Second Law Analysis of Hydromagnetic Couple Stress Fluid Embedded in a Non-Darcian Porous Medium

Abiodun A. Opanuga, *Member, IAENG,* Jacob A. Gbadeyan, and Samuel A. Iyase

Abstract—**This study considers second law analysis of hydromagnetic couple stress fluid through a channel filled with non-Darcian porous medium. We assumed that the fluid exchanges heat with the ambient following Newtonian law. The governing non-linear equations of the fluid flow were formulated and non-dimensionalised. A semi-analytical solution of the dimensionless momentum and energy equations are obtained using Adomian decomposition method (ADM) and differential transform method (DTM). The approximate solutions are utilized to compute the entropy generation and the Bejan numbers. The effects of pertinent flow parameters on the velocity, temperature, entropy generation and the Bejan numbers are presented and discussed graphically.**

Index Terms—**Non-Darcian porous medium; Hydromagnetic; Couple stress fluid; Adomian Decomposition method (ADM); Differential Transform method (DTM); Entropy generation.**

I. Introduction

In recent times, the study of electrically conducting fluid has been the focus of many authors due to its several applications in the industry like geothermal reservoirs, nuclear reactor, marine propulsion, electronic packaging, microelectronic device operations, textile industry, polymer technology, metallurgy, crude oil purification and the cooling of reactors. For instance, Alam et al. [1] discussed viscous heating effect on magnetohydrodynamic heat and mass transfer; and concluded that magnetic field parameter retards local Nusselt number. Turkyilmazohlu [2] presented the effects of thermal radiation on the time-dependent MHD permeable flow with varying viscosity and submitted that magnetic field increases fluid temperature but retards the skin friction. In [3], Adesanya et al. investigated the entropy generation of MHD third grade fluid through porous medium and discovered that magnetic field parameter decreases fluid velocity but increases the temperature. Other interesting studies on MHD flow are in [4-9].

Fluid flows through non-Darcian porous medium abound in many real life scenarios such as geophysical and petrochemical flows. The application of porous media to improve convection heat transfer has been studied by numerous authors like Chauhan et al [10] who investigated slip conditions effects on both forced convection and entropy generation in a circular channel through a highly porous medium. It was submitted that porosity parameter increases the entropy generation but reduces the Bejan number. In a related study, Jha et al. [11] investigated mixed convection in a vertical cylinder filled with porous material. Salah et al. [12] considered MHD flows of second grade fluid in a porous medium and rotating frame. Gbadeyan et al. [13] carried out an investigation on the irreversibility analysis resulting from the effects of partial slippage and couple stresses in a channel filled with highly porous medium. Other contributions on porous medium are [14-15].

However, in most engineering processes, efficient energy utilization has been a great concern due to the continuous exchange of heat between the fluid and its solid boundaries which usually results into great disorderliness, giving rise to increase in entropy generation during convection. Since increase in entropy production measures the destruction of exergy in a system, it is therefore, pertinent to have a critical study of the factors that account for this irreversibility in order to reduce the wastefulness in thermal systems.

After the pioneering work of Bejan [16, 17, 18], numerous researchers have discussed entropy generation under various physical situations. Adesanya et al. [19] presented entropy generation analysis of couple stress fluid in a porous channel using the convective heating boundary conditions. Ajibade et al. [20] studied entropy generation under the effect of suction/injection. Das et al. [21] presented the irreversibility analysis of electrically conducting viscous flow in a porous channel with slip boundary conditions and submitted that increasing value of magnetic field parameter increases loss of useful energy while it decreases heat transfer rate at the lower plate. For other studies on entropy generation see Refs. [22-32].

In all these previous studies, attention has hardly been focused to study the effects of hydromagnetic couple stress fluid on entropy generation rate in a steady flow through a channel occupied by a non-Darcian medium. Such a study is useful and important basically for (i) gaining fundamental understanding of such flows; (ii) the need to ensure entropy minimization in a hydromagnetic couple stress fluid flow; and (iii) possible application of such non-Newtonian fluids in petroleum production, power engineering, movement of biological fluids and food and construction engineering.

The primary motivation for this paper is derived from the above issues which is very important yet unaddressed in the previous papers on the subject [19, 23]. The objective of this study is, therefore, to investigate entropy generation of a hydromagnetic couple stress fluid flow through a channel filled with a non- Darcian medium. To this end, two semi-analytical methods namely: Adomian decomposition method

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(ADM) and the differential transform method (DTM) are used to obtain the solution of the couple fourth order nonlinear set of differential equations. Although various numerical techniques [33-36] are available in literature, the two numerical methods (ADM and DTM) are found to converge rapidly to the exact solution, see Refs.[37,38] and Table 1.

The organization of the rest of this work is as follows; section two presents problem formulation and non-dimensionalization, section three contains the solution to the boundary value problems via ADM. In section four, results are graphically discussed, while in section five the concluding remarks are presented.

