Optimal Pricing and Retail Service Decisions in an Uncertain Supply Chain

Shengju Sang

Abstract—**In a two-stage supply chain composed of one manufacturer and one retailer, the pricing and retail service decisions are researched in an uncertain environment. The manufacturing cost, retail cost and market demand are all characterized as uncertain variables. In consideration of the different market power of the manufacturer and the retailer, three non-cooperative games are analyzed, and their optimal strategies are also obtained by the method of uncertainty theory. Finally, a numerical example is given to compare the optimal strategies in three non-cooperative game models. It shows that the manufacturer and the retailer make the largest expected profits in the Manufacturer-Stackelberg and Retailer-Stackelberg games, respectively, and the customers obtain the highest service level and lowest retail price in the Retailer-Stackelberg game.**

***Index Terms*—supply chain, retail service, uncertain environment, uncertain variable**

I. Introduction

With current dynamic and competitive environment, apart from the pricing, the retail service is also an important factor that influences a consumer's decision to buy a product. The retailer can use the retail service as an effective tool to compete against the direct channel. The retail services provide by the retailer refer to all forms of demand-increasing services, which include presale advice, customer support, in-store advertising and promotions, return service, and technical and shopping assistant, etc [1].

By now, the pricing and service decisions problem has been well researched by both practitioners and scholars. Iyer [2], Tsay and Agrawal [3], Han et al. [4] studied the pricing and service competition problem of two retailers in a traditional supply chain. Xiao and Yang [5] formulated a pricing and service competition model of two supply chains. Xiao and Yang [6] also took the risk sharing rule into consideration and developed a price and service competition model with demand uncertainty. Lu et al. [7], Wu [8] studied the pricing and service decisions problem with two manufacturers and one common retailer. Yan and Pei [9], Dan et al. [10], Wang and Zhao [11] analyzed the pricing and retail service decisions problem in a dual-channel supply chain.

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The literature mentioned above discussed the pricing and service decisions problem with deterministic demand and known operation costs. However, in real world, especially for some new electronic products, the relevant precise dates are difficult to obtain duce to lack of historical data. In this situation, the market base and operation cost can usually be predicted by some experts. Thus, fuzzy set theory proposed by Zadeh [12], is used to deal with the pricing decisions problem of the supply chain by some scholars [13-20]. Some researches [21-22] also studied the pricing and service decisions problem in a fuzzy environment, where the demand was a fuzzy liner function of the selling price and service level.

When we use fuzzy set theory to solve the expert's prediction, there may be some problems. For example, the market demand predicted by experts may be “about 100”. If we measure “about 100” using the fuzzy set theory, we may obtain the market demand is “about 100” with belief degree 1 by possibility measure, and is “not 100” with belief degree 1 as well. That is to say, “about 100” and “not 100” have the same belief degree in possibility measure. It seems that nobody can accept this conclusion. Hence, those imprecise quantities such as “about 100” cannot be quantified by possibility measure, and should not be considered as fuzzy variables. To deal with these problems, Liu [23-24] proposed an uncertainty theory. Later, Ding [25-26] applied the uncertainty theory to solve the inventory problems of the supply chain.

To the best of our knowledge, there is no study that deals with the pricing and retail service decisions problem of the supply chain in an uncertainty environment. Therefore, in this paper, we discuss the pricing and service decisions with a manufacturer and a retailer, in which the market base, price elasticity, service elasticity, investment elasticity, manufacturing cost and retail cost are all characterized as uncertain variables. We mainly discuss the conditions where the manufacturer and the retailer pursue three different power structures, and analyze how the pricing and service are determined in an uncertain environment.

The rest of paper is organized as follows. In section II, the uncertain theory related to this paper is introduced. Section III describes the problem in our models. Section IV develops three non-cooperative games between the manufacturer and the retailer. Section V provides a numerical example to illustrate the results of the proposed models. The last section summarizes the work done in this paper and further research areas.

II. Preliminaries

Definition 1. [24] Let L be a σ-algebra on a nonempty set Γ and M be a set function from L to $[0,1]$. Then M is called an uncertain measure if it satisfies the following four axioms

Axiom 1. (Normality axiom) $M\{\Gamma\}=1$.

Axiom 2. (Duality axiom) $M\{\Lambda\}+M\{\Lambda^c\}=1$, for any event Λ.

Axiom 3. (Subadditivity axiom) For every countable sequence of events $\{\Lambda_i\}$, $i=1,2,\ldots$, we have

$$M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\}\le\sum_{i=1}^{\infty}M\{\Lambda_i\}$$

Axiom 4. (Product axiom) Let $\{\Gamma_k,L_k,M_k\}$ be an uncertainty space, $k=1,2,\ldots$, the product uncertain measure M is an uncertain measure satisfying

$$M\left\{\prod_{i=1}^{\infty}A_k\right\}=\bigcap_{k=1}^{\infty}M_k\{A_k\}$$

where A_k are arbitrarily chosen events from L_k, $k=1,2,\ldots$, respectively.

Definition 2. [23] An uncertain variable is a measurable function ξ from an uncertainty space $\{\Gamma_k,L_k,M_k\}$ to the set of a real number, i.e., for any Borel set B of real numbers, the set

$$\{\xi\in B\}=\{\gamma\in\Gamma \mid \xi(\gamma)\in B\}$$

is an event.

Definition 3. [23] The uncertain variables $\xi_1,\xi_2,\ldots,\xi_n$ are called independent if

$$M\left\{\prod_{i=1}^{n}\{\xi_i\in B_i\}\right\}=\bigcap_{i=1}^{n}M\{\xi_i\in B_i\}$$

for any Borel sets $B_1,B_2,\ldots,B_n$.

