Improved Results on Delay-range-dependent Robust Stability Criteria of Uncertain Neutral Systems with Mixed Interval Time-varying Delays

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***Abstract*—In this paper, we consider the delay-range-dependent robust stability problem for uncertain neutral systems with mixed interval time-varying delays. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties, respectively. The restriction on the derivative of the discrete interval time-varying delay is removed, which means that a fast interval time-varying delay is allowed. By constructing a suitable augmented Lyapunov-Krasovskii functional, mixed model transformation, new improved integral inequalities, Leibniz-Newton formula and utilization of zero equation, new delay-range-dependent robust stability criteria are derived in terms of linear matrix inequalities (LMIs) for the systems. Moreover, we present new delay-range-dependent stability criteria for linear system with non-differentiable interval time-varying delay and nonlinear perturbations. Numerical examples are given to show the effectiveness and less conservativeness of the proposed methods.**

***Index Terms*—uncertain neutral system, Lyapunov-Krasovskii functional, linear matrix inequality, model transformation, interval time-varying delay.**

I. Introduction

The problem of stability analysis for neutral systems has been intensively studied since neutral systems can be found in many industrial systems such as population ecology [9], distributed networks containing lossless transmission lines [1], heat exchangers, robots in contact with rigid environments [27], etc. For interesting research methods, stability criteria for application neutral stochastic systems and neural networks have been discussed in [14], [24], [40]-[42]. The occurrence of the time delays and uncertainties may cause frequently the source of instability or poor performances in various systems. Stability criteria for time-delay systems are generally divided into two classes: delay-independent one and delay-dependent one. Delay-independent stability criteria tend to be more conservative, especially for small size delay, such criteria do not give any information on the size of the delay. On the other hand, delay-dependent stability criteria are concerned with the size of the delay and usually provide a maximal delay size.

Most of the existing for delay-dependent stability criteria are presented by using Lyapunov-Krasovskii approach or Lyapunov-Razumikhin approach. Therefore, the subject of the stability analysis of neutral systems with constants or time-varying delays has attracted considerable attention during the past few decades [2], [3], [6], [17], [22], [23], [28], [29], [31], [33], [45], [49]. Much attention has been paid on stability analysis of the neutral systems with time-varying delays or interval time-varying delays and nonlinear perturbations [4], [5], [18], [19], [20], [30], [34], [36], [38], [47], [48], [52]. Moreover, stability analysis of uncertain neutral systems with time-varying delays has received the attention of a lot of theoreticians in this field over the last few years [6], [11], [13], [21], [28], [31], [36], [39], [43]. In [42], the author has studied the problem of exponential stability analysis for neutral stochastic systems with distributed delays. However, a few results have been obtained for robust stability of uncertain neutral systems with mixed interval time-varying delays. The time-varying delays are assumed to belong to an interval and no restriction on the derivative of the discrete time-varying delay is needed. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties, respectively.

In this paper, the problem of delay-range-dependent stability analysis for neutral systems with non-differentiable discrete interval time-varying delays, neutral interval time-varying delays and nonlinear perturbations is studied. Based on the Lyapunov stability theory, some new delay-range-dependent stability criteria are derived in terms of LMIs for the systems. In order to get less conservative stability criteria and reduce the complexity of its calculation, new improved integral inequalities, Leibniz-Newton formula [13], [34], utilization of zero equation [2], [19], descriptor model transformation [30], [39] and integral inequalities approach [10], [25] are used. Moreover, we consider the problem of delay-range-dependent stability for linear system with non-differentiable interval time-varying delay and nonlinear perturbations. Finally, some illustrative examples are given to show the effectiveness and advantages of the developed method.

II. Problem formulation and preliminaries

We introduce some notations and lemmas that will be used throughout the paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional space with the vector norm $\|\cdot\|$; $\|x\|$ denotes the Euclidean vector norm

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of $x \in R^n$; $R^{n\times r}$ denotes the set $n \times r$ real matrices; A^T denotes the transpose of the matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; matrix A is called semi-positive definite ($A \geq 0$) if $x^TAx \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $x^TAx > 0$ for all $x \neq 0$; matrix B is called semi-negative definite ($B \leq 0$) if $x^TBx \leq 0$, for all $x \in R^n$; B is negative definite ($B < 0$) if $x^TBx < 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$ ($B - A < 0$); $A \geq B$ means $A - B \geq 0$ ($B - A \leq 0$); $C([-\bar{h}, 0], R^n)$ denotes the space of all continuous vector functions mapping $[-\bar{h}, 0]$ into R^n when $\bar{h} = \max\{h_2, r_2\}$, $h_2, r_2 \in R^+$; $*$ represents the elements below the main diagonal of a symmetric matrix.

Consider the neutral system with mixed time-varying delays and nonlinear perturbations of the form

$$\begin{cases} \dot{x}(t) - C\dot{x}(t - r(t)) \\ = Ax(t) + Bx(t - h(t)) + f_1(t, x(t)) \\ +f_2(t, x(t - h(t))) + f_3(t, \dot{x}(t - r(t))), \quad t > 0; \\ x(t_0 + \theta) = \phi(\theta), \\ \dot{x}(t_0 + \theta) = \varphi(\theta), \quad \theta \in [-\bar{h}, 0]; \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state variable, A, B, $C \in R^{n\times n}$ are real constant matrices. $r(t)$ and $h(t)$ are neutral and discrete interval time-varying delays, respectively,

$$0 \leq r_1 \leq r(t) \leq r_2, \quad 0 \leq \dot{r}(t) \leq r_d, \tag{2}$$

$$0 \leq h_1 \leq h(t) \leq h_2, \tag{3}$$

where r_1, r_2, h_1 and h_2 are positive real constants. $\phi(t)$ and $\varphi(t) \in C([-\bar{h}, 0], R^n)$ are initial condition functions with the norm $\|\phi\| = \sup_{s\in[-\bar{h},0]} \|\phi(s)\|$ and $\|\varphi\| = \sup_{s\in[-\bar{h},0]} \|\varphi(s)\|$. The uncertainties $f_1(.)$, $f_2(.)$ and $f_3(.)$ represent the nonlinear parameter perturbations with respect to the current state $x(t)$, the delayed state $x(t - h(t))$ and the neutral delayed state $\dot{x}(t - r(t))$, respectively, and are bounded in magnitude

$$f_1^T(t, x(t))f_1(t, x(t)) \leq \eta^2 x^T(t)x(t), \tag{4}$$

$$f_2^T(t, x(t - h(t)))f_2(t, x(t - h(t))) \leq \rho^2 x^T(t - h(t))x(t - h(t)), \tag{5}$$

$$f_3^T(t, \dot{x}(t - r(t)))f_3(t, \dot{x}(t - r(t))) \leq \gamma^2 \dot{x}^T(t - r(t))\dot{x}(t - r(t)), \tag{6}$$

where η, ρ and γ are given positive real constants. In addition, if the nonlinear perturbations are reduced to be the norm-bounded uncertainties

$$f_1(t, x(t)) = \Delta A(t)x(t), \tag{7}$$

$$f_2(t, x(t - h(t))) = \Delta B(t)x(t - h(t)), \tag{8}$$

$$f_3(t, \dot{x}(t - r(t))) = \Delta C(t)\dot{x}(t - r(t)), \tag{9}$$

then system (1) rewrites to the following system

$$\begin{cases} \dot{x}(t) - [C + \Delta C(t)]\dot{x}(t - r(t)) \\ = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]x(t - h(t)), \\ t > 0; \\ x(t_0 + \theta) = \phi(\theta), \\ \dot{x}(t_0 + \theta) = \varphi(\theta), \quad \theta \in [-\bar{h}, 0]. \end{cases} \tag{10}$$

The uncertain matrices $\Delta A(t)$, $\Delta B(t)$ and $\Delta C(t)$ are norm bounded and can be described as

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) & \Delta C(t) \end{bmatrix} = E\Delta(t) \begin{bmatrix} G_1 & G_2 & G_3 \end{bmatrix}, \tag{11}$$

where E, G_1, G_2 and G_3 are constant matrices with appropriate dimensions. The uncertain matrix $\Delta(t)$ satisfies

$$\Delta(t) = F(t)[I - JF(t)]^{-1}, \tag{12}$$

is said to be admissible where J is known matrix satisfying

$$I - JJ^T > 0. \tag{13}$$

The uncertain matrix $F(t)$ satisfies

$$F(t)^TF(t) \leq I. \tag{14}$$

Lemma 2.1 (Jensen's inequality): For any constant matrix $Q \in R^{n\times n}, Q = Q^T > 0$, positive real constant h, vector function $\dot{x}(t) : [-h_2, 0] \to R^n$ such that the integrations concerned are well defined, then

$$-h\int_{-h}^{0} \dot{x}^T(s + t)Q\dot{x}(s + t)ds \leq -\Big(\int_{-h}^{0} \dot{x}(s + t)ds\Big)^T Q \Big(\int_{-h}^{0} \dot{x}(s + t)ds\Big).$$

Rearranging the term $\int_{-h}^{0} \dot{x}(s + t)ds$ with $x(t) - x(t - h)$, we obtain the following inequality:

$$-h\int_{-h}^{0} \dot{x}^T(s + t)Q\dot{x}(s + t)ds \leq \begin{bmatrix} x(t) \\ x(t - h) \end{bmatrix}^T \begin{bmatrix} -Q & Q \\ * & -Q \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - h) \end{bmatrix}.$$

Lemma 2.2 (Schur complement lemma): Let X, Y, Z be constant matrices of appropriate dimensions with $Y > 0$. Then $X + Z^TY^{-1}Z < 0$ if and only if

$$\begin{bmatrix} X & Z^T \\ * & -Y \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -Y & Z \\ * & X \end{bmatrix} < 0.$$

Lemma 2.3 (Cauchy's inequality): For any constant symmetric positive definite matrix $P \in R^{n\times n}$ and $a, b \in R^n$,

$$\pm 2a^Tb \leq a^TPa + b^TP^{-1}b.$$

Lemma 2.4: [16] Suppose that $\Delta(t)$ is given by (12)-(14). Let M, S and N be real constant matrices of appropriate dimension with $M = M^T$. Then, the inequality

$$M + S\Delta(t)N + N^T\Delta(t)^TS^T < 0,$$

holds if and only if, for any positive real constant δ,

$$\begin{bmatrix} M & S & \delta N^T \\ * & -\delta I & \delta J^T \\ * & * & -\delta I \end{bmatrix} < 0.$$

Lemma 2.5: [25] The following inequality holds for any $a \in R^n$, $b \in R^m$, $N, Y \in R^{n\times m}$, $X \in R^{n\times n}$, and $Z \in R^{m\times m}$,

$$-2a^TNb \leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y - N \\ * & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix},$$

where $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geq 0$.