II. Model Formulation

Consider an electrically conducting couple stress fluid flowing steadily through porous medium. Hot fluid is injected at plate $y = 0$ and sucked off at the upper plate $y = h$ with the same velocity. A uniformly transverse magnetic field is applied in the direction of flow, and the interaction of the induced magnetic field is assumed to be negligible when compared with the interaction of the applied magnetic field. There is an axi-symmetrical exchange of heating between the wall plates and the ambient temperature. The equations for the conservation of momentum, conservation of energy and entropy generation that govern the flow can be written as [19]

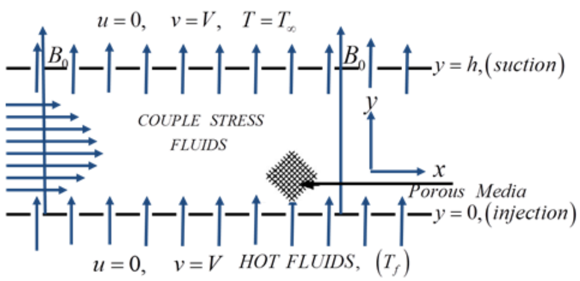

Fig. 1. Schematic diagram of the problem

$$\rho V_0 \frac{du'}{dy'} = -\frac{dp}{dx'} + \mu \frac{d^2u'}{dy'^2} - \eta \frac{d^4u'}{dy'^4} - \sigma B_0^2 u' - \frac{\mu u'}{K} - \frac{bu'^2}{\sqrt{K}} \tag{1}$$

$$\rho C_p v_0 \frac{dT}{dy'} = k\frac{d^2T'}{dy'^2} + \mu\left(\frac{du'}{dy'}\right)^2 + \eta\left(\frac{d^2u'}{dy'^2}\right)^2 + \sigma B_0^2 u'^2 + \frac{\mu u'^2}{K} + \frac{bu'^3}{\sqrt{K}} \tag{2}$$

$$E_G = \frac{k}{T_0^2}\left(\frac{dT'}{dy'}\right)^2 + \frac{\mu}{T_0}\left(\frac{du'}{dy'^2}\right)^2 + \frac{\eta}{T_0}\left(\frac{d^2u'}{dy'^2}\right)^2 + \frac{\sigma B_0^2 u^2}{T_0} + \frac{\mu u'^2}{T_0 K} + \frac{bu'^3}{T_0\sqrt{K}} \tag{3}$$

The boundary conditions are

$$u'(0) = 0 = \frac{d^2u'(0)}{dy^2}, k\frac{dT'(0)}{dy'} = -\gamma_1(T_f - T');$$
$$u'(h) = 0 = \frac{d^2u'(h)}{dy^2}, k\frac{dT'(h)}{dy'} = -\gamma_2(T' - T_0) \tag{4}$$

where u' is the axial velocity, μ is the dynamic viscosity, h is the channel width, ρ is the fluid density, T' is the fluid temperature, T_0 is the initial fluid temperature, T_f is the final fluid temperature, k is the thermal conductivity of the fluid , C_p is the specific heat at constant pressure, v_0 is the constant velocity of fluid suction/injection, σ is the electrical conductivity of the fluid, η is the fluid particle size effect due to couple stresses, K is the porous media permeability, b is the empirical constant in the second order (porous inertia resistance), E_G is the local volumetric entropy generation rate and $\gamma_{1,2}$ are the heat transfer coefficients.
The following dimensionless variables are introduced

$$y = \frac{y'}{h}, u = \frac{u'}{v_0}, \theta = \frac{T' - T_0}{T_f - T_0}, s = \frac{v_0 h}{v}, G = -\frac{h^2 dp}{\mu v_0 dx},$$
$$a^2 = \mu\frac{h^2}{\eta}, Pr = \frac{v\rho C_p}{k}, Br = \frac{\mu v_0}{k(T_f - T_0)},$$
$$Bi_1 = \frac{\gamma_1 h}{k}, Bi_2 = \frac{\gamma_2 h}{k}, N_S = \frac{T_0^2 h^2 E_G}{k(T_f - T_0)^2},$$
$$\Omega = \frac{T_f - T_0}{T_0}, v = \frac{\mu}{\rho}, H^2 = \frac{\sigma B_0^2 h^2}{\rho},$$
$$\beta^2 = \frac{h^2}{K}, \alpha^2 = \frac{bh^2 v_0}{\mu\sqrt{K}} \tag{5}$$

