Fast Estimation of Range and Bearing for a Single Near-field Source without Eigenvalue Decomposition

Hao Chen, Zhaohui Wu, Lu Gan, Xiaoming Liu, and Jinji Ma

Abstract—**In this paper, a fast range and bearing estimation method for a single near-field source is proposed. By constructing the covariance matrix of received data, the phase containing the information of range and the direction of arrival is extracted. Finally, the phase is rewritten as matrix form, and the closed-formed solution of range and bearing of the source can be calculated by least square method. In contrast to high-order statistics based methods, the proposed method can obtain the range and bearing estimation without eigenvalue decomposition and spectral search, which remarkably reduce computational complexity. Simulation results show the proposed method can provide improved estimation accuracy and higher efficiency.**

Index Terms—**Estimation of range and bearing, single source, near-field, linear antenna array.**

I. Introduction

ESTIMATION of range and bearing using array of sensors plays an important roles in microphone array, radar, sonar, and navigation. Uniform linear array is commonly used for target localization and allow for many fast estimation methods. However, the fast estimation methods are majority designed for far-field scenario, which the sources are located far from the array and the range of all sources is infinity. For the near-field sources , the sources are located close to the array, and the received data is a coupling of the direction of arrival (DOA) and range, the source localization is more complicated than the far-field scenario [1][2]. Therefore, the fast estimation methods are no longer applicable.

Many algorithms have been developed for range and bearing estimation of near-field sources, which can be mainly grouped into two categories. On one hand, there are spectral search based methods, especially the multiple signal classification (MUSIC) [3] based methods . Two-dimensional (2-D) MUSIC algorithm was utilized to near-field multiple source localization, which can achieve a sufficiently good localization performance [4]. For a spherical microphone array, spherical harmonics 2-D MUSIC was also developed [5]. However, the computational complexity is tremendous due to the 2D spectral search. To cope with this problem, covariance approximation MUSIC (CA-MUSIC) [6], symmetric partition based MUSIC [7], and two-stage MUSIC [8] were developed, in which using twice one-dimensional (1-D) spectral search to substitute the 2-D search and reducing the computational burden. To some extent, these algorithms can reduce the computational complexity. However, twice 1-D spectral search and the eigenvalue decomposition implementation still consume great computational burden. Reduced-dimensional MUSIC algorithm for near-field sources was proposed recently [9], which can obtain the angle estimation by only 1-D search. On the other hand, high-order estimation of signal parameters via rotational invariance technique (ESPRIT) algorithms are proposed, such as the second-order based method [10], four-order based method [11], and simplified high-order estimation (SHOE) method [12]. In [11], a four-order cumulant based total least square ESPRIT (TLS-ESPRIT) method was proposed for passive localization of near-field sources. High-order based algorithms always accompany with ESPRIT, hence eigenvalue decomposition is inevitable and increasing the computational complexity. In the SHOE, localization for near-field sources can be achieved based on the four-order cumulant and twice 1-D spectral search MUSIC, which is still time consuming.

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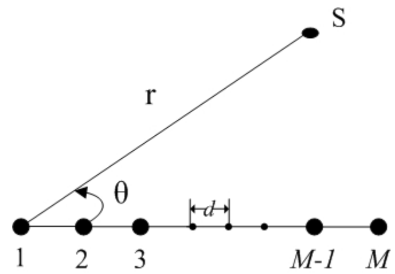

Fig. 1: Near-field ULA configuration.

In order to reduce computational burden, a fast estimation of range and bearing method is developed for a single near-field source. Neither spectrum search nor the eigenvalue decomposition operation is carried out in the proposed algorithm, which can reduce the computational complexity to a great extent. By constructing a correlation function based on the array configuration, the proposed algorithm provides a closed-form solution of the range and bearing based on the least square approach. In addition, the phase ambiguity is also considered. Compared with the SHOE and TLS-ESPRIT algorithm, the proposed algorithm can achieve better estimation accuracy and higher efficiency.

