Determination of Cross-efficiency Considering the Original Efficiency Value of DMUs in DEA Cross-evaluation

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***Abstract*—Data envelopment analysis (DEA) has been extended to cross-efficiency evaluation to provide better discrimination and ranking of decision-making units (DMUs). However, the non-uniqueness of optimal solutions in CCR model damages the usefulness of DEA cross-efficiency evaluation method. To solve this problem, this paper proposes three secondary goal models considering the original efficiency value of DMUs based on the aggressive, benevolent and neutral idea. Finally, the numerical example proves that the proposed models can play a significant role in reducing the number of zero weights for both inputs and outputs in cross-efficiency evaluation.**

***Index Terms*—CCR, DEA Cross-efficiency evaluation, Secondary goal models, Original efficiency value.**

I. Introduction

CCR model proposed by Charnes, Cooper, and Rhodes is a non-parametric method for efficiency evaluation of a group of homogenous decision-making units (DMUs) where multiple inputs are consumed to produce multiple outputs [1]. For its functional ability in efficiency evaluation and identifying the production frontier, it has been widely used for efficiency evaluation of schools, hospitals, colleges in university and so on [2].

Since CCR model allows each DMU to evaluate its efficiency with its most favorable weights, more than one DMU is often evaluated to be DEA efficient and cannot be discriminated further[3]. So, lack of discrimination power is the major drawback that CCR suffers from. To improve the discrimination power of CCR, DEA cross-efficiency evaluation was proposed by Sexton et al. [4]. Unlike the CCR model where each DMU uses its own most favorable weights for efficiency evaluation, it employs both self-evaluation and peer-evaluation. In DEA cross-efficiency evaluation, each DMU will obtain one self-evaluated efficiency based on its most favorable weights and $n-1$ peer-evaluated efficiencies using the most favorable weights of other $n-1$ DMUs. Then all these n efficiencies for each DMU are averaged into a value to be its average cross-efficiency value. Based on the average cross-efficiency value of each DMU, DEA cross-efficiency evaluation can provide a unique rank order to DMUs in most practical situations [5]. Due to its powerful discrimination, DEA cross-efficiency evaluation method has been widely applied in efficiency evaluation of countries in the Olympic Games [6], project ranking and preference voting [7], portfolio selection in Korean stock market [8] and so on.

However, for each DMU the optimal weights solution in CCR model is usually not unique and the problem of non-uniqueness of optimal weights will undermine the usefulness of DEA cross-efficiency evaluation. To solve this problem, Sexton et al. suggested using secondary goal models [4]. Inspired by this idea, many secondary goal models have been proposed. The aggressive (benevolent) model proposed by Doyle and Green minimizes (maximizes) the average cross-efficiency of other DMUs while keeping the efficiency value of DMU under evaluation at its CCR level [9]. Based on the aggressive

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and benevolent ideas, many other scholars offered a series of aggressive and benevolent models [10-12]. In most practical cases, the aggressive and benevolent models will generate different efficiency results and rank orders for DMUs. To avoid the choice difficulty between aggressive and benevolent models, neutral model was introduced [13-15]. Different from the aggressive and benevolent models, neutral model conducted optimal weights selection only from the viewpoint of DMU under evaluation without considering whether it is aggressive or benevolent to other DMUs. The weight-balanced model aims to lessen the difference in weighted data and reduce the number of zero-weights [16]. The models introduced by Wang, Chin, and Wang, Jahanshahloo et al., Ramón, Ruiz, and Sirvent are with similar idea [17-19]. The rank model considers that in some cases pursuing the best ranking is more important than maximizing the individual score [20, 21]. From the goal functions of the models mentioned above, we can acquire that they only consider the impact of selected weights to standard efficiencies of DMUs ignoring their impact to the original efficiencies of DMUs.

In this paper, we will provide a new perspective to understand DEA and construct some new DEA cross-efficiency models. The proposed models here focused on the impact of selected weights to the original efficiency value of DMUs. The rest of the paper unfolds as follows. Section 2 briefly introduces the DEA cross-efficiency evaluation and offers new models. Chapter 3 gives a numerical example, and conclusions are given in Part 4.