Substituting (5) into (1)-((4), the following dimensionless equations are obtained

$$s\frac{du}{dy} = G + \frac{d^2u}{dy^2} - \frac{1}{a}\frac{d^4u}{dy^4} - H^2u - \beta^2 u - \alpha^2 u^2 \tag{6}$$

$$\frac{d^2\theta}{dy^2} = sPr\frac{d\theta}{dy} - Br\left\{\left(\frac{du}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u}{dy^2}\right)^2 + H^2u^2 + \beta^2u^2 + \alpha^2u^3\right\}, \tag{7}$$

$$N_s = \left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega}\left\{\left(\frac{du}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u}{dy^2}\right)^2 + H^2u^2 + \beta^2u^2 + \alpha^2u^3\right\} \tag{8}$$

with the boundary conditions

$$u(0) = 0 = \frac{d^2u(0)}{dy^2}, \frac{d\theta(0)}{dy} = Bi_1(\theta(0) - 1);$$
$$u(1) = 0 = \frac{d^2u(1)}{dy^2}, \frac{d\theta(1)}{dy} = -Bi_2(\theta(1)) \tag{9}$$

where u is the dimensionless velocity,s is the suction/injection parameter, θ is the dimensionless temperature, a is the couple stress parameter, Pr is the Prandtl number, Br is the Brinkman number, Ω is the parameter that measures the temperature difference between the two heat reservoirs, H^2 is the magnetic field parameter, N_s is the dimensionless entropy generation rate, Be and $Bi_{1,2}$ are the Bejan number and Biot numbers respectively and G is the axial pressure gradient, β is the porous media shape parameter and α is the second order porous media resistance parameter.

III. Solution by Adomian Decomposition Method (ADM)

Writing (6) and (7) in integral form yields

$$u(y) = f_1 y + \frac{f_2}{3!}y^3 + a^2\int_0^y\int_0^y\int_0^y\int_0^y\Bigg\{G + \frac{d^2u}{dY^2} - s\frac{du}{dY} + BrH^2u - Br\beta^2u - Br\alpha^2u\Bigg\}dYdYdYdY \quad (10)$$

and

$$\theta(y) = f_3 + f_4(y) + \int_0^y\int_0^y\Bigg\{sPr\frac{d\theta}{dY} - Br\Big(\frac{du}{dY}\Big)^2 - \frac{Br}{a^2}\Big(\frac{d^2u}{dY^2}\Big)^2 + BrH^2u^2 + Br\beta^2u^2 + Br\alpha^2u^3\Bigg\}dYdY \quad (11)$$

Note that the boundary conditions $u(0) = u^{''}(0) = 0$ and f_1, f_2, f_3, f_4 are the parameters to be determined later.

By ADM, an infinite series solution can be defined as

$$u(y) = \sum_{n=0}^{\infty} u_n(y), \theta(y) = \sum_{n=0}^{\infty}\theta_n(y) \quad (12)$$

Now, substituting (12) in (10) and (11) gives

$$\sum_{n=0}^{\infty}u_n(y) = f_1y + \frac{f_2}{3!}y^3 + a^2\int_0^y\int_0^y\int_0^y\int_0^y\Bigg\{G + \sum_{n=0}^{\infty}\frac{d^2u_n}{dY^2} - s\sum_{n=0}^{\infty}\frac{du_n}{dY} - H^2\sum_{n=0}^{\infty}u_n - \beta^2\sum_{n=0}^{\infty}u_n - \alpha^2\sum_{n=0}^{\infty}u_n\Bigg\}dYdYdYdY \quad (13)$$

and

$$\sum_{n=0}^{\infty}\theta_n(y) = f_3 + f_4y + \int_0^y\int_0^y\Bigg\{sPr\sum_{n=0}^{\infty}\frac{d\theta_n}{dY} - Br\Bigg(\sum_{n=0}^{\infty}\frac{du}{dY}\Bigg)^2 - \frac{Br}{a^2}\Bigg(\sum_{n=0}^{\infty}\frac{d^2u}{dY^2}\Bigg)^2 + BrH^2\sum_{n=0}^{\infty}u^2 + Br\beta^2\sum_{n=0}^{\infty}u^2 + Br\alpha^2\sum_{n=0}^{\infty}u^3\Bigg\}dYdY \quad (14)$$

In view of (13) and (14), the zeroth order term can be written as

$$\sum_{n=0}^{\infty}u_0(y) = f_1y + \frac{f_2}{3!}y^3 + a^2\int_0^y\int_0^y\int_0^y\int_0^y\Big\{G\Big\} \quad (15)$$

$$\sum_{n=0}^{\infty}\theta_0(y) = f_3 + f_4y \quad (16)$$

TABLE I
Computation showing convergence of solution when s=0.1,H=a=Br=β=α=Bi_1=Bi_2= G= 1