II. Signal Model

As illustrated in Fig. 1, a single narrowband near-field source impinging on a uniform linear array (ULA) with M identical sensors, in which the distance between two adjacent elements is d and (θ, r) denotes the DOA and range of the source. The received data at the mth sensor can be given by

$$x_m(l) = s(l)e^{j\psi_m} + w_m(l) \tag{1}$$

where $l = 1, 2, \cdots, L$, and L is the number of snapshots, $s(l)$ is the source signal, $w_m(l)$ is the additive Gaussian white sensor noise, ψ_m is the phase shift between the signal received by the first sensor and the m sensor. In near-field scenario, ψ_m can be denoted as [13]

$$\psi_m = \frac{2\pi r}{\lambda}\left(\sqrt{1 + \frac{m^2 d^2}{r^2} - \frac{2md\sin\theta}{r}} - 1\right) \tag{2}$$

where λ is wavelength. For the source is in the Fresnel region, which is satisfying $0.62(D^3/\lambda)^{1/2} < r < 2D^2/\lambda$, with D is the array aperture, by applying the second-order Taylor expansion, the phase difference can be expressed as

$$\psi_m \approx -\frac{2\pi d\sin\theta}{\lambda}m + \frac{\pi d^2\cos^2\theta}{\lambda r}m^2 + O\left(\frac{d^2}{r^2}\right) \tag{3}$$

Neglecting the remainder term $O\left(d^2/r^2\right)$ of Taylor formula, φ_m can be written as

$$\psi_m = \mu m + \delta m^2 + O\left(\frac{d^2}{r^2}\right) \tag{4}$$

where

$$\mu = -\frac{2\pi d}{\lambda}\sin\theta \tag{5}$$

$$\delta = \frac{\pi d^2}{\lambda r}\cos^2\theta \tag{6}$$

Then, the received signal in Eq. (1) can be represented as

$$x_m(l) = s(l)e^{j\left(-\frac{2\pi d}{\lambda}\sin\theta\right)m + j\left(\frac{\pi d^2}{\lambda r}\cos^2\theta\right)m^2} + w_m(l) \tag{7}$$

We make the following assumptions through this paper. 1) Impinging signal is circular. 2) The noise is independent from the source signal.

III. Proposed Algorithm

To estimate the DOA and range, a correction function is firstly constructed. The $(p, q)th$ element in the covariance matrix of received data is

$$\begin{aligned} R_{p,q} &= \mathbf{E}[x_p(l)x_q^*(l)] \\ &= \rho_s^2 e^{j\left(-\frac{2\pi d}{\lambda}\sin\theta\right)(p-q) + j\left(\frac{\pi d^2}{\lambda r}\cos^2\theta\right)\left(p^2-q^2\right)} + \rho_n^2 \end{aligned} \tag{8}$$

where $(\cdot)^*$ represents the complex conjugate, ρ_s^2 and ρ_n^2 are the power of signal and noise, respectively. Due to there is only a single source in the near-filed, we extract the phase of $R_{p,q}$ and obtain

$$\begin{aligned} \sigma_{p,q} &= \left(-\frac{2\pi d}{\lambda}\sin\theta\right)(p-q) + \left(\frac{\pi d^2}{\lambda r}\cos^2\theta\right)\left(p^2-q^2\right) \\ &= -\frac{2\pi d}{\lambda}\left[(p-q)\sin\theta - \left(p^2-q^2\right)\frac{d}{2r}\cos^2\theta\right] \end{aligned} \tag{9}$$

It is known that the high direction finding accuracy can be obtained by large apertures. The lager apertures, the more accurate estimation. From Eq.(3), we can obtain when $d > \lambda/2$, there will be an ambiguity in the phase, which means that the phase difference is larger than 2π. In this case, there will exist at least two direction of arrivals giving rise to the same received data. To guarantee there is no phase ambiguity in $\sigma_{p,q}$, it should satisfy the condition that $d/\lambda \leq 1/4$. Note that the problem of phase ambiguity can also be solved by modulo conversion method [14]. Then, Eq. (9) can be express as

TABLE I: Computation complexity of the methods.