Lemma 2.6: [10] For a positive matrix M, the following inequality holds:

$$-(\alpha - \beta)\int_{\beta}^{\alpha} \dot{x}^T(s)M\dot{x}(s)ds \leq \begin{bmatrix} x(\alpha) \\ x(\beta) \end{bmatrix}^T \begin{bmatrix} -M & M \\ * & -M \end{bmatrix} \begin{bmatrix} x(\alpha) \\ x(\beta) \end{bmatrix}.$$

Lemma 2.7: [10] For a positive matrix M, the following inequality holds:

$$\begin{aligned}
&-\frac{(\alpha-\beta)^2}{2}\int_{\beta}^{\alpha}\int_{s}^{\alpha}x^T(u)Mx(u)duds\\
\leq\ &-\Big(\int_{\beta}^{\alpha}\int_{s}^{\alpha}x(u)duds\Big)^TM\Big(\int_{\beta}^{\alpha}\int_{s}^{\alpha}x(u)duds\Big).
\end{aligned}$$

Lemma 2.8: [10] For a positive matrix M, the following inequality holds:

$$\begin{aligned}
&-\frac{(\alpha-\beta)^3}{6}\int_{\beta}^{\alpha}\int_{s}^{\alpha}\int_{u}^{\alpha}x^T(\lambda)Mx(\lambda)d\lambda duds\\
\leq\ &-\Big(\int_{\beta}^{\alpha}\int_{s}^{\alpha}\int_{u}^{\alpha}x(\lambda)d\lambda duds\Big)^T\\
&\times M\Big(\int_{\beta}^{\alpha}\int_{s}^{\alpha}\int_{u}^{\alpha}x(\lambda)d\lambda duds\Big).
\end{aligned}$$

III. Improved inequalities

Theorem 3.1: For any constant symmetric positive definite matrix $Q\in R^{n\times n}$, $h(t)$ is discrete time-varying delays with (3), vector function $\omega:[-h_2,0]\to R^n$ such that the integrations concerned are well defined,

$$\begin{aligned}
&-(h_2-h_1)\int_{-h_2}^{-h_1}\omega^T(s)Q\omega(s)ds\\
\leq\ &-\int_{-h(t)}^{-h_1}\omega^T(s)dsQ\int_{-h(t)}^{-h_1}\omega(s)ds\\
&-\int_{-h_2}^{-h(t)}\omega^T(s)dsQ\int_{-h_2}^{-h(t)}\omega(s)ds.
\end{aligned}$$

Proof. By Lemma 2.3, it is easy to see that

$$\begin{aligned}
&(h_2-h_1)\int_{-h_2}^{-h_1}\omega^T(s)Q\omega(s)ds\\
=\ &(h_2-h_1)\int_{-h(t)}^{-h_1}\omega^T(s)Q\omega(s)ds\\
&+(h_2-h_1)\int_{-h_2}^{-h(t)}\omega^T(s)Q\omega(s)ds\\
\geq\ &(h(t)-h_1)\int_{-h(t)}^{-h_1}\omega^T(s)Q\omega(s)ds\\
&+(h_2-h(t))\int_{-h_2}^{-h(t)}\omega^T(s)Q\omega(s)ds\\
=\ &\frac{1}{2}\int_{-h(t)}^{-h_1}\int_{-h(t)}^{-h_1}\omega^T(s)Q\omega(s)+\omega^T(\xi)Q\omega(\xi)dsd\xi\\
&+\frac{1}{2}\int_{-h_2}^{-h(t)}\int_{-h_2}^{-h(t)}\omega^T(s)Q\omega(s)+\omega^T(\xi)Q\omega(\xi)dsd\xi\\
\geq\ &\frac{1}{2}\int_{-h(t)}^{-h_1}\int_{-h(t)}^{-h_1}2\omega^T(s){Q^{1/2}}^TQ^{1/2}\omega(\xi)dsd\xi\\
&+\frac{1}{2}\int_{-h_2}^{-h(t)}\int_{-h_2}^{-h(t)}2\omega^T(s){Q^{1/2}}^TQ^{1/2}\omega(\xi)dsd\xi\\
=\ &\int_{-h(t)}^{-h_1}\omega^T(s)dsQ\int_{-h(t)}^{-h_1}\omega(s)ds\\
&+\int_{-h_2}^{-h(t)}\omega^T(s)dsQ\int_{-h_2}^{-h(t)}\omega(s)ds.
\end{aligned}$$

This completes the proof.

Theorem 3.2: For any constant matrices $Q_1,Q_2,Q_3\in R^{n\times n}$, $Q_1\geq 0$, $Q_3>0$, $\begin{bmatrix}Q_1 & Q_2\\ * & Q_3\end{bmatrix}\geq 0$, $h(t)$ is discrete time-varying delays with (3) and vector function $\dot{x}:[-h_2,0]\to R^n$ such that the following integration is well defined,

$$\begin{aligned}
&-(h_2-h_1)\int_{t-h_2}^{t-h_1}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix}Q_1 & Q_2\\ * & Q_3\end{bmatrix}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds\\
\leq\ &\begin{bmatrix}x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\\ \int_{t-h(t)}^{t-h_1}x(s)ds\\ \int_{t-h_2}^{t-h(t)}x(s)ds\end{bmatrix}^T\\
&\times\begin{bmatrix}-Q_3 & Q_3 & 0 & -Q_2^T & 0\\ * & -Q_3-Q_3 & Q_3 & Q_2^T & -Q_2^T\\ * & * & -Q_3 & 0 & Q_2^T\\ * & * & * & -Q_1 & 0\\ * & * & * & * & -Q_1\end{bmatrix}\\
&\times\begin{bmatrix}x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\\ \int_{t-h(t)}^{t-h_1}x(s)ds\\ \int_{t-h_2}^{t-h(t)}x(s)ds\end{bmatrix}.
\end{aligned}$$

Proof. By Theorem 3.1, we have

$$\begin{aligned}
&-(h_2-h_1)\int_{t-h_2}^{t-h_1}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix}Q_1 & Q_2\\ * & Q_3\end{bmatrix}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds\\
\leq\ &-\int_{t-h(t)}^{t-h_1}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}^Tds\begin{bmatrix}Q_1 & Q_2\\ * & Q_3\end{bmatrix}\int_{t-h(t)}^{t-h_1}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds\\
&-\int_{t-h_2}^{t-h(t)}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}^Tds\begin{bmatrix}Q_1 & Q_2\\ * & Q_3\end{bmatrix}\int_{t-h_2}^{t-h(t)}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds\\
=\ &\begin{bmatrix}x(t-h_1)\\ x(t-h(t))\\ \int_{t-h(t)}^{t-h_1}x(s)ds\end{bmatrix}^T\begin{bmatrix}-Q_3 & Q_3 & -Q_2^T\\ * & -Q_3 & Q_2^T\\ * & * & -Q_1\end{bmatrix}\\
&\times\begin{bmatrix}x(t-h_1)\\ x(t-h(t))\\ \int_{t-h(t)}^{t-h_1}x(s)ds\end{bmatrix}\\
&+\begin{bmatrix}x(t-h(t))\\ x(t-h_2)\\ \int_{t-h_2}^{t-h(t)}x(s)ds\end{bmatrix}^T\begin{bmatrix}-Q_3 & Q_3 & -Q_2^T\\ * & -Q_3 & Q_2^T\\ * & * & -Q_1\end{bmatrix}\\
&\times\begin{bmatrix}x(t-h(t))\\ x(t-h_2)\\ \int_{t-h_2}^{t-h(t)}x(s)ds\end{bmatrix}.
\end{aligned}$$

This completes the proof.

Theorem 3.3: Let $x(t)\in R^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any constant matrices $X,M_i\in R^{n\times n},i=1,2,\ldots,5$ and $h(t)$ is discrete time-varying delays with (3),

$$-\int_{t-h_2}^{t-h_1}\dot{x}^T(s)X\dot{x}(s)ds \le \begin{bmatrix} x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}^T \times \begin{bmatrix} M_1+M_1^T & -M_1^T+M_2 & 0\\ * & M_1+M_1^T-M_2-M_2^T & -M_1^T+M_2\\ * & * & -M_2-M_2^T\end{bmatrix} \times \begin{bmatrix} x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix} + (h_2-h_1)\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}^T \times \begin{bmatrix} M_3 & M_4 & 0\\ * & M_3+M_5 & M_4\\ * & * & M_5\end{bmatrix}\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix},$$

where

$$\begin{bmatrix} X & M_1 & M_2\\ * & M_3 & M_4\\ * & * & M_5\end{bmatrix} \ge 0.$$

Proof. From the Leibniz-Newton formula, we obtain

$$0 = x(t-h_1) - x(t-h(t)) - \int_{t-h(t)}^{t-h_1}\dot{x}(s)ds, \qquad (15)$$

$$0 = x(t-h(t)) - x(t-h_2) - \int_{t-h_2}^{t-h(t)}\dot{x}(s)ds. \qquad (16)$$

By (15), the following equation is true for any constant matrices $H_1, H_2 \in R^{n\times n}$

$$\begin{aligned} 0 &= 2\big[x^T(t-h_1) - x^T(t-h(t)) - \int_{t-h(t)}^{t-h_1}\dot{x}^T(t)ds\big] \\ &\quad \times[H_1x(t-h_1)+H_2x(t-h(t))] \\ &= 2x^T(t-h_1)H_1x(t-h_1) \\ &\quad +2x^T(t-h_1)H_2x(t-h(t)) \\ &\quad -2x^T(t-h(t))H_1x(t-h_1) \\ &\quad -2x^T(t-h(t))H_2^Tx(t-h(t)) \\ &\quad -2\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)dsH_1x(t) \\ &\quad -2\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)dsH_2x(t-h(t)) \\ &= \begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T \begin{bmatrix} H_1+H_1^T & -H_1^T+H_2\\ * & -H_2-H_2^T\end{bmatrix} \\ &\quad \times\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix} - 2\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)\begin{bmatrix} H_1 & H_2\end{bmatrix} \\ &\quad \times\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}ds. \end{aligned} \qquad (17)$$

Using Lemma 2.5 with $a = \dot{x}(s)$, $b = \begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}$, $Y = \begin{bmatrix} M_1 & M_2\end{bmatrix}$ and $Z = \begin{bmatrix} M_3 & M_4\\ * & M_5\end{bmatrix}$, we get

$$\begin{aligned} &-2\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)\begin{bmatrix} H_1 & H_2\end{bmatrix}\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}ds \\ &\le \int_{t-h(t)}^{t-h_1}\begin{bmatrix} \dot{x}(s)\\ x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} X & M_1-H_1 & M_2-H_2\\ * & M_3 & M_4\\ * & * & M_5\end{bmatrix} \\ &\quad \times\begin{bmatrix} \dot{x}(s)\\ x(t-h_1)\\ x(t-h(t))\end{bmatrix}ds \\ &= \int_{t-h(t)}^{t-h_1}\dot{x}^T(s)X\dot{x}(s)ds + \begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T \\ &\quad \left(\begin{bmatrix} M_1+M_1^T & -M_1^T+M_2\\ * & -M_2-M_2^T\end{bmatrix} - \begin{bmatrix} H_1+H_1^T & -H_1^T+H_2\\ * & -H_2-H_2^T\end{bmatrix}\right)\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix} \\ &\quad +(h(t)-h_1)\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} M_3 & M_4\\ * & M_5\end{bmatrix} \\ &\quad \times\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix} \\ &\le \int_{t-h(t)}^{t-h_1}\dot{x}^T(s)X\dot{x}(s)ds + \begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T \\ &\quad \left(\begin{bmatrix} M_1+M_1^T & -M_1^T+M_2\\ * & -M_2-M_2^T\end{bmatrix} - \begin{bmatrix} H_1+H_1^T & -H_1^T+H_2\\ * & -H_2-H_2^T\end{bmatrix}\right)\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix} \\ &\quad +(h_2-h_1)\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} M_3 & M_4\\ * & M_5\end{bmatrix} \\ &\quad \times\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}. \end{aligned} \qquad (18)$$