Definition 4. [23] Let ξ be an uncertain variable, and its uncertainty distribution Φ is defined by

$$\Phi(x)=M\{\xi\le x\}$$

for any real number x.

Definition 5. [23] An uncertain variable $\xi=L(a,b)$ is called a linear uncertain variable if it has the following uncertainty distribution

$$\Phi(x)=\begin{cases}0, & x\le a\\ (x-a)/(b-a), & a\le x\le b\\ 1, & x\ge b\end{cases}$$

where a and b are real numbers with $a<b$.

Definition 6. [23] An uncertain variable $\xi=Z(a,b,c)$ is called a zigzag uncertain variable if it has the following uncertainty distribution

$$\Phi(x)=\begin{cases}0, & x<a\\ (x-a)/2(b-a), & a\le x\le b\\ (x+c-2b)/2(c-b), & b<x\le c\\ 1, & x\ge c\end{cases}$$

where a, b and c are real numbers with $a<b<c$.

Lemma 1. [24] Let ξ be an uncertain variable with uncertainty distribution Φ. If the expected value of ξ exists, then

$$E[\xi]=\int_0^1\Phi^{-1}(\alpha)\,\mathrm{d}\alpha$$

where Φ^{-1} is the inverse function of Φ.

Example 1. Let $\xi=L(a,b)$ be a linear uncertain variable. Then, its inverse uncertainty distribution is

$$\Phi^{-1}(\alpha)=a+(b-a)\alpha\ ,\ \alpha\in[0,1]$$

The expected value can be obtained

$$E[\xi]=\int_0^1\big(a+(b-a)\alpha\big)\mathrm{d}\alpha=\frac{a+b}{2}$$

Example 2. Let $\xi=Z(a,b,c)$ be a zigzag uncertain variable. Then, its inverse uncertainty distribution is

$$\Phi^{-1}(\alpha)=\begin{cases}a+2(b-a)\alpha, & 0\le\alpha\le0.5\\ 2b-c+2(c-b)\alpha, & 0.5<\alpha\le1\end{cases}$$

The expected value can be obtained

$$E[\xi]=\int_0^{0.5}\big(a+2(b-a)\alpha\big)\mathrm{d}\alpha+\int_{0.5}^{1}\big(2b-c+2(c-b)\alpha\big)\mathrm{d}\alpha$$
$$=\frac{a+2b+c}{4}$$

Lemma 2. [24] Let $\xi_1,\xi_2,\ldots,\xi_n$ be independent uncertain variables with uncertainty distributions $\Phi_1,\Phi_2,\ldots,\Phi_n$, respectively. A function $f(x_1,x_2,\ldots,x_n)$ is strictly increasing with respected to $x_1,x_2,\ldots,x_m$ and strictly decreasing with respected to $x_{m+1},x_{m+2},\ldots,x_n$. Then the expected value of $\xi=f(\xi_1,\xi_2,\ldots,\xi_n)$ is

$$E[\xi]=\int_0^1 f\left(\Phi_1^{-1}(\alpha),\ldots,\Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha),\ldots,\Phi_n^{-1}(1-\alpha)\right)\mathrm{d}\alpha$$

III. Problem Descriptions

In this paper, we restrict our study on a two stage supply chain pricing and selling service decisions problem which includes one manufacturer and one retailer. The manufacturer produces new products at unit manufacturing cost $\tilde{c}_m$, which is an uncertain variable, and wholesales them to the retailer at unit wholesale price w, which is a decision variable. The retailer in turn retails them to the costumers with unit selling cost $\tilde{c}_r$, which is an uncertain variable. The uncertain market demand q is assumed as a linear function of the retail price p and retail service level s, which is given by

$$q=\tilde{d}-\tilde{\beta}p+\tilde{\gamma}s$$

where $\tilde{d}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are uncertain variables, $\tilde{d}$ denotes the market base, $\tilde{\beta}$ denotes the price elastic coefficient of the demand to retail price, and $\tilde{\gamma}$ denotes the service elastic coefficient of the demand to retail service. As the retail price p can be considered as the total of the wholesale price w and profit margin m, we consider retail price as $p=w+m$. Thus, the market demand for new products can be rewritten as

$$q = \tilde{d} - \tilde{\beta}(w+m) + \tilde{\gamma}s$$

Further, we assume that the marginal cost of the retailer is not affected by the retail service. The cost of achieving retail service level requires fixed investment, which is a quadratic function of the retail service level s. It is given by $\frac{1}{2}\tilde{\lambda}s^2$, where $\tilde{\lambda}$ denotes the investment coefficient, which is an uncertain variable.

In order to obtain the closed-form solutions, we give some assumptions as follows.

Assumptions 1. The manufacturer and the retailer are assumed to be risk neural and maximize their expected profits.

Assumptions 2. The uncertain variables $\tilde{c}_m$, $\tilde{c}_r$, $\tilde{d}$, $\tilde{\beta}$, $\tilde{\gamma}$ and $\tilde{\lambda}$ are assumed nonnegative and mutually independent.

Assumptions 3. We assume that the costs can not exceed the wholesale price and profit margin, and the market demand is positive.