y	Exact solution	ADM Abs Error	DTM Abs Error
0	-2.7756×10^{-17}	2.7756×10^{-17}	2.7756×10^{-17}
0.1	0.003647072	2.93073×10^{-14}	1.28806×10^{-11}
0.2	0.006896387	2.01133×10^{-14}	2.5258×10^{-11}
0.3	0.009436924	6.97446×10^{-14}	3.66252×10^{-11}
0.4	0.011049005	2.86358×10^{-13}	4.6465×10^{-11}
0.5	0.011601677	6.81247×10^{-13}	5.42441×10^{-11}
0.6	0.011051182	1.31362×10^{-12}	5.94085×10^{-11}
0.7	0.009440522	2.25145×10^{-12}	6.13774×10^{-11}
0.8	0.006900099	3.57501×10^{-12}	5.95401×10^{-11}
0.9	0.003649445	5.37918×10^{-12}	5.32521×10^{-11}

while other terms are determined using the recurrence relations

$$\sum_{n=0}^{\infty}u_{n+1}(y) = a^2\int_0^y\int_0^y\int_0^y\int_0^y\Bigg\{\sum_{n=0}^{\infty}\frac{d^2u_n}{dY^2} - s\sum_{n=0}^{\infty}\frac{du_n}{dY} - H^2\sum_{n=0}^{\infty}u_n - \beta^2\sum_{n=0}^{\infty}u_n - \alpha^2\sum_{n=0}^{\infty}u_n\Bigg\}dYdYdYdY \quad (17)$$

$$\sum_{n=0}^{\infty}\theta_{n+1}(y) = a^2\int_0^y\int_0^y\Bigg\{sPr\sum_{n=0}^{\infty}\frac{d\theta_n}{dY} - Br\Bigg(\sum_{n=0}^{\infty}\frac{du}{dY}\Bigg)^2 - \frac{Br}{a^2}\Bigg(\sum_{n=0}^{\infty}\frac{d^2u}{dY^2}\Bigg)^2 + BrH^2\sum_{n=0}^{\infty}u^2 + Br\beta^2\sum_{n=0}^{\infty}u^2 + Br\alpha^2\sum_{n=0}^{\infty}u^3\Bigg\}dYdY \quad (18)$$

The accuracy of the results of these computations can be verified by comparing the approximate solutions obtained via ADM and DTM with the exact solution presented in Table 1.

IV. Entropy Generation

Investigating entropy generation within the flow, according to Bejan [16] the local entropy generation rate as shown in (3) is

$$E_G = \frac{k}{T_0^2}\Big(\frac{dT^{'}}{dy^{'}}\Big)^2 + \frac{\mu}{T_0}\Big(\frac{du^{'}}{dy^{'}}\Big)^2 + \frac{\eta}{T_0}\Big(\frac{d^2u^{'}}{dy^{'2}}\Big)^2 + \frac{\sigma B_0^2u^2}{T_0} + \frac{\mu u^{'2}}{T_0K} + \frac{bu^{'3}}{T_0\sqrt{K}}$$

The first term in the above equation is the irreversibility due to heat transfer, the second and the third terms account for entropy generation due to fluid friction and couple stress respectively while the last three terms represent irreversibility due to the effect of magnetic field and porosity.

The dimensionless form as shown in (8) is given as

$$N_S = \Big(\frac{d\theta}{dy}\Big)^2 + \frac{Br}{\Omega}\Bigg\{\Big(\frac{du}{dy}\Big)^2 + \frac{1}{a^2}\Big(\frac{d^2u}{dy^2}\Big)^2 + H^2u^2 + \beta^2u^2 + \alpha^2u^3\Bigg\}$$

Investigating entropy generation within the flow, let

$$N_1 = \left(\frac{d\theta}{dy}\right)^2, N_2 = \frac{Br}{\Omega}\left\{\left(\frac{du}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u}{dy^2}\right)^2 + H^2u^2 + \beta^2u^2 + \alpha^2u^3\right\} \quad (19)$$

The Bejan number Be = 0 is the irreversibility due to viscous dissipation, couple stress effect, magnetic effect and porous media and Be = 1 is when the irreversibility due to heat transfer dominates the flow. Be = 0.5 indicates that viscous dissipation and heat transfer contribute equally to entropy generation. The Bejan number can be written as

$$Be = \frac{N_1}{N_s} = \frac{1}{1+\Phi}, \Phi = \frac{N_2}{N_1} \quad (20)$$

V. Results and Discussion

Second law analysis of hydromagnetic couple stress fluid through a non-Darcian porous medium has been considered. The graphical results are presented in this section to explain the influence of pertinent parameters on velocity, temperature, entropy generation and Bejan number.

A. *Effects of Parameters Variation on Velocity profiles*

Effect of parameters variation on velocity are shown in Figures 2-6. Fig. 2 depicts the plot of magnetic field parameter on velocity profile; the figure shows that increase in the magnetic field parameter reduces fluid velocity. This can be attributed to the force exerted by the applied magnetic field on fluid particles which clumps the fluid particles together leading to an increase in viscosity and consequently, the drop in fluid velocity. In Figs. 3 and 4, the effects of porous media parameters (β, α) are presented; the graphs reveal that fluid velocity reduces as porous media parameters increase. This can be attributed to the reduction in the porous media permeability (K) of the fluid which reduces the free flow of fluid particles.