Substituting (18) into (17), we obtain

$$\begin{aligned} &-\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)X\dot{x}(s)ds \\ &\le \begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} H_1+H_1^T & -H_1^T+H_2\\ * & -H_2-H_2^T\end{bmatrix} \\ &\quad \times\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix} + \begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T \\ &\quad \times\left(\begin{bmatrix} M_1+M_1^T & -M_1^T+M_2\\ * & -M_2-M_2^T\end{bmatrix} - \begin{bmatrix} H_1+H_1^T & -H_1^T+H_2\\ * & -H_2-H_2^T\end{bmatrix}\right)\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix} \\ &\quad +(h_2-h_1)\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} M_3 & M_4\\ * & M_5\end{bmatrix} \\ &\quad \times\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix} \\ &= \begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} M_1+M_1^T & -M_1^T+M_2\\ * & -M_2-M_2^T\end{bmatrix} \\ &\quad \times\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix} + [h_2-h_1]\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}^T \\ &\quad \times\begin{bmatrix} M_3 & M_4\\ * & M_5\end{bmatrix}\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\end{bmatrix}. \end{aligned} \qquad (19)$$

By (16), the following equation is true for any constant matrices $H_1, H_2 \in R^{n\times n}$

$$0 = 2\big[x^T(t-h(t)) - x^T(t-h_2) - \int_{t-h_2}^{t-h(t)} \dot{x}^T(t)ds\big] \times[H_1x(t-h(t)) + H_2x(t-h_2)].$$

Similarly, we have

$$\begin{aligned} & -\int_{t-h_2}^{t-h(t)} \dot{x}^T(s)X\dot{x}(s)ds \\ \leq & \begin{bmatrix} x(t-h(t)) \\ x(t-h_2) \end{bmatrix}^T \begin{bmatrix} M_1+M_1^T & -M_1^T+M_2 \\ * & -M_2-M_2^T \end{bmatrix} \\ & \times \begin{bmatrix} x(t-h(t)) \\ x(t-h_2) \end{bmatrix} + [h_2-h_1] \begin{bmatrix} x(t-h(t)) \\ x(t-h_2) \end{bmatrix}^T \\ & \times \begin{bmatrix} M_3 & M_4 \\ * & M_5 \end{bmatrix} \begin{bmatrix} x(t-h(t)) \\ x(t-h_2) \end{bmatrix}. \end{aligned} \quad (20)$$

Considering (19) and (20) together, the result is established. This completes the proof.

Remark 3.4: In Theorem 3.1, 3.2 and 3.3, we have modified the method from [8], [32] and [51], respectively.

IV. Main results

A. Delay-range-dependent stability criteria

We will present the robust stability criteria dependent on interval time-varying delays of system (1) via LMIs approach. We introduce the following notations for later use.

$$\sum = \left[\Sigma_{i,j}\right]_{23\times 23}, \quad (21)$$

where $\quad \Sigma_{i,j} = \Sigma_{j,i}^T, \quad i,j = 1,2,3,...,23,$

$$\begin{aligned}
\Sigma_{1,1} &= P_1A + A^TP_1 + W + W^T + Q_1A + A^TQ_1 \\
&\quad +P_3 + P_4 + R_1 + R_4 + (h_1)^2P_9 + (h_2)^2P_{10} \\
&\quad +(h_2-h_1)^2P_{11} - P_{12} + M_1 + M_1^T + h_2M_3 \\
&\quad +(h_1)^2R_{10} + (h_2)^2R_{13} + (h_2-h_1)^2R_{16} \\
&\quad +R_{12} - R_{15} + \frac{(h_1)^4}{4}P_{15} + \frac{(h_2)^4}{4}P_{16} \\
&\quad -(h_1)^2P_{18} - (h_2)^2P_{19} - \frac{(h_1)^4}{4}P_{21} \\
&\quad -\frac{(h_2)^4}{4}P_{22} + h_1C_1 + h_1C_1^T + h_2C_4 \\
&\quad +h_2C_4^T + \epsilon_1\alpha^2I, \\
\Sigma_{1,2} &= P_2 - Q_1 + A^TQ_2 + R_2 + R_5 + (h_1)^2R_{11} \\
&\quad +(h_2)^2R_{14} + (h_2-h_1)^2R_{17}, \\
\Sigma_{1,3} &= P_{12} + R_{12} - h_1C_1 + h_1C_2^T, \\
\Sigma_{1,4} &= \Sigma_{1,5} = \Sigma_{1,6} = 0, \\
\Sigma_{1,7} &= P_1B - W + Q_1B + A^TQ_3 - M_1^T + M_2 \\
&\quad +h_2M_4 + R_{15} - h_2C_4 + h_2C_5^T, \\
\Sigma_{1,8} &= -R_{11}^T, \quad \Sigma_{1,9} = h_1P_{18}, \quad \Sigma_{1,10} = 0, \\
\Sigma_{1,11} &= -R_{14}^T, \quad \Sigma_{1,12} = 0, \quad \Sigma_{1,13} = h_2P_{19}, \\
\Sigma_{1,14} &= 0, \quad \Sigma_{1,15} = -W + Q_4 - h_2C_4 + h_2C_6^T, \\
\Sigma_{1,16} &= \frac{(h_1)^2}{2}P_{21}, \quad \Sigma_{1,17} = \frac{(h_2)^2}{2}P_{22}, \quad \Sigma_{1,18} = 0, \\
\Sigma_{1,19} &= -h_1C_1 + h_1C_3^T, \quad \Sigma_{1,20} = P_1C + Q_1C, \\
\Sigma_{1,21} &= \Sigma_{1,22} = \Sigma_{1,23} = P_1 + Q_1, \\
\Sigma_{2,2} &= (r_2-r_1)Q_5 - 2Q_2 + P_6 + P_7 + R_3 + R_6 \\
&\quad +(h_1)^2P_{12} + h_2P_{13} + (h_2-h_1)P_{14} \\
&\quad +(h_1)^2R_{12} + (h_2)^2R_{15} + (h_2-h_1)^2R_{18} \\
&\quad +\frac{(h_1)^4}{4}P_{18} + \frac{(h_2)^4}{4}P_{19} + \frac{(h_1)^6}{36}P_{21} \\
&\quad +\frac{(h_2)^6}{36}P_{22}, \\
\Sigma_{2,3} &= \Sigma_{2,4} = \Sigma_{2,5} = \Sigma_{2,6} = 0, \\
\Sigma_{2,7} &= -Q_2B - Q_3, \\
\Sigma_{2,8} &= \Sigma_{2,9} = \Sigma_{2,10} = \Sigma_{2,11} = \Sigma_{2,12} = \Sigma_{2,13} = 0, \\
\Sigma_{2,14} &= 0, \quad \Sigma_{2,15} = -Q_4, \\
\Sigma_{2,16} &= \Sigma_{2,17} = \Sigma_{2,18} = \Sigma_{2,19} = 0, \quad \Sigma_{2,20} = Q_2C, \\
\Sigma_{2,21} &= \Sigma_{2,22} = \Sigma_{2,23} = Q_2, \\
\Sigma_{3,3} &= -P_3 + P_5 - R_1 + R_7 - P_{12} + N_1 + N_1^T \\
&\quad +(h_2-h_1)N_3 - R_{12} - R_{18} - (h_2-h_1)^2P_{20} \\
&\quad +\frac{(h_2-h_1)^4}{4}P_{17} - \frac{(h_2-h_1)^4}{4}P_{23} - h_1C_2 \\
&\quad -h_1C_2^T, \\
\Sigma_{3,4} &= -R_2 + R_8, \quad \Sigma_{3,5} = \Sigma_{3,6} = 0, \\
\Sigma_{3,7} &= -N_1^T + N_2 + (h_2-h_1)N_4 + R_{18}, \\
\Sigma_{3,8} &= R_{11}^T, \quad \Sigma_{3,9} = 0, \quad \Sigma_{3,10} = -R_{17}^T, \\
\Sigma_{3,11} &= \Sigma_{3,12} = \Sigma_{3,13} = 0, \\
\Sigma_{3,14} &= (h_2-h_1)P_{20}, \quad \Sigma_{3,15} = \Sigma_{3,16} = \Sigma_{3,17} = 0, \\
\Sigma_{3,18} &= \frac{(h_2-h_1)^2}{2}P_{23}, \quad \Sigma_{3,19} = -h_1C_2 - h_1C_3^T, \\
\Sigma_{3,20} &= \Sigma_{3,21} = \Sigma_{3,22} = \Sigma_{3,23} = 0, \\
\Sigma_{4,4} &= -P_6 + P_8 - R_3 + R_9 + \frac{(h_2-h_1)^4}{4}P_{20} \\
&\quad +\frac{(h_2-h_1)^6}{36}P_{23}, \\
\Sigma_{4,5} &= \Sigma_{4,6} = \Sigma_{4,7} = \Sigma_{4,8} = \Sigma_{4,9} = \Sigma_{4,10} = 0, \\
\Sigma_{4,11} &= \Sigma_{4,12} = \Sigma_{4,13} = \Sigma_{4,14} = \Sigma_{4,15} = \Sigma_{4,16} = 0, \\
\Sigma_{4,17} &= \Sigma_{4,18} = \Sigma_{4,19} = \Sigma_{4,20} = \Sigma_{4,21} = \Sigma_{4,22} = 0, \\
\Sigma_{4,23} &= 0, \\
\Sigma_{5,5} &= -P_4 - P_5 - R_4 - R_7 - M_2 - M_2^T + h_2M_5 \\
&\quad -N_2 - N_2^T + (h_2-h_1)N_5 - R_{15} - R_{18}, \\
\Sigma_{5,6} &= -R_5 - R_8, \\
\Sigma_{5,7} &= -M_1 + M_2^T + h_2M_4^T - N_1 + N_2^T \\
&\quad +(h_2-h_1)N_4^T + R_{15}^T + R_{18}^T, \\
\Sigma_{5,8} &= \Sigma_{5,9} = \Sigma_{5,10} = \Sigma_{5,11} = 0, \\
\Sigma_{5,12} &= R_{14}^T + R_{17}^T, \\
\Sigma_{5,13} &= \Sigma_{5,14} = \Sigma_{5,15} = \Sigma_{5,16} = \Sigma_{5,17} = \Sigma_{5,18} = 0, \\
\Sigma_{5,19} &= \Sigma_{5,20} = \Sigma_{5,21} = \Sigma_{5,22} = \Sigma_{5,23} = 0, \\
\Sigma_{6,6} &= -P_7 - P_8 - R_6 - R_9, \quad \Sigma_{6,7} = \Sigma_{6,8} = 0, \\
\Sigma_{6,9} &= \Sigma_{6,10} = \Sigma_{6,11} = \Sigma_{6,12} = \Sigma_{6,13} = \Sigma_{6,14} = 0, \\
\Sigma_{6,15} &= \Sigma_{6,16} = \Sigma_{6,17} = \Sigma_{6,18} = \Sigma_{6,19} = \Sigma_{6,20} = 0, \\
\Sigma_{6,21} &= \Sigma_{6,22} = \Sigma_{6,23} = 0,
\end{aligned}$$