$M\{w - \tilde{c}_m \le 0\} = 0$, $M\{m - \tilde{c}_r \le 0\} = 0$ and

$M\{\tilde{d} - \tilde{\beta}(w+m) + \tilde{\gamma}s \le 0\} = 0$

Thus, we can get the manufacturer's and retailer's profits as follows

$$\Pi_M = (w - \tilde{c}_m)\left(\tilde{d} - \tilde{\beta}(w+m) + \tilde{\gamma}s\right) \quad (1)$$

$$\Pi_R = (m - \tilde{c}_r)\left(\tilde{d} - \tilde{\beta}(w+m) + \tilde{\gamma}s\right) - \tfrac{1}{2}\tilde{\lambda}s^2 \quad (2)$$

In order to attain the optimal solutions, we should convert the uncertain profits of the manufacturer and the retailer into crisp forms first.

For concise, we define

$$E\left[\tilde{a}^{1-\alpha}\tilde{b}^{\alpha}\right] = \int_0^1 \Phi_{\tilde{a}}^{-1}(1-\alpha)\Phi_{\tilde{b}}^{-1}(\alpha)\,\mathrm{d}\alpha,$$

$$E\left[\tilde{a}^{1-\alpha}\tilde{b}^{1-\alpha}\right] = \int_0^1 \Phi_{\tilde{a}}^{-1}(1-\alpha)\Phi_{\tilde{b}}^{-1}(1-\alpha)\,\mathrm{d}\alpha.$$

where $\Phi_{\tilde{a}}^{-1}$ and $\Phi_{\tilde{b}}^{-1}$ are the reverse uncertainty distribution of the uncertain variables $\tilde{a}$ and $\tilde{b}$, respectively.

Theorem 1. The expected profits of the manufacturer and the retailer can be transformed as follows

$$\begin{aligned} E[\Pi_M] = &-E[\tilde{\beta}]w^2 + \left(E[\tilde{d}] - E[\tilde{\beta}]m + E[\tilde{\gamma}]s + E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\right)w \\ &+ E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]m - E\left[\tilde{c}_m^{1-\alpha}\tilde{\gamma}^{\alpha}\right]s - E\left[\tilde{c}_m^{1-\alpha}\tilde{d}^{\alpha}\right] \quad (3) \end{aligned}$$

$$\begin{aligned} E[\Pi_R] = &-E[\tilde{\beta}]m^2 - \tfrac{1}{2}E[\tilde{\lambda}]s^2 + E[\tilde{\gamma}]ms \\ &+ \left(E[\tilde{d}] - E[\tilde{\beta}]w + E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\right)m \\ &- E\left[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}\right]s + E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]w - E\left[\tilde{c}_r^{1-\alpha}\tilde{d}^{\alpha}\right] \quad (4) \end{aligned}$$

Proof: Let $\tilde{c}_m$, $\tilde{d}$, $\tilde{\beta}$ and $\tilde{\gamma}$ be the positive uncertain variables with uncertainty distributions $\Phi_{\tilde{c}_m}$, $\Phi_{\tilde{d}}$, $\Phi_{\tilde{\beta}}$ and $\Phi_{\tilde{\gamma}}$, respectively. From (1), we can find that $E[\tilde{\Pi}_M]$ is monotone decreasing with $\tilde{c}_m$ and $\tilde{\beta}$, and monotone increasing with $\tilde{d}$ and $\tilde{\gamma}$. Then referring to Lemma 1 and 2, we have

$$\begin{aligned} E[\Pi_M] &= E\left[(w - \tilde{c}_m)\left(\tilde{d} - \tilde{\beta}(w+m) + \tilde{\gamma}s\right)\right] \\ &= \int_0^1 \left[\left(w - \Phi_{\tilde{c}_m}(1-\alpha)\right)\left(\Phi_{\tilde{d}}(\alpha) - \Phi_{\tilde{\beta}}(1-\alpha)(w+m) + \Phi_{\tilde{\gamma}}(\alpha)s\right)\right]\mathrm{d}\alpha \\ &= E[\tilde{d}]w - E[\tilde{\beta}]w(w+m) + E[\tilde{\gamma}]ws - E\left[\tilde{c}_m^{1-\alpha}\tilde{d}^{\alpha}\right] \\ &\quad + E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right](w+m) - E\left[\tilde{c}_m^{1-\alpha}\tilde{\gamma}^{\alpha}\right]s \\ &= -E[\tilde{\beta}]w^2 + \left(E[\tilde{d}] - E[\tilde{\beta}]m + E[\tilde{\gamma}]s + E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\right)w \\ &\quad + E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]m - E\left[\tilde{c}_m^{1-\alpha}\tilde{\gamma}^{\alpha}\right]s - E\left[\tilde{c}_m^{1-\alpha}\tilde{d}^{\alpha}\right] \end{aligned}$$

In the same way, we derive $E[\Pi_R]$ showed as

$$\begin{aligned} E[\Pi_R] &= E\left[(m - \tilde{c}_r)\left(\tilde{d} - \tilde{\beta}(w+m) + \tilde{\gamma}s\right) - \tfrac{1}{2}\tilde{\lambda}s^2\right] \\ &= \int_0^1 \left[\left(m - \Phi_{\tilde{c}_r}(1-\alpha)\right)\left(\Phi_{\tilde{d}}(\alpha) - \Phi_{\tilde{\beta}}(1-\alpha)(w+m) + \Phi_{\tilde{\gamma}}(\alpha)s - \tfrac{1}{2}\Phi_{\tilde{\lambda}}(1-\alpha)s^2\right]\mathrm{d}\alpha \\ &= E[\tilde{d}]m - E[\tilde{\beta}]m(w+m) + E[\tilde{\gamma}]ms - E\left[\tilde{c}_r^{1-\alpha}\tilde{d}^{\alpha}\right] \\ &\quad + E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right](w+m) - E\left[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}\right]s - \tfrac{1}{2}E[\tilde{\lambda}]s^2 \\ &= -E[\tilde{\beta}]m^2 - \tfrac{1}{2}E[\tilde{\lambda}]s^2 + E[\tilde{\gamma}]ms \\ &\quad + \left(E[\tilde{d}] - E[\tilde{\beta}]w + E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\right)m \\ &\quad - E\left[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}\right]s + E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]w - E\left[\tilde{c}_r^{1-\alpha}\tilde{d}^{\alpha}\right] \end{aligned}$$