Methods	Computation complexity
TLS-ESPRIT	$O(4 \times 9(\frac{M}{2})^2 L + \frac{4}{3}(\frac{3M}{2})^3)$
SHOE	$O(9M^2 L + \frac{4}{3}M^3 + WM^2)$
Proposed method	$(M-u)L + (M-u)(L-1) + O(M)$

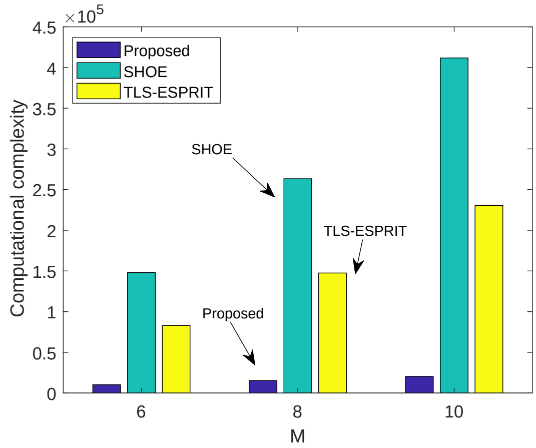

Fig. 2: Comparison of the computation complexity.

$$\sigma_{p,q} = -\frac{2\pi d}{\lambda}\begin{bmatrix} p-q \\ q^2-p^2 \end{bmatrix}^T \begin{bmatrix} \sin\theta \\ \frac{d}{2r}\cos^2\theta \end{bmatrix} \tag{10}$$

where $(\cdot)^T$ represents the transpose. Assume that $p - q = u$, we can express Eq. (10) in matrix form as

$$\boldsymbol{\sigma} = \mathbf{ZB} \tag{11}$$

where

$$\boldsymbol{\sigma} = \left[\sigma_{1,1+u}, \sigma_{2,2+u}, \cdots, \sigma_{M-u,M}\right]^T \tag{12}$$

$$\mathbf{Z} = -\frac{2\pi d}{\lambda}\begin{bmatrix} u & (1+u)^2 - 1^2 \\ u & (2+u)^2 - 2^2 \\ \vdots & \vdots \\ u & M^2 - (M-u)^2 \end{bmatrix} \tag{13}$$

$$\mathbf{B} = \begin{bmatrix} \sin\theta \\ \frac{d}{2r}\cos^2\theta \end{bmatrix} \tag{14}$$

Note that u is a constant, which can be chosen as $1, 2, \cdots, M-1$. To exploit the greatest degree of array aperture, we chose $u = 2$ in all of the simulations.

In practical, assume $\hat{R}_{p,q}$ is the estimate of $R_{p,q}$, and $\hat{\sigma}_{p,q}$ is associated phase, by utilizing the least square method, the estimate of $\hat{\mathbf{B}}$ can be obtained as

$$\hat{\mathbf{B}} = \left[\hat{b}_1, \hat{b}_2\right]^T = \left(\mathbf{Z}^T\mathbf{Z}\right)^{-1}\mathbf{Z}^T\hat{\boldsymbol{\sigma}} \tag{15}$$

where $\hat{\boldsymbol{\sigma}} = [\hat{\sigma}_{1,1+u}, \hat{\sigma}_{2,2+u}, \cdots, \hat{\sigma}_{M-u,M}]^T$. According to Eq. (14), θ and r can be estimated as