$$\begin{aligned}
\Sigma_{7,7} &= Q_3B + B^TQ_3 + M_1 + M_1^T - M_2 - M_2^T \\
&\quad +h_2(M_3 + M_5) + N_1 + N_1^T - N_2 - N_2^T \\
&\quad +(h_2 - h_1)(N_3 + N_5) - R_{15} - R_{15}^T - R_{18} \\
&\quad -R_{18}^T - h_2C_5 - h_2C_5^T + \epsilon_2\beta^2 I, \\
\Sigma_{7,8} &= \Sigma_{7,9} = 0, \quad \Sigma_{7,10} = R_{17}^T, \quad \Sigma_{7,11} = R_{14}^T, \\
\Sigma_{7,12} &= -R_{14}^T - R_{17}^T, \quad \Sigma_{7,13} = \Sigma_{7,14} = 0, \\
\Sigma_{7,15} &= -Q_4 - h_2C_5 - h_2C_6^T, \\
\Sigma_{7,16} &= \Sigma_{7,17} = \Sigma_{7,18} = \Sigma_{7,19} = 0, \quad \Sigma_{7,20} = Q_3C, \\
\Sigma_{7,21} &= \Sigma_{7,22} = \Sigma_{7,23} = Q_3, \\
\Sigma_{8,8} &= -P_9 - P_{10} - R_{10}, \quad \Sigma_{8,9} = 0, \\
\Sigma_{8,10} &= -P_{10}, \quad \Sigma_{8,11} = 0, \quad \Sigma_{8,12} = -P_{10}, \\
\Sigma_{8,13} &= \Sigma_{8,14} = \Sigma_{8,15} = \Sigma_{8,16} = \Sigma_{8,17} = \Sigma_{8,18} = 0, \\
\Sigma_{8,19} &= \Sigma_{8,20} = \Sigma_{8,21} = \Sigma_{8,22} = \Sigma_{8,23} = 0, \\
\Sigma_{9,9} &= -P_{18}, \quad \Sigma_{9,10} = \Sigma_{9,11} = \Sigma_{9,12} = \Sigma_{9,13} = 0, \\
\Sigma_{9,14} &= \Sigma_{9,15} = \Sigma_{9,16} = \Sigma_{9,17} = \Sigma_{9,18} = \Sigma_{9,19} = 0, \\
\Sigma_{9,20} &= \Sigma_{9,21} = \Sigma_{9,22} = \Sigma_{9,23} = 0, \\
\Sigma_{10,10} &= -P_{10} - P_{11} - R_{16}, \quad \Sigma_{10,11} = 0, \\
\Sigma_{10,12} &= -P_{10}, \quad \Sigma_{10,13} = \Sigma_{10,14} = \Sigma_{10,15} = 0, \\
\Sigma_{10,16} &= \Sigma_{10,17} = \Sigma_{10,18} = \Sigma_{10,19} = \Sigma_{10,20} = 0, \\
\Sigma_{10,21} &= \Sigma_{10,22} = \Sigma_{10,23} = 0, \quad \Sigma_{11,11} = -R_{13}, \\
\Sigma_{11,12} &= \Sigma_{11,13} = \Sigma_{11,14} = \Sigma_{11,15} = \Sigma_{11,16} = 0, \\
\Sigma_{11,17} &= \Sigma_{11,18} = \Sigma_{11,19} = \Sigma_{11,20} = \Sigma_{11,21} = 0, \\
\Sigma_{11,22} &= \Sigma_{11,23} = 0, \\
\Sigma_{12,12} &= -P_{10} - P_{11} - R_{13} - R_{16}, \\
\Sigma_{12,13} &= \Sigma_{12,14} = \Sigma_{12,15} = \Sigma_{12,16} = \Sigma_{12,17} = 0, \\
\Sigma_{12,18} &= \Sigma_{12,19} = \Sigma_{12,20} = \Sigma_{12,21} = \Sigma_{12,22} = 0, \\
\Sigma_{12,23} &= 0, \quad \Sigma_{13,13} = -P_{19}, \\
\Sigma_{13,14} &= \Sigma_{13,15} = \Sigma_{13,16} = \Sigma_{13,17} = \Sigma_{13,18} = 0, \\
\Sigma_{13,19} &= \Sigma_{13,20} = \Sigma_{13,21} = \Sigma_{13,22} = \Sigma_{13,23} = 0, \\
\Sigma_{14,14} &= -P_{20}, \quad \Sigma_{14,15} = \Sigma_{14,16} = \Sigma_{14,17} = 0, \\
\Sigma_{14,18} &= \Sigma_{14,19} = \Sigma_{14,20} = 0, \\
\Sigma_{14,21} &= \Sigma_{14,22} = \Sigma_{14,23} = 0, \\
\Sigma_{15,15} &= -2Q_4 - h_2C_6 - h_2C_6^T, \\
\Sigma_{15,16} &= \Sigma_{15,17} = \Sigma_{15,18} = \Sigma_{15,19} = \Sigma_{15,20} = 0, \\
\Sigma_{15,21} &= \Sigma_{15,22} = \Sigma_{15,23} = 0, \\
\Sigma_{16,16} &= -P_{15} - P_{21}, \quad \Sigma_{16,17} = \Sigma_{16,18} = \Sigma_{16,19} = 0, \\
\Sigma_{16,20} &= \Sigma_{16,21} = \Sigma_{16,22} = \Sigma_{16,23} = 0, \\
\Sigma_{17,17} &= -P_{16} - P_{22}, \quad \Sigma_{17,18} = \Sigma_{17,19} = \Sigma_{17,20} = 0, \\
\Sigma_{17,21} &= \Sigma_{17,22} = \Sigma_{17,23} = 0, \\
\Sigma_{18,18} &= -P_{17} - P_{23}, \quad \Sigma_{18,19} = \Sigma_{18,20} = \Sigma_{18,21} = 0, \\
\Sigma_{18,22} &= \Sigma_{18,23} = 0, \\
\Sigma_{19,19} &= -h_1C_3 - h_1C_3^T, \quad \Sigma_{19,20} = \Sigma_{19,21} = 0, \\
\Sigma_{19,22} &= \Sigma_{19,23} = 0, \\
\Sigma_{20,20} &= -(r_2 - r_1)(1 - r_d)Q_5 + \epsilon_3\gamma^2 I, \\
\Sigma_{20,21} &= \Sigma_{20,22} = \Sigma_{20,23} = 0, \\
\Sigma_{21,21} &= -\epsilon_1 I, \quad \Sigma_{21,22} = \Sigma_{21,23} = 0, \\
\Sigma_{22,22} &= -\epsilon_2 I, \quad \Sigma_{22,23} = 0, \quad \Sigma_{23,23} = -\epsilon_3 I.
\end{aligned}$$

Theorem 4.1: The system (1) is asymptotically stable, if there exist positive definite symmetric matrices Q_5, R_{13}, R_{15}, R_{16}, R_{18}, P_i, $i = 1, 2, ..., 23$, any appropriate dimensional matrices G, Q_k, M_j, N_j, R_l, $k = 1, 2, ..., 4, j = 1, 2, .., 5, l = 1, 2, ..., 18$ and positive real constants ϵ_1, ϵ_2 and ϵ_3 satisfying the following LMIs

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0, \tag{22}$$

$$\begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} > 0, \tag{23}$$

$$\begin{bmatrix} R_7 & R_8 \\ * & R_9 \end{bmatrix} > 0, \tag{24}$$

$$\begin{bmatrix} R_{10} & R_{11} \\ * & R_{12} \end{bmatrix} > 0, \tag{25}$$

$$\begin{bmatrix} R_{13} & R_{14} \\ * & R_{15} \end{bmatrix} > 0, \tag{26}$$

$$\begin{bmatrix} R_{16} & R_{17} \\ * & R_{18} \end{bmatrix} > 0, \tag{27}$$

$$\begin{bmatrix} P_{13} & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \geq 0, \tag{28}$$

$$\begin{bmatrix} P_{14} & N_1 & N_2 \\ * & N_3 & N_4 \\ * & * & N_5 \end{bmatrix} \geq 0, \tag{29}$$

$$\sum < 0. \tag{30}$$

Proof. Firstly, we rewrite the system (1) in the following descriptor system

$$\begin{aligned}
\dot{x}(t) &= z(t), \qquad (31) \\
0 &= -z(t) + Ax(t) + Bx(t - h(t)) + f_1(t, x(t)) \\
&\quad +f_2(t, x(t - h(t))) + f_3(t, z(t - r(t))) \\
&\quad +Cz(t - r(t)). \qquad (32)
\end{aligned}$$

By utilizing the following zero equation, we obtain

$$0 = Gx(t) - Gx(t - h(t)) - G\int_{t-h(t)}^{t} \dot{x}(s)ds, \tag{33}$$

where $G \in R^{n\times n}$ will be chosen to guarantee the asymptotic stability of the system (1). By (33), the systems (31) and (32) can be represented in the form of the descriptor delayed system

$$\begin{aligned}
\dot{x}(t) &= z(t) + Gx(t) - Gx(t - h(t)) \\
&\quad -G\int_{t-h(t)}^{t} z(s)ds, \qquad (34) \\
0 &= -z(t) + Ax(t) + Bx(t - h(t)) + f_1(t, x(t)) \\
&\quad +f_2(t, x(t - h(t))) + f_3(t, z(t - r(t))) \\
&\quad +Cz(t - r(t)). \qquad (35)
\end{aligned}$$