IV. MODELS ANALYSIS

In this section, we analyze the manufacturer and retailer how to set their optimal solutions when they pursue different power structures in an uncertain supply chain. We mainly discuss the conditions where they pursue three non-cooperative games: the manufacturer leads the supply chain, the retailer leads the supply chain, and they have the same power.

A. Manufacturer-Stackelberg game

Under the MS (Manufacturer-Stackelberg) game, the manufacturer obtains the supply chain power and leads the retailer. That is, firstly, the manufacturer sets the wholesale price w by using the retailer's reaction functions. Then, the retailer sets the profit margin m and retail service level s so as to maximize his expected profit. Thus, the MS game model can be given as follows

$$\begin{cases} \max\limits_{w} E[\Pi_M] = E\left[(w - \tilde{c}_m)\left(\tilde{d} - \tilde{\beta}(w+m^*) + \tilde{\gamma}s^*\right)\right] \\ \text{s.t.} \\ M\{w - \tilde{c}_m \le 0\} = 0 \\ \text{where } m^* \text{ and } s^* \text{ solve the following problem} \\ \max\limits_{m,s} E[\Pi_R] = E\left[(m - \tilde{c}_r)\left(\tilde{d} - \tilde{\beta}(w+m) + \tilde{\gamma}s\right) - \tfrac{1}{2}\tilde{\lambda}s^2\right] \\ \text{s.t.} \\ M\{m - \tilde{c}_r \le 0\} = 0 \\ M\{\tilde{d} - \tilde{\beta}(w+m) + \tilde{\gamma}s \le 0\} = 0 \end{cases} \quad (5)$$

We first obtain the optimal decisions of the retailer.

Theorem 2. In the MS game model, if $M\{m^*(w) - \tilde{c}_r \le 0\} = 0$, $M\{\tilde{d} - \tilde{\beta}(w + m^*(w)) + \tilde{\gamma}s^*(w) \le 0\} = 0$ and $2E[\tilde{\beta}]E[\tilde{\lambda}]$

$-\left(E\left[\tilde{\gamma}\right]\right)^2>0$ hold, the optimal reaction functions $m^*\left(w\right)$ and $s^*\left(w\right)$ of the retailer can be given by considering the wholesale price made earlier by the manufacturer

$$m^*\left(w\right)=A_1-A_3w \tag{5}$$

$$s^*\left(w\right)=A_2-A_4w \tag{6}$$

where

$$A_1=\frac{E\left[\tilde{\lambda}\right]\left(E\left[\tilde{d}\right]+E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\right)-E\left[\tilde{\gamma}\right]E\left[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}\right]}{2E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\left(E\left[\tilde{\gamma}\right]\right)^2},$$

$$A_2=\frac{E\left[\tilde{\gamma}\right]\left(E\left[\tilde{d}\right]+E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\right)-2E\left[\tilde{\beta}\right]E\left[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}\right]}{2E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\left(E\left[\tilde{\gamma}\right]\right)^2},$$

$$A_3=\frac{E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]}{2E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\left(E\left[\tilde{\gamma}\right]\right)^2},$$

$$A_4=\frac{E\left[\tilde{\beta}\right]E\left[\tilde{\gamma}\right]}{2E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\left(E\left[\tilde{\gamma}\right]\right)^2}.$$

Proof. Referring to (4), we can get the first order derivatives of $E\left[\Pi_R\right]$ to m and s as follows

$$\frac{\partial E\left[\Pi_R\right]}{\partial m}=-2E\left[\tilde{\beta}\right]m+E\left[\tilde{\gamma}\right]s+E\left[\tilde{d}\right]-E\left[\tilde{\beta}\right]w+E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right] \tag{9}$$

$$\frac{\partial E\left[\Pi_R\right]}{\partial s}=-2E\left[\tilde{\lambda}\right]s+E\left[\tilde{\gamma}\right]m-E\left[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}\right] \tag{10}$$

Then, the second order derivatives of $E\left[\Pi_R\right]$ to m and s can be shown as

$$\frac{\partial^2 E\left[\Pi_R\right]}{\partial m^2}=-2E\left[\tilde{\beta}\right],\quad \frac{\partial^2 E\left[\Pi_R\right]}{\partial m\partial s}=E\left[\tilde{\gamma}\right],$$

$$\frac{\partial^2 E\left[\Pi_R\right]}{\partial s^2}=-E\left[\tilde{\lambda}\right],\quad \frac{\partial^2 E\left[\Pi_R\right]}{\partial s\partial m}=E\left[\tilde{\gamma}\right]$$

Thus, the Hessian matrix can be obtained

$$H=\begin{vmatrix}\frac{\partial^2 E[\Pi_R]}{\partial m^2} & \frac{\partial^2 E[\Pi_R]}{\partial m\partial s}\\ \frac{\partial^2 E[\Pi_R]}{\partial s\partial m} & \frac{\partial^2 E[\Pi_R]}{\partial s^2}\end{vmatrix}=\begin{vmatrix}-2E\left[\tilde{\beta}\right] & E\left[\tilde{\gamma}\right]\\ E\left[\tilde{\gamma}\right] & -E\left[\tilde{\lambda}\right]\end{vmatrix}$$

Note that the Hessian matrix is negative definite, since $\tilde{\beta}$ and $\tilde{\lambda}$ are nonnegative uncertain variables, and $2E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\left(E\left[\tilde{\gamma}\right]\right)^2>0$. Consequently, $E\left[\Pi_R\right]$ is strictly jointly concave in m and s.

