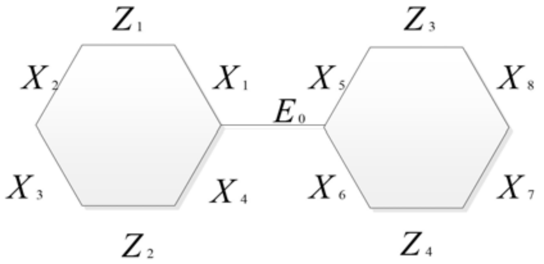

Fig. 3. The center hexagons of $NS_1[n]$.

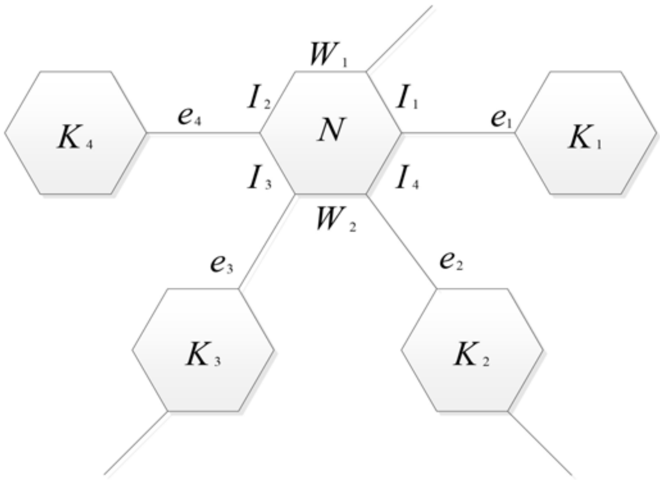

Fig. 4. The detail structure for a branch of $NS_1[n]$.

If $e=uv=E_j^i$ ($1\le i\le n$, $1\le j\le 2^{i+1}$), we have $n_u(e)=30\cdot(2^{n-i+1}-1)$ and $m_u(e)=35\cdot 2^{n-i+1}-36$; If $e=uv=E_0$, we obtain $n_u(E_0)=15\cdot 2^{n+2}-18$ and $m_u(E_0)=70\cdot 2^n-64$; If $e=uv$ is an edge from two center hexagons (see Figure 3 for more detials), then for $e=X_1=a_1b_1,\cdots,X_8=a_8b_8$ in the center hexagons N_1^0 and N_2^0, we infer $n_{a_t}(x_t)=30\cdot 2^n-27$ and $m_{a_t}(x_t)=35\cdot 2^n-33$ for $1\le t\le 8$, and $n_{c_s}(z_s)=60\cdot 2^n-57$, $m_{c_s}(z_s)=70\cdot 2^n-68$ for $z_1=c_1d_1,\cdots,z_4=c_4d_4$, and $1\le s\le 4$. For any hexagon N in the i-th branch $NS_1[n]$ (see Figure 4 for more details), by setting $e=uv\in N$, we have $n_{g_t}(l_t)=60\cdot 2^{n-i}-39$ and $m_{g_t}(l_t)=70\cdot 2^{n-i}-39$ for $l_1=g_1h_1,\cdots,l_4=g_4h_4$, $1\le t\le 4$, and $n_{r_1}(w_1)=n_{r_2}(w_2)=30\cdot 2^{n-i}-15$, $m_{r_1}(w_1)=m_{r_2}(w_2)=36\cdot 2^{n-i}-20$ for $w_1=r_1s_1$, $w_2=r_2s_2$.

Moreover, we deduce $n_u(e)=3$ and $m_u(e)=2$ for any edge in the hexagons of K_1 and K_4, and $n_u(e)=30\cdot 2^{n-i}-27$ and $m_u(e)=36\cdot 2^{n-i}-34$ for any edge in the hexagons of K_2 and K_3. And, we get $n_{u_1}(e_1)=n_{u_4}(e_4)=m_{u_1}(e_1)=m_{u_4}(e_4)=6$ for $e_1=u_1v_1$ and $e_4=u_4v_4$, and $n_{u_2}(e_2)=n_{u_3}(e_3)=30\cdot 2^{n-i}-24$, $m_{u_2}(e_2)=m_{u_3}(e_3)=36\cdot 2^{n-i}-30$ for $e_2=u_2v_2$ and $e_3=u_3v_3$.

In view of discussing above, we get the following conclusions.

$$Sz_v^*(NS_1[n])=(60\cdot 2^n-54)(60\cdot 2^n-54)+$$
$$\sum_{i=1}^{n}2^{i+1}(120\cdot 2^n-30\cdot 2^{n-i+1}-78)(30\cdot 2^{n-i+1}-30)$$
$$+8(30\cdot 2^n-27)(90\cdot 2^n-81)$$
$$+4(60\cdot 2^n-57)(60\cdot 2^n-51)$$
$$+4\sum_{i=1}^{n}2^{i+1}(120\cdot 2^n-60\cdot 2^{n-i}-69)(60\cdot 2^{n-i}-39)$$
$$+2\sum_{i=1}^{n}2^{i+1}(120\cdot 2^n-30\cdot 2^{n-i}-93)(30\cdot 2^{n-i}-15)$$
$$+48(2^n-1)[3\cdot(120\cdot 2^n-111)]$$
$$+12\sum_{i=1}^{n}2^{i+1}(120\cdot 2^n-30\cdot 2^{n-i}-81)(30\cdot 2^{n-i}-27)$$
$$+8(2^n-1)[6\cdot(120\cdot 2^n-114)]$$
$$+2\sum_{i=1}^{n}2^{i+1}(120\cdot 2^n-30\cdot 2^{n-i}-84)(30\cdot 2^{n-i}-24)$$
$$=-284400\cdot 4^n+187200n\cdot 4^n-75600n\cdot 2^n$$
$$+415944\cdot 2^n-131184.$$

$$Sz_e^*(NS_1[n])=(70\cdot 2^n-64+\frac{1}{2})(70\cdot 2^n-64+\frac{1}{2})+$$
$$\sum_{i=1}^{n}2^{i+1}(140\cdot 2^n-35\cdot 2^{n-i+1}-92+\frac{1}{2})(35\cdot 2^{n-i+1}-36+\frac{1}{2})$$
$$+8(35\cdot 2^n-32)(105\cdot 2^n-95)+4(70\cdot 2^n-67)(70\cdot 2^n-60)$$
$$+4\sum_{i=1}^{n}2^{i+1}(140\cdot 2^n-70\cdot 2^{n-i}-81)(70\cdot 2^{n-i}-46)$$
$$+2\sum_{i=1}^{n}2^{i+1}(140\cdot 2^n-36\cdot 2^{n-i}-108)(36\cdot 2^{n-i}-19)$$
$$+48(2^n-1)[3\cdot(140\cdot 2^n-130)]$$
$$+12\sum_{i=1}^{n}2^{i+1}(140\cdot 2^n-36\cdot 2^{n-i}-94)(36\cdot 2^{n-i}-33)$$
$$+8(2^n-1)[(6+\frac{1}{2})\cdot(140\cdot 2^n-134+\frac{1}{2})]$$
$$+2\sum_{i=1}^{n}2^{i+1}(140\cdot 2^n-36\cdot 2^{n-i}-98+\frac{1}{2})(36\cdot 2^{n-i}-30+\frac{1}{2})$$
$$=508899\cdot 2^n-\frac{763347}{4}-102752\cdot 2^n\cdot n-272424\cdot 4^n$$
$$-45236\cdot 8^n+259280\cdot 4^n\cdot n.$$

$$Szs^*(G)=\{(-284400\cdot 4^n+187200n\cdot 4^n-75600n\cdot 2^n$$
$$+415944\cdot 2^n-131184)/(140\cdot 2^n-127)\}^{\frac{1}{2}}.$$

$$ABC_2(NS_1[n])=\frac{\sqrt{120\cdot 2^n-110}}{60\cdot 2^n-54}+$$

$$\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{120\cdot 2^{n}-110}{(120\cdot 2^{n}-30\cdot 2^{n-i+1}-78)(30\cdot 2^{n-i+1}-30)}}$$
$$+8\sqrt{\frac{120\cdot 2^{n}-110}{(30\cdot 2^{n}-27)(90\cdot 2^{n}-81)}}$$
$$+4\sqrt{\frac{120\cdot 2^{n}-110}{(60\cdot 2^{n}-57)(60\cdot 2^{n}-51)}}$$
$$+4\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{120\cdot 2^{n}-110}{(120\cdot 2^{n}-60\cdot 2^{n-i}-69)(60\cdot 2^{n-i}-39)}}$$
$$+2\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{120\cdot 2^{n}-110}{(120\cdot 2^{n}-30\cdot 2^{n-i}-93)(30\cdot 2^{n-i}-15)}}$$
$$+48(2^{n}-1)\sqrt{\frac{120\cdot 2^{n}-110}{3\cdot(120\cdot 2^{n}-111)}}$$
$$+12\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{120\cdot 2^{n}-110}{(120\cdot 2^{n}-30\cdot 2^{n-i}-81)(30\cdot 2^{n-i}-27)}}$$
$$+8(2^{n}-1)\sqrt{\frac{120\cdot 2^{n}-110}{6\cdot(120\cdot 2^{n}-114)}}$$
$$+2\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{120\cdot 2^{n}-110}{(120\cdot 2^{n}-30\cdot 2^{n-i}-84)(30\cdot 2^{n-i}-24)}}.$$

$$ABC_{3}(NS_{1}[n])=\frac{\sqrt{140\cdot 2^{n}-130}}{70\cdot 2^{n}-64}+$$
$$\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{140\cdot 2^{n}-130}{(140\cdot 2^{n}-35\cdot 2^{n-i+1}-92)(35\cdot 2^{n-i+1}-36)}}$$
$$+8\sqrt{\frac{140\cdot 2^{n}-131}{(35\cdot 2^{n}-33)(105\cdot 2^{n}-96)}}+4\sqrt{\frac{140\cdot 2^{n}-131}{(70\cdot 2^{n}-68)(70\cdot 2^{n}-61)}}$$
$$+4\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{140\cdot 2^{n}-131}{(140\cdot 2^{n}-70\cdot 2^{n-i}-82)(70\cdot 2^{n-i}-47)}}$$
$$+2\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{140\cdot 2^{n}-131}{(140\cdot 2^{n}-36\cdot 2^{n-i}-109)(36\cdot 2^{n-i}-20)}}$$
$$+48(2^{n}-1)\sqrt{\frac{140\cdot 2^{n}-131}{2\cdot(140\cdot 2^{n}-131)}}$$
$$+12\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{140\cdot 2^{n}-131}{(140\cdot 2^{n}-36\cdot 2^{n-i}-95)(36\cdot 2^{n-i}-34)}}$$
$$+8(2^{n}-1)\sqrt{\frac{140\cdot 2^{n}-130}{6\cdot(140\cdot 2^{n}-134)}}$$
$$+2\sum_{i=1}^{n}2^{i+1}\sqrt{\frac{140\cdot 2^{n}-130}{(140\cdot 2^{n}-36\cdot 2^{n-i}-98)(36\cdot 2^{n-i}-30)}}$$

