A Manufacturer Stackelberg Game in Price Competition Supply Chain under a Fuzzy Decision Environment

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Abstract—**In a two-echelon supply chain models composed of a manufacturer acting as the leader and two retailers acting as followers under a fuzzy decision environment. The parameters of demand function and manufacturing cost are all characterized as fuzzy variables. Two retailers are assumed to act in collusion and the optimum policy of the expected value and chance-constrained programming models are derived. Finally, numerical examples are presented to illustrate the theoretical underpinning of proposed models. It is shown that in fuzzy models, the confidence level of the profits for supply chain members affects the final optimal solutions.**

Index Terms—**Supply chain, price competition, game theory, fuzzy theory**

I. INTRODUCTION

IN today's highly competitive market, more and more firms realize that price is important behavior and competing firms often carry a price war to attract customers. In supply chain competition, retailers compete with each other on determining their retail prices and order quantities to maximize their profits.

There is a large body of literatures that deals with price competition in supply chain. Choi [1] used the linear and constant elasticity demand functions to study the price competition in a two-manufacture and one-retailer supply chain with two Stagckelberg and one Nash games. Ingene and Parry [2] considered the coordination of the supply chain with two retailers competing in price. Yang and Zhou [3] investigated two duopolistic retailers' three kinds of competitive behaviors: Cournot, Collusion and Stackelberg. Sang [4] researched the pricing and retail service decisions in a two-stage supply chain composed of one manufacturer and one retailer under an uncertain environment. Xiao and Qi [5] studied the coordination models of cost and demand disruptions for a supply chain with two competing retailers. Yao et al. [6] investigated a revenue sharing contract for coordinating a supply chain comprising one manufacturer and two competing retailers. They showed that the intensity of competition between the retailers leaded to a higher efficiency, but it would hurt the retailers themselves. Anderson and Bao [7] considered n supply chains price competing with a linear demand function. Farahat and Perakis[8] studied the efficiency of price competition among multi-product firms in differentiated oligopolies. Zhao and Chen [9] investigated a coordination mechanism of a supply chain that consists of one supplier and duopoly retailers from the perspective of operating uncertainty. Choi and Fredj [10] studied pricing strategies in a market channel composed of one national brand manufacturer and two retailers. Wang et al. [11] studied a markup contract for coordinating a supply chain comprising two competitive manufacturers and a common dominant retailer. Kawakatsu et al. [12] discussed a quantity discount problem between a single wholesaler and two retailers.

All studies mentioned above discussed the price competition models under a crisp environment, such as a linear or probabilistic market demand and known production cost. However, in real world, especially for some new products, the relevant precise date or probabilities are not possible to get due to lack of history data. Moreover, in today's highly competitive market, shorter and shorter product life cycles make the useful statistical data less and less available. Thus, the fuzzy set theory, rather than the traditional probability theory is well suited to the supply chain problem.

In recent years, more and more researchers have applied the fuzzy sets theory and technique to develop and solve the supply chain models problem. Huang and Huang [13] studied price coordination problem in a three-echelon supply chain composed of a single supplier, a single manufacturer and a single retailer. Xu and Zhai [14-15] assumed the demand to be a triangular fuzzy number and dealt with the newsboy problem in a two stage supply chain. Zhou et al. [16] considered two-echelon supply chain operations in a fuzzy environment which composed of one manufacturer and one retailer. Wei and Zhao [17] considered a fuzzy closed-loop supply chain with retailer's competition. Ye and Li [18] developed a Stackelberg model with fuzzy demand. Recently, Zhao et al. [19] considered a two-stage supply chain where two different manufacturers competed to sell substitutable products through a common retailer. Wei and Zhao [20] investigated the decisions of reverse channel choice in a fuzzy closed-loop supply chain. Zhao et al. [21] studied a distribution system in which two manufacturers competition under service and price supplied two substitutable products to one common retailer in fuzzy environments. Yu et al. [22] developed the joint optimal

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price-inventory decisions in a fuzzy price-setting newsvendor model. Sang [23] investigated supply chain contracts with a supplier and multiple competing retailers in a fuzzy demand environment.

In this paper, we will concentrate on price competition, where a manufacturer who sell his product to two retailers under a fuzzy decision environment. We also perform sensitivity analysis of the confidence level of the profits for supply chain members of the models.

The rest of paper is organized as follows. In section2, the fuzzy set theory in our models is described. Section3 is the problem descriptions. Section4 develops the fuzzy two-echelon supply chain models with a manufacturer and two competitive retailers. Section5 provides numerical examples to illustrate the result of the proposed models. The last section summarizes the work done in this paper and further research areas.

II. Preliminaries

This section begins with some concepts and properties of fuzzy variables, which will be used in the rest of the paper. Let ξ be a fuzzy variable on a possibility space $\left(\Theta, P(\Theta), \mathrm{Pos}\right)$ (for the concept of the possibility space, see Nahmias [24]), where Θ is a universe, $P(\Theta)$ is the power set of Θ and Pos is a possibility measure defined on $P(\Theta)$.

Definition 1 (Liu [25]) A fuzzy variable ξ is said to be nonnegative, if $\mathrm{Pos}\{\xi < 0\} = 0$.

Definition 2 (Liu [25]) Let ξ be a fuzzy variable and $\alpha \in (0,1]$. Then $\xi_\alpha^L = \inf\{r \mid \mathrm{Pos}\{\xi \le r\} \ge \alpha\}$ and $\xi_\alpha^R = \sup\{r \mid \mathrm{Pos}\{\xi \ge r\} \ge \alpha\}$ are called the α-pessimistic value and the α-optimistic value of ξ.

Definition 3 The fuzzy set $\xi = (a,b,c)$, where $a < b < c$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_\xi(x) == \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & \text{if } a_1 \le x \le a_2, \\ \dfrac{a_3-x}{a_3-a_2}, & \text{if } a_2 \le x \le a_3, \\ 0, & \text{otherwise}. \end{cases}$$

where a_1 and a_3 are the lower limit and upper limit respectively of the triangular fuzzy number ξ .The triangular fuzzy number ξ is called the positive triangular fuzzy number if $a > 0$.

Example 1 Let ξ= (a, b, c) be a triangular fuzzy variable, then its α-pessimistic value and α-optimistic value are respectively

$\xi_\alpha^L = b\alpha + a(1-\alpha)$, And $\xi_\alpha^U = b\alpha + c(1-\alpha)$.