Setting (9) and (10) to zero, the first order conditions can be shown as

$$-2E\left[\tilde{\beta}\right]m+E\left[\tilde{\gamma}\right]s+E\left[\tilde{d}\right]-E\left[\tilde{\beta}\right]w+E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]=0 \tag{11}$$

$$-2E\left[\tilde{\lambda}\right]s+E\left[\tilde{\gamma}\right]m-E\left[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}\right]=0 \tag{12}$$

Solving (11) and (12), we obtain (6) and (7).
The proof of Theorem 2 is completed.

After knowing the retailer's reaction functions, the manufacturer would use them to maximize his expected profit by choosing the wholesale price.

Theorem 3. In the MS game model, if $M\left\{w^*-\tilde{c}_m\le 0\right\}=0$, $M\left\{m^*-\tilde{c}_r\le 0\right\}=0$, $M\left\{\tilde{d}-\tilde{\beta}\left(w^*+m^*\right)+\tilde{\gamma}s^*\le 0\right\}=0$, and $2E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\left(E\left[\tilde{\gamma}\right]\right)^2>0$ hold, the optimal solutions of the manufacturer and the retailer are

$$w^*=\frac{E\left[\tilde{d}\right]+E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]-E\left[\tilde{\beta}\right]A_1+E\left[\tilde{\gamma}\right]A_2}{2E\left[\tilde{\beta}\right]A_3}-\frac{E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]A_3-E\left[\tilde{c}_m^{1-\alpha}\tilde{\gamma}^{\alpha}\right]A_4}{2E\left[\tilde{\beta}\right]A_3} \tag{13}$$

$$m^*=A_1-A_3w^* \tag{14}$$

$$s^*=A_2-A_4w^* \tag{15}$$

where A_1, A_2, A_3 and A_4 are constants defined in Theorem 2.

Proof. Substituting $m^*\left(w\right)$ and $s^*\left(w\right)$ into (3), we can get the expected profit of the manufacturer $E\left[\Pi_M\right]$ as follows

$$\begin{aligned}E\left[\Pi_M\right]=&-E\left[\tilde{\beta}\right]A_3w^2+\Big(E\left[\tilde{d}\right]+E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]-E\left[\tilde{\beta}\right]A_1\\&+E\left[\tilde{\gamma}\right]A_2-E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]A_3+E\left[\tilde{c}_m^{1-\alpha}\tilde{\gamma}^{\alpha}\right]A_4\Big)w\\&+E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]A_1-E\left[\tilde{c}_m^{1-\alpha}\tilde{\gamma}^{\alpha}\right]A_2-E\left[\tilde{c}_m^{1-\alpha}\tilde{d}^{\alpha}\right]\end{aligned} \tag{16}$$

From (16), the first order derivative of $E\left[\Pi_M\right]$ to w can be shown as

$$\begin{aligned}\frac{\mathrm{d}\,E\left[\Pi_M\right]}{\mathrm{d}\,w}=&-2E\left[\tilde{\beta}\right]A_3w+E\left[\tilde{d}\right]+E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]-E\left[\tilde{\beta}\right]A_1\\&+E\left[\tilde{\gamma}\right]A_2-E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]A_3+E\left[\tilde{c}_m^{1-\alpha}\tilde{\gamma}^{\alpha}\right]A_4\end{aligned} \tag{17}$$

Then, the second order derivative of $E\left[\Pi_M\right]$ to w can be shown as

$$\frac{\mathrm{d}^2\,E\left[\Pi_M\right]}{\mathrm{d}\,w^2}=-2E\left[\tilde{\beta}\right]A_3$$

Note that the second order derivative of $E\left[\Pi_M\right]$ is negative definite, since $\tilde{\beta}$ is a nonnegative uncertain variable, and $A_3>0$. Consequently, $E\left[\Pi_M\right]$ is strictly concave in w.

Setting (17) to zero, the first order condition can be shown as

$$\begin{aligned}&-2E\left[\tilde{\beta}\right]A_3w+E\left[\tilde{d}\right]+E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]-E\left[\tilde{\beta}\right]A_1\\&+E\left[\tilde{\gamma}\right]A_2-E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]A_3+E\left[\tilde{c}_m^{1-\alpha}\tilde{\gamma}^{\alpha}\right]A_4=0\end{aligned} \tag{18}$$

Solving (18), we obtain (13).

Substituting w^* into (6) and (7), we obtain (14) and (15).
The proof of Theorem 3 is completed.