$$GA_{2}^{\gamma}(NS_{1}[n])=1+$$
$$\sum_{i=1}^{n}2^{i+1}\left(\frac{\sqrt{(120\cdot 2^{n}-30\cdot 2^{n-i+1}-78)(30\cdot 2^{n-i+1}-30)}}{60\cdot 2^{n}-54}\right)^{\gamma}$$
$$+8\left(\frac{\sqrt{(30\cdot 2^{n}-27)(90\cdot 2^{n}-81)}}{60\cdot 2^{n}-54}\right)^{\gamma}$$
$$+4\left(\frac{\sqrt{(60\cdot 2^{n}-57)(60\cdot 2^{n}-51)}}{60\cdot 2^{n}-54}\right)^{\gamma}$$
$$+4\sum_{i=1}^{n}2^{i+1}\left(\frac{\sqrt{(120\cdot 2^{n}-60\cdot 2^{n-i}-69)(60\cdot 2^{n-i}-39)}}{60\cdot 2^{n}-54}\right)^{\gamma}$$
$$+2\sum_{i=1}^{n}2^{i+1}\left(\frac{\sqrt{(120\cdot 2^{n}-30\cdot 2^{n-i}-93)(30\cdot 2^{n-i}-15)}}{60\cdot 2^{n}-54}\right)^{\gamma}$$
$$+48(2^{n}-1)\left(\frac{\sqrt{3\cdot(120\cdot 2^{n}-111)}}{60\cdot 2^{n}-54}\right)^{\gamma}$$
$$+12\sum_{i=1}^{n}2^{i+1}\left(\frac{\sqrt{(120\cdot 2^{n}-30\cdot 2^{n-i}-81)(30\cdot 2^{n-i}-27)}}{60\cdot 2^{n}-54}\right)^{\gamma}$$
$$+8(2^{n}-1)\left(\frac{\sqrt{6\cdot(120\cdot 2^{n}-114)}}{60\cdot 2^{n}-54}\right)^{\gamma}$$
$$+2\sum_{i=1}^{n}2^{i+1}\left(\frac{\sqrt{(120\cdot 2^{n}-30\cdot 2^{n-i}-84)(30\cdot 2^{n-i}-24)}}{60\cdot 2^{n}-54}\right)^{\gamma}.$$

$$GA_{3}^{\gamma}(NS_{1}[n])=1+$$
$$\sum_{i=1}^{n}2^{i+1}\left(\frac{\sqrt{(140\cdot 2^{n}-35\cdot 2^{n-i+1}-92)(35\cdot 2^{n-i+1}-36)}}{70\cdot 2^{n}-64}\right)^{\gamma}$$
$$+8\left(\frac{2\sqrt{(35\cdot 2^{n}-33)(105\cdot 2^{n}-96)}}{140\cdot 2^{n}-129}\right)^{\gamma}$$
$$+4\left(\frac{2\sqrt{(70\cdot 2^{n}-68)(70\cdot 2^{n}-61)}}{140\cdot 2^{n}-129}\right)^{\gamma}$$
$$+4\sum_{i=1}^{n}2^{i+1}\left(\frac{2\sqrt{(140\cdot 2^{n}-70\cdot 2^{n-i}-82)(70\cdot 2^{n-i}-47)}}{140\cdot 2^{n}-129}\right)^{\gamma}$$
$$+2\sum_{i=1}^{n}2^{i+1}\left(\frac{2\sqrt{(140\cdot 2^{n}-36\cdot 2^{n-i}-109)(36\cdot 2^{n-i}-20)}}{140\cdot 2^{n}-129}\right)^{\gamma}$$
$$+48(2^{n}-1)\left(\frac{2\sqrt{2\cdot(140\cdot 2^{n}-131)}}{140\cdot 2^{n}-129}\right)^{\gamma}$$
$$+12\sum_{i=1}^{n}2^{i+1}\left(\frac{2\sqrt{(140\cdot 2^{n}-36\cdot 2^{n-i}-95)(36\cdot 2^{n-i}-34)}}{140\cdot 2^{n}-129}\right)^{\gamma}$$
$$+8(2^{n}-1)\left(\frac{\sqrt{6\cdot(140\cdot 2^{n}-134)}}{70\cdot 2^{n}-64}\right)^{\gamma}$$
$$+2\sum_{i=1}^{n}2^{i+1}\left(\frac{\sqrt{(140\cdot 2^{n}-36\cdot 2^{n-i}-98)(36\cdot 2^{n-i}-30)}}{70\cdot 2^{n}-64}\right)^{\gamma}.$$

$$GA_{2}(NS_{1}[n])=1+$$
$$\sum_{i=1}^{n}2^{i}\frac{\sqrt{(120\cdot 2^{n}-30\cdot 2^{n-i+1}-78)(30\cdot 2^{n-i+1}-30)}}{30\cdot 2^{n}-27}$$

$$+\frac{4\sqrt{(30\cdot 2^n-27)(90\cdot 2^n-81)}}{30\cdot 2^n-27}$$
$$+\frac{2\sqrt{(60\cdot 2^n-57)(60\cdot 2^n-51)}}{30\cdot 2^n-27}$$
$$+4\sum_{i=1}^{n}\frac{2^i\sqrt{(120\cdot 2^n-60\cdot 2^{n-i}-69)(60\cdot 2^{n-i}-39)}}{30\cdot 2^n-27}$$
$$+2\sum_{i=1}^{n}\frac{2^i\sqrt{(120\cdot 2^n-30\cdot 2^{n-i}-93)(30\cdot 2^{n-i}-15)}}{30\cdot 2^n-27}$$
$$+24(2^n-1)\frac{\sqrt{3\cdot(120\cdot 2^n-111)}}{30\cdot 2^n-27}$$
$$+12\sum_{i=1}^{n}\frac{2^i\sqrt{(120\cdot 2^n-30\cdot 2^{n-i}-81)(30\cdot 2^{n-i}-27)}}{30\cdot 2^n-27}$$
$$+4(2^n-1)\frac{\sqrt{6\cdot(120\cdot 2^n-114)}}{30\cdot 2^n-27}$$
$$+2\sum_{i=1}^{n}\frac{2^i\sqrt{(120\cdot 2^n-30\cdot 2^{n-i}-84)(30\cdot 2^{n-i}-24)}}{30\cdot 2^n-27}.$$

$$GA_3(NS_1[n])=1+$$
$$\sum_{i=1}^{n}\frac{2^i\sqrt{(140\cdot 2^n-35\cdot 2^{n-i+1}-92)(35\cdot 2^{n-i+1}-36)}}{35\cdot 2^n-32}$$
$$+\frac{16\sqrt{(35\cdot 2^n-33)(105\cdot 2^n-96)}}{140\cdot 2^n-129}$$
$$+\frac{8\sqrt{(70\cdot 2^n-68)(70\cdot 2^n-61)}}{140\cdot 2^n-129}$$
$$+4\sum_{i=1}^{n}\frac{2^{i+2}\sqrt{(140\cdot 2^n-70\cdot 2^{n-i}-82)(70\cdot 2^{n-i}-47)}}{140\cdot 2^n-129}$$
$$+2\sum_{i=1}^{n}\frac{2^{i+2}\sqrt{(140\cdot 2^n-36\cdot 2^{n-i}-109)(36\cdot 2^{n-i}-20)}}{140\cdot 2^n-129}$$
$$+(2^n-1)\frac{96\sqrt{2\cdot(140\cdot 2^n-131)}}{140\cdot 2^n-129}$$
$$+12\sum_{i=1}^{n}\frac{2^{i+2}\sqrt{(140\cdot 2^n-36\cdot 2^{n-i}-95)(36\cdot 2^{n-i}-34)}}{140\cdot 2^n-129}$$
$$+4(2^n-1)\frac{\sqrt{6\cdot(140\cdot 2^n-134)}}{35\cdot 2^n-32}$$
$$+2\sum_{i=1}^{n}2^i\frac{\sqrt{(140\cdot 2^n-36\cdot 2^{n-i}-98)(36\cdot 2^{n-i}-30)}}{35\cdot 2^n-32}.$$

$$Sz_{ve}(NS_1[n])=\frac{1}{2}\{2(60\cdot 2^n-54)(70\cdot 2^n-64)+$$
$$\sum_{i=1}^{n}2^{i+1}((120\cdot 2^n-30\cdot 2^{n-i+1}-78)(140\cdot 2^n-35\cdot 2^{n-i+1}-92)$$
$$+(30\cdot 2^{n-i+1}-30)(35\cdot 2^{n-i+1}-36))$$
$$+8((30\cdot 2^n-27)(35\cdot 2^n-33)+(90\cdot 2^n-81)(105\cdot 2^n-96))$$
$$+4((60\cdot 2^n-57)(70\cdot 2^n-68)+(60\cdot 2^n-51)(70\cdot 2^n-61))$$
$$+4\sum_{i=1}^{n}2^{i+1}((120\cdot 2^n-60\cdot 2^{n-i}-69)(140\cdot 2^n-70\cdot 2^{n-i}-82)$$
$$+(60\cdot 2^{n-i}-39)(70\cdot 2^{n-i}-47))$$
$$+2\sum_{i=1}^{n}2^{i+1}((120\cdot 2^n-30\cdot 2^{n-i}-93)(140\cdot 2^n-36\cdot 2^{n-i}-109)$$
$$+(30\cdot 2^{n-i}-15)(36\cdot 2^{n-i}-20))$$
$$+48(2^n-1)((3)(2)+(120\cdot 2^n-111)(140\cdot 2^n-131))$$
$$+12\sum_{i=1}^{n}2^{i+1}((120\cdot 2^n-30\cdot 2^{n-i}-81)(140\cdot 2^n-36\cdot 2^{n-i}-95)$$
$$+(30\cdot 2^{n-i}-27)(36\cdot 2^{n-i}-34))$$
$$+8(2^n-1)(6(6)+(120\cdot 2^n-114)(140\cdot 2^n-134))$$
$$+2\sum_{i=1}^{n}2^{i+1}((120\cdot 2^n-30\cdot 2^{n-i}-84)(140\cdot 2^n-36\cdot 2^{n-i}-98)$$
$$+(30\cdot 2^{n-i}-24)(36\cdot 2^{n-i}-30))\}$$
$$=2479104\cdot 2^n-718986+88164\cdot 2^n\cdot n-2973840\cdot 4^n$$
$$+1214280\cdot 8^n\text{-}220320\cdot 4^n\cdot n.$$