Moreover, Fig. 5 represents the plot of couple stress inverse parameter on velocity profile. As observed from the plot, increase in couple stress inverse parameter increases the velocity profile. It means that couple stress parameter will eventually reduce fluid velocity due to increased viscosity of the fluid. In Fig.6 the effect of pressure gradient on fluid velocity is described, the plot reveals that fluid velocity is accelerated as the values of (G) increases.

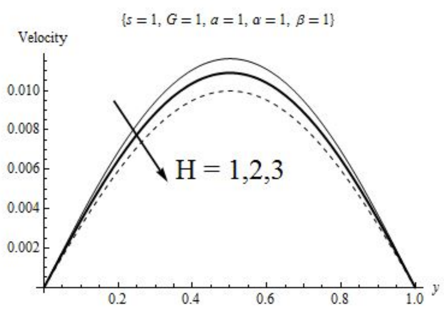

Fig. 2. Effect of magnetic field parameter (H^2) on velocity profile

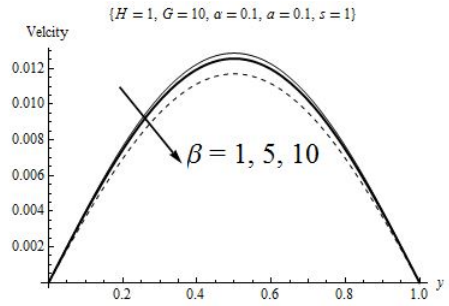

Fig. 3. Effect of porous media shape parameter (β) on velocity profile

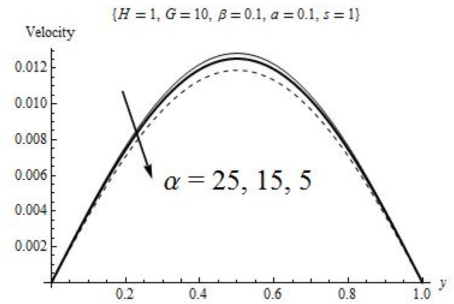

Fig. 4. Effect of second order media shape factor parameter (α) on velocity profile

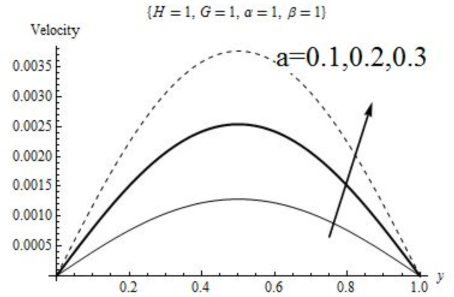

Fig. 5. Effect of couple stress inverse parameter (a) on velocity profile

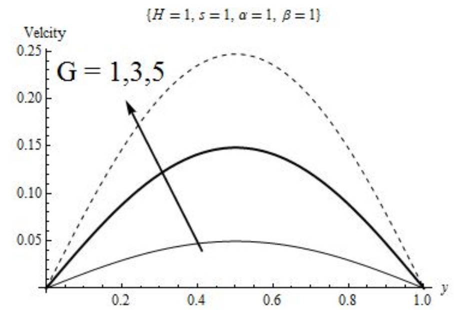

Fig. 6. Effect of pressure gradient (G) on velocity profile

B. *Effects of Parameters Variation Temperature profiles*

The influence of parameters variation on fluid temperature are displayed in plots 7-11. Fig. 7 displays the graph of magnetic field parameter variation on fluid temperature. It is observed that fluid temperature increases with increase in magnetic field parameter. This is due to an increase in heat source from the Ohmic heating present in the flow; this enhances transfer of heat to the boundaries. Furthermore, Fig. 8 indicates that as porous media shape parameter rises in value fluid temperature is enhanced. The reduction in the porous media permeability of the fluid is responsible for the rise in temperature.

The influence of the inverse of couple stress parameter on temperature is shown in Fig. 9. It is noticed from the graph that as couple stress parameter increases the temperature of the fluid drops. The implication of this is that increase in couple stresses enhances fluid thickness that is, the dynamic viscosity increases with a corresponding increase in the temperature. Fig. 10 displays the effect of lower Biot number (Bi_1) on fluid temperature. As seen from the graph, the temperature increases as convective heating from the lower wall increases, while the trend is reversed in Fig. 11 with upper Biot number (Bi_2) due to the cooling effect.