Proposition 1 (Liu and Liu [26] and Zhao et al. [27]) Let ξ and η be two nonnegative independent fuzzy variables. Then, for any $\alpha \in (0,1]$

(a) $\left(\xi+\eta\right)_\alpha^L = \xi_\alpha^L + \eta_\alpha^L$ and $\left(\xi+\eta\right)_\alpha^U = \xi_\alpha^U + \eta_\alpha^U$;

(b) if λ >0, $\left(\lambda\xi\right)_\alpha^L = \lambda\xi_\alpha^L$ and $\left(\lambda\xi\right)_\alpha^U = \lambda\xi_\alpha^U$;

(c) $(\xi\cdot\eta)_\alpha^L = \xi_\alpha^L \cdot \eta_\alpha^L$ and $(\xi\cdot\eta)_\alpha^U = \xi_\alpha^U \cdot \eta_\alpha^U$;

(d) $\left(\xi-\eta\right)_\alpha^L = \xi_\alpha^L - \eta_\alpha^U$ and $\left(\xi-\eta\right)_\alpha^U = \xi_\alpha^U - \eta_\alpha^L$.

Proposition 2 (Liu and Liu [26]) Let ξ be a fuzzy variable with the finite expected value $E[\xi]$, Then we have

$$E\left[\xi\right] = \tfrac{1}{2}\int_0^1 \left(\xi_\alpha^L + \xi_\alpha^U\right) \mathrm{d}\alpha .$$

Proposition 3 (Liu and Liu [26]) Let ξ and η be two independent fuzzy variables with finite expected values. Then for any real numbers a and b, we have

$$E\left[a\xi + b\eta\right] = aE\left[\xi\right] + bE\left[\eta\right].$$

Example 2 Let ξ= (a, b, c) be a triangular fuzzy variable, then its expected value $E[\xi]$ is

$$\mathrm{E}[\xi] = \frac{1}{2}\int_0^1 (\mathrm{b}\alpha + \mathrm{a}(1-\alpha) + \mathrm{ba} + \mathrm{c}(1-\alpha))\mathrm{d}\alpha = \frac{\mathrm{a}+2\mathrm{b}+\mathrm{c}}{4}$$

Definition 4 Let ξ and η be two nonnegative independent fuzzy variables, $\xi > \eta$ if and only if for any $\alpha \in (0,1]$, $\xi_\alpha^L > \eta_\alpha^L$ and $\xi_\alpha^U > \eta_\alpha^U$.

Definition 5 Let ξ and η be two nonnegative independent fuzzy variables, if $\xi > \eta$, then $E[\xi]>E[\eta]$.

III. Problem Descriptions

This paper considers a two-echelon supply chain consisting of a manufacturer selling his product to two competitive retailers, who in turn retail it to the customers. The interaction between two echelons is assumed that the manufacturer acts as a leader and sets a united wholesale price to the two retailers, then the two competitive retailers share a common sharing respond independently by setting the sale price and the corresponding order quantity. The notations used in this paper are given as follows:

p_i the sale price charged to customers by retailer i, $i = 1,2$;

w the wholesale price per unit charged to the retailers by the manufacturer;

$\tilde{c}$ unit manufacturing cost;

$\tilde{Q}_i$ deterministic demand faced by retailer i or quantity ordered by retailer i;

Π_{R_i} the fuzzy profit for retailer i, $i = 1,2$;

Π_{M} the fuzzy profit for manufacturer.

We assume that retailer i faces the similar market demand function, which is given by

$$\tilde{Q}_i = \tilde{D} - \tilde{\beta}p_i + \tilde{\gamma}p_j, \quad i = 1,2, \quad j = 3-i .$$

where $\tilde{D}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are positive and independent fuzzy variables. The parameter $\tilde{D}$ represents the market base, the parameter $\tilde{\beta}$ represents the measure of sensitivity of retailer-i's sales to changes of the retailer-j's price and the parameter $\tilde{\gamma}$ represents the degree of substitutability between retailers and reflects the impacts of the marketing mix decision of retailer on customer demand. The parameters $\tilde{\beta}$ and $\tilde{\gamma}$ are assumed to satisfy $\tilde{\beta} > \tilde{\gamma}$ and $\tilde{\beta}_\alpha^L > \tilde{\gamma}_\alpha^U$. Since there is no negative demand in the real world we assume $\mathrm{Pos}\left\{\tilde{D} - \tilde{\beta} p_i + \tilde{\gamma} p_j < 0\right\} = 0$. The quantity ordered by retailer i can be expressed as $Q_i = E\left[\tilde{Q}_i\right]$. $\tilde{Q}_i$ is called a fuzzy liner demand function in this paper. Let the cost $\tilde{c}$ be a positive fuzzy variable and be independent of parameters $\tilde{D}$, $\tilde{\beta}$ and $\tilde{\gamma}$.

The fuzzy profits of the manufacturer and the retailer i $(i = 1, 2)$ can be expressed as

$$\Pi_{\mathrm{M}}\left(w, p_i\right) = \sum_{i=1}^{2}\left(w - \tilde{c}\right)\left(\tilde{D} - \tilde{\beta} p_i + \tilde{\gamma} p_j\right),$$
$$\Pi_{\mathrm{R}_i}\left(p_i\right) = \left(p_i - w\right)\left(\tilde{D} - \tilde{\beta} p_i + \tilde{\gamma} p_j\right),\ i = 1, 2,\ \ j = 3 - i.$$

IV. Fuzzy supply chain models in price competition

In this section, we develop the fuzzy two-echelon supply chain models with a manufacturer and two competitive retailers, which can tell both the manufacture and the retailers how to make their decisions when the duopolistic retailers acting in collusion in a fuzzy decision environment. In this condition, two competition retailers agree to act in union in order to maximize their total fuzzy expected profits, hence, the fuzzy optimal model in this condition can be formulate as below:

$$\begin{cases} \max\limits_{w} E\left[\Pi_M\left(w, p_i^*(w)\right)\right] = E\left[\sum_{i=1}^{2}\left(w - \tilde{c}\right)\left(\tilde{D} - \tilde{\beta} p_i^*(w) + \tilde{\gamma} p_j^*(w)\right)\right] \\ \text{s.t.} \\ \mathrm{Pos}\left\{w - \tilde{c} < 0\right\} = 0 \\ p_i^* = \arg\max \sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right] \\ \quad \begin{cases} \max\limits_{p_i} \sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right] = \sum_{i=1}^{2} E\left[\left(p_i - w\right)\left(\tilde{D} - \tilde{\beta} p_i + \tilde{\gamma} p_j\right)\right] \\ \text{s.t.} \\ \mathrm{Pos}\left\{\tilde{D} - \tilde{\beta} p_i + \tilde{\gamma} p_j < 0\right\} = 0 \\ p_i \geq w \end{cases} \\ i = 1, 2,\ \ j = 3 - i. \end{cases} \tag{1}$$