B. Retailer-Stackelberg game

Under the RS (Retailer-Stackelberg) game, the retailer obtains the supply chain power and leads the manufacturer. That is, firstly, the retailer sets the profit margin m and retail service level s by using the manufacturer's reaction function. Then, the manufacturer sets the wholesale price w so as to maximize his expected profit. Thus, the RS game model can be given as follows

$$\begin{cases} \max\limits_{m,s} E[\Pi_R] = E\left[(m-\tilde{c}_r)\left(\tilde{d}-\tilde{\beta}\left(w^{**}+m\right)+\tilde{\gamma}s\right)-\frac{1}{2}\tilde{\lambda}s^2\right] \\ \text{s.t.} \\ \mathcal{M}\{m-\tilde{c}_r \le 0\}=0 \\ \text{where } w^{**} \text{ solves the following problem} \\ \max\limits_{w} E[\Pi_M] = E\left[(w-\tilde{c}_m)\left(\tilde{d}-\tilde{\beta}(w+m)+\tilde{\gamma}s\right)\right] \\ \text{s.t.} \\ \mathcal{M}\{w-\tilde{c}_m \le 0\}=0 \\ \mathcal{M}\left\{\tilde{d}-\tilde{\beta}(w+m)+\tilde{\gamma}s \le 0\right\}=0 \end{cases} \tag{19}$$

Theorem 4. In the RS game model, if $\mathcal{M}\left\{w^{**}(m,s) -\tilde{c}_m \le 0\right\}=0$ and $\mathcal{M}\left\{\tilde{d}-\tilde{\beta}\left(w^{**}(m,s)+m\right)+\tilde{\gamma}s \le 0\right\}=0$ hold, the optimal reaction function $w^{**}(m,s)$ of the manufacturer can be given by considering the profit margin and retail service made earlier by the retailer

$$w^{**}(m,s)=\frac{E\left[\tilde{d}\right]-E\left[\tilde{\beta}\right]m+E[\tilde{\gamma}]s+E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]}{2E\left[\tilde{\beta}\right]} \tag{20}$$

Proof. Referring to Eq. (3), the first order derivative of $E[\Pi_M]$ to w can be shown as

$$\frac{\mathrm{d}E[\Pi_M]}{\mathrm{d}w}=-2E\left[\tilde{\beta}\right]w+E\left[\tilde{d}\right]-E\left[\tilde{\beta}\right]m+E[\tilde{\gamma}]s +E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right] \tag{21}$$

Then, the second order derivative of $E[\Pi_M]$ to w can be shown as

$$\frac{\mathrm{d}^2 E[\Pi_M]}{\mathrm{d}w^2}=-2E\left[\tilde{\beta}\right]$$

Note that the second order derivative of $E[\Pi_M]$ is negative definite, since $\tilde{\beta}$ is a nonnegative uncertain variable. Consequently, $E[\Pi_M]$ is strictly concave in w.

Setting (21) to zero, the first order condition can be shown as

$$-2E\left[\tilde{\beta}\right]w+E\left[\tilde{d}\right]-E\left[\tilde{\beta}\right]m+E[\tilde{\gamma}]s+E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]=0 \tag{22}$$

Solving (22), we obtain (20).

The proof of Theorem 4 is completed.

Theorem 5. In the RS game model, if $\mathcal{M}\left\{w^{**}-\tilde{c}_m \le 0\right\}=0$, $\mathcal{M}\left\{m^{**}-\tilde{c}_r \le 0\right\}=0$, $\mathcal{M}\left\{\tilde{d}-\tilde{\beta}\left(w^{**}+m^{**}\right)+\tilde{\gamma}s^{**} \le 0\right\}=0$, and $E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\frac{1}{4}\left(E[\tilde{\gamma}]\right)^2>0$ hold, the optimal solutions of the retailer and the manufacturer are

$$m^{**}=\frac{E\left[\tilde{\lambda}\right]B_1-\frac{1}{2}E[\tilde{\gamma}]B_2}{E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\frac{1}{4}\left(E[\tilde{\gamma}]\right)^2} \tag{23}$$

$$s^{**}=\frac{\frac{1}{2}E[\tilde{\gamma}]B_1-E\left[\tilde{\beta}\right]B_2}{E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\frac{1}{4}\left(E[\tilde{\gamma}]\right)^2} \tag{24}$$

$$w^{**}=\frac{E\left[\tilde{d}\right]-E\left[\tilde{\beta}\right]m^{**}+E[\tilde{\gamma}]s^{**}+E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]}{2E\left[\tilde{\beta}\right]} \tag{25}$$

where

$$B_1=\frac{1}{2}\left(E\left[\tilde{d}\right]+E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]-E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\right),$$

$$B_2=E\left[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}\right]-\frac{E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]E[\tilde{\gamma}]}{2E\left[\tilde{\beta}\right]}.$$

Proof. Substituting $w^{**}(m,s)$ into (4), the expected profit of the retailer $E[\Pi_R]$ can be shown as

$$\begin{aligned} E[\Pi_R]=&-\frac{1}{2}E\left[\tilde{\beta}\right]m^2-\frac{1}{2}E\left[\tilde{\lambda}\right]s^2+\frac{1}{2}E[\tilde{\gamma}]ms \\ &+\frac{1}{2}\left(E\left[\tilde{d}\right]+E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]-E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\right)m \\ &-\left(E\left[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}\right]-\frac{E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]E[\tilde{\gamma}]}{2E\left[\tilde{\beta}\right]}\right)s \\ &+\frac{E\left[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\left(E\left[\tilde{d}\right]+E\left[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\right)}{2E\left[\tilde{\beta}\right]} \\ &-E\left[\tilde{c}_r^{1-\alpha}\tilde{d}^{\alpha}\right] \end{aligned} \tag{26}$$

From (26), we can get the first order derivatives of $E[\Pi_R]$ to m and s as follows

$$\frac{\partial E[\Pi_R]}{\partial m}=-E\left[\tilde{\beta}\right]m+\frac{1}{2}E[\tilde{\gamma}]s+B_1 \tag{27}$$

$$\frac{\partial E[\Pi_R]}{\partial s}=-E\left[\tilde{\lambda}\right]s+\frac{1}{2}E[\tilde{\gamma}]m-B_2 \tag{28}$$