$$Sz_{ev}(NS_1[n])=\frac{1}{2}\{2(60\cdot 2^n-54)(70\cdot 2^n-64)+$$
$$\sum_{i=1}^{n}2^{i+1}((120\cdot 2^n-30\cdot 2^{n-i+1}-78)(35\cdot 2^{n-i+1}-36)$$
$$+(30\cdot 2^{n-i+1}-30)(140\cdot 2^n-35\cdot 2^{n-i+1}-92))$$
$$+8((30\cdot 2^n-27)(105\cdot 2^n-96)+(90\cdot 2^n-81)(35\cdot 2^n-33))$$
$$+4((60\cdot 2^n-57)(70\cdot 2^n-61)+(60\cdot 2^n-51)(70\cdot 2^n-68))$$
$$+4\sum_{i=1}^{n}2^{i+1}((120\cdot 2^n-60\cdot 2^{n-i}-69)(70\cdot 2^{n-i}-47)$$
$$+(60\cdot 2^{n-i}-39)(140\cdot 2^n-70\cdot 2^{n-i}-82))$$
$$+2\sum_{i=1}^{n}2^{i+1}((120\cdot 2^n-30\cdot 2^{n-i}-93)(36\cdot 2^{n-i}-20)$$
$$+(30\cdot 2^{n-i}-15)(140\cdot 2^n-36\cdot 2^{n-i}-109))$$
$$+48(2^n-1)((3)(140\cdot 2^n-131)+(120\cdot 2^n-111)(2))$$
$$+12\sum_{i=1}^{n}2^{i+1}((120\cdot 2^n-30\cdot 2^{n-i}-81)(36\cdot 2^{n-i}-34)$$
$$+(30\cdot 2^{n-i}-27)(140\cdot 2^n-36\cdot 2^{n-i}-95))$$
$$+8(2^n-1)(6(140\cdot 2^n-134)+6(120\cdot 2^n-114))$$
$$+2\sum_{i=1}^{n}2^{i+1}((120\cdot 2^n-30\cdot 2^{n-i}-84)(36\cdot 2^{n-i}-30)$$
$$+(30\cdot 2^{n-i}-24)(140\cdot 2^n-36\cdot 2^{n-i}-98))\}$$
$$=-220626+100400\cdot 8^n+667452\cdot 2^n$$
$$+168960\cdot 4^n\cdot n\text{-}53256\cdot 2^n\cdot n-546960\cdot 4^n.$$

$$Sz(NS_1[n],x)=x^{(60\cdot 2^n-54)(60\cdot 2^n-54)}+$$

$$\sum_{i=1}^{n} 2^{i+1} x^{(120\cdot 2^n - 30\cdot 2^{n-i+1} - 78)(30\cdot 2^{n-i+1} - 30)}$$
$$+8x^{(30\cdot 2^n - 27)(90\cdot 2^n - 81)} + 4x^{(60\cdot 2^n - 57)(60\cdot 2^n - 51)}$$
$$+4\sum_{i=1}^{n} 2^{i+1} x^{(120\cdot 2^n - 60\cdot 2^{n-i} - 69)(60\cdot 2^{n-i} - 39)}$$
$$+2\sum_{i=1}^{n} 2^{i+1} x^{(120\cdot 2^n - 30\cdot 2^{n-i} - 93)(30\cdot 2^{n-i} - 15)}$$
$$+48(2^n - 1)x^{3\cdot(120\cdot 2^n - 111)}$$
$$+12\sum_{i=1}^{n} 2^{i+1} x^{(120\cdot 2^n - 30\cdot 2^{n-i} - 81)(30\cdot 2^{n-i} - 27)}$$
$$+8(2^n - 1)x^{6\cdot(120\cdot 2^n - 114)}$$
$$+2\sum_{i=1}^{n} 2^{i+1} x^{(120\cdot 2^n - 30\cdot 2^{n-i} - 84)(30\cdot 2^{n-i} - 24)}.$$

$$Sz_e(NS_1[n], x) = x^{(70\cdot 2^n - 64)(70\cdot 2^n - 64)} +$$
$$\sum_{i=1}^{n} 2^{i+1} x^{(140\cdot 2^n - 35\cdot 2^{n-i+1} - 92)(35\cdot 2^{n-i+1} - 36)}$$
$$+8x^{(35\cdot 2^n - 33)(105\cdot 2^n - 96)} + 4x^{(70\cdot 2^n - 68)(70\cdot 2^n - 61)}$$
$$+4\sum_{i=1}^{n} 2^{i+1} x^{(140\cdot 2^n - 70\cdot 2^{n-i} - 82)(70\cdot 2^{n-i} - 47)}$$
$$+2\sum_{i=1}^{n} 2^{i+1} x^{(140\cdot 2^n - 36\cdot 2^{n-i} - 109)(36\cdot 2^{n-i} - 20)}$$
$$+48(2^n - 1)x^{2\cdot(140\cdot 2^n - 131)}$$
$$+12\sum_{i=1}^{n} 2^{i+1} x^{(140\cdot 2^n - 36\cdot 2^{n-i} - 95)(36\cdot 2^{n-i} - 34)}$$
$$+8(2^n - 1)x^{6\cdot(140\cdot 2^n - 134)}$$
$$+2\sum_{i=1}^{n} 2^{i+1} x^{(140\cdot 2^n - 36\cdot 2^{n-i} - 98)(36\cdot 2^{n-i} - 30)}.$$

$$Sz_T(NS_1[n]) = (130\cdot 2^n - 118)^2 +$$
$$\sum_{i=1}^{n} 2^{i+1}(260\cdot 2^n - 65\cdot 2^{n-i+1} - 170)(65\cdot 2^{n-i+1} - 66)$$
$$+8(65\cdot 2^n - 60)(195\cdot 2^n - 177)$$
$$+4(130\cdot 2^n - 125)(130\cdot 2^n - 112)$$
$$+4\sum_{i=1}^{n} 2^{i+1}(260\cdot 2^n - 130\cdot 2^{n-i} - 151)(130\cdot 2^{n-i} - 86)$$
$$+2\sum_{i=1}^{n} 2^{i+1}(260\cdot 2^n - 66\cdot 2^{n-i} - 202)(66\cdot 2^{n-i} - 35)$$
$$+240(2^n - 1)(260\cdot 2^n - 242)$$
$$+12\sum_{i=1}^{n} 2^{i+1}(260\cdot 2^n - 66\cdot 2^{n-i} - 176)(66\cdot 2^{n-i} - 61)$$
$$+96(2^n - 1)(260\cdot 2^n - 248)$$
$$+2\sum_{i=1}^{n} 2^{i+1}(260\cdot 2^n - 66\cdot 2^{n-i} - 182)(66\cdot 2^{n-i} - 54)$$
$$= -945344\cdot 4^n + 1767160\cdot 2^n - 666396$$
$$-354680n\cdot 2^n + 887120n\cdot 4^n - 154196\cdot 8^n.$$

$$PI(NS_1[n], x) = (120(2^n - 1) + 12)x^{140\cdot 2^n - 129}$$
$$+(20(2^n - 1) + 1)x^{140\cdot 2^n - 128}.$$
$$PI_v(NS_1[n], x) = (140\cdot 2^n - 127)x^{120\cdot 2^n - 108}.$$
$$PI_v(NS_1[n]) = 16800\cdot 4^n - 30360\cdot 2^n + 13716.$$

IV. DISTANCE-BASED INDICES OF SECOND FAMILY OF NANOSTAR DENDRIMERS

Now, we consider the distance-based indices of the second family of nanostar dendrimers $NS_2[n]$. The detailed structure of this kind of nanostar dendrimers can be referred to Mirzargar [23]. We follow the notations from Mirzargar [23] and several new conclusions are obtained.

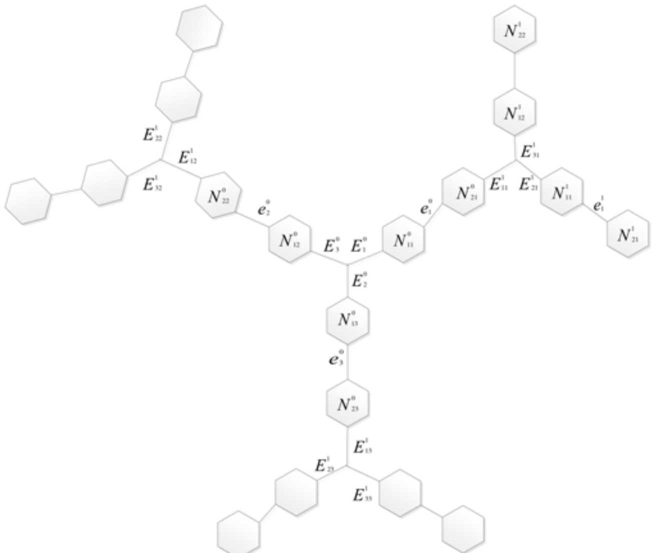

Fig. 5. The detail structure of $NS_2[1]$

The structure of this kind of Nanostar dendrimers can be referred to Figure 2. By analyzing of graph structure, we check that $|V(NS_2[n])| = 75\cdot 2^n - 38$ and $|E(NS_2[n])| = 87\cdot 2^n - 45$. Set $E_1^0 = a_1b_1$, $E_2^0 = a_2b_2$, $E_3^0 = a_3b_3$, we have $n_{a_r}(E_r^0) = 25\cdot 2^n - 13$ and $m_{a_r}(E_r^0) = 29\cdot 2^{n+1} - 16$ for $1 \le r \le 3$. Considering $e=uv= E_{jk}^i$ for $j \in \{1,2,3\}$, $i \in \{1,\cdots,n\}$ and $1 \le k \le 3\cdot 2^{i-1}$. We yield $n_a(e) = n_c(e) = 25\cdot 2^{n-i} - 13$ and $m_a(e) = m_c(e) = 29\cdot 2^{n-i} - 16$ for $e=ab= E_{2k}^i$ and $e=cd= E_{3k}^i$, and $n_g(e) = 50\cdot 2^{n-i} - 25$, $m_g(e) = 58\cdot 2^{n-i} - 30$ for $e=gh= E_{1k}^i$.