II. Cross-Efficiency Evaluation and Newly Proposed Models

Suppose that there are n DMU_S to be evaluated where m inputs are consumed to produce s outputs. The inputs and outputs value of $DMU_j(j=1,\ldots,n)$ are denoted by $x_{ij}(i=1,\ldots,m)$ and $y_{rj}(r=1,\ldots,s)$. The ratio $\sum_{r=1}^{s}u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij}$ denotes the efficiency value of DMU_j. The efficiency value of DMU_k under CCR is calculated by model (1), Where $DMU_k \in \{DMU_1,\ldots,DMU_n\}$ is the decision-making unit (DMU) under evaluation, the inputs and outputs weights

$$\begin{aligned} &\text{Maximize} \quad \theta_{kk}=\sum_{r=1}^{s}u_{rk}y_{rk} \\ &\text{Subject to} \quad \sum_{i=1}^{m}v_{ik}x_{ik}=1 \\ &\qquad \sum_{r=1}^{s}u_{rk}y_{rj}-\sum_{i=1}^{m}v_{ik}x_{ij}\le 0,\ j=1,\ldots,n, \\ &\qquad u_{rk},v_{ik}\ge 0,\ r=1,\ldots,s;\ i=1,\ldots,m \end{aligned} \tag{1}$$

are denoted by $v_{ik}(i=1,\ldots,m)$ and $u_{rk}(r=1,\ldots,s)$. If $u_{rk}^*(r=1,\ldots,s)$ and $v_{ik}^*(i=1,\ldots,m)$ are the optimal solution to the above CCR model, the $\theta_{kk}^*=\sum_{r=1}^{s}u_{rk}^* y_{rk}$ will be the CCR efficiency value of DMU_k. If θ_{kk}^* is equal to 1, the DMU_k will be referred to as DEA efficient; Otherwise, it will be non-DEA efficient. $\theta_{jk}=\sum_{r=1}^{s}u_{rk}^* y_{rj} / \sum_{i=1}^{m}v_{ik}^* x_{ij}$ is referred to as a cross-efficiency of DMU_j and reflects the peer evaluation of DMU_k to DMU_j $(j=1,\ldots,n,\ j\ne k)$.

CCR model (1) is solved for each of n DMU_S respectively. As a result, there will be n sets of inputs and outputs weights available for n DMUs and based on them, each DMU will gain n efficiency values which include 1 self-evaluated efficiency value and $n-1$ peer-evaluated efficiency values, which form a cross-efficiency matrix shown in table 1. They are usually aggregated by equal weights to obtain average cross-efficiency (ACE) value for each DMU, and based on which the DMU_S can be fully ranked.

TABLE I
CROSS-EFFICIENCY MATRIX

DMU	Target DMU				Average Cross-Efficiency
	1	2	$\ldots$	n	
1	θ_{11}	θ_{12}	$\ldots$	θ_{1n}	$\frac{1}{n}\sum_{k=1}^{n}\theta_{1k}$
2	θ_{21}	θ_{22}	$\ldots$	θ_{2n}	$\frac{1}{n}\sum_{k=1}^{n}\theta_{2k}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	θ_{n1}	θ_{n2}	$\ldots$	θ_{nn}	$\frac{1}{n}\sum_{k=1}^{n}\theta_{nk}$

It is noticed that the optimal solution in the CCR model (1) may be not unique that will damage the usefulness of cross-efficiency evaluation. To handle this problem, Sexton et al. [4] introduce the concept of secondary goal model. Inspired by this idea, many secondary goal models have been proposed. Among them, the aggressive, benevolent and neutral models are widely used. Their formulations are as stated by the model (2), (3) and (4). The aggressive (benevolent) model minimizes (maximizes) the average efficiency value of other DMUs while keeping the efficiency value of DMU under evaluation at its CCR efficiency. The neutral model selects the unique set of weights only from the viewpoint of DMU under evaluation without considering whether it is aggressive or benevolent to other DMUs. The above neutral model searches for a set of input and output weights to maximize its efficiency as a whole and at the same time to make its each output being as efficient as possible to produce sufficient efficiency as an individual [13]. From the formulations of the above models, it is clearly shown that the weights in models need to assure the efficiency values of all DMUs not more than 1. That means the efficiency results calculated by the CCR model are standard values and the existing DEA cross-efficiency models selected the weights only considering their impact to standard efficiency values of DMUs ignoring their impact to the original efficiencies of DMUs. Different from them, we proposed models from an original efficiency viewpoint. Under the original efficiency perspective, the above CCR model will be reformulated to be model (5). Different from the model (1) the weights in model (5) are denoted by new letters. It means if there exists no less than one set of weights meeting the restraint in above model that can make the efficiency value of DMU_k maximal among all DMUs, the DMU_k will be DEA efficient. We let μ_r be equal to $u_r \Big/ \sum_{r=1}^{s} u_r$, v_i be equal to $v_i \Big/ \sum_{i=1}^{m} v_i$ and