Theorem 1 Let $\sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right]$ be the total fuzzy expected values of the profits for two retailers. A wholesale price w chosen by the manufacturer is fixed. If $\mathrm{Pos}\left\{\tilde{D} - \tilde{\beta} p_1^*\left(w\right) + \tilde{\gamma} p_2^*\left(w\right) < 0\right\} = 0$ and $w < \dfrac{E\left[\tilde{D}\right]}{E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]}$, then the optimal response functions $p_1^*\left(w\right)$ and $p_2^*\left(w\right)$ of the retailer 1 and 2 are

$$p_1^*\left(w\right) = p_2^*\left(w\right) = \frac{E\left[\tilde{D}\right]}{2\left(E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right)} + \frac{1}{2} w. \tag{2}$$

Proof. Note that fuzzy variables $\tilde{D}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are positive and independent with each other. By Proposition 3, we have

$$\sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right] = -E\left[\tilde{\beta}\right] p_1^2 - E\left[\tilde{\beta}\right] p_2^2 + 2E\left[\tilde{\gamma}\right] p_1 p_2$$
$$+ \left(E\left[\tilde{D}\right] + \left(E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right) w\right)\left(p_1 + p_2\right) - 2E\left[\tilde{D}\right] w. \tag{3}$$

From (3), we can get the first-order derivatives of $\sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right]$ with respect to p_1 and p_2 as follows:

$$\frac{\partial}{\partial p_1} \sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right] = -2E\left[\tilde{\beta}\right] p_1 + 2E\left[\tilde{\gamma}\right] p_2 + E\left[\tilde{D}\right] + \left(E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right) w, \tag{4}$$

$$\frac{\partial}{\partial p_2} \sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right] = -2E\left[\tilde{\beta}\right] p_2 + 2E\left[\tilde{\gamma}\right] p_1 + E\left[\tilde{D}\right] + \left(E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right) w. \tag{5}$$

Therefore the Hessian matrix of $\sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right]$ is

$$\mathrm{H} = \begin{bmatrix} -2E\left[\tilde{\beta}\right] & 2E\left[\tilde{\gamma}\right] \\ 2E\left[\tilde{\gamma}\right] & -2E\left[\tilde{\beta}\right] \end{bmatrix}. \tag{6}$$

Note that the Hessian matrix of $\sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right]$ is negative definite, since $\tilde{\beta}$, $\tilde{\gamma}$ are positive fuzzy variables and $\tilde{\beta} > \tilde{\gamma}$. Consequently, $\sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right]$ is jointly concave in p_1 and p_2. Hence, the optimal response functions $p_1^*\left(w\right)$ and $p_2^*(w)$ of the retailer 1 and 2 are can be obtained by solving $\dfrac{\partial}{\partial p_1} \sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right] = 0$ and $\dfrac{\partial}{\partial p_2} \sum_{i=1}^{2} E\left[\Pi_{R_i}\left(p_i\right)\right] = 0$, which give (2).

The poof of Theorem 1 is completed.

Having the information about the decisions of the retailers, the manufacturer would then use those to maximize his fuzzy expected profit. So, we get the following Theorem.

Theorem 2 Let $E\left[\Pi_{\mathrm{M}}\left(w, p_i^*\left(w\right)\right)\right]$ be the fuzzy expected value of the profit for manufacturer. If $\mathrm{Pos}\left\{w^* - \tilde{c} < 0\right\} = 0$, $E\left[\tilde{D}\right] > E\left[\tilde{\beta}\right] - \frac{1}{2}\int_0^1\left(\tilde{c}_\alpha^L \gamma_\alpha^U + \tilde{c}_\alpha^U \gamma_\alpha^L\right)\mathrm{d}\alpha$ and $\mathrm{Pos}\left\{\tilde{D} - \tilde{\beta}\, p_i^* + p_j^* < 0\right\} = 0$, $i = 1, 2,\ j = 3 - i$, the optimal solutions $\left(w^*, p_1^*, p_2^*\right)$ of model (1) are

$$w^* = \frac{2E\left[\tilde{D}\right] + 2E\left[\tilde{c}\tilde{\beta}\right] - \int_0^1\left(\tilde{c}_\alpha^L \tilde{\gamma}_\alpha^U + \tilde{c}_\alpha^U \tilde{\gamma}_\alpha^L\right)\mathrm{d}\alpha}{4\left(E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right)}, \tag{7}$$

$$p_1^* = p_2^* = \frac{6E\left[\tilde{D}\right] + 2E\left[\tilde{c}\tilde{\beta}\right] - \int_0^1\left(\tilde{c}_\alpha^L \tilde{\gamma}_\alpha^U + \tilde{c}_\alpha^U \tilde{\gamma}_\alpha^L\right)\mathrm{d}\alpha}{8\left(E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right)}. \tag{8}$$

Proof. Note that fuzzy variables $\tilde{c}$, $\tilde{D}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are positive and independent with each other. By Propositions 1 and 2, we have

$$E\left[\Pi_M\left(w, p_i^*\left(w\right)\right)\right] = \frac{1}{2}\int_0^1\left[\left(w - \tilde{c}_\alpha^U\right)\left(\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^U p_1^*\left(w\right) + \tilde{\gamma}_\alpha^L p_2^*\left(w\right)\right) + \left(w - \tilde{c}_\alpha^L\right)\left(\tilde{D}_\alpha^U - \tilde{\beta}_\alpha^L p_1^*\left(w\right) + \tilde{\gamma}_\alpha^U p_2^*\left(w\right)\right)\right]\mathrm{d}\alpha$$

$$+\tfrac{1}{2}\int_0^1\Big[\big(w-\tilde{c}_\alpha^U\big)\big(\tilde{D}_\alpha^L-\tilde{\beta}_\alpha^U p_2^*(w)+\tilde{\gamma}_\alpha^L p_1^*(w)\big)$$
$$+\big(w-\tilde{c}_\alpha^L\big)\big(\tilde{D}_\alpha^U-\tilde{\beta}_\alpha^L p_2^*(w)+\tilde{\gamma}_\alpha^U p_1^*(w)\big)\Big]\mathrm{d}\alpha. \quad (9)$$

