On the White Noise of the Price of Stocks related to the Option Prices from the Black-Scholes Equation

A. Kananthai †, T. Kraiwiradechachai*‡

Abstract— **In this paper, we study the white noise from the stock model and obtained some interesting properties. Moreover such white noise can be applied to the black-scholes equation in the form of white noise and obtained the option price of such equation. We also found the kernel which has interesting properties.**

Keywords: Black-Scholes equation, white noise, kernel

1 Introduction

We know that the white noise is the cause of the fluctuation of the price of stock. In the past, the white noise has not been computed properly from the stock model. Fortunately, we can compute such white noise by using the idea of generalized function or distribution theory. When we get the valued of white noise we can understand how much the fluctuation of any kind of stock. Moreover we know the interesting properties such as the tempered distribution and a generalized stochastic process and also the Gaussian normal distribution. Moreover, we have applied such white noise to the Black-Scholes equation in the form of white noise. It is well known that the Black-Scholes equation plays an important role in financial mathematics, particularly in finding the option price of the stock market. In this work we start with the stock model

$$ds = \mu s dt + \sigma s dB \tag{1}$$

where s is the price of stock at the time t, μ is the drift of stock, σ is the volatility of stock and B is the Brownian motion. From (1) we define the white noise $\xi = \dfrac{dB}{dt}$. Actually, $\dfrac{dB}{dt}$ does not exist in the classical sense or Newtonian sense. But it has meaning in the distributional sense that is in the space of tempered distribution. By applying the Ito's formula and the tempered distribution to (1) we obtained the white noise ξ in the form

$$\xi = \frac{1}{t\sigma} ln\left(\frac{s}{s_0}\right) - \frac{\mu}{\sigma} + \frac{\sigma}{2} \tag{2}$$

where s_0 is the price of stock at $t = 0$. We can relate (2) to the Black-Scholes equation which is given by

$$\frac{\partial u(s,t)}{\partial t} + rs\frac{\partial u(s,t)}{\partial s} + \frac{\sigma^2}{2}s^2\frac{\partial^2 u(s,t)}{\partial s^2} - ru(s,t) = 0 \tag{3}$$

with the terminal condition

$$u(s,t) = (s-p)^+ \tag{4}$$

(see [1], pp(637-654)) for $0 \le t \le T$ where $u(s,t)$ is the option price at time t, σ is the volatility of stock and p is the strike price. Let $u(s,t) = V(\xi,t)$ where ξ is given by (2). Then (3) is transformed to the equation

$$\frac{\partial V(\xi,t)}{\partial t} + \frac{1}{2t^2}\frac{\partial^2 V(\xi,t)}{\partial \xi^2} + \left(\frac{r}{t\sigma} - \frac{\sigma}{2t}\right)\frac{\partial V(\xi,t)}{\partial \xi} - rV(\xi,t) = 0 \tag{5}$$

with the condition

$$\frac{\partial V(\xi,t)}{\partial t} = f(\xi), 0 \le t \le 1 \tag{6}$$

where $f(\xi)$ is the given generalized function. We obtain the solution $V(\xi,t)$ of (5) with (6) in the convolution form

$$V(\xi,t) = K(\xi,t) * f(\xi) \tag{7}$$

where

$$K(\xi,t) = \sqrt{\frac{t}{2\pi(1-t)}}\exp^{-(1-t)r}\exp\left[\frac{-t(\xi - (\frac{r}{\sigma} - \frac{\sigma}{2})\ln t)^2}{2(1-t)}\right] \tag{8}$$

is the kernel (see [2], pp 1627-1634)

2 Preliminary Notes

Recall the stock model

$$ds = \mu s dt + \sigma s dB \tag{9}$$

*Manuscript received August 04, 2017; revised November 16, 2017. This work was supported in part by Naresuan University under Grant R2559C068 The authors are very grateful to the referee for their valuable suggestions and comments that improved the paper.This project was partially completed while the first author visit to Department of Mathematics, Naresuan University.