$$\hat{\theta} = \arcsin(\hat{w}_1) \tag{16}$$

$$\hat{r} = \frac{d\cos^2\hat{\theta}}{2\hat{w}_2} \tag{17}$$

To analysis the estimation efficiency of the proposed method, we compare the computation complexity of the proposed algorithm with the TLS-ESPRIT based algorithm and the SHOE. As shown in Table 1, for a single source, the main computation load of the TLS-ESPRIT lies in constructing cumulant matrices and performing the EVD. Summing these two operations, the total computation load of the TLS-ESPRIT is $O(4\times 9(\frac{M}{2})^2L+\frac{4}{3}(\frac{3M}{2})^3)$. While for the SHOE, twice 1-D spectrum search is needed. For W points spectrum search, the total computation load of the SHOE is roughly $O(9M^2L + \frac{4}{3}M^3 + WM^2)$. In comparison, the proposed algorithm needs neither constructing cumulant matrices nor performing the EVD, it only requires to formulate multiply operation in Eq. (8), and LS calculation in Eq. (15), and the total computation complexity is $(M-u)L+(M-u)(L-1)$. Fig. 2 shows the computational complexity of the above three methods with $L = 256$, $u = 2$, and the search step is $0.1°$, from which we can obtain that the proposed method has a much higher computational efficiency than the TLS-ESPRIT based algorithm and SHOE.

The advantages of the proposed method are as follow:

1) Without performing spectral search and eigenvalue decomposition, the proposed method has higher computational efficiency than the TLS-ESPRIT based algorithm and SHOE. 2) The proposed algorithm can provide closed-form solution of the range and bearing, and the has sufficiently good estimation accuracy.

IV. Cramer Rao lower bound

The Cramer Rao lower bound (CRLB) is an important reference to weigh the performance of an estimation algorithm. The inverse of the Fisher information matrix (FIM) is the CRLB, which bounds the error variance of the estimation from the radar system. According to the signal and system models, we can describe the FIM as

$$\mathbf{FIM} = \frac{2L}{\sigma_n^2}\left\{\mathrm{Re}\left[\left(\mathbf{D}^H\mathbf{\Pi}_A^{\perp}\mathbf{D}\right)\odot\mathbf{R}_S^T\right]\right\} \tag{18}$$

where $\mathbf{D} = [\mathbf{D}_\theta, \mathbf{D}_r]$ with $\mathbf{D}(\theta) = \frac{\partial\mathbf{A}(\theta,r)}{\partial r}$, $\mathbf{D}(r) = \frac{\partial\mathbf{A}(\theta,r)}{\partial\theta}$, and $\mathbf{A} = [e^{j\psi_1}, e^{j\psi_2}, \cdots, e^{j\psi_M}]^T$. Moreover, $\mathbf{\Pi}_A^{\perp} = \mathbf{I} - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$, $\mathbf{R}_S^T = \mathbf{S}\mathbf{S}^H/L$, and $\mathbf{S} = [s(1), s(2), \cdots, s(L)]$. Define $\mathbf{F}$ indicates the FIM for target, which can be written as

$$\mathbf{F}(\theta, r) = \begin{bmatrix} \mathbf{F}_{\theta\theta} & \mathbf{F}_{\theta r} \\ \mathbf{F}_{r\theta} & \mathbf{F}_{rr} \end{bmatrix} \tag{19}$$

where

$$\mathbf{F}_{f,g} = \frac{2L}{\sigma_n^2}\left\{\mathrm{Re}\left[\left((\mathbf{D}(f))^H\mathbf{\Pi}_A^{\perp}(\mathbf{D}(g))\right)\odot\mathbf{R}_S^T\right]\right\} \tag{20}$$

and $(f, g) \in \{\theta, r\}$

V. Simulation results

In this section, several simulations are carried out to verify the performance of the proposed algorithm, which are also compared with the TLS-ESPRIT [11], the SHOE [12], and the CRLB [15]. For the simulations, a ULA with $M = 9$ and where K=500 is the number of Monte Carlo trials, $\hat{x}_i$ is the estimated DOA or range of the kth trail, and x is the corresponding real value. $d = \lambda/4$ is utilized, and the noise