Substituting $p_1^*(w)$ and $p_2^*(w)$ in (2) into (9), we can get

$$E\left[\Pi_M\left(w,p_i^*(w)\right)\right]=-\left(E\left[\tilde{\beta}\right]-E\left[\tilde{\gamma}\right]\right)w^2$$
$$+\left(E\left[\tilde{D}\right]+E\left[\tilde{c}\tilde{\beta}\right]-\tfrac{1}{2}\int_0^1\left(\tilde{c}_\alpha^L\gamma_\alpha^U+\tilde{c}_\alpha^U\gamma_\alpha^L\right)\mathrm{d}\alpha\right)w$$
$$+\frac{2E\left[\tilde{D}\right]E\left[\tilde{c}\tilde{\beta}\right]-E\left[\tilde{D}\right]\int_0^1\left(\tilde{c}_\alpha^L\gamma_\alpha^U+\tilde{c}_\alpha^U\gamma_\alpha^L\right)\mathrm{d}\alpha}{2\left(E\left[\tilde{\beta}\right]-E\left[\tilde{\gamma}\right]\right)}$$
$$-\int_0^1\left(\tilde{c}_\alpha^L\tilde{D}_\alpha^U+\tilde{c}_\alpha^U\tilde{D}_\alpha^L\right)\mathrm{d}\alpha. \quad (10)$$

Thus, from (10), we can get the first-order and second-order derivatives of $E\left[\Pi_M\left(w,p_i^*(w)\right)\right]$ with respect to w are as follows:

$$\frac{\partial}{\partial w}E\left[\Pi_M\left(w,p_i^*(w)\right)\right]=-2\left(E\left[\tilde{\beta}\right]-E\left[\tilde{\gamma}\right]\right)w$$
$$+E\left[\tilde{D}\right]+E\left[\tilde{c}\tilde{\beta}\right]-\tfrac{1}{2}\int_0^1\left(\tilde{c}_\alpha^L\gamma_\alpha^U+\tilde{c}_\alpha^U\gamma_\alpha^L\right)\mathrm{d}\alpha, \quad (11)$$
$$\frac{\partial^2}{\partial w^2}E\left[\Pi_M\left(w,p_i^*(w)\right)\right]=-2\left(E\left[\tilde{\beta}\right]-E\left[\tilde{\gamma}\right]\right). \quad (12)$$

Note that the second-order derivative of $E\left[\Pi_M\right]$ is negative definite, since $\tilde{\beta}$, $\tilde{\gamma}$ are positive fuzzy variables and $\tilde{\beta}>\tilde{\gamma}$. Consequently, $E\left[\Pi_M\right]$ is concave in w. Hence, the optimal wholesale price of manufacturer can be obtained by solving $\frac{\partial}{\partial w}E\left[\Pi_M\left(w,p_i^*(w)\right)\right]=0$, which give (7).

Substituting w^* in (7) into (2), we can get (8).
The poof of Theorem 2 is completed.

Combining (7) and (8) with (3) and (9) will easily yield the optimal fuzzy expected profits for retailer i and manufacturer, $i=1,2$.

The chance-constrained programming, which was introduced by Liu and Iwamura [28-29], plays an important role in modeling fuzzy decision systems. Its basic ideal is to optimize some critical value with a given confidence level subject to some chance constraints. Motivated by this ideal, the following maximax chance-constrained programming model for the two-echelon supply chain can be formulated in the collusion solution:

$$\begin{cases}\max\limits_{w}\bar{\Pi}_M\\ \text{s.t.}\\ \text{Pos}\left\{\sum_{i=1}^2(w-\tilde{c})\left(\tilde{D}-\tilde{\beta}p_i^*(w)+\tilde{\gamma}p_j^*(w)\right)\geq\bar{\Pi}_M\right\}\geq\alpha\\ \text{Pos}\left\{w-\tilde{c}<0\right\}=0\\ p_i^*=\arg\max\sum_{i=1}^2\bar{\Pi}_{R_i}\\ \quad\begin{cases}\max\limits_{p_i}\sum_{i=1}^2\bar{\Pi}_{R_i}\\ \text{s.t.}\\ \text{Pos}\left\{(p_i-w)\left(\tilde{D}-\tilde{\beta}p_i+\tilde{\gamma}p_j\right)\geq\sum_{i=1}^2\bar{\Pi}_{R_i}\right\}\geq\alpha\\ \text{Pos}\left(\tilde{D}-\tilde{\beta}p_i+\tilde{\gamma}p_j<0\right)=0\\ p_i\geq w\\ i=1,2,\quad j=3-i.\end{cases}\end{cases} \quad (13)$$

where α is a predetermined confidence level of the profits for the manufacture and the retailers. For each fixed feasible p_i, $\sum_{i=1}^2\bar{\Pi}_{R_i}$ should be the total maximum value of the profit function for retailers, which $\sum_{i=1}^2\bar{\Pi}_{R_i}(p_i)$ achieves with at least possibility α, and $\bar{\Pi}_M$ should be maximum value of the profit function for manufacture, which $\Pi_M\left(w,p_i^*(w)\right)$ achieves with at least possibility α. Clearly, the model (13) can be transformed into the following model (14) in which the manufacture and the retailers try to maximize their optimal α-optimistic profits $\left(\Pi_M\left(w,p_i^*(w)\right)\right)_\alpha^U$ and $\sum_{i=1}^2\left(\Pi_{R_i}(p_i)\right)_\alpha^U$ by selecting the best pricing strategies, respectively

$$\begin{cases}\max\limits_{w}\Pi_M\left(w,p_i^*(w)\right)_\alpha^U=\sum_{i=1}^2\left((w-\tilde{c})\left(\tilde{D}-\tilde{\beta}p_i^*(w)+\tilde{\gamma}p_j^*(w)\right)\right)_\alpha^U\\ \text{s.t.}\\ \text{Pos}\left\{w-\tilde{c}<0\right\}=0\\ p_i^*=\arg\max\sum_{i=1}^2\left(\Pi_{R_i}(p_i)\right)_\alpha^U\\ \quad\begin{cases}\max\limits_{p_i}\sum_{i=1}^2\left(\Pi_{R_i}(p_i)\right)_\alpha^U=\sum_{i=1}^2\left((p_i-w)\left(\tilde{D}-\tilde{\beta}p_i+\tilde{\gamma}p_j\right)\right)_\alpha^U\\ \text{s.t.}\\ \text{Pos}\left\{\tilde{D}-\tilde{\beta}p_i+\tilde{\gamma}p_j<0\right\}=0\\ p_i\geq w\end{cases}\\ i=1,2,\quad j=3-i.\end{cases} \quad (14)$$