Construct a Lyapunov-Krasovskii functional candidate for the system (1) of the form

$$V(t) = \sum_{i=1}^{12} V_i(t), \tag{36}$$

where

$$\begin{aligned}
V_1(t) &= x^T(t)P_1x(t),\\
V_2(t) &= x^T(t)P_2x(t)\\
&= \begin{bmatrix} x(t)\\ z(t)\\ x(t-h(t))\\ \int_{t-h(t)}^t z(s)ds \end{bmatrix}^T \begin{bmatrix} I&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix}\\
&\quad\times \begin{bmatrix} P_2&0&0&0\\0&0&0&0\\0&0&0&0\\Q_1&Q_2&Q_3&Q_4 \end{bmatrix} \begin{bmatrix} x(t)\\ z(t)\\ x(t-h(t))\\ \int_{t-h(t)}^t z(s)ds \end{bmatrix},\\
V_3(t) &= \int_{t-h_1}^t x^T(s)P_3x(s)ds + \int_{t-h_2}^t x^T(s)P_4x(s)ds\\
&\quad+\int_{t-h_2}^{t-h_1} x^T(s)P_5x(s)ds,\\
V_4(t) &= \int_{t-h_1}^t z^T(s)P_6z(s)ds + \int_{t-h_2}^t z^T(s)P_7z(s)ds\\
&\quad+\int_{t-h_2}^{t-h_1} z^T(s)P_8z(s)ds,\\
V_5(t) &= \int_{t-h_1}^t \begin{bmatrix} x(s)\\ z(s)\end{bmatrix}^T \begin{bmatrix} R_1&R_2\\ *&R_3\end{bmatrix}\begin{bmatrix} x(s)\\ z(s)\end{bmatrix} ds\\
&\quad+\int_{t-h_2}^t \begin{bmatrix} x(s)\\ z(s)\end{bmatrix}^T \begin{bmatrix} R_4&R_5\\ *&R_6\end{bmatrix}\begin{bmatrix} x(s)\\ z(s)\end{bmatrix} ds\\
&\quad+\int_{t-h_2}^{t-h_1} \begin{bmatrix} x(s)\\ z(s)\end{bmatrix}^T \begin{bmatrix} R_7&R_8\\ *&R_9\end{bmatrix}\begin{bmatrix} x(s)\\ z(s)\end{bmatrix} ds,\\
V_6(t) &= h_1\int_{-h_1}^0\int_{t+s}^t x^T(\theta)P_9x(\theta)d\theta ds\\
&\quad+h_2\int_{-h_2}^0\int_{t+s}^t x^T(\theta)P_{10}x(\theta)d\theta ds\\
&\quad+(h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^t x^T(\theta)P_{11}x(\theta)d\theta ds,\\
V_7(t) &= h_1\int_{-h_1}^0\int_{t+s}^t z^T(\theta)P_{12}z(\theta)d\theta ds\\
&\quad+\int_{-h_2}^0\int_{t+s}^t z^T(\theta)P_{13}z(\theta)d\theta ds\\
&\quad+\int_{-h_2}^{-h_1}\int_{t+s}^t z^T(\theta)P_{14}z(\theta)d\theta ds,\\
V_8(t) &= h_1\int_{-h_1}^0\int_{t+s}^t \begin{bmatrix} x(\theta)\\ z(\theta)\end{bmatrix}^T\\
&\quad\times\begin{bmatrix} R_{10}&R_{11}\\ *&R_{12}\end{bmatrix}\begin{bmatrix} x(\theta)\\ z(\theta)\end{bmatrix} d\theta ds\\
&\quad+h_2\int_{-h_2}^0\int_{t+s}^t \begin{bmatrix} x(\theta)\\ z(\theta)\end{bmatrix}^T\\
&\quad\times\begin{bmatrix} R_{13}&R_{14}\\ *&R_{15}\end{bmatrix}\begin{bmatrix} x(\theta)\\ z(\theta)\end{bmatrix} d\theta ds\\
&\quad+(h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^t \begin{bmatrix} x(\theta)\\ z(\theta)\end{bmatrix}^T\\
&\quad\times\begin{bmatrix} R_{16}&R_{17}\\ *&R_{18}\end{bmatrix}\begin{bmatrix} x(\theta)\\ z(\theta)\end{bmatrix} d\theta ds,\\
V_9(t) &= \frac{(h_1)^2}{2}\int_{t-h_1}^t\int_s^t\int_u^t x^T(\lambda)P_{15}x(\lambda)d\lambda du ds\\
&\quad+\frac{(h_2)^2}{2}\int_{t-h_2}^t\int_s^t\int_u^t x^T(\lambda)P_{16}x(\lambda)d\lambda du ds\\
&\quad+\frac{(h_2-h_1)^2}{2}\int_{t-h_2}^{t-h_1}\int_s^{t-h_1}\int_u^{t-h_1} x^T(\lambda)\\
&\quad\times P_{17}x(\lambda)d\lambda du ds,\\
V_{10}(t) &= \frac{(h_1)^2}{2}\int_{t-h_1}^t\int_s^t\int_u^t z^T(\lambda)P_{18}z(\lambda)d\lambda du ds\\
&\quad+\frac{(h_2)^2}{2}\int_{t-h_2}^t\int_s^t\int_u^t z^T(\lambda)P_{19}z(\lambda)d\lambda du ds\\
&\quad+\frac{(h_2-h_1)^2}{2}\int_{t-h_2}^{t-h_1}\int_s^{t-h_1}\int_u^{t-h_1} z^T(\lambda)\\
&\quad\times P_{20}z(\lambda)d\lambda du ds,\\
V_{11}(t) &= \frac{(h_1)^3}{6}\int_{t-h_1}^t\int_s^t\int_u^t\int_\lambda^t z^T(\theta)\\
&\quad\times P_{21}z(\theta)d\theta d\lambda du ds\\
&\quad+\frac{(h_2)^3}{6}\int_{t-h_2}^t\int_s^t\int_u^t\int_\lambda^t z^T(\theta)\\
&\quad\times P_{22}z(\theta)d\theta d\lambda du ds\\
&\quad+\frac{(h_2-h_1)^3}{6}\int_{t-h_2}^{t-h_1}\int_s^{t-h_1}\int_u^{t-h_1}\int_\lambda^{t-h_1}\\
&\quad\times z^T(\theta)P_{23}z(\theta)d\theta d\lambda du ds,\\
V_{12}(t) &= (r_2-r_1)\int_{t-r(t)}^t z^T(s)Q_5z(s)ds.
\end{aligned}$$

The time derivative of $V(t)$ along the trajectory of system (34)-(35) is given by

$$\dot{V}(t)=\sum_{i=1}^{12}\dot{V}_i(t). \quad (37)$$

The time derivatives of $V_1(t)$ and $V_2(t)$ are calculated as

$$\begin{aligned}
\dot{V}_1(t) &= 2x^T(t)P_1\dot{x}(t)\\
&= 2x^T(t)P_1\Big[Ax(t)+Bx(t-h(t))\\
&\quad+Cz(t-h_1(t))+f_1(t,x(t))\\
&\quad+f_2(t,x(t-h(t)))+f_3(t,z(t-r(t)))\Big]\\
&= 2x^T(t)P_1Ax(t)+2x^T(t)P_1Bx(t-h(t))\\
&\quad+2x^T(t)P_1Cz(t-r(t))\\
&\quad+2x^T(t)P_1f_1(t,x(t))\\
&\quad+2x^T(t)P_1f_2(t,x(t-h(t)))\\
&\quad+2x^T(t)P_1f_3(t,z(t-r(t))), \quad (38)\\
\dot{V}_2(t) &= \begin{bmatrix} x(t)\\ z(t)\\ x(t-h(t))\\ \int_{t-h(t)}^t z(s)ds \end{bmatrix}^T \begin{bmatrix} P_2^T&0&0&Q_1^T\\0&0&0&Q_2^T\\0&0&0&Q_3^T\\0&0&0&Q_4^T \end{bmatrix}\\
&\quad\times \begin{bmatrix} I^T&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix} \begin{bmatrix} \dot{x}(t)\\ \dot{z}(t)\\ \dot{x}(t-h(t))\\ \int_{t-h(t)}^t \dot{z}(s)ds \end{bmatrix}
\end{aligned}$$

$$
\begin{aligned}
= \;& 2x^T(t)P_2\Big[z(t)+Gx(t)-Gx(t-h(t)) \\
& -G\int_{t-h(t)}^{t} z(s)ds\Big] \\
& +2x^T(t)Q_1\Big[-z(t)+Ax(t)+Bx(t-h(t)) \\
& +C\dot{x}(t-r(t))+f_1(t,x(t)) \\
& +f_2(t,x(t-h(t)))+f_3(t,\dot{x}(t-r(t)))\Big] \\
& +2z^T(t)Q_2\Big[-z(t)+Ax(t)+Bx(t-h(t)) \\
& +C\dot{x}(t-r(t))+f_1(t,x(t)) \\
& +f_2(t,x(t-h(t)))+f_3(t,\dot{x}(t-r(t)))] \\
& +2x^T(t-h(t))Q_3\Big[-z(t)+Ax(t) \\
& +Bx(t-h(t))+C\dot{x}(t-r(t)) \\
& +f_1(t,x(t))+f_2(t,x(t-h(t))) \\
& +f_3(t,\dot{x}(t-r(t)))\Big] \\
& +2\int_{t-h(t)}^{t} z^T(s)dsQ_4\Big[x(t)-x(t-h(t)) \\
& -\int_{t-h(t)}^{t} z(s)ds\Big]. \qquad (39)
\end{aligned}
$$

Differentiating $V_3(t)$ and $V_4(t)$, we have

$$
\begin{aligned}
\dot{V}_3(t) &= x^T(t)P_3x(t)-x^T(t-h_1)P_3x(t-h_1) \\
&\quad +x^T(t)P_4x(t)-x^T(t-h_2)P_4x(t-h_2) \\
&\quad +x^T(t-h_1)P_5x(t-h_1) \\
&\quad -x^T(t-h_2)P_5x(t-h_2), \qquad (40) \\
\dot{V}_4(t) &= z^T(t)P_6z(t)-z^T(t-h_1)P_6z(t-h_1) \\
&\quad +z^T(t)P_7z(t)-z^T(t-h_2)P_7z(t-h_2) \\
&\quad +z^T(t-h_1)P_8z(t-h_1) \\
&\quad -z^T(t-h_2)P_8z(t-h_2). \qquad (41)
\end{aligned}
$$

Taking the time derivative of $V_5(t)$, we obtain

$$
\begin{aligned}
\dot{V}_5(t) &= \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \\
&\quad -\begin{bmatrix} x(t-h_1) \\ z(t-h_1) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \begin{bmatrix} x(t-h_1) \\ z(t-h_1) \end{bmatrix} \\
&\quad +\begin{bmatrix} x(t) \\ z(t) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \\
&\quad -\begin{bmatrix} x(t-h_2) \\ z(t-h_2) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \begin{bmatrix} x(t-h_2) \\ z(t-h_2) \end{bmatrix} \\
&\quad +\begin{bmatrix} x(t-h_1) \\ z(t-h_1) \end{bmatrix}^T \begin{bmatrix} R_7 & R_8 \\ R_8^T & R_9 \end{bmatrix} \begin{bmatrix} x(t-h_1) \\ z(t-h_1) \end{bmatrix} \\
&\quad -\begin{bmatrix} x(t-h_2) \\ z(t-h_2) \end{bmatrix}^T \begin{bmatrix} R_7 & R_8 \\ R_8^T & R_9 \end{bmatrix} \begin{bmatrix} x(t-h_2) \\ z(t-h_2) \end{bmatrix}. \qquad (42)
\end{aligned}
$$

Using Lemma 2.1 and Theorem 3.1, we calculate $V_6(t)$ as

$$
\begin{aligned}
\dot{V}_6(t) &= h_1^2x^T(t)P_9x(t)-h_1\int_{t-h_1}^{t} x^T(s)P_9x(s)ds \\
&\quad +h_2^2x^T(t)P_{10}x(t)-h_2\int_{t-h_2}^{t} x^T(s)P_{10}x(s)ds \\
&\quad +(h_2-h_1)^2x^T(t)P_{11}x(t) \\
&\quad -(h_2-h_1)\int_{t-h_2}^{t-h_1} x^T(s)P_{11}x(s)ds \\
&\le h_1^2x^T(t)P_9x(t)-\int_{t-h_1}^{t} x^T(s)ds \\
&\quad \times P_9\int_{t-h_1}^{t} x(s)ds+h_2^2x^T(t)P_{10}x(t) \\
&\quad -\Big[\int_{t-h_1}^{t} x^T(s)ds+\int_{t-h(t)}^{t-h_1} x^T(s)ds \\
&\quad +\int_{t-h_2}^{t-h(t)} x^T(s)ds\Big]P_{10}\Big[\int_{t-h_1}^{t} x(s)ds \\
&\quad +\int_{t-h(t)}^{t-h_1} x(s)ds+\int_{t-h_2}^{t-h(t)} x(s)ds\Big] \\
&\quad +(h_2-h_1)^2x^T(t)P_{11}x(t) \\
&\quad -\int_{t-h(t)}^{t-h_1} x^T(s)dsP_{11}\int_{t-h(t)}^{t-h_1} x(s)ds \\
&\quad -\int_{t-h_2}^{t-h(t)} x^T(s)dsP_{11}\int_{t-h_2}^{t-h(t)} x(s)ds.
\end{aligned}
$$