Then, the second order derivatives of $E[\Pi_R]$ to m and s can be shown as

$$\frac{\partial^2 E[\Pi_R]}{\partial m^2}=-E\left[\tilde{\beta}\right],\quad \frac{\partial^2 E[\Pi_R]}{\partial m\partial s}=\frac{1}{2}E[\tilde{\gamma}],$$

$$\frac{\partial^2 E[\Pi_R]}{\partial s^2}=-E\left[\tilde{\lambda}\right],\quad \frac{\partial^2 E[\Pi_R]}{\partial s\partial m}=\frac{1}{2}E[\tilde{\gamma}].$$

Thus, the Hessian matrix can be obtained

$$H=\begin{vmatrix} \frac{\partial^2 E[\Pi_R]}{\partial m^2} & \frac{\partial^2 E[\Pi_R]}{\partial m\partial s} \\ \frac{\partial^2 E[\Pi_R]}{\partial s\partial m} & \frac{\partial^2 E[\Pi_R]}{\partial s^2} \end{vmatrix}=\begin{vmatrix} -E\left[\tilde{\beta}\right] & \frac{1}{2}E[\tilde{\gamma}] \\ \frac{1}{2}E[\tilde{\gamma}] & -E\left[\tilde{\lambda}\right] \end{vmatrix}$$

Note that the Hessian matrix is negative definite, since $\tilde{\beta}$ and $\tilde{\lambda}$ are nonnegative uncertain variables, and $E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\frac{1}{4}\left(E[\tilde{\gamma}]\right)^2>0$. Consequently, $E[\Pi_R]$ is strictly jointly concave in m and s.

Setting (27) and (28) to zero, the first order conditions can be shown as

$$-E\left[\tilde{\beta}\right]m+\frac{1}{2}E[\tilde{\gamma}]s+B_1=0 \tag{29}$$

$$-E\left[\tilde{\lambda}\right]s+\frac{1}{2}E[\tilde{\gamma}]m-B_2=0 \tag{30}$$

Solving (29) and (30), we obtain (23) and (24).

Substituting m^{**} and s^{**} into (20), we obtain (25).

The proof of Theorem 5 is completed.

C. Vertical-Nash game

Under the VN (Vertical-Nash) game, the manufacturer and the retailer have the same market power. That is, the manufacturer determines his wholesale price w, and the retailer makes his profit margin m and retail service s simultaneously and independently, so as to maximize their expected profits. Thus, the VN game model can be given as

$$\begin{cases} \max_{w} E[\Pi_M] = E\left[(w-\tilde{c}_m)\left(\tilde{d}-\tilde{\beta}(w+m)+\tilde{\gamma}s\right)\right] \\ \max_{m,s} E[\Pi_R] = E\left[(m-\tilde{c}_r)\left(\tilde{d}-\tilde{\beta}(w+m)+\tilde{\gamma}s\right)-\frac{1}{2}\tilde{\lambda}s^2\right] \\ \text{s.t.} \\ M\{w-\tilde{c}_m \le 0\}=0 \\ M\{m-\tilde{c}_r \le 0\}=0 \\ M\{\tilde{d}-\tilde{\beta}(w+m)+\tilde{\gamma}s \le 0\}=0 \end{cases} \quad (31)$$

Theorem 6. In the VN game model, if $M\{w^{***}-\tilde{c}_m \le 0\}=0$, $M\{m^{***}-\tilde{c}_r \le 0\}=0$ and $M\{\tilde{d}-\tilde{\beta}(w^{***}+m^{***})+\tilde{\gamma}s^{***} \le 0\}=0$ hold, the optimal solutions of the manufacturer and the retailer are

$$w^{***} = \frac{E[\tilde{d}]-E[\tilde{\beta}]A_1+E[\tilde{\gamma}]A_2+E[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}]}{2E[\tilde{\beta}]-E[\tilde{\beta}]A_3+E[\tilde{\gamma}]A_4} \quad (32)$$

$$m^{***} = A_1 - A_3 w^{***} \quad (33)$$

$$s^{***} = A_2 - A_4 w^{***} \quad (34)$$

where A_1, A_2, A_3 and A_4 are constants defined in Theorem 2.

Proof. Note that $E[\Pi_M]$ is strictly concave in w, and $E[\Pi_R]$ is strictly jointly concave in m and s.

Then, the first order conditions of $E[\Pi_M]$ and $E[\Pi_R]$ can be shown as

$$-2E[\tilde{\beta}]w+E[\tilde{d}]-E[\tilde{\beta}]m+E[\tilde{\gamma}]s+E[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}]=0 \quad (35)$$

$$-2E[\tilde{\beta}]m+E[\tilde{\gamma}]s+E[\tilde{d}]-E[\tilde{\beta}]w+E[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}]=0 \quad (36)$$

$$-2E[\tilde{\lambda}]s+E[\tilde{\gamma}]m-E[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}]=0 \quad (37)$$

Solving (35), (36) and (37), we obtain (32), (33) and (34). The proof of Theorem 6 is completed.

V. Numerical Example

Owing to the complicated forms of the solutions, we conduct a numerical example to compare the optimal solutions under three different game models. Due to lack of the historical date, the market base $\tilde{d}$, price elasticity $\tilde{\beta}$, service elasticity $\tilde{\gamma}$, investment elasticity $\tilde{\lambda}$, costs of the manufacturer and the retailer $\tilde{c}_m$ and $\tilde{c}_r$ are predicted by the experiences of the experts showed in Table I.