Theorem 3 Let $\sum_{i=1}^2\left(\Pi_{R_i}(p_i)\right)_\alpha^U$ be the total α-optimistic value of the profit for two competitive retailers. A wholesale price w chosen by the manufacturer is fixed. If $\text{Pos}\left\{\tilde{D}-\tilde{\beta}p_1^*(w)+\tilde{\gamma}p_2^*(w)<0\right\}=0$ and $w<\dfrac{\tilde{D}_\alpha^U}{\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^U}$, then the optimal response functions $p_1^*(w)$ and $p_2^*(w)$ of the retailer 1 and 2 are

$$p_1^*(w)=p_2^*(w)=\frac{\tilde{D}_\alpha^U}{2\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^U\right)}+\frac{1}{2}w. \quad (15)$$

Proof. Note that fuzzy variables $\tilde{D}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are positive and independent with each other. By Proposition 1, we have

$$\sum_{i=1}^2\left(\pi_{R_i}(p_i)\right)_\alpha^U=-\tilde{\beta}_\alpha^L p_1^2-\tilde{\beta}_\alpha^L p_2^2+2\tilde{\gamma}_\alpha^U p_1p_2$$
$$+\left(\tilde{\beta}_\alpha^L w-\tilde{\gamma}w+\tilde{D}_\alpha^U\right)(p_1+p_2)-2\tilde{D}_\alpha^U w. \quad (16)$$

From (16), we can get the first-order derivatives of $\sum_{i=1}^2\left(\Pi_{R_i}(p_i)\right)_\alpha^U$ with respect to p_1 and p_2 as follows:

$$\frac{\partial}{\partial p_1}\sum_{i=1}^2\left(\Pi_{R_i}(p_i)\right)_\alpha^U=-2\tilde{\beta}_\alpha^L p_1+2\tilde{\gamma}_\alpha^U p_2+\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}\right)w+\tilde{D}_\alpha^U, \quad (17)$$
$$\frac{\partial}{\partial p_2}\sum_{i=1}^2\left(\Pi_{R_i}(p_i)\right)_\alpha^U=-2\tilde{\beta}_\alpha^L p_2+2\tilde{\gamma}_\alpha^U p_1+\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}\right)w+\tilde{D}_\alpha^U. \quad (18)$$

Therefore the Hessian matrix of $\sum_{i=1}^2\left(\Pi_{R_i}(p_i)\right)_\alpha^U$ is

$$\mathrm{H}=\begin{bmatrix}-2\tilde{\beta}_{\alpha}^{L} & 2\tilde{\gamma}_{\alpha}^{U}\\ 2\tilde{\gamma}_{\alpha}^{U} & -2\tilde{\beta}_{\alpha}^{L}\end{bmatrix}. \tag{19}$$

Note that the Hessian matrix of $\sum_{i=1}^{2}\left(\Pi_{R_i}\left(p_i\right)\right)_{\alpha}^{U}$ is negative definite, since $\tilde{\beta}$, $\tilde{\gamma}$ are positive fuzzy variables and $\tilde{\beta}_{\alpha}^{L}>\tilde{\gamma}_{\alpha}^{U}$. Consequently, $\sum_{i=1}^{2}\left(\Pi_{R_i}\left(p_i\right)\right)_{\alpha}^{U}$ is jointly concave in p_1 and p_2 . Hence, the optimal response functions $p_1^*\left(w\right)$ and $p_2^*\left(w\right)$ of the retailer 1 and 2 are can be obtained by solving $\frac{\partial}{\partial p_1}\sum_{i=1}^{2}\left(\Pi_{R_i}\left(p_i\right)\right)_{\alpha}^{U}=0$ and $\frac{\partial}{\partial p_2}\sum_{i=1}^{2}\left(\Pi_{R_i}\left(p_i\right)\right)_{\alpha}^{U}=0$, which give (15).

The poof of Theorem 3 is completed.

Having the information about the decisions of the retailers, the manufacturer would then use those to maximize his α-optimistic value of the profit. So, we get the following Theorem 4.

Theorem 4 Let $\left(\Pi_M\left(w,p_i^*\left(w\right)\right)\right)_{\alpha}^{U}$ be the α-optimistic value of the profit for manufacturer. If $\mathrm{Pos}\left\{w^*-\tilde{c}<0\right\}=0$, $\tilde{D}_{\alpha}^{U}>\tilde{c}_{\alpha}^{L}\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)$ and $\mathrm{Pos}\left\{\tilde{D}-\tilde{\beta}p_i^*+\tilde{\gamma}p_j^*<0\right\}=0$ $i=1,2$, $j=3-i$. The optimal solutions $\left(w^*,p_1^*,p_2^*\right)$ of model (13) are

$$w^*=\frac{\tilde{D}_{\alpha}^{U}+\tilde{c}_{\alpha}^{L}\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)}{2\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)}, \tag{20}$$

$$p_1^*=p_2^*=\frac{3\tilde{D}_{\alpha}^{U}+\tilde{c}_{\alpha}^{L}\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)}{4\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)}. \tag{21}$$

Proof. Note that fuzzy variables $\tilde{c}$, $\tilde{D}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are positive and independent with each other. By Proposition 1, we have

$$\begin{aligned}\left(\Pi_M\left(w,p_i^*\left(w\right)\right)\right)_{\alpha}^{U}&=\left(w-\tilde{c}_{\alpha}^{L}\right)\left(\tilde{D}_{\alpha}^{U}-\tilde{\beta}_{\alpha}^{L}p_1^*(w)+\tilde{\gamma}_{\alpha}^{U}p_2^*(w)\right)\\&\quad+\left(w-\tilde{c}_{\alpha}^{L}\right)\left(\tilde{D}_{\alpha}^{U}-\tilde{\beta}_{\alpha}^{L}p_2^*\left(w\right)+\tilde{\gamma}_{\alpha}^{U}p_1^*\left(w\right)\right).\end{aligned} \tag{22}$$

Substituting $p_1^*(w)$ and $p_2^*(w)$ in (15) into (22), we can get

$$\begin{aligned}\left(\Pi_M\left(w,p_i^*\left(w\right)\right)\right)_{\alpha}^{U}&=-\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)w^2\\&\quad+\left(\tilde{D}_{\alpha}^{U}+\tilde{c}_{\alpha}^{L}\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)\right)w-\tilde{c}_{\alpha}^{L}\tilde{D}_{\alpha}^{U}.\end{aligned} \tag{23}$$

Thus, we can get the first-order and second-order derivatives of $\left(\Pi_M\left(w,p_i^*\left(w\right)\right)\right)_{\alpha}^{U}$ with respect to w are as follows:

$$\begin{aligned}\frac{\partial}{\partial w}\left(\Pi_M\left(w,p_i^*\left(w\right)\right)\right)_{\alpha}^{U}&=-2\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)w\\&\quad+\tilde{D}_{\alpha}^{U}+\tilde{c}_{\alpha}^{L}\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right),\end{aligned} \tag{24}$$

$$\frac{\partial^2}{\partial w^2}\left(\Pi_M\left(w,p_i^*\left(w\right)\right)\right)_{\alpha}^{U}=-2\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right). \tag{25}$$

Note that the second-order derivative of $\left(\Pi_M\right)_{\alpha}^{U}$ is negative definite, since $\tilde{\beta}$, $\tilde{\gamma}$ are positive fuzzy variables and $\tilde{\beta}_{\alpha}^{L}>\tilde{\gamma}_{\alpha}^{U}$. Consequently, $\left(\Pi_M\left(w,p_i^*\left(w\right)\right)\right)_{\alpha}^{U}$ is concave in w. Hence, the optimal wholesale price of manufacturer can be obtained by solving $\frac{\partial}{\partial w}\left(\Pi_M\left(w,p_i^*\left(w\right)\right)\right)_{\alpha}^{U}=0$, which give (20).