†A. Kananthai (e-mail: malamnka@gmail.com) is with the Department of Mathematics, Faculty of science, Chiangmai University, Chiangmai 50200, Thailand.

‡T. Kraiwiradechachai (corresponding author e-mail: tanyongk@nu.ac.th) is with the Research center for Academic Excellence in Mathematics, Department of Mathematics, Faculty of science, Naresuan University, Phitsanulok 65000, Thailand.

(Advance online publication: 28 May 2018)

or

$$ds = \mu s dt + \sigma s \dot{B}(t) dt$$

where $\dot{B}(t) = \dfrac{dB}{dt}$ is the white noise denoted by $\xi(t) = \dot{B}(t)$. Apply the Ito's formula to (9), we obtain

$$\int_0^t d(\ln s(\tau)) = \left(\mu - \frac{\sigma^2}{2}\right) \int_0^t d\tau + \sigma \int_0^t \dot{B}(\tau) d\tau$$

where $0 \le \tau \le t$ Thus

$$\ln s(t) - \ln s_0 = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma \int_0^t \xi(\tau) d\tau \tag{10}$$

where $\xi(\tau) = \dot{B}(\tau)$ and $s(0) = s_0$. Since $\dot{B}(t) = \dfrac{dB}{dt}$ does not exist in classical sense or Newtonian sense. But it can be show that $\xi(t) = \dot{B}(t)$ is a tempered distribution, that is $\xi \in \acute{\mathrm{S}}(\mathcal{R})$- the space of tempered distribution (see [3], pp 6-8). Thus for any testing function $\varphi \in \mathrm{S}(\mathcal{R})$-the Schwartz space, define $\langle \xi, \varphi \rangle = \int_0^t \xi(\tau)\varphi(\tau) d\tau$.
Thus, from (10)

$$\ln \frac{s(t)}{s_0} = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma \int_0^t \frac{\varphi(\tau)}{\varphi(\tau)} \xi(\tau) d\tau$$

for $\varphi(\tau) \neq 0$. Now $\xi(\tau) \in \acute{\mathrm{S}}(\mathcal{R})$, also $\dfrac{1}{\varphi(\tau)}\xi(\tau) \in \acute{\mathrm{S}}(\mathcal{R})$.
Let $F(\tau) = \dfrac{1}{\varphi(\tau)}\xi(\tau)$ thus

$$\int_0^t \frac{\xi(\tau)}{\varphi(\tau)} = \int_0^t F(\tau)\varphi(\tau) d\tau.$$

Since $F(\tau)\varphi(\tau)$ is a smooth function of τ. By mean value theorem, there exist τ^* for $0 \le \tau^* \le t$ such that

$$\begin{aligned}
&\int_0^t F(\tau)\varphi(\tau) d\tau \\
&\qquad = F(\tau^*)\varphi(\tau^*) \int_0^t d\tau \\
&\qquad = F(\tau^*)\varphi(\tau^*) t \\
&\qquad = \frac{\xi(\tau^*)}{\varphi(\tau^*)} \varphi(\tau^*) t \\
&\qquad = \xi(\tau^*) t
\end{aligned}$$

Thus from (10)

$$\ln \frac{s(t)}{s_0} = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma \xi(\tau^*) t.$$

Thus

$$\xi = \frac{1}{t\sigma} \ln\left(\frac{s}{s_0}\right) - \frac{\mu}{\sigma} + \frac{\sigma}{2} \tag{11}$$

for $t \neq 0$. By changing the variable s to ξ from (11) and let $u(s,t) = V(\xi,t)$ we have

$$\begin{aligned}
\frac{\partial u}{\partial s} &= \frac{\partial V}{\partial s} \\
&= \frac{\partial V}{\partial \xi}\frac{\partial \xi}{\partial s} \\
&= \frac{1}{t\sigma s}\frac{\partial V}{\partial \xi},
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2 u}{\partial s^2} &= \frac{\partial^2 V}{\partial s^2} \\
&= \frac{\partial}{\partial \xi}\left(\frac{1}{t\sigma s}\frac{\partial V}{\partial \xi}\right)\frac{\partial \xi}{\partial s} \\
&= \frac{1}{t^2\sigma^2 s^2}\frac{\partial^2 V}{\partial \xi^2} - \frac{1}{t\sigma s^2}\frac{\partial V}{\partial \xi}.
\end{aligned}$$