From Lemma 2.1 and Theorem 3.3, an upper bound of $V_7(t)$ can be obtained as

$$
\begin{aligned}
\dot{V}_7(t) &\le h_1^2z^T(t)P_{12}z(t)+h_2z^T(t)P_{13}z(t) \\
&\quad +(h_2-h_1)z^T(t)P_{14}z(t) \\
&\quad -\int_{t-h_1}^{t} \dot{x}^T(s)dsP_{12}\int_{t-h_1}^{t} \dot{x}(s)ds \\
&\quad +\begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}^T \begin{bmatrix} \Phi & \Gamma & 0 \\ \Psi & \Pi & \Gamma \\ 0 & \Psi & \Theta \end{bmatrix} \\
&\quad \times \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix} + h_2 \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} M_3 & M_4 & 0 \\ M_4^T & M_3+M_5 & M_4 \\ 0 & M_4^T & M_5 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix} \\
&\quad +\begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}^T \begin{bmatrix} \Phi' & \Gamma' & 0 \\ \Psi' & \Pi' & \Gamma' \\ 0 & \Psi' & \Theta' \end{bmatrix} \\
&\quad \times \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix} \\
&\quad +(h_2-h_1) \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} N_3 & N_4 & 0 \\ N_4^T & N_3+N_5 & N_4 \\ 0 & N_4^T & N_5 \end{bmatrix} \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}, \qquad (43)
\end{aligned}
$$

where $\Phi=M_1+M_1^T$, $\Psi=-M_1+M_2^T$, $\Gamma=-M_1^T+M_2$, $\Pi=M_1+M_1^T-M_2-M_2^T$, $\Theta=-M_2-M_2^T$, $\Phi'=N_1+N_1^T$, $\Psi'=-N_1+N_2^T$, $\Gamma'=-N_1^T+N_2$, $\Pi'=N_1+N_1^T-N_2-N_2^T$, $\Theta'=-N_2-N_2^T$. From the Theorem 3.2, we have

$$\begin{aligned}\dot{V}_8(t) \leq\ & h_1^2\begin{bmatrix}x(t)\\z(t)\end{bmatrix}^T\begin{bmatrix}R_{10} & R_{11}\\R_{11}^T & R_{12}\end{bmatrix}\begin{bmatrix}x(t)\\z(t)\end{bmatrix} \\ & +h_2^2\begin{bmatrix}x(t)\\z(t)\end{bmatrix}^T\begin{bmatrix}R_{13} & R_{14}\\R_{14}^T & R_{15}\end{bmatrix}\begin{bmatrix}x(t)\\z(t)\end{bmatrix} \\ & +(h_2-h_1)^2\begin{bmatrix}x(t)\\z(t)\end{bmatrix}^T\begin{bmatrix}R_{16} & R_{17}\\R_{17}^T & R_{18}\end{bmatrix}\begin{bmatrix}x(t)\\z(t)\end{bmatrix} \\ & -\begin{bmatrix}\int_{t-h_1}^t x(s)ds\\x(t)-x(t-h_1)\end{bmatrix}^T\begin{bmatrix}R_{10} & R_{11}\\R_{11}^T & R_{12}\end{bmatrix} \\ & \times\begin{bmatrix}\int_{t-h_1}^t x(s)ds\\x(t)-x(t-h_1)\end{bmatrix}+\begin{bmatrix}x(t)\\x(t-h(t))\\x(t-h_2)\\\int_{t-h(t)}^t x(s)ds\\\int_{t-h_2}^{t-h(t)} x(s)ds\end{bmatrix}^T \\ & \times\begin{bmatrix}-R_{15} & R_{15} & 0 & -R_{14}^T & 0\\R_{15}^T & R'_{15} & R_{15} & R_{14}^T & -R_{14}^T\\0 & R_{15}^T & -R_{15} & 0 & R_{14}^T\\-R_{14} & R_{14} & 0 & -R_{13} & 0\\0 & -R_{14} & R_{14} & 0 & -R_{13}\end{bmatrix} \\ & \times\begin{bmatrix}x(t)\\x(t-h(t))\\x(t-h_2)\\\int_{t-h(t)}^t x(s)ds\\\int_{t-h_2}^{t-h(t)} x(s)ds\end{bmatrix}+\begin{bmatrix}x(t-h_1)\\x(t-h(t))\\x(t-h_2)\\\int_{t-h(t)}^{t-h_1} x(s)ds\\\int_{t-h_2}^{t-h(t)} x(s)ds\end{bmatrix}^T \\ & \times\begin{bmatrix}-R_{18} & R_{18} & 0 & -R_{17}^T & 0\\R_{18}^T & R'_{18} & R_{18} & R_{17}^T & -R_{17}^T\\0 & R_{18}^T & -R_{18} & 0 & R_{17}^T\\-R_{17} & R_{17} & 0 & -R_{16} & 0\\0 & -R_{17} & R_{17} & 0 & -R_{16}\end{bmatrix} \\ & \times\begin{bmatrix}x(t-h_1)\\x(t-h(t))\\x(t-h_2)\\\int_{t-h(t)}^{t-h_1} x(s)ds\\\int_{t-h_2}^{t-h(t)} x(s)ds\end{bmatrix}, \qquad (44)\end{aligned}$$

where $R'_{15}=-R_{15}-R_{15}^T$, $R'_{18}=-R_{18}-R_{18}^T$. By Lemma 2.7, we obtain $\dot{V}_9(t)$ and $\dot{V}_{10}(t)$ as follows

$$\begin{aligned}\dot{V}_9(t) \leq\ & \frac{h_1^4}{4}x^T(t)P_{15}x(t)-\int_{t-h_1}^t\int_u^t x^T(\lambda)d\lambda du \\ & \times P_{15}\int_{t-h_1}^t\int_u^t x(\lambda)d\lambda du+\frac{h_2^4}{4}x^T(t)P_{16}x(t) \\ & -\int_{t-h_2}^t\int_u^t x^T(\lambda)d\lambda du \\ & \times P_{16}\int_{t-h_2}^t\int_u^t x(\lambda)d\lambda du \\ & +\frac{(h_2-h_1)^4}{4}x^T(t-h_1)P_{17}x(t-h_1) \\ & -\int_{t-h_2}^{t-h_1}\int_u^t x^T(\lambda)d\lambda du \\ & \times P_{17}\int_{t-h_2}^{t-h_1}\int_u^t x(\lambda)d\lambda du, \qquad (45)\end{aligned}$$

$$\begin{aligned}\dot{V}_{10}(t) \leq\ & \frac{h_1^4}{4}z^T(t)P_{18}z(t)-\int_{t-h_1}^t\int_u^t z^T(\lambda)d\lambda du \\ & \times P_{18}\int_{t-h_1}^t\int_u^t z(\lambda)d\lambda du \\ & +\frac{h_2^4}{4}z^T(t)P_{19}z(t)-\int_{t-h_2}^t\int_u^t z^T(\lambda)d\lambda du \\ & \times P_{19}\int_{t-h_2}^t\int_u^t z(\lambda)d\lambda du \\ & +\frac{(h_2-h_1)^4}{4}z^T(t-h_1)P_{20}z(t-h_1) \\ & -\int_{t-h_2}^{t-h_1}\int_u^t z^T(\lambda)d\lambda du \\ & \times P_{20}\int_{t-h_2}^{t-h_1}\int_u^t z(\lambda)d\lambda du \\ =\ & \frac{h_1^4}{4}z^T(t)P_{18}z(t) \\ & -\Big[h_1x^T(t)-\int_{t-h_1}^t x^T(u)du\Big] \\ & \times P_{18}\Big[h_1x(t)-\int_{t-h_1}^t x(u)du\Big] \\ & +\frac{h_2^4}{4}z^T(t)P_{19}z(t) \\ & -\Big[h_2x^T(t)-\int_{t-h_2}^t x^T(u)du\Big] \\ & \times P_{19}\Big[h_2x(t)-\int_{t-h_2}^t x(u)du\Big] \\ & +\frac{(h_2-h_1)^4}{4}z^T(t-h_1)P_{20}z(t-h_1) \\ & -\Big[(h_2-h_1)x^T(t-h_1)-\int_{t-h_2}^{t-h_1} x^T(u)du\Big] \\ & \times P_{20}\Big[(h_2-h_1)x(t-h_1)-\int_{t-h_2}^{t-h_1} x(u)du\Big]. \qquad (46)\end{aligned}$$

By Lemma 2.8 and calculating $\dot{V}_{11}(t)$, we have

$$\begin{aligned}\dot{V}_{11}(t) \leq\ & \frac{h_1^6}{36}z^T(t)P_{21}z(t)+\frac{h_2^6}{36}z^T(t)P_{22}z(t) \\ & +\frac{(h_2-h_1)^6}{36}z^T(t-h_1)P_{23}z(t-h_1) \\ & -\int_{t-h_1}^t\int_u^t\int_\lambda^t z^T(\theta)d\theta d\lambda du \\ & \times P_{21}\int_{t-h_1}^t\int_u^t\int_\lambda^t z(\theta)d\theta d\lambda du \\ & -\int_{t-h_2}^t\int_u^t\int_\lambda^t z^T(\theta)d\theta d\lambda du \\ & \times P_{22}\int_{t-h_2}^t\int_u^t\int_\lambda^t z(\theta)d\theta d\lambda du\end{aligned}$$

$$
\begin{aligned}
&-\int_{t-h_2}^{t-h_1}\int_{u}^{t-h_1}\int_{\lambda}^{t-h_1} z^T(\theta)d\theta d\lambda du \\
&\times P_{23}\int_{t-h_2}^{t-h_1}\int_{u}^{t-h_1}\int_{\lambda}^{t-h_1} z(\theta)d\theta d\lambda du \\
= &\frac{h_1^6}{36}z^T(t)P_{21}z(t)+\frac{h_2^6}{36}z^T(t)P_{22}z(t) \\
&+\frac{(h_2-h_1)^6}{36}z^T(t-h_1)P_{23}z(t-h_1) \\
&-\Big[\frac{h_1^2}{2}x^T(t)-\int_{t-h_1}^{t}\int_{u}^{t}x^T(\lambda)d\lambda du\Big] \\
&\times P_{21}\Big[\frac{h_1^2}{2}x(t)-\int_{t-h_1}^{t}\int_{u}^{t}x(\lambda)d\lambda du\Big] \\
&-\Big[\frac{h_2^2}{2}x^T(t)-\int_{t-h_2}^{t}\int_{u}^{t}x^T(\lambda)d\lambda du\Big] \\
&\times P_{22}\Big[\frac{h_2^2}{2}x(t)-\int_{t-h_2}^{t}\int_{u}^{t}x(\lambda)d\lambda du\Big] \\
&-\Big[\frac{(h_2-h_1)^2}{2}x^T(t-h_1) \\
&-\int_{t-h_2}^{t-h_1}\int_{u}^{t-h_1}x^T(\lambda)d\lambda du\Big] \\
&\times P_{23}\Big[\frac{(h_2-h_1)^2}{2}x(t-h_1) \\
&-\int_{t-h_2}^{t-h_1}\int_{u}^{t-h_1}x(\lambda)d\lambda du\Big]. \qquad (47)
\end{aligned}
$$