Substituting w^* in (20) into (15), we can get (21).

The poof of Theorem 4 is completed.

Combining (20) and (21) with (16) and (23) will easily yield the optimal α-optimistic value of profits for the two competition retailers and manufacturer as follows

$$\left(\Pi_{R_1}\left(p_i\right)\right)_{\alpha}^{U}=\left(\Pi_{R_2}\left(p_i\right)\right)_{\alpha}^{U}=\frac{\left(\tilde{c}_{\alpha}^{L}\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)-\tilde{D}_{\alpha}^{U}\right)^2}{16\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)}, \tag{26}$$

$$\left(\Pi_M\left(w,p_i^*\left(w\right)\right)\right)_{\alpha}^{U}=\frac{\left(\tilde{c}_{\alpha}^{L}\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)-\tilde{D}_{\alpha}^{U}\right)^2}{4\left(\tilde{\beta}_{\alpha}^{L}-\tilde{\gamma}_{\alpha}^{U}\right)}. \tag{27}$$

The minimax chance-constrained programming model for the two-echelon supply chain when two retailers act the collusion solution can also be formulated as bellow:

$$\begin{cases}\max\limits_{w}\min\limits_{\bar{\Pi}_M}\bar{\Pi}_M\\ \text{s.t.}\\ \mathrm{Pos}\left\{\sum_{i=1}^{2}\left(w-\tilde{c}\right)\left(\tilde{D}-\tilde{\beta}p_i^*\left(w\right)+\tilde{\gamma}p_j^*\left(w\right)\right)\le\bar{\Pi}_M\right\}\ge\alpha\\ \mathrm{Pos}\left\{w-\tilde{c}<0\right\}=0\\ p_i^*=\arg\max\min\limits_{\sum_{i=1}^{2}\bar{\Pi}_{R_i}}\sum_{i=1}^{2}\bar{\Pi}_{R_i}\\ \begin{cases}\max\limits_{p_i}\min\limits_{\sum_{i=1}^{2}\bar{\Pi}_{R_i}}\sum_{i=1}^{2}\bar{\Pi}_{R_i}\\ \text{s.t.}\\ \mathrm{Pos}\left\{\left(p_i-w\right)\left(\tilde{D}-\tilde{\beta}p_i+\tilde{\gamma}p_j\right)\le\sum_{i=1}^{2}\bar{\pi}_{\mathrm{R}_i}\right\}\ge\alpha\\ \mathrm{Pos}\left(\tilde{D}-\tilde{\beta}p_i+\tilde{\gamma}p_j<0\right)=0\\ p_i\ge w\\ i=1,2,\quad j=3-i.\end{cases}\end{cases} \tag{28}$$

Where α is a predetermined confidence level of the profits for the manufacture and the retailers. For each fixed feasible p_i, $\sum_{i=1}^{2}\bar{\Pi}_{R_i}$ should be the total minimum value of the profit function for retailers, which $\sum_{i=1}^{2}\bar{\Pi}_{R_i}\left(p_i\right)$ achieves with at least possibility α, and $\bar{\Pi}_M$ should be minimum value of the profit function for manufacture, which $\Pi_M\left(w,p_i^*\left(w\right)\right)$ achieves with at least possibility α. The model (28) can be transformed into the following model (29) in which the manufacture and two competition retailers try to maximize their optimal α-pessimistic profits $\left(\Pi_M\left(w,p_i^*\left(w\right)\right)\right)_{\alpha}^{L}$ and $\sum_{i=1}^{2}\left(\Pi_{R_i}\left(p_i\right)\right)_{\alpha}^{L}$ by selecting the best pricing strategies, respectively

$$\begin{cases} \max\limits_{w} \Pi_M\left(w, p_i^*(w)\right)_\alpha^L = \sum_{i=1}^2 \left((w-\tilde{c})\left(\tilde{D}-\tilde{\beta}p_i^*(w)+\tilde{\gamma}p_j^*(w)\right)\right)_\alpha^L \\ \text{s.t.} \\ \text{Pos}\{w-\tilde{c}<0\}=0 \\ p_i^* = \arg\max \sum_{i=1}^2 \left(\Pi_{R_i}(p_i)\right)_\alpha^L \\ \quad \begin{cases} \max\limits_{p_i} \sum_{i=1}^2 \left(\Pi_{R_i}(p_i)\right)_\alpha^L = \sum_{i=1}^2 \left((p_i-w)\left(\tilde{D}-\tilde{\beta}p_i+\tilde{\gamma}p_j\right)\right)_\alpha^L \\ \text{s.t.} \\ \text{Pos}\left\{\tilde{D}-\tilde{\beta}p_i+\tilde{\gamma}p_j<0\right\}=0 \\ p_i \ge w \end{cases} \\ i=1,2, \quad j=3-i. \end{cases} \tag{29}$$

Theorem 5 Let $\left(\Pi_{R_i}(p_i)\right)_\alpha^L$ and $\left(\Pi_M\left(w, p_i^*(w)\right)\right)_\alpha^L$ be the optimal α-pessimistic value of the profits for retailer i and manufacturer. If $\text{Pos}\{w^*-\tilde{c}<0\}=0$, $\tilde{D}_\alpha^L > \tilde{c}_\alpha^U\left(\tilde{\beta}_\alpha^U-\tilde{\gamma}_\alpha^L\right)$ and $\text{Pos}\left\{\tilde{D}-\tilde{\beta}p_i^*+\tilde{\gamma}p_j^*<0\right\}=0$, $i=1,2,\ j=3-i$. The optimal solutions $\left(w^*, p_1^*, p_2^*\right)$ of model (28) are

$$w^* = \frac{\tilde{D}_\alpha^L + \tilde{c}_\alpha^U\left(\tilde{\beta}_\alpha^U-\tilde{\gamma}_\alpha^L\right)}{2\left(\tilde{\beta}_\alpha^U-\tilde{\gamma}_\alpha^L\right)}, \tag{30}$$

$$p_1^* = p_2^* = \frac{3\tilde{D}_\alpha^L + \tilde{c}_\alpha^U\left(\tilde{\beta}_\alpha^U-\tilde{\gamma}_\alpha^L\right)}{4\left(\tilde{\beta}_\alpha^U-\tilde{\gamma}_\alpha^L\right)}. \tag{31}$$

Proof. Similar to the proof of Theorem 4.