Thus (3) is transformed to

$$\frac{\partial V(\xi,t)}{\partial t} + \frac{1}{2t^2}\frac{\partial^2 V(\xi,t)}{\partial \xi^2} + \left(\frac{r}{t\sigma} - \frac{\sigma}{2t}\right)\frac{\partial V(\xi,t)}{\partial \xi} - rV(\xi,t) = 0 \tag{12}$$

where $0 < t \le T$ with the terminal condition of (4) and (11)

$$V(\xi,T) = \left(s_0 \exp\left[(\mu - \frac{\sigma^2}{2})T + \sigma T\xi\right] - p\right)^+. \tag{13}$$

Let $f(\xi) = \left(s_0 \exp\left[(\mu - \frac{\sigma^2}{2})T + \sigma T\xi\right] - p\right)^+$ thus

$$V(\xi,T) = f(\xi) \tag{14}$$

Definition 2.1 *Let $f(x)$ is a locally integrable function. The Fourier transform $\widehat{f}(\omega)$ of f(x)is definition by*

$$\widehat{f}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega x} f(x) dx \tag{15}$$

and the inverse Fourier transform of $\widehat{f(\omega)}$ also defined by

$$f(x) = \mathcal{F}^{-1}\widehat{f}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega x}\widehat{f}(\omega) d\omega \tag{16}$$

3 Main Results

Theorem 3.1 *The equation given by (12) with the terminal condition given by (14) has a solution $V(\xi,t) = K(\xi,t) * f(\xi)$ in the convolution form, where*

$$K(\xi,t) = \sqrt{\frac{tT}{2\pi(T-t)}} e^{-(T-t)r} exp\left[\frac{-tT\left(\left(\frac{r}{\sigma} - \frac{\sigma}{2}\right)\ln\frac{T}{t} + \xi\right)^2}{2(T-t)}\right]$$

is the kernel of (12)

Proof. Take the Fourier transform defined by (15) to (12), we obtain

$$\frac{\partial\widehat{V}(\omega,t)}{\partial t}-\frac{\omega^2}{2t^2}\widehat{V}(\omega,t)+\frac{1}{t}\Big(\frac{r}{\sigma}-\frac{\sigma}{2}\Big)i\omega\widehat{V}(\omega,t)-r\widehat{V}(\omega,t)=0.$$

Thus

$$\widehat{V}(\omega,t)=C(\omega)e^{rt}e^{-\dfrac{\omega^2}{2t}-i\omega\Big(\dfrac{r}{\sigma}-\dfrac{\sigma}{2}\Big)lnt}$$

and from (14),

$$\widehat{V}(\omega,t)=\widehat{f}(\omega)$$

and

$$\widehat{V}(\omega,T)=C(\omega)e^{rT}e^{-\dfrac{\omega^2}{2T}-i\omega\Big(\dfrac{r}{\sigma}-\dfrac{\sigma}{2}\Big)lnT}$$

Thus

$$\begin{aligned}C(\omega)&=\frac{\widehat{f}(\omega)}{e^{rT-\dfrac{\omega^2}{2T}-i\omega(\dfrac{r}{\sigma}-\dfrac{\sigma}{2})\ln T}}\\&=\widehat{f}(\omega)e^{-rT+\dfrac{\omega^2}{2T}+i\omega(\dfrac{r}{\sigma}-\dfrac{\sigma}{2})\ln T}\end{aligned}$$