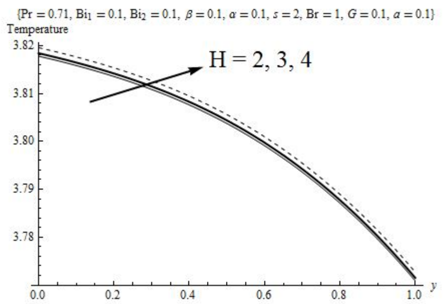

Fig. 7. Effect of magnetic field parameter (H^2) on temperature profile

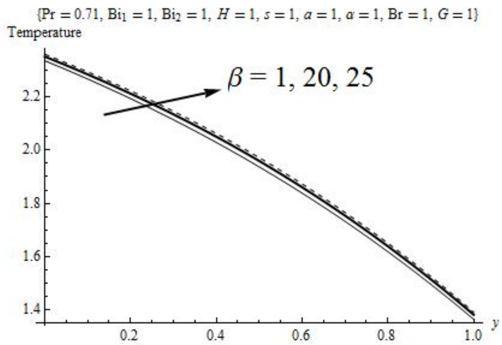

Fig. 8. Effect of porous media shape parameter (β) on temperature profile

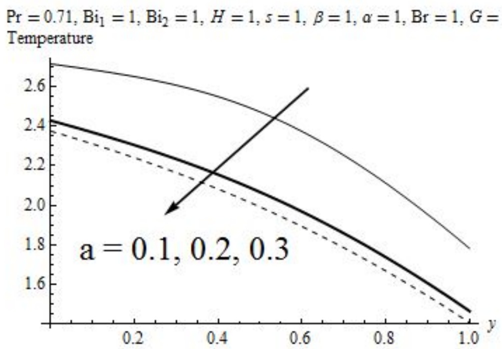

Fig. 9. Effect of couple stresses (a) on temperature profile

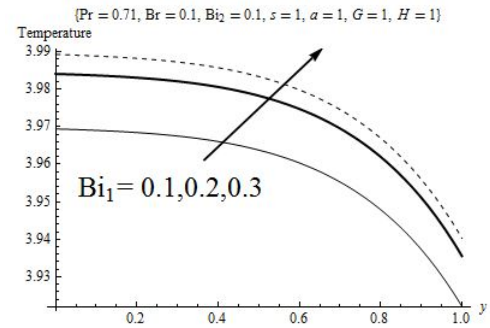

Fig. 10. Effect of convective heating (Bi_1) on temperature profile

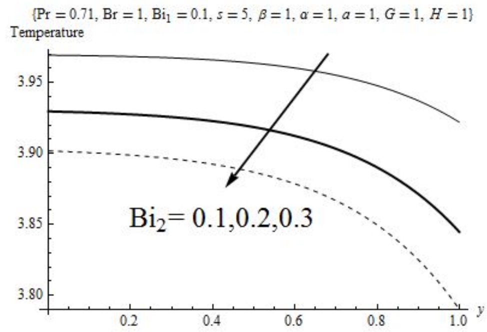

Fig. 11. Effect of convective cooling (Bi_2) on temperature profile

C. Effects of Parameter Variation on Entropy Generation

In this section the effect of parameters variation on entropy generation rate in Figs. 12-18 are presented. In Fig. 12, the influence of magnetic field parameter on entropy generation is depicted. It is indicated in the graph that entropy generation is enhanced with increase in magnetic field parameter. This can be traced to Fig. 2 which shows that fluid velocity decreases with increased Hartman number caused by clumping of fluid particle. Furthermore, it is shown in Fig. 7 that fluid temperature rises as Hartman number is increased due to increased heat transfer to the boundaries from Ohmic heating. The effect of these is the significant rise in entropy generation displayed in Fig. 11. In Figs. 13 and 14, the effects of porous shape parameters on entropy generation are displayed. The plots indicate that increase in porous shape parameters reduces entropy generation. This is clearly shown in Figs. 3 and 4 that fluid velocity reduces with increase in porosity parameters; the drop in fluid velocity reduces the random movement of fluid particles and consequently the reduction in entropy generation rate.

Moreover, in Fig. 15 the graph displays the influence of couple stress inverse parameter on the entropy generation rate. It is revealed that entropy generation rises with increase in couple stress inverse parameter (a). This implies that couple stresses reduce entropy generation due to the reduction in random movement of fluid particles. The drop in the randomness of fluid particles is clearly revealed in Fig. 5 which shows that fluid velocity decreases with increase in couple stresses. Effect of pressure gradient on entropy production is shown in Fig.16, it is depicted from the plot that a rise in pressure gradient enhances entropy generation rate because of the significant rise in fluid velocity (see Fig. 6). Finally, Figs. 17 and 18 show similar result, the plots depict the effect of Biot numbers on entropy generation. As observed from the figures, entropy generation increases considerably across the channel as both lower and upper Biot numbers increase in values.