We get $n_u(e) = 25\cdot 2^{n-i} - 19$ and $m_u(e) = 29\cdot 2^{n-i} - 23$ for $e=uv= e_j^i$ where $i \in \{1,\cdots,n\}$ and $1 \le j \le 3\cdot 2^i$. Considering the edges of hexagons N_{jk}^i for

$j\in\{1,2\}$, $i\in\{1,\cdots,n\}$ and $1\le k\le 3\cdot 2^{i-1}$. We deduce $n_u(e)=25\cdot 2^{n-i}-16$ and $m_u(e)=29\cdot 2^{n-i}-20$ if $e=uv$ is an edge from hexagon N_{1k}^i, and $n_u(e)=25\cdot 2^{n-i}-22$, $m_u(e)=29\cdot 2^{n-i}-27$ if $e=uv$ is an edge from hexagon N_{2k}^i.

In view of above discussing, we have the following facts.

$Sz_v^*(NS_2[n])$

$$=3(25\cdot 2^n-13)(50\cdot 2^n-25)$$
$$+6\sum_{i=1}^n 2^{i-1}(25\cdot 2^{n-i}-13)(75\cdot 2^n-25\cdot 2^{n-i}-25)$$
$$+3\sum_{i=1}^n 2^{i-1}(25\cdot 2^{n-i}-25)(75\cdot 2^n-25\cdot 2^{n-i}-13)$$
$$+3\sum_{i=0}^n 2^{i}(25\cdot 2^{n-i}-19)(75\cdot 2^n-25\cdot 2^{n-i}-19)$$
$$+18\sum_{i=0}^n 2^{i}(25\cdot 2^{n-i}-16)(75\cdot 2^n-25\cdot 2^{n-i}-22)$$
$$+18\sum_{i=0}^n 2^{i}(25\cdot 2^{n-i}-22)(75\cdot 2^n-25\cdot 2^{n-i}-16)$$
$$=-18081+125712\cdot 2^n+84375n\cdot 4^n-100125\cdot 4^n-2700n\cdot 2^n.$$

$Sz_e^*(NS_2[n])$

$$=3(29\cdot 2^n-16+\frac{1}{2})(58\cdot 2^n-30+\frac{1}{2})$$
$$+6\sum_{i=1}^n 2^{i-1}(29\cdot 2^{n-i}-16+\frac{1}{2})(87\cdot 2^n-29\cdot 2^{n-i}-30+\frac{1}{2})$$
$$+3\sum_{i=1}^n 2^{i-1}(58\cdot 2^{n-i}-30+\frac{1}{2})(87\cdot 2^n-58\cdot 2^{n-i}-16+\frac{1}{2})$$
$$+3\sum_{i=0}^n 2^{i}(29\cdot 2^{n-i}-23+\frac{1}{2})(87\cdot 2^n-29\cdot 2^{n-i}-23+\frac{1}{2})$$
$$+18\sum_{i=0}^n 2^{i}(29\cdot 2^{n-i}-20+\frac{2}{2})(87\cdot 2^n-29\cdot 2^{n-i}-27+\frac{2}{2})$$
$$+18\sum_{i=0}^n 2^{i}(29\cdot 2^{n-i}-27+\frac{2}{2})(87\cdot 2^n-29\cdot 2^{n-i}-20+\frac{2}{2})$$
$$=-\frac{155295}{2}\cdot 4^n+\frac{518361}{4}\cdot 2^n-\frac{88185}{4}+113535\cdot 4^n\cdot n\text{-}20184\cdot 8^n.$$

$$Szs^*(NS_2[n])=((-18081+125712\cdot 2^n+84375n\cdot 4^n-100125\cdot 4^n-2700n\cdot 2^n)/(87\cdot 2^n-45))^{\frac{1}{2}}.$$

$ABC_2(NS_2[n])$

$$=3\sqrt{\frac{75\cdot 2^n-40}{(25\cdot 2^n-13)(50\cdot 2^n-25)}}$$
$$+6\sum_{i=1}^n 2^{i-1}\sqrt{\frac{75\cdot 2^n-40}{(25\cdot 2^{n-i}-13)(75\cdot 2^n-25\cdot 2^{n-i}-25)}}$$
$$+3\sum_{i=1}^n 2^{i-1}\sqrt{\frac{75\cdot 2^n-40}{(25\cdot 2^{n-i}-25)(75\cdot 2^n-25\cdot 2^{n-i}-13)}}$$
$$+3\sum_{i=0}^n 2^{i}\sqrt{\frac{75\cdot 2^n-40}{(25\cdot 2^{n-i}-19)(75\cdot 2^n-25\cdot 2^{n-i}-19)}}$$
$$+18\sum_{i=0}^n 2^{i}\sqrt{\frac{75\cdot 2^n-40}{(25\cdot 2^{n-i}-16)(75\cdot 2^n-25\cdot 2^{n-i}-22)}}$$
$$+18\sum_{i=0}^n 2^{i}\sqrt{\frac{75\cdot 2^n-40}{(25\cdot 2^{n-i}-22)(75\cdot 2^n-25\cdot 2^{n-i}-16)}}.$$

$ABC_3(NS_2[n])$

$$=3\sqrt{\frac{87\cdot 2^n-48}{(29\cdot 2^n-16)(58\cdot 2^n-30)}}$$
$$+6\sum_{i=1}^n 2^{i-1}\sqrt{\frac{87\cdot 2^n-48}{(29\cdot 2^{n-i}-16)(87\cdot 2^n-29\cdot 2^{n-i}-30)}}$$
$$+3\sum_{i=1}^n 2^{i-1}\sqrt{\frac{87\cdot 2^n-48}{(58\cdot 2^{n-i}-30)(87\cdot 2^n-58\cdot 2^{n-i}-16)}}$$
$$+3\sum_{i=0}^n 2^{i}\sqrt{\frac{87\cdot 2^n-48}{(29\cdot 2^{n-i}-23)(87\cdot 2^n-29\cdot 2^{n-i}-23)}}$$
$$+18\sum_{i=0}^n 2^{i}\sqrt{\frac{87\cdot 2^n-49}{(29\cdot 2^{n-i}-20)(87\cdot 2^n-29\cdot 2^{n-i}-27)}}$$
$$+18\sum_{i=0}^n 2^{i}\sqrt{\frac{87\cdot 2^n-49}{(29\cdot 2^{n-i}-27)(87\cdot 2^n-29\cdot 2^{n-i}-20)}}.$$

$GA_2^{\gamma}(NS_2[n])$

$$=3(\frac{2\sqrt{(25\cdot 2^n-13)(50\cdot 2^n-25)}}{75\cdot 2^n-38})^{\gamma}$$
$$+6\sum_{i=1}^n 2^{i-1}(\frac{2\sqrt{(25\cdot 2^{n-i}-13)(75\cdot 2^n-25\cdot 2^{n-i}-25)}}{75\cdot 2^n-38})^{\gamma}$$
$$+3\sum_{i=1}^n 2^{i-1}(\frac{2\sqrt{(25\cdot 2^{n-i}-25)(75\cdot 2^n-25\cdot 2^{n-i}-13)}}{75\cdot 2^n-38})^{\gamma}$$
$$+3\sum_{i=0}^n 2^{i}(\frac{2\sqrt{(25\cdot 2^{n-i}-19)(75\cdot 2^n-25\cdot 2^{n-i}-19)}}{75\cdot 2^n-38})^{\gamma}$$
$$+18\sum_{i=0}^n 2^{i}(\frac{2\sqrt{(25\cdot 2^{n-i}-16)(75\cdot 2^n-25\cdot 2^{n-i}-22)}}{75\cdot 2^n-38})^{\gamma}$$
$$+18\sum_{i=0}^n 2^{i}(\frac{2\sqrt{(25\cdot 2^{n-i}-22)(75\cdot 2^n-25\cdot 2^{n-i}-16)}}{75\cdot 2^n-38})^{\gamma}.$$

$GA_3^{\gamma}(NS_2[n])$

$$=3(\frac{2\sqrt{(29\cdot 2^n-16)(58\cdot 2^n-30)}}{87\cdot 2^n-46})^{\gamma}$$

$$+6\sum_{i=1}^{n}2^{i-1}(\frac{2\sqrt{(29\cdot 2^{n-i}-16)(87\cdot 2^{n}-29\cdot 2^{n-i}-30)}}{87\cdot 2^{n}-46})^{\gamma}$$
$$+3\sum_{i=1}^{n}2^{i-1}(\frac{2\sqrt{(58\cdot 2^{n-i}-30)(87\cdot 2^{n}-58\cdot 2^{n-i}-16)}}{87\cdot 2^{n}-46})^{\gamma}$$
$$+3\sum_{i=0}^{n}2^{i}(\frac{2\sqrt{(29\cdot 2^{n-i}-23)(87\cdot 2^{n}-29\cdot 2^{n-i}-23)}}{87\cdot 2^{n}-46})^{\gamma}$$
$$+18\sum_{i=0}^{n}2^{i}(\frac{2\sqrt{(29\cdot 2^{n-i}-20)(87\cdot 2^{n}-29\cdot 2^{n-i}-27)}}{87\cdot 2^{n}-47})^{\gamma}$$
$$+18\sum_{i=0}^{n}2^{i}(\frac{2\sqrt{(29\cdot 2^{n-i}-27)(87\cdot 2^{n}-29\cdot 2^{n-i}-20)}}{87\cdot 2^{n}-47})^{\gamma}.$$