Thus

$$\begin{aligned}&\widehat{V}(\omega,t)\\&=e^{(-T-t)r}exp\Big((\frac{1}{2T}-\frac{1}{2t})\omega^2+(\ln T-\ln t)i\omega\big(\frac{r}{\sigma}-\frac{\sigma}{2}\big)\Big)\widehat{f}(\omega),\end{aligned}$$

as a solution of (12) for $0<t\le T$. Now

$$|\widehat{V}(\omega,t)|\le|e^{-(T-t)r}||e^{\Big(\dfrac{1}{2T}-\dfrac{1}{2t}\Big)\omega^2}||\widehat{f}(\omega)|$$

Let $M=\max|\widehat{f}(\omega)|$. Now $|e^{-(T-t)r}|$ and$|e^{-\dfrac{1}{2}(\dfrac{1}{t}-\dfrac{1}{T}\omega^2}|$ are bounded, thus

$$|\widehat{V}(\omega,t)|\le|e^{-(T-t)r}||e^{-\dfrac{1}{2}\Big(\dfrac{1}{T}-\dfrac{1}{t}\Big)\omega^2}|M\le K$$

is bounded. Thus the inversion

$$\begin{aligned}&V(\xi,t)\\&=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega\xi}\widehat{V}(\omega,t)d\omega\\&=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega\xi}e^{-(T-t)r}\\&exp\Big[-\frac{1}{2}\Big(\frac{1}{t}-\frac{1}{T}\Big)\omega^2-\ln\frac{T}{t}i\omega\Big(\frac{r}{\sigma}-\frac{\sigma}{2}\Big)\Big]\widehat{f}(\omega)d\omega\\&=\frac{1}{2\pi}e^{-(T-t)r}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{(\xi-y)i\omega}\\&exp\Big[-\frac{1}{2}\Big(\frac{1}{t}-\frac{1}{T}\Big)\omega^2-\ln\frac{T}{t}i\omega\Big(\frac{r}{\sigma}-\frac{\sigma}{2}\Big)\Big]f(y)\\&dyd\omega\\&=\frac{1}{2\pi}e^{-(T-t)r}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\\&exp\Big[-\frac{(T-t)}{2tT}\Big[\omega^2-2tT\Big(\frac{\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2})+\xi-y}{T-t}\Big)i\omega\Big]\Big]\\&f(y)dyd\omega\\&=\frac{1}{2\pi}e^{-(T-t)r}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\\&exp\Big[-\frac{(T-t)}{2tT}\Big[\omega-tT\Big(\frac{\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2})+\xi-y}{T-t}\Big)i\Big)^2\\&-tT\Big(\frac{\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2})+\xi-y}{2(T-t)}\Big)^2\Big]f(y)dyd\omega\end{aligned}$$

Put $u=\sqrt{\dfrac{(T-t)}{2tT}}\Big(\omega-\dfrac{itT(\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2})+\xi-y)}{T-t}\Big)$, then $du=\sqrt{\dfrac{(T-t)}{2tT)}}d\omega$ or $d\omega=\sqrt{\dfrac{2tT}{T-t}}du$. Thus

$$\begin{aligned}V(\xi,t)&=\frac{1}{2\pi}e^{-(T-t)r}\int_{-\infty}^{\infty}e^{-u^2}du\sqrt{\frac{2tT}{T-t}}\\&\int_{-\infty}^{\infty}\exp\left[-\frac{-tT(\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2})\xi-y)^2}{2(T-t)}\right]f(y)dy\\&=\frac{1}{2\pi}\sqrt{\frac{2tT}{T-t}}e^{-(T-t)r}\sqrt{\pi}\\&\int_{-\infty}^{\infty}\exp\left[-\frac{-tT(\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2})\xi-y)^2}{2(T-t)}\right]f(y)dy,\end{aligned}$$

since $\int_{-\infty}^{\infty} e^{-u^2}du = \sqrt{\pi}$.
Thus

$$V(\xi,t) = \sqrt{\frac{tT}{2\pi(T-t)}}e^{-(T-t)r} \int_{-\infty}^{\infty} exp\Big[\frac{-tT(\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2})+\xi-y)^2}{2(T-t)}\Big]f(y)dy.$$