The optimal α-pessimistic value of profits for the two competition retailers and manufacturer are as follows

$$\left(\Pi_{R_1}(p_i)\right)_\alpha^L = \left(\Pi_{R_2}(p_i)\right)_\alpha^L = \frac{\left(\tilde{c}_\alpha^U\left(\tilde{\beta}_\alpha^U-\tilde{\gamma}_\alpha^L\right)-\tilde{D}_\alpha^L\right)^2}{16\left(\tilde{\beta}_\alpha^U-\tilde{\gamma}_\alpha^L\right)}, \tag{32}$$

$$\left(\Pi_M\left(w, p_i^*(w)\right)\right)_\alpha^L = \frac{\left(\tilde{c}_\alpha^U\left(\tilde{\beta}_\alpha^U-\tilde{\gamma}_\alpha^L\right)-\tilde{D}_\alpha^L\right)^2}{4\left(\tilde{\beta}_\alpha^U-\tilde{\gamma}_\alpha^L\right)}. \tag{33}$$

Remark when α=1, it is clear the manufacturing cost $\tilde{c}$, the market base $\tilde{D}$, the demand change rate $\tilde{\beta}$ and the degree of substitutability between retailers $\tilde{\gamma}$ degenerate into crisp real numbers, the main result in Theorems 4 and 5 can degenerate into:

$$w^* = \frac{D+c(\beta-\gamma)}{2(\beta-\gamma)}, \tag{34}$$

$$p_1^* = p_2^* = \frac{3D+c(\beta-\gamma)}{4(\beta-\gamma)}. \tag{35}$$

There are just the conventional results in crisp solution.

V. Numerical Example

In this section, we present a numerical example which is aimed at illustrating the computational process of the fuzzy supply chain models established in previous section. We will also perform sensitivity analysis of the parameter α of these models. Here, we consider that $\tilde{D}$ is about 600, $\tilde{D}=(580,600,620)$, $\tilde{\beta}$ is about 20, $\tilde{\beta}=(19,20,21)$, $\tilde{\gamma}$ is about 5, $\tilde{\gamma}=(4,5,6)$ and $\tilde{c}$ is about 10, $\tilde{c}=(9,10,11)$ respectively.

Moreover, the α-optimistic values and α-pessimistic values of $\tilde{D}$, $\tilde{\beta}$, $\tilde{\gamma}$ and $\tilde{c}$ are as follows

$$\tilde{D}_\alpha^L = 580+20\alpha,\ \tilde{D}_\alpha^U = 620-20\alpha,$$
$$\tilde{\beta}_\alpha^L = 19+\alpha,\ \tilde{\beta}_\alpha^U = 21-\alpha,$$
$$\tilde{\gamma}_\alpha^L = 4+\alpha,\ \tilde{\gamma}_\alpha^U = 6-\alpha,$$
$$\tilde{c}_\alpha^L = 9+\alpha,\ \tilde{c}_\alpha^U = 11-\alpha.$$

The expected values of parameters are

$$E\left[\tilde{D}\right] = \frac{580+2\times600+620}{4} = 600,$$
$$E\left[\tilde{\beta}\right] = \frac{19+2\times20+21}{4} = 20,$$
$$E\left[\tilde{\gamma}\right] = \frac{4+2\times5+6}{4} = 5,$$
$$E\left[\tilde{c}\right] = \frac{9+2\times10+11}{4} = 10.$$

We can calculate that

$$E\left[\tilde{c}\tilde{\beta}\right] = \tfrac{1}{2}\int_0^1 \left(\tilde{c}_\alpha^L\tilde{\beta}_\alpha^L + \tilde{c}_\alpha^U\tilde{\beta}_\alpha^U\right)\mathrm{d}\alpha = \frac{601}{3},$$
$$\int_0^1 \left(\tilde{c}_\alpha^L\tilde{\gamma}_\alpha^U + \tilde{c}_\alpha^U\tilde{\gamma}_\alpha^L\right)\mathrm{d}\alpha = \frac{298}{3},$$
$$\int_0^1 \left(\tilde{c}_\alpha^L\tilde{D}_\alpha^U + \tilde{c}_\alpha^U\tilde{D}_\alpha^L\right)\mathrm{d}\alpha = \frac{35960}{3}.$$

Based on the above analysis, the optimal expected values, α-optimistic values and α-pessimistic values for the fuzzy supply chain models above can be listed in Table I.

TABLE I
OPTIMAL EQUILIBRIUM VALUE OF THE PARAMETERS FOR DIFFERENT α IN FUZZY SUPPLY CHAIN

	α	p_i^*	w^*	$E\left[\Pi_{R_i}\right]^*$	$E\left[\Pi_M\right]^*$
Expected value	—	32.51	25.02	842.63	3431.67
α-optimistic	1.00	32.50	25.00	843.75	3375.00
value	0.95	32.74	25.14	859.51	3439.23
	0.90	32.98	25.29	876.11	3504.43
	0.85	33.23	25.44	892.65	3570.61
	0.80	33.48	25.58	909.45	3637.80
	0.75	33.73	25.74	926.50	3706.00
α-pessimistic	1.00	32.50	25.00	843.75	3375.00
value	0.95	32.26	24.86	827.93	3311.72
	0.90	32.03	24.72	812.35	3249.38
	0.85	31.80	24.58	796.99	3187.96
	0.80	31.57	24.45	781.86	3127.45
	0.75	31.35	24.32	766.96	3067.82

Based on the results showed in Table I, we find:

(a) The 3th and 9th rows in Table Ⅰ show the solutions for fuzzy models at α=1, which are just the results in crisp case.

(b) Because of dominating, the expected values, α-optimistic values and α-pessimistic values of the profits for manufacturer are more than that of the total profits for two retailers. It indicates that the actor who is the leader in the supply chain holds advantage in obtaining the higher expected profits. Moreover, the profits of retailer 1 are equal to those of retailer 2 in the Collusion solution.