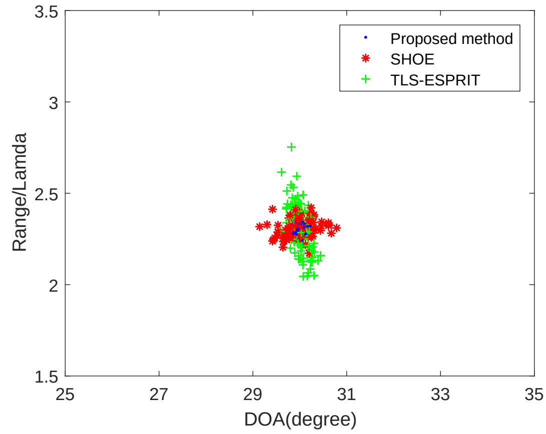

Fig. 3: Scatter plot for $\theta = 30°$ and $r = 2.3\lambda$.

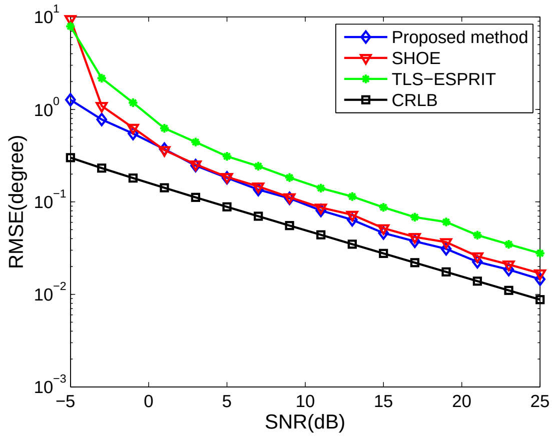

Fig. 4: RMSE versus the SNR of DOA estimation.

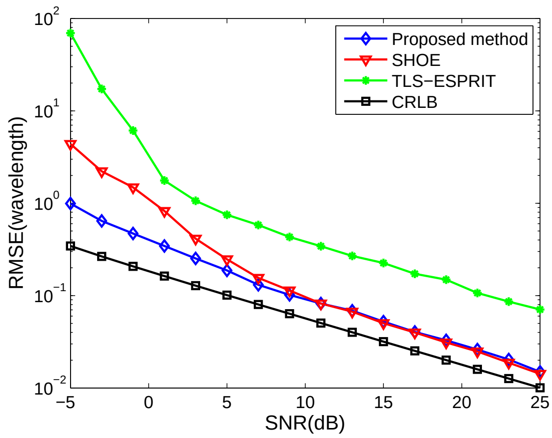

Fig. 5: RMSE versus the SNR of range estimation.

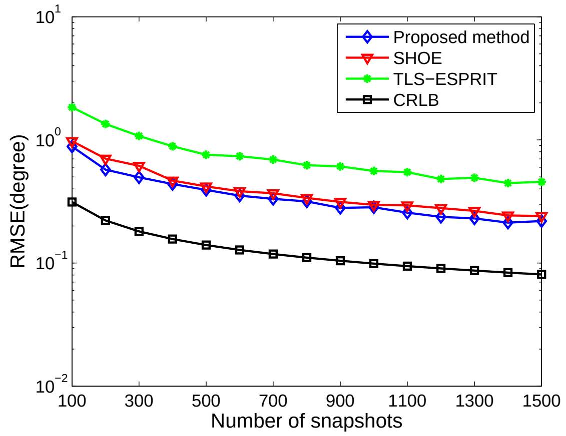

Fig. 6: RMSE versus the number of snapshots of DOA estimation.