Actually $K(\xi,t)$ is the kernel of (12).
Thus $V(\xi,t) = K(\xi,t) * f(\xi)$ in the convolution form.
Now,$K(\xi,t) = \sqrt{\frac{1}{2\pi\frac{(T-t)}{tT}}}e^{-(T-t)r}$

$$\exp\left[\frac{-\left(\ln\frac{T}{t}\left(\frac{r}{\sigma}-\frac{\sigma}{2}\right)+\xi\right)^2}{2\frac{(T-t)}{tT}}\right].$$

Thus $K(\xi,t)$ is a Gaussian function or normal distribution with mean $e^{-(T-t)r}\left(\frac{\sigma}{2}-\frac{r}{\sigma}\right)\ln\frac{T}{t}$ and variance $e^{-2(T-t)r\frac{(T-t)}{tT}}$. Thus $K(\xi,t) * f(\xi)$ we need to show that as $t \to T, V(\xi,t) = \delta(\xi) * f(\xi) = f(\xi)$ that (14) holds. That means $\lim_{t\to T} K(\xi,t) = \delta(\xi)$ where $\delta(\xi)$ is the Dirac-delta distribution. Moreover, we see that the kernel $K(\xi,t)$ involving the white noise ξ which causes the fluctuation of the price of stock as mentioned before. Actually, such kernel plays the significant role for find the particular solution of the nonhomogeneous differential equation. For example, given the nonhomogeneous differential equation $Lu(x) = f(x)$ where L is the partial differential operator, then we can find the particular solution $u(x) = K(x) * f(x)$ where $K(x)$ is the kernel of such equation.

Corollary 3.2 *From $V(\xi,t) = K(\xi,t) * f(\xi), 0 < t \le T$. We obtain the following conditions.*

(i) $V(\xi,T) = f(\xi)$ *that is the terminal condition.*

(ii) $\lim_{t\to 0^+} V(\xi,t) = 0$.

Proof. (i) We need to show that $\lim_{t\to T} K(\xi,t) = \delta(\xi)$. Now,

$$K(\xi,t) = \sqrt{\frac{tT}{2\pi(T-t)}}e^{-(T-t)}exp\Big[\frac{-tT(\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2})+\xi)^2}{2(T-t)}\Big].$$

Thus

$$\lim_{t\to T} K(\xi,t) = \lim_{t\to T} e^{-(T-t)} \lim_{t\to T}\sqrt{\frac{tT}{2\pi(T-t)}} exp\Big[\frac{-tT(\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2})+\xi)^2}{2(T-t)}\Big]. = 1\cdot\delta(\xi) = \delta(\xi).$$

Thus $V(\xi,t) = \delta(\xi) * f(\xi) = f(\xi)$.
(ii) We have $\lim_{t\to 0^+} V(\xi,t) = \lim_{t\to o^+} K(t,\xi,t) * f(\xi)$. By applying L'Hospital rule we obtain $\lim_{t\to 0^+} K(\xi,t) = 0$. Thus

$$\lim_{t\to 0^+} V(\xi,t) = 0 * f(\xi) = 0.$$

Now, we have

$$V(\xi,t) = u(s,t) = u\left(s_0\exp\left[(\mu-\frac{\sigma^2}{2})t+\sigma t\xi\right],t\right)$$

but from (3.2), $V(\xi,0) = 0$ it follows that $V(\xi,0) = u(s_0,0) = 0$. Now consider the option price at $t = 0, s = s_0$ and we have $u(s_0,0) = 0$ which is not really appear in the real world. Actually when $t = 0, s = s_0$ the option price $u(s_0,0)$ need not be zero. This can be conclude that the initial condition $V(\xi,0)$ of (12) mat be different from the initial condition $u(s_0,0)$ of (3).

Theorem 3.3 *Properties of $K(\xi,t)$.*

(i) $K(\xi,T)$ *satisfies equation (12)*

(ii) $K(\xi,T)$ *is a tempered distribution, that is* $K(\xi,T) \in \mathcal{S}'(R)$.