$GA_2(NS_2[n])$
$$=\frac{6\sqrt{(25\cdot 2^{n}-13)(50\cdot 2^{n}-25)}}{75\cdot 2^{n}-38}$$
$$+6\sum_{i=1}^{n}\frac{2^{i}\sqrt{(25\cdot 2^{n-i}-13)(75\cdot 2^{n}-25\cdot 2^{n-i}-25)}}{75\cdot 2^{n}-38}$$
$$+3\sum_{i=1}^{n}\frac{2^{i}\sqrt{(25\cdot 2^{n-i}-25)(75\cdot 2^{n}-25\cdot 2^{n-i}-13)}}{75\cdot 2^{n}-38}$$
$$+3\sum_{i=0}^{n}\frac{2^{i+1}\sqrt{(25\cdot 2^{n-i}-19)(75\cdot 2^{n}-25\cdot 2^{n-i}-19)}}{75\cdot 2^{n}-38}$$
$$+18\sum_{i=0}^{n}\frac{2^{i+1}\sqrt{(25\cdot 2^{n-i}-16)(75\cdot 2^{n}-25\cdot 2^{n-i}-22)}}{75\cdot 2^{n}-38}$$
$$+18\sum_{i=0}^{n}\frac{2^{i+1}\sqrt{(25\cdot 2^{n-i}-22)(75\cdot 2^{n}-25\cdot 2^{n-i}-16)}}{75\cdot 2^{n}-38}.$$

$GA_3(NS_2[n])$
$$=\frac{6\sqrt{(29\cdot 2^{n}-16)(58\cdot 2^{n}-30)}}{87\cdot 2^{n}-46}$$
$$+6\sum_{i=1}^{n}\frac{2^{i}\sqrt{(29\cdot 2^{n-i}-16)(87\cdot 2^{n}-29\cdot 2^{n-i}-30)}}{87\cdot 2^{n}-46}$$
$$+3\sum_{i=1}^{n}\frac{2^{i}\sqrt{(58\cdot 2^{n-i}-30)(87\cdot 2^{n}-58\cdot 2^{n-i}-16)}}{87\cdot 2^{n}-46}$$
$$+3\sum_{i=0}^{n}\frac{2^{i+1}\sqrt{(29\cdot 2^{n-i}-23)(87\cdot 2^{n}-29\cdot 2^{n-i}-23)}}{87\cdot 2^{n}-46}$$
$$+18\sum_{i=0}^{n}\frac{2^{i+1}\sqrt{(29\cdot 2^{n-i}-20)(87\cdot 2^{n}-29\cdot 2^{n-i}-27)}}{87\cdot 2^{n}-47}$$
$$+18\sum_{i=0}^{n}\frac{2^{i+1}\sqrt{(29\cdot 2^{n-i}-27)(87\cdot 2^{n}-29\cdot 2^{n-i}-20)}}{87\cdot 2^{n}-47}.$$

$Sz_{ve}(NS_2[n])$
$$=\frac{1}{2}\{3((25\cdot 2^{n}-13)(29\cdot 2^{n}-16)$$
$$+(50\cdot 2^{n}-25)(58\cdot 2^{n}-30))$$
$$+6\sum_{i=1}^{n}2^{i-1}((25\cdot 2^{n-i}-13)(29\cdot 2^{n-i}-16)$$
$$+(75\cdot 2^{n}-25\cdot 2^{n-i}-25)(87\cdot 2^{n}-29\cdot 2^{n-i}-30))$$
$$+3\sum_{i=1}^{n}2^{i-1}((25\cdot 2^{n-i}-25)(58\cdot 2^{n-i}-30)$$
$$+(75\cdot 2^{n}-25\cdot 2^{n-i}-13)(87\cdot 2^{n}-58\cdot 2^{n-i}-16))$$
$$+3\sum_{i=0}^{n}2^{i}((25\cdot 2^{n-i}-19)(29\cdot 2^{n-i}-23)$$
$$+(75\cdot 2^{n}-25\cdot 2^{n-i}-19)(87\cdot 2^{n}-29\cdot 2^{n-i}-23))$$
$$+18\sum_{i=0}^{n}2^{i}((25\cdot 2^{n-i}-16)(29\cdot 2^{n-i}-20)$$
$$+(75\cdot 2^{n}-25\cdot 2^{n-i}-22)(87\cdot 2^{n}-29\cdot 2^{n-i}-27))$$
$$+18\sum_{i=0}^{n}2^{i}((25\cdot 2^{n-i}-22)(29\cdot 2^{n-i}-27)$$
$$+(75\cdot 2^{n}-25\cdot 2^{n-i}-16)(87\cdot 2^{n}-29\cdot 2^{n-i}-20))\}$$
$$= \quad -20637+117564\cdot 2^{n}\text{-}300\cdot 2^{n}\cdot n+\frac{1202775}{4}\cdot 8^{n}$$
$$-\frac{384975}{4}\cdot 4^{n}\cdot n-\frac{1497729}{4}\cdot 4^{n}.$$

$Sz_{ev}(NS_2[n])$
$$=\frac{1}{2}\{3((25\cdot 2^{n}-13)(58\cdot 2^{n}-30)$$
$$+(50\cdot 2^{n}-25)(29\cdot 2^{n}-16))$$
$$+6\sum_{i=1}^{n}2^{i-1}((25\cdot 2^{n-i}-13)(87\cdot 2^{n}-29\cdot 2^{n-i}-30)$$
$$+(75\cdot 2^{n}-25\cdot 2^{n-i}-25)(29\cdot 2^{n-i}-16))$$
$$+3\sum_{i=1}^{n}2^{i-1}((25\cdot 2^{n-i}-25)(87\cdot 2^{n}-58\cdot 2^{n-i}-16)$$
$$+(75\cdot 2^{n}-25\cdot 2^{n-i}-13)(58\cdot 2^{n-i}-30))$$
$$+3\sum_{i=0}^{n}2^{i}((25\cdot 2^{n-i}-19)(87\cdot 2^{n}-29\cdot 2^{n-i}-23)$$
$$+(75\cdot 2^{n}-25\cdot 2^{n-i}-19)(29\cdot 2^{n-i}-23))$$
$$+18\sum_{i=0}^{n}2^{i}((25\cdot 2^{n-i}-16)(87\cdot 2^{n}-29\cdot 2^{n-i}-27)$$
$$+(75\cdot 2^{n}-25\cdot 2^{n-i}-22)(29\cdot 2^{n-i}-20))$$
$$+18\sum_{i=0}^{n}2^{i}((25\cdot 2^{n-i}-22)(87\cdot 2^{n}-29\cdot 2^{n-i}-20)$$
$$+(75\cdot 2^{n}-25\cdot 2^{n-i}-16)(29\cdot 2^{n-i}-27))\}$$
$$=-19617+113856\cdot 2^{n}-\frac{777}{2}\cdot 2^{n}\cdot n-\frac{67425}{4}\cdot 8^{n}$$
$$+97875\cdot 4^{n}\cdot n-\frac{275865}{4}\cdot 4^{n}.$$

$$Sz_T(NS_2[n])$$
$$=3(54\cdot 2^n-29)(108\cdot 2^n-55)$$
$$+6\sum_{i=1}^{n}2^{i-1}(54\cdot 2^{n-i}-29)(162\cdot 2^n-54\cdot 2^{n-i}-55)$$
$$+3\sum_{i=1}^{n}2^{i-1}(83\cdot 2^{n-i}-55)(162\cdot 2^n-83\cdot 2^{n-i}-29)$$
$$+3\sum_{i=0}^{n}2^{i}(54\cdot 2^{n-i}-42)(162\cdot 2^n-54\cdot 2^{n-i}-42)$$
$$+18\sum_{i=0}^{n}2^{i}(54\cdot 2^{n-i}-36)(162\cdot 2^n-54\cdot 2^{n-i}-49)$$
$$+18\sum_{i=0}^{n}2^{i}(54\cdot 2^{n-i}-49)(162\cdot 2^n-54\cdot 2^{n-i}-36)$$
$$=-278559\cdot 4^n+456831\cdot 2^n-78366$$
$$+393660n\cdot 4^n-67959\cdot 8^n-4125n\cdot 2^{n-1}.$$

$$PI_v(NS_2[n],x)=(87\cdot 2^n-45)x^{75\cdot 2^n-38}.$$

$$PI_v(NS_2[n])=6525\cdot 4^n-6681\cdot 2^n+1710.$$

V. DISTANCE-BASED INDICES OF THIRD FMAILY OF NANOSTAR DENDRIMERS

In what follows, we consider the distance-based indices of the last family of Nanostar dendrimers $NS_3[n]$. The detailed structure of this kind of Nanostar dendrimers can be referred to [24]. By means of table 1 in Ashrafi and Karbasioun [24] and the definitions of indices, we present the following conclusions.

$$Sz_v(NS_3[n],x)$$
$$=\sum_{i=1}^{n}3\cdot 2^{i+1}x^{(2^{n-i+4}-8)(48\cdot 2^{n+1}-2^{n-i+4}+34)}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}x^{(2^{n-i+4}-9)(48\cdot 2^{n+1}-2^{n-i+4}+35)}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}x^{(2^{n-i+4}-10)(48\cdot 2^{n+1}-2^{n-i+4}+36)}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+3}x^{(2^{n-i+3}-5)(48\cdot 2^{n+1}-2^{n-i+3}+31)}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}x^{(2^{n-i+4}-13)(48\cdot 2^{n+1}-2^{n-i+4}+39)}$$
$$+6x^{(2^{n+4}-2)(48\cdot 2^{n+1}-2^{n+4}+28)}+3x^{(2^{n+5}+8)(48\cdot 2^{n+1}-2^{n+5}+18)}$$
$$+3x^{(2^{n+5}+2)(48\cdot 2^{n+1}-2^{n+5}+24)}+3x^{(2^{n+5}+1)(48\cdot 2^{n+1}-2^{n+5}+25)}$$
$$+3x^{2^{n+1}(48\cdot 2^{n+1}-2^{n+5}+26)}+3x^{(2^{n+1}-1)(48\cdot 2^{n+1}-2^{n+5}+27)}$$
$$+3x^{(2^{n+5}-2)(48\cdot 2^{n+1}-2^{n+5}+28)}+6x^{(2^{n+4}-8)(48\cdot 2^{n+1}-2^{n+4}+34)}$$
$$+6x^{(2^{n+4}-9)(48\cdot 2^{n+1}-2^{n+4}+35)}+6x^{(2^{n+4}-10)(48\cdot 2^{n+1}-2^{n+4}+36)}$$
$$+24x^{(2^{n+3}-5)(48\cdot 2^{n+1}-2^{n+3}+31)}+12x^{(2^{n+4}-13)(48\cdot 2^{n+1}-2^{n+4}+39)}$$
$$+18x^{(2^{n+5}+2)(48\cdot 2^{n+1}-2^{n+5}+24)}+36x^{(2^{n+4}-5)(48\cdot 2^{n+1}-2^{n+4}+31)}$$
$$+4x^{48\cdot 2^{n+1}+25}.$$