(c) The α-optimistic values of the optimal pricing strategies and optimal profits for the retailer i and the manufacturer decrease with the increasing of the confidence level α. With the increasing of the confidence level α, the α-pessimistic values of the optimal pricing strategies and the profits for the retailer i and the manufacture will increase.

VI. Conclusions

This paper proposes a fuzzy model for two-echelon supply chain management, where two competitive retailers pursue the Collusion solution. The pricing solutions for manufacturer and two retailers in expected value and chance-constrained programming models are provided. We find that the proposed fuzzy models can be reduced to the crisp models and the confidence level of the profits for the manufactures and the retailer affects the final optimal solutions. Our study mainly concentrates on one manufacture and two competing retailers when the fuzzy demand function is linear. Therefore other forms of fuzzy demand function and with multiple competitive retailers or manufacturers are the important directions for the future research.

References

[1] S.C. Choi, "Price competition in a channel structure with a common retailer", *Marketing Science*, vol. 10, no 4, pp. 271–296, 1991.

[2] C.A. Ingene, and M.E. Parry, "Channel coordination when retailers compete", *Marketing Science*, vol. 14, no 4, pp. 360–377, 1995.

[3] S. Yang, and Y. Zhou, "Two-echelon supply chain models: Considering duopolistic retailers' different competitive behaviors". *International Journal of Production Economics*, vol. 103, no 1, pp. 104–116, 2006.

[4] Shengju Sang, "Optimal Pricing and Retail Service Decisions in an Uncertain Supply Chain," *IAENG International Journal of Applied Mathematics, vol.* 46, no.2, pp268-274, 2016.

[5] T. Xiao, and X. Qi, "Price competition, cost and demand disruptions and coordination of a supply chain with one manufacturer and two competing retailers", *Omega*, vol. 36,no 5, pp. 741–753, 2008.

[6] Z. Yao, S.C.H. Leung, and K.K. Lai, "Manufacturer's revenue-sharing contract and retail competition", *European Journal of Operational Research*, vol. 186, no2, pp. 637–651, 2008.

[7] E. Anderson, and Y. Bao, "Price competition with integrated and decentralized supply chains", *European Journal of Operational Research*, vol. 200, no 1, pp. 227–234, 2010.

[8] A. Farahat, and G. Perakis, "On the efficiency of price competition", *Operations Research Letters*, vol. 39, no 6, pp. 414–418, 2011.

[9] Q. Zhao, and H. Chen, "Coordination of a supply chain including duopoly retailers under supply-chain ripple effect by an operating uncertainty: price competition", *Advances in Information Sciences and Service Sciences*, vol. 4, no 16, pp. 23 – 31, 2012.

[10] S. Choi, and K. Fredj, "Price competition and store competition: Store brands vs. national brand", *European Journal of Operational Research*, vol. 225, no 1, pp. 532–538, 2013.

[11] C.J. Wang, A.M. Wang, and Y.Y. Wang, "Markup pricing strategies between a dominant retailer and competitive manufacturers", *Computers & Industrial Engineering*, vol. 64, no 1, pp. 235–246, 2013.

[12] H. Kawakatsu, T. Homma and K. Sawada, "An optimal quantity discount policy for deteriorating items with a single wholesaler and two retailers" *IAENG International Journal of Applied Mathematics*, vol.43, no 2, pp. 81-86, 2013.

[13] Y. Huang and G. Q. Huang, "Price competition and coordination in a multi-echelon supply chain", *Engineering Letters*, vol.18, no 4, pp.399-405, 2010.

[14] R. Xu, and X. Zhai, "Optimal models for single-period supply chain problems with fuzzy demand". *Information Sciences*, vol.178, no 17, pp. 3374–3381, 2008.

[15] R. Xu, and X. Zhai, "Analysis of supply chain coordination under fuzzy demand in a two-stage supply chain". *Applied Mathematical Modeling*, vol. 34, no 1, pp. 129–139, 2010.

[16] C. Zhou, R. Zhao, and W. Tang, "Two-echelon supply chain games in a fuzzy environment". *Computers & Industrial Engineering*", vol.55, no 2, pp. 390–405, 2008.

[17] J. Wei, and J. Zhao, "Pricing decisions with retail competition in a fuzzy closed-loop supply chain". *Expert Systems with Applications*, vol. 38, no 9, pp. 11209–11216, 2011.

[18] F. Ye, and Y. Li, "A Stackelberg single-period supply chain inventory model with weighted possibilistic mean values under fuzzy environment". *Applied Soft Computing*, vol. 11, no 8, pp. 5519–5527, 2011.

[19] J. Zhao, W. Tang, and J. Wei, "Pricing decision for substitutable products with retail competition in a fuzzy environment". *International Journal of Production Economics*, vol. 135, no 1, pp.144-153, 2012.

[20] J. Wei, and J. Zhao, "Reverse channel decisions for a fuzzy closed-loop supply chain". *Applied Mathematical Modelling*, vol. 37, no 3, pp. 1502-1513, 2013.

[21] J. Zhao, W. Liu, and J. Wei, "Competition under manufacturer service and price in fuzzy environments". *Knowledge-Based Systems*, vol. 50, no 2013, pp. 121-133, 2013.

[22] Y. Yu, J. Zhu, and C. Wang, "A newsvendor model with fuzzy price-dependent demand". *Applied Mathematical Modelling*, vol. 37, no 5, pp. 2644-2661, 2013.

[23] S. Sang, "Supply Chain Contracts with Multiple Retailers in a Fuzzy Demand Environment". *Mathematical Problems in Engineering*, vol. 2013, no 2013, pp. 1-12, 2013.

[24] S. Nahmias, "Fuzzy variables". *Fuzzy Sets and Systems*, vol. 1, no 2, pp. 97–110, 1978.

[25] B. Liu, *Theory and practice of uncertain programming*, Physica-Verlag, Heidelberg, 2002.

[26] B. Liu, and Y. Liu, "Expected value of fuzzy variable and fuzzy expected value model", *IEEE Transactions on Fuzzy Systems*, vol. 10, no 4, pp. 445–450, 2006.

[27] R. Zhao, W. Tang, and H. Yun, "Random fuzzy Renewal process". *European Journal of Operation Research*, vol. 169, no 1, pp. 189–201, 2006.

[28] B. Liu, and K. Iwamura, "Chance constrained programming with fuzzy parameters", *Fuzzy Sets and Systems*, vol. 94, no 2, pp. 227–237, 1998.

[29] B. Liu, and K. Iwamura, "A note on chance constrained programming with fuzzy coefficients", *Fuzzy Sets and Systems*, vol. 100, no 1, pp. 229–233, 1998.