$$
\begin{aligned}
&\text{Minimize} \quad \sum_{r=1}^{s} u_{rk} \left(\sum_{j=1,\, j\neq k}^{n} y_{rj} \right) \\
&\text{Subject to} \sum_{i=1}^{m} v_{ik} \left(\sum_{j=1,\, j\neq k}^{n} x_{ij} \right) = 1 \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rk} - \theta_{kk}^{*} \sum_{i=1}^{m} v_{ik} x_{ik} = 0 \qquad\qquad (2) \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \quad j = 1, \ldots, n, \ j \neq k, \\
&\qquad u_{rk} \ge 0, \quad r = 1, \ldots, s, \\
&\qquad v_{ik} \ge 0, \quad i = 1, \ldots, m,
\end{aligned}
$$

$$
\begin{aligned}
&\text{Maximize} \quad \sum_{r=1}^{s} u_{rk} \left(\sum_{j=1,\, j\neq k}^{n} y_{rj} \right) \\
&\text{Subject to} \sum_{i=1}^{m} v_{ik} \left(\sum_{j=1,\, j\neq k}^{n} x_{ij} \right) = 1 \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rk} - \theta_{kk}^{*} \sum_{i=1}^{m} v_{ik} x_{ik} = 0 \qquad\qquad (3) \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \quad j = 1, \ldots, n, \ j \neq k, \\
&\qquad u_{rk} \ge 0, \quad r = 1, \ldots, s, \\
&\qquad v_{ik} \ge 0, \quad i = 1, \ldots, m,
\end{aligned}
$$

$$
\begin{aligned}
&\text{Maximize} \quad \delta = \underset{r\in\{1,\ldots,s\}}{\text{minimum}} \left\{ \frac{u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \right\} \\
&\text{subject to} \quad \theta_{kk}^{*} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \\
&\theta_{jk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1, \quad j = 1, \ldots, n, \ j \neq k, \qquad\qquad (4) \\
&\qquad u_{rk} \ge 0, \quad r = 1, \ldots, s, \\
&\qquad v_{ik} \ge 0, \quad i = 1, \ldots, m,
\end{aligned}
$$

$$
\begin{aligned}
&\text{Maximize} \quad \theta_{kk} = \frac{\sum_{r=1}^{s} \mu_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \Big/ \underset{j=1,\cdots,n}{\text{Maximize}} \left\{ \frac{\sum_{r=1}^{s} \mu_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \right\} \\
&\text{Subject to} \quad \sum_{r=1}^{s} \mu_{rk} = 1, \\
&\qquad \sum_{i=1}^{m} v_{ik} = 1, \qquad\qquad (5) \\
&\mu_{rk} \ge 0, \quad r = 1, \cdots, s, \\
&v_{ik} \ge 0 \quad i = 1, \cdots, m,
\end{aligned}
$$

the model (5) will be transformed to be the linear model (1) through Charnes-Cooper transformation. The two models are equivalent. Because the weights of two models are one-to-one correspondence, accordingly the original efficiency values and standard values are also one-to-one correspondence. Considering the original efficiency values, the DMU under evaluation will minimize (maximize) the average original efficiency value of other DMUs while selecting unique set of weights among many weights solutions in CCR model based on aggressive (benevolent) idea. The DMU under evaluation will maximize its original efficiency value based on neutral idea. The modeling mechanism is more transparent and more accessible to understand than that considering standard efficiency values. Inspired by this, the formulations of newly proposed aggressive, benevolent and neutral models are illustrated by model (6), (7) and (8) respectively. The goal functions of the above models concern the original efficiencies of DMUs.

$$Minimize \quad \frac{1}{n-1}\times\frac{\sum_{i=1}^{m} v_i}{\sum_{r=1}^{s} u_r}\times\sum_{j=1, j\neq k}^{n}\theta_{jk}$$
$$subject\ to\ \ \theta_{kk}^{*} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}}$$
$$\theta_{jk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1,\ \ j = 1,\ldots,n,\ j\neq k, \tag{6}$$
$$u_{rk} \ge 0,\ \ r = 1,\ldots,s,$$
$$v_{ik} \ge 0,\ \ i = 1,\ldots,m,$$