$$ABC_2(NS_3[n])$$
$$=\sum_{i=1}^{n}3\cdot 2^{i+1}\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n-i+4}-8)(48\cdot 2^{n+1}-2^{n-i+4}+34)}}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n-i+4}-9)(48\cdot 2^{n+1}-2^{n-i+4}+35)}}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n-i+4}-10)(48\cdot 2^{n+1}-2^{n-i+4}+36)}}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+3}\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n-i+3}-5)(48\cdot 2^{n+1}-2^{n-i+3}+31)}}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n-i+4}-13)(48\cdot 2^{n+1}-2^{n-i+4}+39)}}$$
$$+6\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+4}-2)(48\cdot 2^{n+1}-2^{n+4}+28)}}$$
$$+3\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+5}+8)(48\cdot 2^{n+1}-2^{n+5}+18)}}$$
$$+3\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+5}+2)(48\cdot 2^{n+1}-2^{n+5}+24)}}$$
$$+3\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+5}+1)(48\cdot 2^{n+1}-2^{n+5}+25)}}$$
$$+3\sqrt{\frac{48\cdot 2^{n+1}-2^{n+5}+2^{n+1}+24}{2^{n+1}(48\cdot 2^{n+1}-2^{n+5}+26)}}$$
$$+3\sqrt{\frac{48\cdot 2^{n+1}+2^{n+1}-2^{n+5}+24}{(2^{n+1}-1)(48\cdot 2^{n+1}-2^{n+5}+27)}}$$
$$+3\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+5}-2)(48\cdot 2^{n+1}-2^{n+5}+28)}}$$
$$+6\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+4}-8)(48\cdot 2^{n+1}-2^{n+4}+34)}}$$
$$+6\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+4}-9)(48\cdot 2^{n+1}-2^{n+4}+35)}}$$
$$+6\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+4}-10)(48\cdot 2^{n+1}-2^{n+4}+36)}}$$
$$+24\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+3}-5)(48\cdot 2^{n+1}-2^{n+3}+31)}}$$
$$+12\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+4}-13)(48\cdot 2^{n+1}-2^{n+4}+39)}}$$
$$+18\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+5}+2)(48\cdot 2^{n+1}-2^{n+5}+24)}}$$
$$+36\sqrt{\frac{48\cdot 2^{n+1}+24}{(2^{n+4}-5)(48\cdot 2^{n+1}-2^{n+4}+31)}}$$

$$+4\sqrt{\frac{48\cdot 2^{n+1}+24}{48\cdot 2^{n+1}+25}}$$

$GA_2^{\gamma}(NS_3[n])$

$$=\sum_{i=1}^{n}3\cdot 2^{i+1}(\frac{\sqrt{(2^{n-i+4}-8)(48\cdot 2^{n+1}-2^{n-i+4}+34)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}(\frac{\sqrt{(2^{n-i+4}-9)(48\cdot 2^{n+1}-2^{n-i+4}+35)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}(\frac{\sqrt{(2^{n-i+4}-10)(48\cdot 2^{n+1}-2^{n-i+4}+36)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+3}(\frac{\sqrt{(2^{n-i+3}-5)(48\cdot 2^{n+1}-2^{n-i+3}+31)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}(\frac{\sqrt{(2^{n-i+4}-13)(48\cdot 2^{n+1}-2^{n-i+4}+39)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+6(\frac{\sqrt{(2^{n+4}-2)(48\cdot 2^{n+1}-2^{n+4}+28)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+3(\frac{\sqrt{(2^{n+5}+8)(48\cdot 2^{n+1}-2^{n+5}+18)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+3(\frac{\sqrt{(2^{n+5}+2)(48\cdot 2^{n+1}-2^{n+5}+24)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+3(\frac{\sqrt{(2^{n+5}+1)(48\cdot 2^{n+1}-2^{n+5}+25)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+3(\frac{\sqrt{2^{n+1}(48\cdot 2^{n+1}-2^{n+5}+26)}}{24\cdot 2^{n+1}-2^{n+4}+2^{n}+13})^{\gamma}$$
$$+3(\frac{\sqrt{(2^{n+1}-1)(48\cdot 2^{n+1}-2^{n+5}+27)}}{24\cdot 2^{n+1}+2^{n}-2^{n+4}+13})^{\gamma}$$
$$+3(\frac{\sqrt{(2^{n+5}-2)(48\cdot 2^{n+1}-2^{n+5}+28)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+6(\frac{\sqrt{(2^{n+4}-8)(48\cdot 2^{n+1}-2^{n+4}+34)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+6(\frac{\sqrt{(2^{n+4}-9)(48\cdot 2^{n+1}-2^{n+4}+35)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+6(\frac{\sqrt{(2^{n+4}-10)(48\cdot 2^{n+1}-2^{n+4}+36)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+24(\frac{\sqrt{(2^{n+3}-5)(48\cdot 2^{n+1}-2^{n+3}+31)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+12(\frac{\sqrt{(2^{n+4}-13)(48\cdot 2^{n+1}-2^{n+4}+39)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+18(\frac{\sqrt{(2^{n+5}+2)(48\cdot 2^{n+1}-2^{n+5}+24)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+36(\frac{\sqrt{(2^{n+4}-5)(48\cdot 2^{n+1}-2^{n+4}+31)}}{24\cdot 2^{n+1}+13})^{\gamma}$$
$$+4(\frac{\sqrt{48\cdot 2^{n+1}+25}}{24\cdot 2^{n+1}+13})^{\gamma}.$$

$GA_2(NS_3[n])$

$$=\sum_{i=1}^{n}3\cdot 2^{i+1}\frac{\sqrt{(2^{n-i+4}-8)(48\cdot 2^{n+1}-2^{n-i+4}+34)}}{24\cdot 2^{n+1}+13}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}\frac{\sqrt{(2^{n-i+4}-9)(48\cdot 2^{n+1}-2^{n-i+4}+35)}}{24\cdot 2^{n+1}+13}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}\frac{\sqrt{(2^{n-i+4}-10)(48\cdot 2^{n+1}-2^{n-i+4}+36)}}{24\cdot 2^{n+1}+13}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+3}\frac{\sqrt{(2^{n-i+3}-5)(48\cdot 2^{n+1}-2^{n-i+3}+31)}}{24\cdot 2^{n+1}+13}$$
$$+\sum_{i=1}^{n}3\cdot 2^{i+1}\frac{\sqrt{(2^{n-i+4}-13)(48\cdot 2^{n+1}-2^{n-i+4}+39)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{6\sqrt{(2^{n+4}-2)(48\cdot 2^{n+1}-2^{n+4}+28)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{3\sqrt{(2^{n+5}+8)(48\cdot 2^{n+1}-2^{n+5}+18)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{3\sqrt{(2^{n+5}+2)(48\cdot 2^{n+1}-2^{n+5}+24)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{3\sqrt{(2^{n+5}+1)(48\cdot 2^{n+1}-2^{n+5}+25)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{3\sqrt{2^{n+1}(48\cdot 2^{n+1}-2^{n+5}+26)}}{24\cdot 2^{n+1}-2^{n+4}+2^{n}+13}$$
$$+\frac{3\sqrt{(2^{n+1}-1)(48\cdot 2^{n+1}-2^{n+5}+27)}}{24\cdot 2^{n+1}+2^{n}-2^{n+4}+13}$$
$$+\frac{3\sqrt{(2^{n+5}-2)(48\cdot 2^{n+1}-2^{n+5}+28)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{6\sqrt{(2^{n+4}-8)(48\cdot 2^{n+1}-2^{n+4}+34)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{6\sqrt{(2^{n+4}-9)(48\cdot 2^{n+1}-2^{n+4}+35)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{6\sqrt{(2^{n+4}-10)(48\cdot 2^{n+1}-2^{n+4}+36)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{24\sqrt{(2^{n+3}-5)(48\cdot 2^{n+1}-2^{n+3}+31)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{12\sqrt{(2^{n+4}-13)(48\cdot 2^{n+1}-2^{n+4}+39)}}{24\cdot 2^{n+1}+13}$$

$$+\frac{18\sqrt{(2^{n+5}+2)(48\cdot 2^{n+1}-2^{n+5}+24)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{36\sqrt{(2^{n+4}-5)(48\cdot 2^{n+1}-2^{n+4}+31)}}{24\cdot 2^{n+1}+13}$$
$$+\frac{4\sqrt{48\cdot 2^{n+1}+25}}{24\cdot 2^{n+1}+13}.$$

$$PI_v(NS_3[n],x)$$
$$=(96\cdot 2^n+34)x^{48\cdot 2^{n+1}+26}+6x^{48\cdot 2^{n+1}-2^{n+5}+2^{n+1}+26}.$$
$$PI_v(NS_3[n])=9216\cdot 4^n+6156\cdot 2^n+1040.$$

VI. Supplemental Results

The purpose of this section is to present more findings on the distance-based indices of several important molecular structures in chemical and pharmacal areas.

A. $TC_4C_8(R)$ Nanotorus

The aim of this subsection is to determine the Szeged polynomial, vertex PI polynomial, second ABC index and second GA related indices of $TC_4C_8(R)$ nanotorus. We assume that the integer variables *p* and *q* in this subsection are denoted by the number of rhombs in each row and the number of rhombs in each column, respectively. From a chemical perspective, $TC_4C_8(R)$ is a kind of rhombic nanotorus $R[p,q]$, where the unit rhombic cell of the rhomb-octagonal lattice *R* has been selected with the four vertices (see Fig. 6 as the two dimensional version of $R[p,q]$).

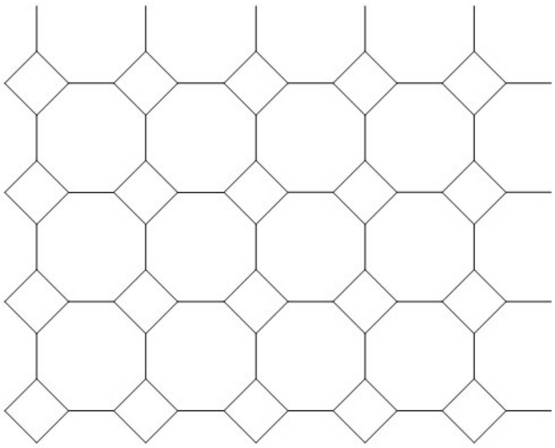

Fig. 6. The 2D version of $R[p,q]$.