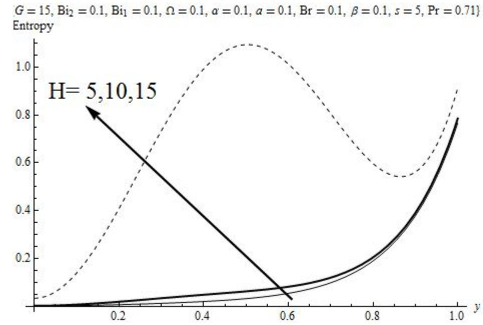

Fig. 12. Effect of magnetic field parameter (H^2) on entropy generation rate

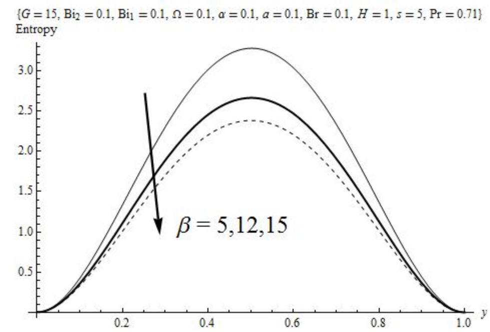

Fig. 13. Effect of porous media shape factor parameter (β) on entropy generation rate

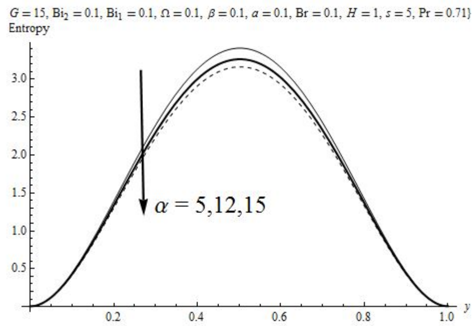

Fig. 14. Effect of second order media shape factor parameter (α) on entropy generation rate

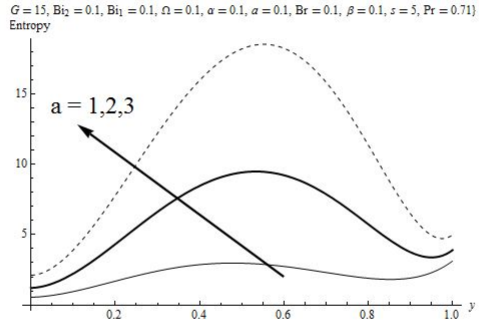

Fig. 15. Effect of couple stresses (a) on entropy generation rate

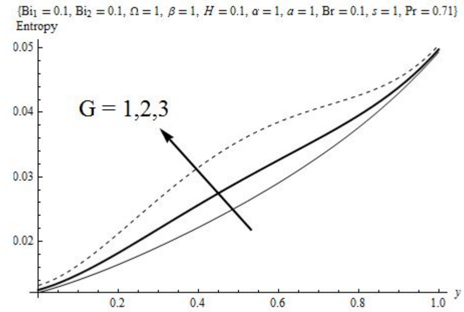

Fig. 16. Effect of pressure gradient (G) on entropy generation rate

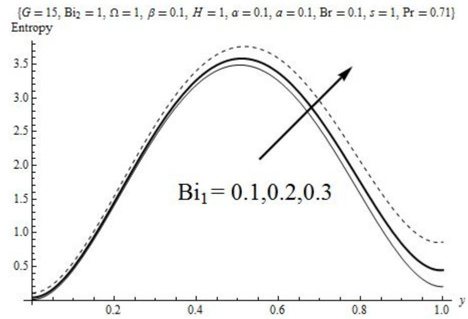

Fig. 17. Effect of convective heating (Bi_1) on entropy generation rate

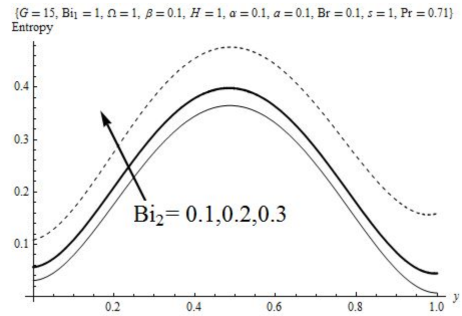

Fig. 18. Effect of convective cooling (Bi_2) on entropy generation rate

D. Effects of Parameter Variation on Bejan Number

Influence of parameters variation on Bejan number are presented in Figs. 19-25. Figs. 19-25 display the plots of variation in magnetic field parameter, porous media shape parameter, couple stresses, Biot numbers, Brinkman number and Prandtl number respectively on Bejan number. The plots indicate that increase in these parameters increases Bejan number. However, in Fig. 16 Prandtl number decreases Bejan number slightly at the lower wall. The results indicate the dominance of irreversibility due to heat transfer in the middle and upper walls across the channel.