$$Maximize \quad \frac{1}{n-1}\times\frac{\sum_{i=1}^{m} v_i}{\sum_{r=1}^{s} u_r}\times\sum_{j=1, j\neq k}^{n}\theta_{jk}$$
$$subject\ to\ \ \theta_{kk}^{*} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}}$$
$$\theta_{jk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1,\ \ j = 1,\ldots,n,\ j\neq k, \tag{7}$$
$$u_{rk} \ge 0,\ \ r = 1,\ldots,s,$$
$$v_{ik} \ge 0,\ \ i = 1,\ldots,m,$$

$$Maximize \quad \frac{\left(\sum_{r=1}^{s} u_{rk} y_{rk} \Big/ \sum_{r=1}^{s} u_{rk}\right)}{\left(\sum_{i=1}^{m} v_{ik} x_{ik} \Big/ \sum_{i=1}^{m} v_{ik}\right)}$$
$$Subject\ to \quad \theta_{kk}^{*} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}}$$
$$\theta_{jk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1,\ \ j = 1,\ldots,n,\ j\neq k, \tag{8}$$
$$u_{rk} \ge 0,\ \ r = 1\cdots,s$$
$$v_{ik} \ge 0,\ \ i = 1\cdots,m$$

III. NUMERICAL EXAMPLE

This part will provide a numerical example to demonstrate the efficiency results difference between the proposed models and the models considering standard efficiency value of DMUs.

Numerical example: Efficiency evaluation of seven departments in a university [22]. Seven academic departments are needed to be evaluated with three inputs and three outputs. The inputs are number of academic staff, academic staff salaries in thousands of pounds and support staff salaries in thousands of pounds. The outputs are number of undergraduate students, number of postgraduate students, number of research papers. Table 2 shows the input and output data and CCR results of DMUs. From it, it is clearly shown that six of seven are DEA efficient and efficient DMUs cannot be further distinguished. Table 3 shows the efficiency evaluation results through aggressive, benevolent, neutral and proposed models. The results are shown to be different which can indicate the difference between considering original efficiency values and considering standard values when constructing DEA cross-efficiency secondary goal models. To further illustrate their difference, table 4 to 9 shows the unique set of weights selected by the models and they are obviously different. Moreover, it also clearly shows that the weights selected by aggressive, benevolent and neutral models contain many zero weights. Meantime, the zero weights chosen by the neutral model are only in inputs part. That means many inputs and outputs information are ignored when generating cross-efficiencies through aggressive, benevolent and neutral models. This situation can result in the ultimate efficiency results to be unreasonable. From table 7 to 9, it clearly shows the proposed models can significantly reduce the number of zero weights. That indicates the efficiency results generated by the proposed models to be more reasonable because in reality each input or output is critical and none of them can be ignored.

TABLE II

INPUTS AND OUTPUTS DATA AND CCR VALUES

DMUs	Inputs			Outputs			CCR
	x_1	x_2	x_3	y_1	y_2	y_3	
1	12	400	20	65	35	17	1
2	19	750	70	139	41	40	1
3	42	1500	70	225	68	75	1
4	15	600	100	90	12	17	0.8197
5	45	2000	250	253	145	130	1
6	19	730	50	132	45	45	1
7	41	2350	600	305	159	97	1

IV. CONCLUSION

Aiming at solving the problem of non-uniqueness of optimal weights in DEA cross-efficiency evaluation, we propose three DEA cross-efficiency models considering the original efficiency values of DMUs based on aggressive, benevolent and neutral notions.

The proposed models bring at least three contributions to DEA. Firstly, the concept of original efficiency values of DMUs is incorporated into DEA cross-efficiency evaluation. Secondly, the modeling mechanism of proposed models is more precise. Thirdly, the proposed models can significantly reduce the number of zero weights, so the efficiency results generated by them are more reasonable. About the further research direction based on this paper, the interesting readers can enrich the models considering the original efficiency values of DMUs based on other ideas such as the idea of the rank model and so on.

