Permanence and Uniform Asymptotical Stability in Consumer-Resource System with Mutual Interference on Time Scales

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Abstract—**This paper is concerned with a consumer-resource system described by a mutual interference model with feedback controls on time scale. By using time scale calculus theory and differential inequality, some verifiable conditions are obtained for the permanence of the above system. Further, by some important inequalities, the uniform asymptotical stability of the model have been studied. An example and numerical remarks are provided to illustrate the main results of this paper. Finally, a conclusion is also given to discuss how the feedback control and mutual interference of the system influence the permanence.**

Index Terms—**Consumer-resource; permanence; feedback control; mutual interference; asymptotical stability.**

I. Introduction

IT is well known that the dynamic relationship between predator and prey has been extensively studied in both economy and mathematical ecology by several scholars[1-7]. However, there were few papers considering the mutual interference until Hassell introduced it as constant $m(0 < m \leq 1)$ in 1971, see paper [8-10] for more details. On the basis of the results in[8], further research was done by Feng Zhang in[11], the system as follows:

$$\begin{cases} x'(t) = x(t)[a(t) - b(t)x^{\theta}(t)] - a(t)x(t)y^{m}(t), \\ y'(t) = y(t)[d(t) - e(t)\frac{y(t)}{x(t)}], \end{cases} \tag{1.1}$$

where $a(t)$ and $d(t)$ stand for the intrinsic growth rates of the prey and the predator at time t, respectively, $\frac{a(t)}{b(t)}$ is the carrying capacity of the prey in the absence of predation, $e(t)$ is a measure of the food quantity that prey provides and converted to predator birth, $0 < m, \theta \leq 1$.

On the other hand, the consumer-resource(C-R) interaction, which has become a central principle for understanding interspecific interactions, play a very important role and received much attention [12-16]. Motivated by the above reasons and based on the system (1.1), we present a specific consumer-resource system described by a model with mutual interference and feedback control:

$$\begin{cases} N_1'(t) &= N_1(t)[a(t) - b(t)N_1^{\theta}(t) - c(t)N_2^{m}(t)] \\ & \quad -\alpha(t)u(t), \\ u'(t) &= -f_1(t)u(t) + p_1(t)N_1(t-\tau), \\ N_2'(t) &= N_2(t)[d(t) - e(t)\frac{N_2(t)}{N_1(t)}] - \beta(t)v(t), \\ v'(t) &= -f_2(t)v(t) + p_2(t)N_2(t-\tau), \end{cases} \tag{1.2}$$

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where $N_1(t)$ and $N_2(t)$ represent the population density of the resource and the consumer at time t, respectively. $a(t)$ and $d(t)$ denote the intrinsic growth rate of the resource and consumer, $\frac{a(t)}{b(t)}$ stands for its carrying capacity when in isolation from the resource, $c(t)N_1(t)$ represents the decrease in the growth of the resource$e(t)$ is a measurement of consumption of resource, $0 < m, \theta \leq 1$, $u(t)$ and $v(t)$ denote the feedback control variables. If $m = 1$, there is no mutual interference between the consumer and resource, hence in this paper we just consider the system with $0 < m < 1$. But any economy models are set up in either continuous or discrete model. So, time scale approach is more flexible and realistic. Motivated by the above reasons and based on the system (1.2), we present a specific consumer-resource system described by a model with mutual interference and feedback control on time scale:

$$\begin{cases} N_1^{\Delta}(t) &= a(t) - b(t)\exp\{\theta N_1(t)\} \\ & \quad -c(t)\exp\{mN_2(t)\} - \alpha(t)u(t), \\ u^{\Delta}(t) &= -f_1(t)u(t) + p_1(t)\exp\{N_1(t-\tau)\}, \\ N_2^{\Delta}(t) &= d(t) - e(t)\exp\{N_2(t) - N_1(t)\} - \beta(t)v(t), \\ v^{\Delta}(t) &= -f_2(t)v(t) + p_2(t)\exp\{N_2(t-\tau)\}, \end{cases} \tag{1.3}$$

where all the variables have the same meaning of the system (1.2) and are rd-continuous positive. Clearly, if we choose $\mathbb{T} = \mathbb{R}$, then the system (1.3) can be reduced to (1.2). System (1.3) satisfies the initial values:

$$\begin{cases} N_1(s) = \varphi(s) \geq 0, s \in [-\tau, 0)_T, \\ \varphi \in C_{rd}([-\tau, 0)_T, R), \varphi(0) > 0; \\ N_2(s) = \phi(s) \geq 0, s \in [-\tau, 0)_T, \\ \phi \in C_{rd}([-\tau, 0)_T, R), \phi(0) > 0; \\ u(0) > 0, v(0) > 0. \end{cases}$$

This article is organized as follows: Section 2 provides some definitions and notations, we show the permanence of the system (1.3) in section 3, Finally we give some examples.

II. Preliminaries

In this section, we shall first recall some basic definitions and lemmas on time scales, which can be found in [17].

Definition 1. ([17]) A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$, the set $\mathbb{T}$ inherits the standard topology of $\mathbb{R}$.

Definition 2. ([17]) The forward and the backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$, and the graininess $\mu : \mathbb{T} \to \mathbb{R}^{+} = [0, \infty)$ are defined, respecting, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\},$$

$$\rho(t) = \sup\{s \in \mathbb{T} : s < t\},$$

$$\mu(t) = \sigma(t) - t \ \ for \ \ t \in \mathbb{T}.$$

If $\sigma(t) = t$, then t is called right-dense (otherwise: right-scattered), and if $\rho(t) = t$, then t is called left-dense (otherwise: left-scattered).