(iii) $K(\xi,T) > 0$ *for* $0 < t \le T$.

(iv) $e^{(T-t)r}\int_{-\infty}^{\infty} K(\xi,T)d\xi = 1$.

(v) $\lim_{t\to 1} K(\xi,T) = \delta(\xi)$.

(vi) $K(\xi,T)$ *is Guassian distribution with mean* $e^{(T-t)r}\left(\frac{\sigma}{2}-\frac{r}{\sigma}\right)\ln\frac{T}{t}$ *and variance* $e^{-2(T-t)r}\frac{(T-t)}{tT}$. *That is* $K(\xi,T)$ *is* $\left(e^{(T-t)r}\left(\frac{\sigma}{2}-\frac{r}{\sigma}\right)\ln\left(\frac{T}{t}\right), e^{-2(T-t)r}\frac{(T-t)}{tT}\right)$.

Proof. (i) By computing directly,$K(\xi,T)$ satisfies (12).
(ii) Since $K(\xi,T)$ is a Gaussian function and $K(\xi,T) \in \mathcal{L}(R)$- the space of integrable function on the real R. It follows that $K(\xi,T)$ is N a tempered distribution.

(iii) $K(\xi,T)>0$ for $0<t\leq T$ is obvious.
(iv)

$$e^{(T-t)r}\int_{-\infty}^{\infty}K(\xi,t)d\xi$$
$$=e^{(T-t)r}\int_{-\infty}^{\infty}\sqrt{\frac{tT}{2\pi(T-t)}}e^{-(T-t)r}$$
$$exp\Big[\frac{-tT\Big(\Big(\frac{r}{\sigma}-\frac{\sigma}{2}\Big)\ln\frac{T}{t}+\xi\Big)^2}{2(T-t)}\Big]d\xi$$
$$=\sqrt{\frac{tT}{2\pi(T-t)}}\int_{-\infty}^{\infty}exp\Big[\frac{-tT\Big(\Big(\frac{r}{\sigma}-\frac{\sigma}{2}\Big)\ln\frac{T}{t}+\xi\Big)^2}{2(T-t)}\Big]d\xi.$$

Let $u=\sqrt{\frac{tT}{2\pi(T-t)}}[\xi+(\frac{r}{\sigma}-\frac{\sigma}{2})\ln\frac{T}{t}$,
then $du=\sqrt{\frac{tT}{2\pi(T-t)}}d\xi$ or $d\xi=\sqrt{\frac{2(T-t)}{tT}}du.$
Thus

$$e^{(T-t)r}\int_{-\infty}^{\infty}K(\xi,t)d\xi$$
$$=\sqrt{\frac{tT}{2\pi(T-t}}\int_{-\infty}^{\infty}e^{-u^2}\sqrt{\frac{2(T-t)}{tT}}du$$
$$=\sqrt{\frac{tT}{2\pi(T-t}}\sqrt{\frac{2(T-t)}{tT}}\sqrt{\pi}$$
$$=1$$

(v) $\lim_{t\to 1}K(\xi,T)=\delta(\xi)$ by Corollary (3.2)
(vi) Since

$$K(\xi,t)$$
$$=\sqrt{\frac{tT}{2\pi(T-t)}}e^{-(T-t)r}\exp\left[\frac{-tT\Big(\Big(\frac{r}{\sigma}-\frac{\sigma}{2}\Big)\ln\frac{T}{t}+\xi\Big)^2}{2(T-t)}\right]$$
$$=e^{-(T-t)r}\sqrt{\frac{1}{2\pi}\Big(\frac{1}{t}-\frac{1}{T}\Big)}\exp\left[\frac{-\Big(\xi-\Big(\frac{\sigma}{2}-\frac{r}{\sigma}\Big)\ln\frac{T}{t}\Big)^2}{2\Big(\frac{1}{t}-\frac{1}{T}\Big)}\right]$$