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TABLE III

EFFICIENCY EVALUATION RESULTS THROUGH THE MODELS

DMUs	Aggressive	Benevolent	Neutral	Model (6)	Model (7)	Model (8)
1	0.8788 (1)	0.9442 (3)	0.9362 (2)	0.8919(2)	0.8350(3)	0.8302 (3)
2	0.7219 (4)	0.9486 (2)	0.9026 (3)	0.8529(3)	0.8879(2)	0.8458 (2)
3	0.7301 (3)	0.7827 (6)	0.7763 (6)	0.7770(5)	0.8268(4)	0.7991 (4)
4	0.4018 (7)	0.6160 (7)	0.5649 (7)	0.5067(7)	0.5648(7)	0.5226 (7)
5	0.6259 (5)	0.8534 (5)	0.8272 (5)	0.8171(4)	0.7122(5)	0.7366 (5)
6	0.8126 (2)	0.9801 (1)	0.9493 (1)	0.9363(1)	0.9579(1)	0.9309 (1)
7	0.5966 (6)	0.8992 (4)	0.8552 (4)	0.6766(6)	0.6591(6)	0.6515 (6)

TABLE IV

WEIGHTS SELECTED BY AGGRESSIVE MODEL

DMUs	x_1	x_2	x_3	y_1	y_2	y_3
1	0	0	0.0009	0	0.0005	0
2	0	0.0001	0.0001	0.0007	0	0
3	0	0	0.0009	0.0003	0	0
4	0.0054	5.3E-06	0	0.0008	0	0
5	0.0043	0	0.0004	0	0	0.0023
6	0.0010	0	0.0007	0	0	0.0012
7	0.0066	0	0	0	0.0017	0

TABLE V

WEIGHTS SELECTED BY BENEVOLENT MODEL

DMUs	x_1	x_2	x_3	y_1	y_2	y_3
1	0.0020	8.1E-05	0	0.0003	0.0009	0.0003
2	0.0029	6.6E-05	0	0.0006	0.0006	0
3	0	2.9E-05	0.0007	0	0.0002	0.0010
4	0.0054	5.3E-06	0	0.0008	0	3.2E-11
5	0.0025	0.0001	0	0.0004	0.0011	0.0004
6	0.0021	8.4E-05	0	0.0003	0.0009	0.0003
7	0.0025	0.0001	0	0.0004	0.0012	0.0004

TABLE VI

WEIGHTS SELECTED BY NEUTRAL MODEL

DMUs	x_1	x_2	x_3	y_1	y_2	y_3
1	0	0.0017	0.0162	0.0056	0.0095	0.0196
2	0.0371	0.0004	0	0.0053	0.0032	0.0033
3	0	0	0.0143	0.0008	0.0026	0.0085
4	0.0642	6.29E-05	0	0.0091	1.60E-10	1.13E-10
5	0.0108	0.0003	0	0.0013	0.0023	0.0026
6	0	0.0014	0	0.0025	0.0074	0.0074
7	0.0174	0.0001	0	0.0011	0.0021	0.0034

TABLE VII

WEIGHTS SELECTED BY MODEL (6)

DMUs	x_1	x_2	x_3	y_1	y_2	y_3
1	0.1724	0.1061	0.3219	0.1847	1.0442	0.1951
2	0.0000	0.2259	0.0000	1.0915	0.2854	0.1497
3	0.0004	7.7997	165.7457	0.0002	0.0006	310.6892
4	0.2648	0.0731	0.0018	0.4274	0.0045	0.0073
5	0.7942	0.5732	0.0048	0.1530	3.0743	5.3753
6	1.4935	1.0691	0.4550	3.9806	2.9201	3.8835
7	2.1224	0.0566	0.0046	0.0542	1.1923	0.1716

TABLE VIII

WEIGHTS SELECTED BY MODEL (7)

DMUs	x_1	x_2	x_3	y_1	y_2	y_3
1	1.5088	0.0252	1.0635	0.6674	0.2683	0.0005
2	1.5577	0.0559	0.0061	0.4302	0.1793	0.1195
3	0.1732	0.1108	3.3974	1.5306	0.1822	0.7264
4	4.3444	0.0043	0.0000	0.6172	0.0000	0.0001
5	1.9763	0.0208	0.0084	0.0043	0.3013	0.6754
6	1.9357	0.0012	0.7566	0.4728	0.0915	0.1997
7	3.5260	0.0203	0.0028	0.5313	0.1954	0.0082

TABLE IX

WEIGHTS SELECTED BY MODEL (8)

DMUs	x_1	x_2	x_3	y_1	y_2	y_3
1	0.5233	0.0868	0.8751	0.1983	1.1951	0.2820
2	1.1679	0.0878	0.0004	0.5685	0.1449	0.0775
3	0.1871	0.0134	1.3189	0.0635	0.2586	1.1791
4	1.6167	0.0016	0	0.2297	0	0
5	2.9180	0.0224	0.0287	0.0579	0.3724	0.8820
6	9.8724	0.0063	4.2159	2.8648	0.3116	0.2404
7	2.9744	0.0104	0.0017	0.3479	0.0497	0.3441

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