TABLE I
THE UNCERTAIN VARIABLES

Parameter	Linguistic description	Distribution
Market base $\tilde{d}$	About 200	$Z=(150,200,250)$
Price elasticity $\tilde{\beta}$	About 10	$Z=(8,10,12)$
Service elasticity $\tilde{\gamma}$	About 8	$Z=(7,8,9)$
Investment elasticity $\tilde{\lambda}$	Between 9 and 11	$L=(9,11)$
Manufacturing cost $\tilde{c}_m$	Between 7 and 9	$L=(7,9)$
Retail cost $\tilde{c}_r$	Between 1 and 3	$L=(1,3)$

From Table I, we obtain

$$E[\tilde{d}] = \frac{150+2\times200+250}{4} = 200,$$

$$E[\tilde{\beta}] = \frac{8+2\times10+12}{4} = 10, \quad E[\tilde{\gamma}] = \frac{7+2\times8+9}{4} = 8,$$

$$E[\tilde{\lambda}] = \frac{9+11}{2} = 10, \quad E[\tilde{c}_m] = \frac{7+9}{2} = 8, \quad E[\tilde{c}_r] = \frac{1+3}{2} = 2,$$

$$E[\tilde{c}_r^{1-\alpha}\tilde{d}^{\alpha}] = \int_0^1 \Phi_{\tilde{c}_r}^{-1}(1-\alpha)\Phi_{\tilde{d}}^{-1}(\alpha)\,\mathrm{d}\alpha = \frac{1150}{3},$$

$$E[\tilde{c}_r^{1-\alpha}\tilde{\beta}^{1-\alpha}] = \int_0^1 \Phi_{\tilde{c}_r}^{-1}(1-\alpha)\Phi_{\tilde{\beta}}^{-1}(1-\alpha)\,\mathrm{d}\alpha = \frac{62}{3},$$

$$E[\tilde{c}_r^{1-\alpha}\tilde{\gamma}^{\alpha}] = \int_0^1 \Phi_{\tilde{c}_r}^{-1}(1-\alpha)\Phi_{\tilde{\gamma}}^{-1}(\alpha)\,\mathrm{d}\alpha = \frac{47}{3},$$

$$E[\tilde{c}_m^{1-\alpha}\tilde{d}^{\alpha}] = \int_0^1 \Phi_{\tilde{c}_m}^{-1}(1-\alpha)\Phi_{\tilde{d}}^{-1}(\alpha)\,\mathrm{d}\alpha = \frac{4850}{3},$$

$$E[\tilde{c}_m^{1-\alpha}\tilde{\beta}^{1-\alpha}] = \int_0^1 \Phi_{\tilde{c}_m}^{-1}(1-\alpha)\Phi_{\tilde{\beta}}^{-1}(1-\alpha)\,\mathrm{d}\alpha = \frac{232}{3},$$

$$E[\tilde{c}_m^{1-\alpha}\tilde{\gamma}^{\alpha}] = \int_0^1 \Phi_{\tilde{c}_m}^{-1}(1-\alpha)\Phi_{\tilde{\gamma}}^{-1}(\alpha)\,\mathrm{d}\alpha = \frac{195}{3}.$$

Based on the above analysis, we present the optimal solutions of the manufacturer and the retailer in the MS, RS and VN games in Table II.

TABLE II
THE OPTIMAL SOLUTIONS WITH DIFFERENT POWER STRUCTURES

	w	m	s	p	$E[\Pi_M]$	$E[\Pi_R]$	$E[\Pi_{SC}]$
MS	12.99	5.75	3.03	18.74	114.24	122.29	236.53
RS	10.79	8.18	2.53	18.97	15.48	186.97	202.45
VN	12.08	6.42	3.57	18.50	108.17	158.79	266.96

Based on the results showed in Table II, we find:

(1) The wholesale price w is the highest in the MS case when the manufacturer has more power, followed by the VN and then the RS cases. The retailer sets the highest retail price in the RS case, and the lowest in the MS case. The retail service s is the lowest in the RS case this is because under this case the full costs of service are afforded by the retailer. The profit margin of the retailer m is the highest in the RS case, followed by the VN and then the RS cases.

(2) The manufacturer obtains the largest expected profit in the MS case, and the smallest in the RS case. On the other hand, the retailer obtains the largest expected profit in the RS case, and the smallest in the MS case. It indicates that the actor who is the leader in the supply chain takes advantage in making the higher expected profit. That is, the expected profits of the manufacturer and the retailer are the largest when they have the leadership. In addition, the supply chain system obtains the largest expected profit in the VN case, followed by the MS and then the RS cases.

(3) The VN case is a preferred policy for customers, this is because in this case the retail service level s is the highest and the retail price p is the lowest.

VI. Conclusion

This paper considers a supply chain pricing and retail service decisions problem in an uncertain environment, where the manufacturer and the retailer pursue three different kinds of scenarios: Manufacturer-Stackelberg, Retailer-Stackelberg and Vertical-Nash games. The models in our research contain three decision variables, wholesale price, profit margin and retail service level, six uncertain variables, market base, price elasticity, service elasticity, investment elasticity, manufacturing cost and retail cost, which is truly representative of the electronic industry. Our study mainly focus on one manufacturer and one retailer in a two stage supply chain, therefore, the pricing and service decisions with multiple competitive manufacturers and retailers are the important directions for the future research.

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