In the following context of this subsection, for convenience, we always use $TC_4C_8(R)$ to denote $TC_4C_8(R)[p,q]$. Several articles contributed to this molecular structure from the standpoint of mathematics. Mehranian et al., [26] presented the revised Szeged index of $TC_4C_8(R)$ nanotorus. Yousefi and Ashrafi [27] applied a 3-dimensional matrix trick for determining the number of vertices with a fixed distance from a fixed vertex in a $TC_4C_8(R)$ nanotorus. Ashrafi and Yousefi [28] obtained the Szeged index of $TC_4C_8(R/S)$ nanotori by means of algebraic technology.

Assume that the parameter *p* is odd. It is easy to check that the edge set of $TC_4C_8(R)$ can be divided into three subsets (see Fig. 7 for the description of these three kinds of edges): E_1, E_2 and E_3. Here, E_1, E_2 and E_3 are the set of horizontal edges, vertical edges and rhombs edges, respectively.

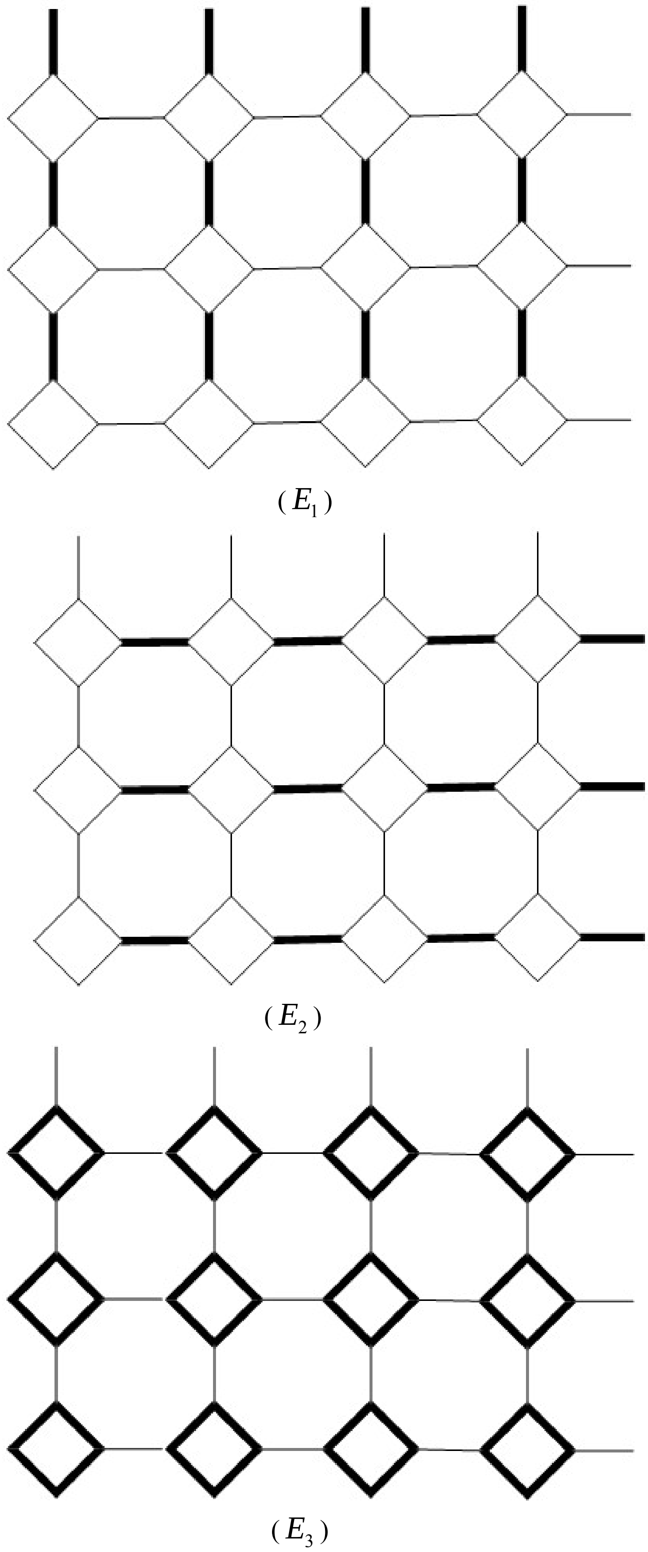

Fig. 7. Three kinds of edges in $TC_4C_8(R)$.

By computation and deduction in view of three classes of edges, we verify that there is different value of $n_u(e)$ and

$n_v(e)$ according to different relationship between p and q. Specifically, we infer the following conclusions which can be divided into four situations:

- $p < q,\ q \equiv 0 (\mathrm{mod}\ 2),\ p \equiv 1 (\mathrm{mod}\ 2)$.

$$Sz_v(TC_4C_8(R),x)$$
$$= pqx^{4p^2q^2} + pqx^{(2pq-2q+p)^2} + 4pqx^{2pq(2pq-p)},$$
$$PI_v(TC_4C_8(R),x)$$
$$= pqx^{4pq} + pqx^{2(2pq-2q+p)} + 4pqx^{4pq-p},$$
$$PI_v(TC_4C_8(R)) = 24p^2q^2 - 4q^2p - 2p^2q,$$
$$ABC_2(TC_4C_8(R)) = \frac{\sqrt{4pq-2}}{2}$$
$$+\frac{pq\sqrt{4pq-4q+2p-2}}{2pq-2q+p} + 4pq\sqrt{\frac{4pq-p-2}{2pq(2pq-p)}},$$
$$GA_2(TC_4C_8(R)) = 2pq + pq\frac{8\sqrt{2pq(2pq-p)}}{4pq-p},$$
$$GA_2^{\gamma}(TC_4C_8(R)) = 2pq + 4pq\left(\frac{2\sqrt{2pq(2pq-p)}}{4pq-p}\right)^{\gamma}.$$

- $p \ge q,\ q \equiv 0 (\mathrm{mod}\ 2),\ p \equiv 1 (\mathrm{mod}\ 2)$.

$$Sz_v(TC_4C_8(R),x)$$
$$= pqx^{4p^2q^2} + pqx^{(2pq-q)^2} + 4pqx^{2pq(2pq-q)},$$
$$PI_v(TC_4C_8(R),x)$$
$$= pqx^{4pq} + pqx^{4pq-2q} + 4pqx^{4pq-q},$$
$$PI_v(TC_4C_8(R)) = 24p^2q^2 - 6q^2p,$$
$$ABC_2(TC_4C_8(R)) = \frac{\sqrt{4pq-2}}{2}$$
$$+\frac{pq\sqrt{4pq-2q-2}}{2pq-q} + 4pq\sqrt{\frac{4pq-q-2}{2pq(2pq-q)}},$$
$$GA_2(TC_4C_8(R)) = 2pq + pq\frac{8\sqrt{2pq(2pq-q)}}{4pq-q},$$
$$GA_2^{\gamma}(TC_4C_8(R)) = 2pq + 4pq\left(\frac{2\sqrt{2pq(2pq-q)}}{4pq-q}\right)^{\gamma}.$$

- $p \ge q,\ q \equiv 1 (\mathrm{mod}\ 2),\ p \equiv 1 (\mathrm{mod}\ 2)$.

$$Sz_v(TC_4C_8(R),x) = pqx^{(2pq-2p+q)^2} + 5pqx^{(2pq-q)^2},$$
$$PI_v(TC_4C_8(R),x) = pqx^{4pq-4p+2q} + 5pqx^{4pq-2q},$$
$$PI_v(TC_4C_8(R)) = 24p^2q^2 - 4p^2q - 8q^2p,$$
$$ABC_2(TC_4C_8(R)) = \frac{pq\sqrt{4pq-4p+2q-2}}{2pq-2p+q}$$
$$+\frac{5pq\sqrt{4pq-2q-2}}{2pq-q},$$
$$GA_2(TC_4C_8(R)) = GA_2^{\gamma}(TC_4C_8(R)) = 6pq.$$

- $p < q,\ q \equiv 1 (\mathrm{mod}\ 2),\ p \equiv 1 (\mathrm{mod}\ 2)$.

$$Sz_v(TC_4C_8(R),x) = pqx^{(2pq-2q+p)^2} + 5pqx^{(2pq-p)^2},$$
$$PI_v(TC_4C_8(R),x) = pqx^{4pq-4q+2p} + 5pqx^{4pq-2p},$$
$$PI_v(TC_4C_8(R)) = 24p^2q^2 - 8p^2q - 4q^2p,$$
$$ABC_2(TC_4C_8(R)) = \frac{pq\sqrt{4pq-4q+2p-2}}{2pq-2q+p}$$
$$+\frac{5pq\sqrt{4pq-2p-2}}{2pq-p},$$
$$GA_2(TC_4C_8(R)) = GA_2^{\gamma}(TC_4C_8(R)) = 6pq.$$

B. H-naphtalenic Nanotubes

In this subsection, we consider the distance-based indices of H-Naphtalenic nanotubes. The phenylenic net is a widely appeared molecular structure in compounds and drugs which is consisted with the sequences $C_6, C_6, C_4, C_6, \cdots$ and $C_6, \cdots, C_6, C_6, C_4, C_6, C_6$. Readers can refer to Fig. 8 on the two classes of naphthy networks. In the following context of this subsection, we use $\mathrm{NPHX}[2m, 2n]$ to denote the H-Naphtalenic nanotubes.

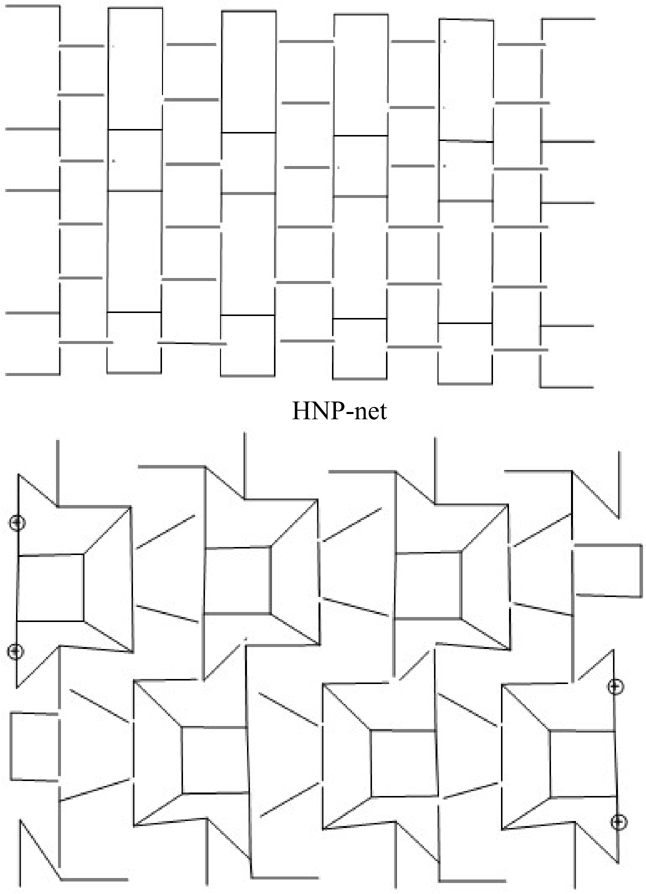

Fig. 8. Two classes of naphthylenic net.