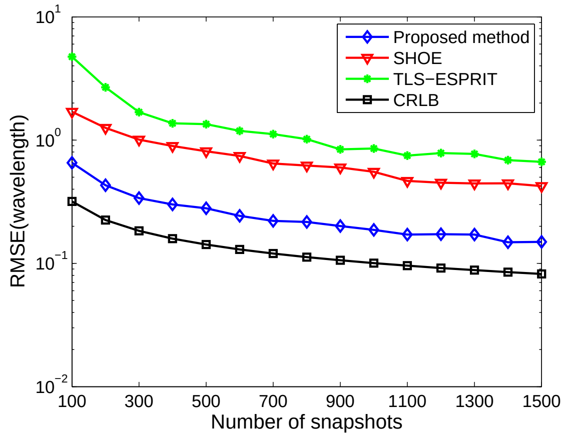

Fig. 7: RMSE versus the number of snapshots of range estimation.

is additive white Gaussian noise. A single source is located at $(\theta, r) = (30^\circ, 2.3\lambda)$. We define the root mean square error (RMSE) of the estimate DOA and range as

$$RMSE = \sqrt{\frac{1}{K}\sum_{k=1}^{K}(\hat{x}_k - x)^2} \tag{21}$$

Fig. 3 shows the scatter plot for $L = 300$, the signal-to-noise ratio (SNR) is 10 dB, the number of Monte Carlo is 200. We can obtain that the proposed methods holds a better localization accuracy than the TLS-ESPRIT method and SHOE.

As shown in Figs. 4 and 5, the RMSE of the DOA and range against the SNR, respectively. The number of snapshots is set as $L = 300$, and the SNR is varying from -5 to 25 dB. From these figures, we can see that the proposed algorithm has a lower RMSE than the SHOE and TLS-ESPRIT algorithm for both DOA and range estimation. Note that the estimate performance is distinctly enhanced at low SNR, which is ascribed to the proposed algorithm directly extract phase operation to restrain the influence of noise.

As depicted in Figs. 6 and 7, we analysis the RMSE of the DOA and range versus the number of snapshots. The SNR is fixed as 0 dB, and the number of snapshots is varying from 100 to 1500. For the DOA estimation, although the proposed

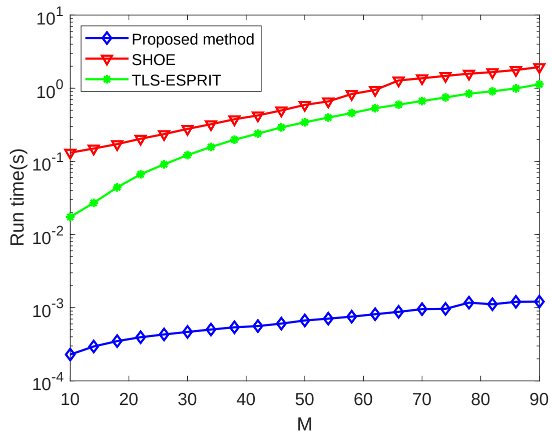

Fig. 8: Run time versus the number of sensors.

algorithm exhibits similar RMSE to the SHOE, it is superior to that of the TLS-ESPRIT. For the range estimation, the RMSE of the proposed method is lower than that of the other two algorithms.

To demonstrate the computational efficiency, the simulation time of the proposed algorithm is compared with that of the SHOE and the TLS-ESPRIT. The number of snapshots is fixed at 256, and the SNR is 20, $M = 10$. The simulations are carried out at MATLAB platform with a PC of Inter(R) Core(TM) i5-4440 CPU and 8G RAM. The results are averaged over 500 runs. For the search interval is 0.01°, the computation consume of the SHOE, the TLS-ESPRIT, and the proposed algorithm are 0.3029, 0.0113, and 1.3879×10^{-4} s, respectively. The runtime versus sensor number is depicted in Fig. 8, in which we can see that the proposed algorithm has much lower time consuming than the SHOE and the TLS-ESPRIT.

VI. Conclusion

A fast estimation of range and bearing algorithm is proposed for near-field source in this paper. To utilize the phase performance of received data for the single source, a correlation function matrix is constructed. The phase contains information about the range and DOA, which can be expressed as matrix multiply form. The closed-form solution of range and DOA can be obtained by least square method for the matrix. In addition, the CLRB of estimation is also derived. Compared with the TLS-ESPRIT and SHOE, the proposed method has higher accuracy and efficiency for escaping from spectral search and eigenvalue decomposition. Simulation results verify the performance advantages of the proposed method.

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