Calculating $\dot{V}_{12}(t)$ leads to

$$
\begin{aligned}
\dot{V}_{12}(t) &= (r_2-r_1)\dot{x}^T(t)Q_5\dot{x}(t) \\
&\quad -(r_2-r_1)(1-\dot{r}(t))\dot{x}^T(t-r(t)) \\
&\quad \times Q_5\dot{x}(t-r(t)) \\
&\le (r_2-r_1)\dot{x}^T(t)Q_5\dot{x}(t) \\
&\quad -(r_2-r_1)(1-r_d)\dot{x}^T(t-r(t)) \\
&\quad \times Q_5\dot{x}(t-r(t)). \qquad (48)
\end{aligned}
$$

From the Leibniz-Newton formula, the following equations are true for any real constant matrices C_i, $i=1,2,...,6$ with appropriate dimensions

$$
\begin{aligned}
&2h_1\Big[x^T(t)C_1+x^T(t-h_1)C_2+\int_{t-h_1}^{t}z^T(s)dsC_3\Big] \\
&\times\Big[x(t)-x(t-h_1)-\int_{t-h_1}^{t}z(s)ds\Big]=0, \qquad (49)
\end{aligned}
$$

$$
\begin{aligned}
&2h_2\Big[x^T(t)C_4+x^T(t-h(t))C_5+\int_{t-h(t)}^{t}z^T(s)dsC_6\Big] \\
&\times\Big[x(t)-x(t-h(t))-\int_{t-h(t)}^{t}z(s)ds\Big]=0. \qquad (50)
\end{aligned}
$$

From (4), (5) and (6), we obtain for any positive real constants $\epsilon_1,\epsilon_2,\epsilon_3$,

$$
\epsilon_1\Big(\alpha^2x^T(t)x(t)-f_1^T(t,x(t))f_1(t,x(t))\Big)\ge 0, \qquad (51)
$$

$$
\begin{aligned}
&\epsilon_2\Big(\beta^2x^T(t-h(t))x(t-h(t)) \\
&-f_2^T(t,x(t-h(t)))f_2(t,x(t-h(t)))\Big)\ge 0, \qquad (52)
\end{aligned}
$$

$$
\begin{aligned}
&\epsilon_3\Big(\gamma^2\dot{x}^T(t-r(t))\dot{x}(t-r(t)) \\
&-f_3^T(t,\dot{x}(t-r(t)))f_3(t,\dot{x}(t-r(t)))\Big)\ge 0. \qquad (53)
\end{aligned}
$$

According to (37)-(53), it is straightforward to see that

$$
\dot{V}(t)\le\zeta^T(t)\sum\zeta(t), \qquad (54)
$$

where $\zeta^T(t)=\Big[x^T(t),z^T(t),x^T(t-h_1),z^T(t-h_1),$ $x^T(t-h_2),z^T(t-h_2),x^T(t-h(t)),\int_{t-h_1}^{t}x^T(s)ds,$ $\int_{t-h_1}^{t}x^T(u)du,\int_{t-h(t)}^{t-h_1}x^T(s)ds,\int_{t-h(t)}^{t}x^T(s)ds,$ $\int_{t-h_2}^{t-h(t)}x^T(s)ds,\int_{t-h_2}^{t}x^T(u)du,\int_{t-h_2}^{t-h_1}x^T(u)du,$ $\int_{t-h(t)}^{t}z^T(s)ds,\int_{t-h_1}^{t}\int_u^t x^T(\lambda)d\lambda du,\int_{t-h_2}^{t}\int_u^t x^T(\lambda)d\lambda du,$ $\int_{t-h_2}^{t-h_1}\int_u^{t-h_1}x^T(\lambda)d\lambda du,\int_{t-h_1}^{t}z^T(s)ds,z(t-r(t)),$ $f_1(t,x(t)),f_2(t,x(t-h(t))),f_3(t,z(t-r(t)))\Big].$

If the conditions (22)-(30) hold, then (54) implies that there exists $\delta>0$ such that $\dot{V}(t)\le-\delta\|x(t)\|^2$. Therefore, system (1) is asymptotically stable. The proof of theorem is complete.

If C=$f_1(t,x(t))$=$f_2(t,x(t-h(t)))$=$f_3(t,\dot{x}(t-r(t)))$=0, then system (1) reduce to the following system

$$
\begin{cases}
\dot{x}(t)=Ax(t)+Bx(t-h(t)), \quad t>0; \\
x(t_0+\theta)=\phi(\theta), \quad \dot{x}(t_0+\theta)=\varphi(\theta), \\
\quad\theta\in[-h_2,0].
\end{cases} \qquad (55)
$$

Take the Lyapunov-Krasovskii functional as

$$
V(t)=\sum_{i=1}^{11}V_i(t), \qquad (56)
$$

where $V_1(t)$ to $V_{11}(t)$ are defined in Theorem 4.1. According to Theorem 4.1, we can obtain delay-range-dependent stability criteria of system (55). We introduce the following notations for later use.

$$
\widetilde{\sum}=\big[\tilde{\Sigma}_{i,j}\big]_{19\times19}, \qquad (57)
$$

where $\tilde{\Sigma}_{i,j}=\tilde{\Sigma}_{j,i}^T=\Sigma_{i,j}$, $i,j=1,2,3,...,19$, except

$$
\begin{aligned}
\tilde{\Sigma}_{1,1} &= P_1A+A^TP_1+W+W^T+Q_1A+A^TQ_1 \\
&\quad +P_3+P_4+R_1+R_4+(h_1)^2P_9+(h_2)^2P_{10} \\
&\quad +(h_2-h_1)^2P_{11}-P_{12}+M_1+M_1^T+h_2M_3 \\
&\quad +(h_1)^2R_{10}+(h_2)^2R_{13}+(h_2-h_1)^2R_{16} \\
&\quad +R_{12}-R_{15}+\frac{(h_1)^4}{4}P_{15}+\frac{(h_2)^4}{4}P_{16} \\
&\quad -(h_1)^2P_{18}-(h_2)^2P_{19}-\frac{(h_1)^4}{4}P_{21}-\frac{(h_2)^4}{4}P_{22} \\
&\quad +h_1C_1+h_1C_1^T+h_2C_4+h_2C_4^T, \\
\tilde{\Sigma}_{2,2} &= -2Q_2+P_6+P_7+R_3+R_6+(h_1)^2P_{12} \\
&\quad +h_2P_{13}+(h_2-h_1)P_{14}+(h_1)^2R_{12}+(h_2)^2R_{15} \\
&\quad +(h_2-h_1)^2R_{18}+\frac{(h_1)^4}{4}P_{18}+\frac{(h_2)^4}{4}P_{19} \\
&\quad +\frac{(h_1)^6}{36}P_{21}+\frac{(h_2)^6}{36}P_{22}, \\
\tilde{\Sigma}_{7,7} &= Q_3B+B^TQ_3+M_1+M_1^T-M_2-M_2^T \\
&\quad +h_2(M_3+M_5)+N_1+N_1^T-N_2-N_2^T \\
&\quad +(h_2-h_1)(N_3+N_5)-R_{15}-R_{15}^T-R_{18} \\
&\quad -R_{18}^T-h_2C_5-h_2C_5^T.
\end{aligned}
$$

Corollary 4.2: The system (55) is asymptotically stable, if there exist positive definite symmetric matrices R_{13}, R_{15},

R_{16}, R_{18}, P_i, $i = 1, 2, ..., 23$, any appropriate dimensional matrices G, Q_k, M_j, N_j, R_l, $k = 1, 2, ..., 4, j = 1, 2, .., 5, l = 1, 2, ..., 18$, satisfying the following LMIs:

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0, \tag{58}$$

$$\begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} > 0, \tag{59}$$

$$\begin{bmatrix} R_7 & R_8 \\ * & R_9 \end{bmatrix} > 0, \tag{60}$$

$$\begin{bmatrix} R_{10} & R_{11} \\ * & R_{12} \end{bmatrix} > 0, \tag{61}$$

$$\begin{bmatrix} R_{13} & R_{14} \\ * & R_{15} \end{bmatrix} > 0, \tag{62}$$

$$\begin{bmatrix} R_{16} & R_{17} \\ * & R_{18} \end{bmatrix} > 0, \tag{63}$$

$$\begin{bmatrix} P_{13} & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \geq 0, \tag{64}$$

$$\begin{bmatrix} P_{14} & N_1 & N_2 \\ * & N_3 & N_4 \\ * & * & N_5 \end{bmatrix} \geq 0, \tag{65}$$

$$\widetilde{\sum} < 0. \tag{66}$$

If C=$f_3(t, \dot{x}(t - r(t)))$=0, then system (1) reduce to the following system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t - h(t)) + f_1(t, x(t)) \\ \quad +f_2(t, x(t - h(t))), \quad t > 0; \\ x(t_0 + \theta) = \phi(\theta), \quad \dot{x}(t_0 + \theta) = \varphi(\theta), \\ \quad \theta \in [-\bar{h}, 0]. \end{cases} \tag{67}$$

Take the Lyapunov-Krasovskii functional in (56). According to Theorem 4.1, we can obtain delay-range-dependent stability criteria of system (67). We introduce the following notations for later use.

$$\widehat{\sum} = [\hat{\Sigma}_{i,j}]_{21 \times 21}, \tag{68}$$

where $\quad \hat{\Sigma}_{i,j} = \hat{\Sigma}^T_{j,i} = \Sigma_{i,j}, \quad i, j = 1, 2, 3, ..., 21,$ except

$$\begin{aligned} \hat{\Sigma}_{1,20} &= \hat{\Sigma}_{1,21} = P_1 + Q_1, \quad \hat{\Sigma}_{2,20} = \hat{\Sigma}_{2,21} = Q_2, \\ \hat{\Sigma}_{7,20} &= \hat{\Sigma}_{7,21} = Q_3, \quad \hat{\Sigma}_{20,20} = -\epsilon_1 I, \\ \hat{\Sigma}_{20,21} &= 0, \quad \hat{\Sigma}_{21,21} = -\epsilon_2 I. \end{aligned}$$

Corollary 4.3: The system (67) is asymptotically stable, if there exist positive definite symmetric matrices R_{13}, R_{15}, R_{16}, R_{18}, P_i, $i = 1, 2, ..., 23$, any appropriate dimensional matrices G, Q_k, M_j, N_j, R_l, $k = 1, 2, ..., 4, j = 1, 2, .., 5, l = 1, 2, ..., 18$ and positive real constants ϵ_1 and ϵ_2 satisfying the following LMIs:

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0, \tag{69}$$

$$\begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} > 0, \tag{70}$$

$$\begin{bmatrix} R_7 & R_8 \\ * & R_9 \end{bmatrix} > 0, \tag{71}$$

$$\begin{bmatrix} R_{10} & R_{11} \\ * & R_{12} \end{bmatrix} > 0, \tag{72}$$

$$\begin{bmatrix} R_{13} & R_{14} \\ * & R_{15} \end{bmatrix} > 0, \tag{73}$$

$$\begin{bmatrix} R_{16} & R_{17} \\ * & R_{18} \end{bmatrix} > 0, \tag{74}$$

$$\begin{bmatrix} P_{13} & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \geq 0, \tag{75}$$

$$\begin{bmatrix} P_{14} & N_1 & N_2 \\ * & N_3 & N_4 \\ * & * & N_5 \end{bmatrix} \geq 0, \tag{76}$$

$$\widehat{\sum} < 0. \tag{77}$$

B. Delay-range-dependent robust stability criteria

According to Theorem 4.1, we can obtain delay-range-dependent robust asymptotic stability criteria of system (10). We introduce the following notations for later use.

$$\overline{\sum} = [\bar{\Sigma}_{i,j}]_{20 \times 20}, \tag{78}$$

where $\quad \bar{\Sigma}_{i,j} = \bar{\Sigma}^T_{j,i} = \Sigma_{i,j}, \quad i, j = 1, 2, 3, ..., 20,$ except

$$\begin{aligned} \bar{\Sigma}_{1,1} &= P_1A + A^TP_1 + W + W^T + Q_1A + A^TQ_1 \\ &\quad +P_3 + P_4 + R_1 + R_4 + (h_1)^2P_9 + (h_2)^2P_{10} \\ &\quad +(h_2 - h_1)^2P_{11} - P_{12} + M_1 + M_1^T \\ &\quad +h_2M_3 + (h_1)^2R_{10} + (h_2)^2R_{13} + (h_2 - h_1)^2 \\ &\quad \times R_{16} + R_{12} - R_{15} + \frac{(h_1)^4}{4}P_{15} + \frac{(h_2)^4}{4}P_{16} \\ &\quad -(h_1)^2P_{18} - (h_2)^2P_{19} - \frac{(h_1)^4}{4}P_{21} - \frac{(h_2)^4}{4} \\ &\quad \times P_{22} + h_1C_1 + h_1C_1^T + h_2C_4 + h_2C_4^T, \\ \bar{\Sigma}_{7,7} &= Q_3B + B^TQ_3 + M_1 + M_1^T - M_2 - M_2^T \\ &\quad +h_2(M_3 + M_5) + N_1 + N_1^T - N_2 - N_2^T \\ &\quad +(h_2 - h_1)(N_3 + N_5) - R_{15} - R_{15}^T - R_{18} \\ &\quad -R_{18}^T - h_2C_5 - h_2C_5^T, \\ \bar{\Sigma}_{20,20} &= -(r_2 - r_1)(1 - r_d)Q_5, \\ S^T &= [E^T(P_1 + Q_1) \quad E^TQ_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad E^TQ_3 \\ &\quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \\ N &= [G_1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad G_2 \quad 0 \quad 0 \quad 0 \quad 0 \\ &\quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad G_3]. \end{aligned}$$

Theorem 4.4: The system (10) is robustly asymptotically stable, if there exist positive definite symmetric matrices Q_5, R_{13}, R_{15}, R_{16}, R_{18}, P_i, $i = 1, 2, ..., 23$, any appropriate dimensional matrices G, Q_k, M_j, N_j, R_l, $k = 1, ..., 4, j = 1, 2, .., 5, l = 1, 2, ..., 18$ and positive constant δ satisfying the following LMIs:

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0, \tag{79}$$

$$\begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} > 0, \tag{80}$$

$$\begin{bmatrix} R_7 & R_8 \\ * & R_9 \end{bmatrix} > 0, \tag{81}$$

$$\begin{bmatrix} R_{10} & R_{11} \\ * & R_{12} \end{bmatrix} > 0, \tag{82}$$

$$\begin{bmatrix} R_{13} & R_{14} \\ * & R_{15} \end{bmatrix} > 0, \tag{83}$$

$$\begin{bmatrix} R_{16} & R_{17} \\ * & R_{18} \end{bmatrix} > 0, \tag{84}$$

$$\begin{bmatrix} P_{13} & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \geq 0, \tag{85}$$

$$\begin{bmatrix} P_{14} & N_1 & N_2 \\ * & N_3 & N_4 \\ * & * & N_5 \end{bmatrix} \geq 0, \tag{86}$$

$$\begin{bmatrix} \overline{\sum} & S & \delta N^T \\ * & -\delta I & \delta J^T \\ * & * & -\delta I \end{bmatrix} < 0. \tag{87}$$

Proof. Replacing A, B and C in (30) with $A = A + E\Delta(t)G_1$, $B = B + E\Delta(t)G_2$ and $C = C + E\Delta(t)G_3$, respectively, we find that condition (30) is equivalent to the following condition

$$\overline{\sum} + S\Delta(t)N + N^T\Delta(t)^T S^T < 0, \tag{88}$$

where $\overline{\sum}$ are define in (78). By using Lemma (2.4), we can find that (88) is equivalent to the LMIs as follows

$$\begin{bmatrix} \overline{\sum} & S & \delta N^T \\ * & -\delta I & \delta J^T \\ * & * & -\delta I \end{bmatrix} < 0, \tag{89}$$

where δ is positive real constant. From Theorem (4.1) and conditions (79)-(87), system (10) is robustly asymptotically stable. The proof of theorem is complete.

V. Numerical examples

Example 5.1 Consider the system (55) with the following system matrices, which is considered in [10], [15], [37], [50] :

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}. \tag{90}$$

By using the LMI Toolbox in MATLAB (with accuracy 0.01) for application of Corollary 4.2 to system (55) with (90), the maximum upper bounds h_2 for asymptotic stability of Example 5.1 is listed in the comparison in Table I, for different values of h_1. We can see that our results in Corollary 4.2 are much less conservative than those obtained in [10], [15], [37], [50].

Example 5.2 Consider the system (67) with the following system matrices, which is considered in [7], [35], [46] :

$$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}. \tag{91}$$

TABLE I
UPPER BOUNDS OF TIME DELAY h_2 FOR DIFFERENT CONDITIONS FOR EXAMPLE 5.1.

Method	h_1	1	2	3	4
Sun et al. (2010) [37]	h_2	1.6198	2.4884	3.3403	4.3424
Zhu et al. (2010) [50](m=4)	h_2	1.7228	2.5608	3.4542	4.3787
Liu et al. (2012) [15]	h_2	1.7753	2.6134	3.5046	4.4271
Kwon et al. (2013) [10]	h_2	1.8446	2.6344	3.5124	4.4304
Corollary 4.2	h_2	1.7990	2.7970	3.7920	4.7603

By using the LMI Toolbox in MATLAB (with accuracy 0.01) for Corollary 4.3 to system (67) with (91), we can compare the maximum allowable bound h_2 for guaranteeing asymptotic stability of the system in Table II. This example shows that the stability criterion in this paper gives much less conservative results than those obtained in [7], [35], [46].

TABLE II
UPPER BOUNDS OF TIME DELAY h_2 FOR DIFFERENT CONDITIONS FOR EXAMPLE 5.2.

Method	h_1	0.5	1
$\alpha = 0, \beta = 0.1$			
Zhang et al. (2011) [46]	h_2	1.442	1.543
Ramakrishnan et al. (2011) [35]	h_2	1.558	1.760
Hui et al. (2015) [7]	h_2	1.824	1.9930
Corollary 4.3	h_2	2.2240	2.3210
$\alpha = 0.1, \beta = 0.1$			
Zhang et al. (2011) [46]	h_2	1.284	1.408
Ramakrishnan et al. (2011) [35]	h_2	1.384	1.532
Hui et al. (2015) [7]	h_2	1.524	1.6380
Corollary 4.3	h_2	2.2240	2.3210

Example 5.3 Consider the system (1) with the following system matrices, which is considered in [5], [20] and [26]:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad \alpha = 0.1, \quad \beta = \gamma = 0.05.$$

By using the LMI Toolbox in MATLAB (with accuracy 0.01) for Theorem 4.1 to system (1) with above parameters, one can obtain the maximum upper bounds of the time delay h_2 under different values of r_d and h_1 as shown in Table III. Our results in Theorem 4.1 are much less conservative than those presented in [5], [20] and [26].

TABLE III
UPPER BOUNDS OF TIME DELAY h_2 FOR DIFFERENT CONDITIONS FOR EXAMPLE 5.3.

Method	h_1	0.5	1
$r(t) = r_d = 0$			
Lakshmanan et al. (2011) [20]	h_2	4.7392	5.0992
Cheng et al. (2013) [5]	h_2	5.2164	5.9862
Mohajerpoor et al. (2016) [26]	h_2	6.3114	7.0600
Theorem 4.1	h_2	12.1065	12.0056
Method	h_1	1	10
$r_d = 0.6$			
Lakshmanan et al. (2011) [20]	h_2	4.6391	13.2500
Cheng et al. (2013) [5]	h_2	5.0052	14.3261
Mohajerpoor et al. (2016) [26]	h_2	6.5010	16.1001
Theorem 4.1	h_2	11.1260	17.0790

Example 5.4 Consider the system (10) with the following

system matrices, which is considered in [13], [44] and [48]:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \quad \Delta A(t) = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix},$$
$$\Delta B(t) = \begin{bmatrix} \gamma_3 & 0 \\ 0 & \gamma_4 \end{bmatrix}, \quad \Delta C(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

where $0 \leq |c| \leq 1$, and $\gamma_i = 1, 2, ..., 4$ are unknown parameter satisfying $|\gamma_1| \leq 1.6, |\gamma_2| \leq 0.05, |\gamma_3| < 0.1$, and $|\gamma_4| < 0.3$. By using the LMI Toolbox in MATLAB (with accuracy 0.01) for Theorem 4.4 to system (10) with above matrices, the maximum upper bounds h_2 for robust stability of Example 5.4 is listed in the comparison in Table IV, for $h_1 = 0.5$. Our results are better than those given in [13], [44] and [48].

TABLE IV
UPPER BOUNDS OF TIME DELAY h_2 FOR DIFFERENT CONDITIONS FOR EXAMPLE 5.4.

Method		h_1=0.5
Yu and Lien (2008) [48]	h_2	0.793
Kwon et al. (2008) [13]	h_2	0.894
Weera and Niamsup (2011) [44]	h_2	0.951
Theorem 4.4	h_2	1.1996

VI. CONCLUSION

The problem of robust stability for uncertain neutral systems with mixed interval time-varying delays was studied. The restriction on the derivative of the discrete interval time-varying delay is removed. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties, respectively. By constructing a suitable augmented Lyapunov-Krasovskii functional, mixed model transformation, new improved integral inequalities, Leibniz-Newton formula and utilization of zero equation, new delay-range-dependent robust stability criteria are derived in terms of LMIs for the neutral systems. Moreover, we present new delay-range-dependent stability criteria for linear system with non-differentiable interval time-varying delay and nonlinear perturbations. Numerical examples have shown significant improvements over some existing results.

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