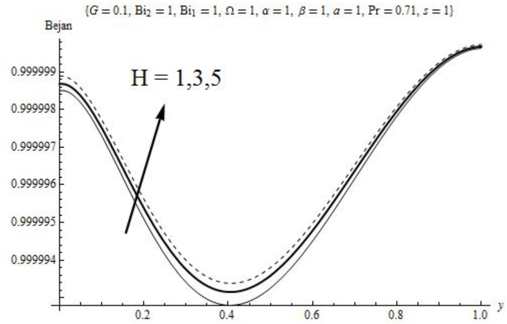

Fig. 19. Effect of magnetic field parameter (H^2) on Bejan number

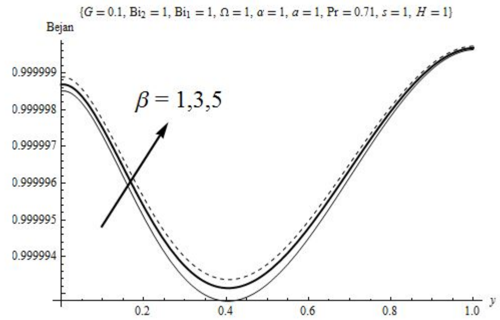

Fig. 20. Effect of porous media shape parameter (β) on Bejan number

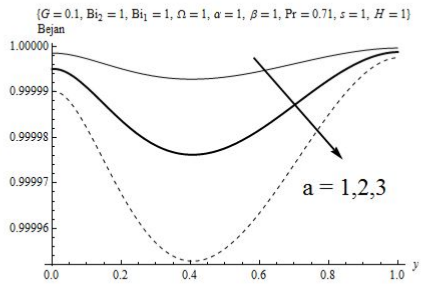

Fig. 21. Effect of couple stress inverse (a) on Bejan number

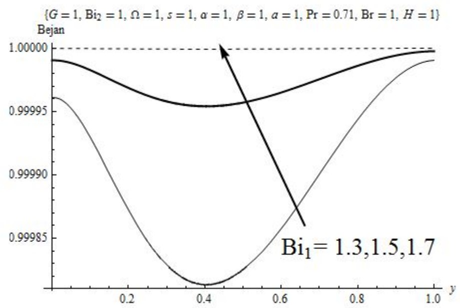

Fig. 22. Effect of convective heating (Bi_1) on Bejan number

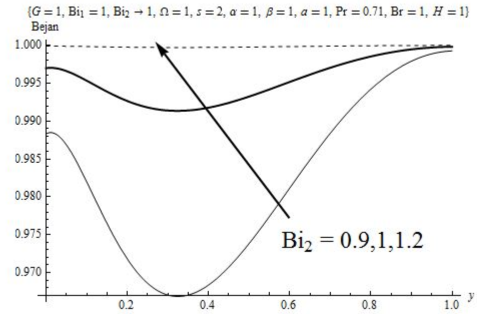

Fig. 23. Effect of convective cooling (Bi_2) on Bejan number

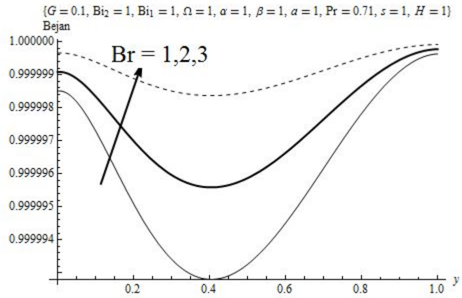

Fig. 24. Effect of Brinkman number (Br) on Bejan number

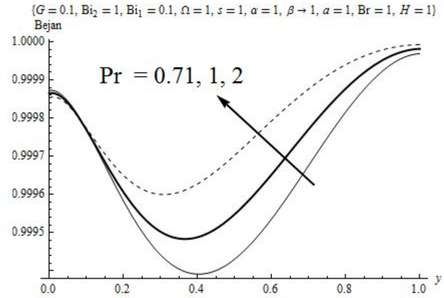

Fig. 25. Effect of Prandtl number (Pr) on Bejan number

VI. Conclusions

Second law analysis of hydromagnetic couple stress fluid through a channel filled with non-Darcian porous medium has been investigated. The non-linear governing equations of momentum and energy are solved numerically by ADM and DTM. The results are used to compute the non-dimensional entropy generation and Bejan number. Conclusions of the study are as follows:

- Fluid velocity decreases with increase in magnetic field parameter, porosity parameters and couple stresses while pressure gradient accelerates fluid velocity;
- there is a rise in fluid temperature with increase in magnetic field parameter, porous media shape parameter, couple stresses and convective heating parameter while fluid temperature reduces as convective cooling parameter increases in value;
- magnetic field parameter, pressure gradient and Biot numbers enhance entropy production but the entropy generation rate reduced with increase in porosity and couple stress parameters;
- there is an increase in Bejan number in the middle and upper walls of the channel with increase in porous me-

dia shape parameter, magnetic field parameter, couple stresses, Biot numbers, Brinkman number and Prandtl number; and
- entropy generation due to heat transfer dominates the flow at the middle and upper walls of the channel.

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