Several advances have been obtained on the theoretical analysis of H-naphtalenic nanotubes. Yazdani and Bahrami [29] studied the topological descriptors of H-naphtalenic nanotubes. Yazdani and Bahrami [30] raised the edge PI index of H-naphtalenic nanotubes and nanotori. Siddiqui and Imran [31] proofed that the metric dimension of

H-Naphtalenic is not finite but their partition dimension is of three.

Now, we consider the Szeged polynomial, vertex PI polynomial, second ABC and second GA related indices of $\mathrm{NPHX}[2m,2n]$ according to the relationship between *m* and *n*.

$$PI_v(\mathrm{NPHX}[2m,2n],x)$$
$$=(12n^2+24m^2-12m+6m(2|m-n|-1))x^{40mn},$$
$$PI_v(\mathrm{NPHX}[2m,2n])$$
$$=40mn(12n^2+24m^2-12m+6m(2|m-n|-1)).$$

- $m\geq n$.

$$Sz_v(\mathrm{NPHX}[2m,2n],x)=\sum_{i=1}^{2m}6nx^{10n(i+1)(40mn-10n(i+1))}$$
$$+6m\sum_{i=1}^{2|m-n|-1}x^{4mn(i+1)(40mn-4mn(i+1))}$$
$$+12m\sum_{i=1}^{2m-1}x^{5n(2i+m)(40mn-5n(2i+m))},$$

$$ABC_2(\mathrm{NPHX}[2m,2n])$$
$$=\sum_{i=1}^{2m}6n\sqrt{\frac{20mn-1}{10n(i+1)(20mn-5n(i+1))}}$$
$$+6m\sum_{i=1}^{2|m-n|-1}\sqrt{\frac{20mn-1}{4mn(i+1)(20mn-2mn(i+1))}}$$
$$+12m\sum_{i=1}^{2m-1}\sqrt{\frac{40mn-2}{5n(2i+m)(40mn-5n(2i+m))}},$$

$$GA_2(\mathrm{NPHX}[2m,2n])$$
$$=\sum_{i=1}^{2m}\frac{3\sqrt{n(i+1)(4mn-n(i+1))}}{m}$$
$$+\sum_{i=1}^{2|m-n|-1}\frac{3\sqrt{2m(i+1)(20m-2m(i+1))}}{5}$$
$$+\sum_{i=1}^{2m-1}3\sqrt{(2i+m)(8m-(2i+m))},$$

$$GA_2^{\gamma}(\mathrm{NPHX}[2m,2n])$$
$$=\sum_{i=1}^{2m}6n\left(\frac{\sqrt{(i+1)(4m-(i+1))}}{2m}\right)^{\gamma}$$
$$+6m\sum_{i=1}^{2|m-n|-1}\left(\frac{\sqrt{(i+1)(10-(i+1))}}{5}\right)^{\gamma}$$
$$+12m\sum_{i=1}^{2m-1}\left(\frac{\sqrt{(2i+m)(8m-(2i+m))}}{4m}\right)^{\gamma}.$$

- $m<n$.

$$Sz_v(\mathrm{NPHX}[2m,2n],x)=\sum_{i=1}^{2m}6nx^{10n(i+1)(40mn-10n(i+1))}$$
$$+2\sum_{i=1}^{2|m-n|-1}3mx^{2(2mn-1)(i+1)(40mn-2(2mn-1)(i+1))}$$
$$+2\sum_{i=1}^{2m-1}6mx^{5n(2i+m)(40mn-5n(2i+m))},$$

$$ABC_2(\mathrm{NPHX}[2m,2n])$$
$$=\sum_{i=1}^{2m}6n\sqrt{\frac{20mn-1}{10n(i+1)(20mn-5n(i+1))}}$$
$$+6m\sum_{i=1}^{2|m-n|-1}\sqrt{\frac{20mn-1}{(2mn-1)(i+1)(40mn-4mn(i+1))}}$$
$$+12m\sum_{i=1}^{2m-1}\sqrt{\frac{40mn-2}{5n(2i+m)(40mn-5n(2i+m))}},$$

$$GA_2(\mathrm{NPHX}[2m,2n])$$
$$=\sum_{i=1}^{2m}\frac{3\sqrt{n(i+1)(4mn-n(i+1))}}{m}$$
$$+\sum_{i=1}^{2|m-n|-1}\frac{3\sqrt{(2mn-1)(i+1)(20mn-2mn(i+1))}}{5n}$$
$$+\sum_{i=1}^{2m-1}3\sqrt{(2i+m)(8m-(2i+m))},$$

$$GA_2^{\gamma}(\mathrm{NPHX}[2m,2n])$$
$$=\sum_{i=1}^{2m}6n\left(\frac{\sqrt{2(i+1)(8m-2(i+1))}}{5m}\right)^{\gamma}$$
$$+6m\sum_{i=1}^{2|m-n|-1}\left(\frac{\sqrt{(2mn-1)(i+1)(20mn-2mn(i+1))}}{10mn}\right)^{\gamma}$$
$$+12m\sum_{i=1}^{2m-1}\left(\frac{\sqrt{(2i+m)(8m-(2i+m))}}{4m}\right)^{\gamma}.$$

C. *An Infinite Array of* 1,3-*Adamantane*

In this subsection, we consider another molecular structure of infinite array, and the Szeged polynomial, vertex PI polynomial, second GA related indices and second ABC index of 1,3-adamantane array A_n (see Fig. 9 for more details on its molecular structure) are manifested. There are a few articles studying on this graph, Behroozpoor et al., [32] presented the Wiener index and edge version of Wiener index of 1,3-Adamantane Array.

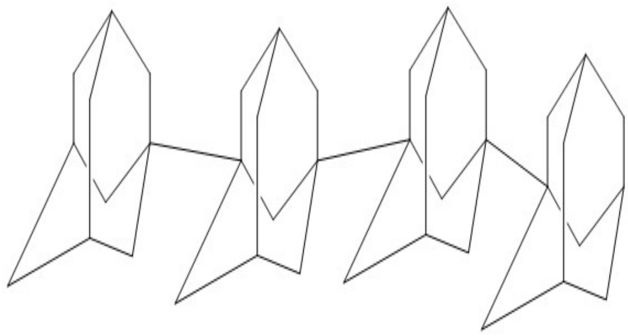

Fig. 9. 1,3-Adamantane Array.

By means of the structure analysis and computing, we get

$$Sz_v(A_n,x)=\sum_{i=1}^{n-1}x^{100i(n-i)}+3\sum_{i=1}^{n}x^{(10(n-i)+6)(10(i-1)+4)}$$

$$+3\sum_{i=1}^{n}x^{(10(n-i)+4)(10(i-1)+6)}+3\sum_{i=1}^{n}x^{4(10(n-1)+6)},$$

$$PI_v(A_n,x)=(13n-1)x^{10n},$$

$$PI_v(A_n)=130n^2-10n,$$

$$ABC_2(A_n)=\sum_{i=1}^{n-1}\sqrt{\frac{5n-1}{50i(n-i)}}$$

$$+3\sum_{i=1}^{n}\sqrt{\frac{5n-1}{(5(n-i)+3)(10(i-1)+4)}}$$

$$+3\sum_{i=1}^{n}\sqrt{\frac{5n-1}{(5(n-i)+2)(10(i-1)+6)}}+3\sum_{i=1}^{n}\sqrt{\frac{5n-1}{2(10(n-1)+6)}},$$

$$GA_2(A_n)=\sum_{i=1}^{n-1}\frac{2\sqrt{i(n-i)}}{n}$$

$$+3\sum_{i=1}^{n}\frac{\sqrt{(10(n-i)+6)(10(i-1)+4)}}{5n}$$

$$+3\sum_{i=1}^{n}\frac{\sqrt{(10(n-i)+4)(10(i-1)+6)}}{5n}$$

$$+3\sum_{i=1}^{n}\frac{\sqrt{4(10(n-1)+6)}}{5n},$$

$$GA_2^{\gamma}(A_n)=\sum_{i=1}^{n-1}\left(\frac{2\sqrt{i(n-i)}}{n}\right)^{\gamma}$$

$$+3\sum_{i=1}^{n}\left(\frac{\sqrt{(10(n-i)+6)(10(i-1)+4)}}{5n}\right)^{\gamma}$$

$$+3\sum_{i=1}^{n}\left(\frac{\sqrt{(10(n-i)+4)(10(i-1)+6)}}{5n}\right)^{\gamma}$$

$$+3\sum_{i=1}^{n}\left(\frac{\sqrt{4(10(n-1)+6)}}{5n}\right)^{\gamma}.$$

VII. Conclusion

Combinatorial chemistry is a new powerful technology in molecular recognition and drug design. It is a wet-laboratory methodology purposed to massively parallel screening of chemical compounds for the founding of compounds that have certain biological activities. The power of trick draws from the interaction between computational modeling and experimental design.

In this paper, we discuss the distance-based indices (such as the second and the third atom bond connectivity index, the second and the third geometric-arithmetic index, vertex-edge Szeged index, edge-vertex Szeged index, and total Szeged index et al.), and several conclusions are presented to determine parts of these indices. Furthermore, we obtain the exact expression of Szeged polynomial, vertex PI polynomial and index, second ABC index and second GA related indices of the following three molecular structures as additional conclusion: $TC_4C_8(R)$ Nanotorus, H-Naphtalenic nanotubes and 1,3-adamantane array. The result achieved in our paper illustrates the promising prospects of application for chemistry, medicine and pharmaceutical.

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