Thus $K(\xi,t)$ is a Gaussian distribution with

$$\text{mean}=E\left(e^{-(T-t)r}\sqrt{\frac{1}{2\pi\Big(\frac{1}{t}-\frac{1}{T}\Big)}}\right.$$
$$\left.\exp\left[\frac{-\Big(\xi-\Big(\frac{\sigma}{2}-\frac{r}{\sigma}\Big)\ln\frac{T}{t}\Big)^2}{2\Big(\frac{1}{t}-\frac{1}{T}\Big)}\right]\right)$$
$$=e^{-(T-t)r}\Big(\frac{\sigma}{2}-\frac{r}{\sigma}\Big)\ln\frac{T}{t},$$

where E is expectation.
And

$$\text{variance}=V\left(e^{-(T-t)r}\sqrt{\frac{1}{2\pi\Big(\frac{1}{t}-\frac{1}{T}\Big)}}\right.$$
$$\left.\exp\left[\frac{-(\xi-(\frac{\sigma}{2}-\frac{r}{\sigma})\ln\frac{T}{t})^2}{2\Big(\frac{1}{t}-\frac{1}{T}\Big)}\right]\right)$$
$$=e^{-(T-t)r}V\left(\sqrt{\frac{1}{2\pi\Big(\frac{1}{t}-\frac{1}{T}\Big)}}\right.$$
$$\left.\exp\left[\frac{-(\xi-(\frac{\sigma}{2}-\frac{r}{\sigma})\ln\frac{T}{t})^2}{2\Big(\frac{1}{t}-\frac{1}{T}\Big)}\right]\right)$$
$$=e^{-2(T-t)r}\Big(\frac{1}{t}-\frac{1}{T}\Big)$$

where V is variance.
Thus $K(\xi,T)$ is

$$\Big(e^{(T-t)r}\Big(\frac{\sigma}{2}-\frac{r}{\sigma}\Big)\ln\Big(\frac{T}{t}\Big),e^{-2(T-t)r}\frac{(T-t)}{tT}\Big).$$

Note : The solution $V(\xi.t)$ of (12) is called the option price in the white noise form where the white noise ξ can be computed from (11) , now

$$V(\xi,t)=\sqrt{\frac{tT}{2\pi(T-t)}}e^{-(T-t)r}$$
$$\int_{-\infty}^{\infty}\exp\left[\frac{-tT(\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2})+\xi-y)^2}{2(T-t)}\right]f(y)dy.$$

or

$$V(\xi,t)e^{-(T-t)r}=\sqrt{\frac{tT}{2\pi(T-t)}}$$
$$\exp\left[\frac{-tT(\ln\frac{T}{t}(\frac{r}{\sigma}-\frac{\sigma}{2}+\xi-y)^2}{2(T-t)}\right]*f(\xi)$$

The left hand side of the above equation is the value of money that the option price $V(\xi,t)$ put in the Bank with the riskless interest r at the time $T-t(0\leq t\leq T)$.

4 Conclusion

It is well known that the volatility σ causes the fluctuation of the price of stock. But there is another factor that

called the white noise ξ which is not really well known. Such white noise ξ also cause the fluctuation of the price of stock. We are succeeded in formulating the white noise ξ given in (2). Such white noise ξ is helpful for the investor to estimate the expected return of the price of stock for trading.
Moreover, we can relate such white noise ξ to the Black-Scholes equation which is the important area of studying the option prices of the stock market.

Acknowledgment

The authors would like to thank Naresuan University for financial support.

References

[1] F. Black and M. Scholes, The pricing of options and Corporate Liabilities, Journal of Political Economy, Vol. 81, No.3(1973), 637-654.

[2] A. Kananthai, *On the Kernel of the Black-Scholes Equation in form of White Noise*, Applied Mathematical Science, Vol. 4(2010), 1627-1634.

[3] H. M. Kuo, *White Noise Distribution Theory*, CRC Press, Boca Raton, 1996.

[4] I. M. Gel'fand and G. E. Shilov, Generalized Functions, *Generalized Functions*, Vol. 1(1964).