Definition 3. ([17]) A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exists (finite) at left-dense point in $\mathbb{T}$. The set of rd-continuous function $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 4. ([17]) Assume $f : \mathbb{T} \to \mathbb{R}$ is a function and let $t \in \mathbb{T}$. Then we define $f^{\Delta}(t)$ to be the number (provided it exists) with the property that given any $\epsilon > 0$, for all $s \in U$, there is a neighborhood U of t such that

$$|f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s)| \leq \epsilon|\sigma(t) - s|$$

In this case, $f^{\Delta}(t)$ is called the delta (or Hilger) derivative of f at t. Moreover, f is said to be delta or Hilger differentiable on $\mathbb{T}$ if $f^{\Delta}(t)$ exists for all $t \in \mathbb{T}$. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that are delta-differentiable and those delta-derivative are rd-continuous functions is denoted by $C^1_{rd} = C^1_{rd}(\mathbb{T}) = C^1_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 5. ([17]) A function $F : \mathbb{T} \to \mathbb{R}$ is called a antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^{\Delta}(t) = f(t)$ for all $t, s \in \mathbb{T}$. Then we write

$$\int_r^s f(t)\Delta t = F(s) - F(r).$$

Lemma 1. ([17]) *Every rd-continuous function has an antiderivative and every continuous function is rd-continuous.*

Lemma 2. ([17]) *Assume* $-a \in \mathbb{R}^+, x(t_0) > 0, t_0 \in \mathbb{T}$.

(i) *If* $x^{\Delta}(t) \leq b - ax(t)$, *when* $t \geq t_0$, *one has*

$$x(t) \leq x(t_0)e_{(-a)}(t, t_0) + \frac{b}{a}(1 - e_{(-a)}(t, t_0)),$$

particularly, when $a > 0, b > 0$, *one has*

$$\lim_{t \to +\infty} \sup x(t) \leq \frac{b}{a};$$

(ii) *If* $x^{\Delta}(t) \geq b - ax(t)$, *when* $t \geq t_0$, *one has*

$$x(t) \geq x(t_0)e_{(-a)}(t, t_0) + \frac{b}{a}(1 - e_{(-a)}(t, t_0)),$$

particularly, when $a > 0, b > 0$, *one has*

$$\lim_{t \to +\infty} \inf x(t) \geq \frac{b}{a}.$$

Let f be a continuous bounded function on $\mathbb{T}$ and set $f^u = \sup_{t\in\mathbb{T}} f(t), f^l = \inf_{t\in\mathbb{T}} f(t)$, one assumes the following hypothesis holds:

- $(H1)$ $a(t), b(t), c(t), d(t), e(t), f_i(t), p_i(t)\alpha(t), \beta(t)$ are all positive and rd-continuous$(i = 1, 2)$;
- $(H2)$ $d^u + e^l(M_1 - 1) > 0$;
- $(H3)$ $a^u - b^l > 0$.

Definition 6. For any solution $(N_1(t), u(t), N_2(t), v(t))^T$ of system (1.3), exist $m_i, l_i, M_i, L_i > 0(i = 1, 2)$, such that:

$$m_1 \leq \lim_{t\to+\infty} \inf N_1(t) \leq \lim_{t\to+\infty} \sup N_1(t) \leq M_1,$$

$$m_2 \leq \lim_{t\to+\infty} \inf N_2(t) \leq \lim_{t\to+\infty} \sup N_2(t) \leq M_2,$$

$$l_1 \leq \lim_{t\to+\infty} \inf u(t) \leq \lim_{t\to+\infty} \sup u(t) \leq L_1,$$

$$l_2 \leq \lim_{t\to+\infty} \inf v(t) \leq \lim_{t\to+\infty} \sup v(t) \leq L_2.$$

III. PERMANENCE

This section studies the permanence of system (1.3), for convenience we put:

$$M_1 = \frac{a^u - b^l}{\theta b^l}, \quad L_1 = \frac{p_1^u \exp\{M_1\}}{f_1^l},$$

$$M_2 = \frac{d^u + e^l(M_1 - 1)}{e^l}, \quad L_2 = \frac{p_2^u \exp\{M_2\}}{f_2^l}$$

$$m_1 = \frac{1}{\theta} \ln \frac{a^l - c^u \exp\{mM_2\} - \alpha^u L_1}{b^u}, \quad l_1 = \frac{p_1^l \exp\{m_1\}}{f_1^u}$$

$$m_2 = m_1 + \ln \frac{d^l - \beta^u L_2}{e^u}, \quad l_2 = \frac{p_2^l m_2}{f_2^u}.$$

Theorem 1. *Suppose the system (1.3) satisfies the conditions* $(H1)$-$(H3)$ *and the following conditions:*
$(H4)$ $a^l - c^u \exp\{mM_2\} - \alpha^u L_1 > 0$,
$(H5)$ $d^l - \beta^u L_2 > 0$,
then the system (1.3) is permanence.

Proof: (1)From the first equation of the system (1.3), it follows that

$$\begin{aligned} N_1^{\Delta}(t) &\leq a(t) - b(t)\exp\{\theta N_1(t)\} \\ &\leq a(t) - b(t)(1 + \theta N_1(t)) \\ &\leq a^u - b^l - \theta b^l N_1(t). \end{aligned} \tag{3.1}$$

Applying lemma 2.2 and condition (H3) to (3.1), we obtain

$$\lim_{t\to+\infty} \sup N_1(t) \leq \frac{a^u - b^l}{\theta b^l} = M_1. \tag{3.2}$$

From (3.2), we know that there exists a $t_1 \in \mathbb{T}$ enough large such that$N_1(t) \leq M_1, t \geq t_1 > 0$. Then, there exists $t_2 = t_1 + \tau$ such that, for any $t \geq t_2 > 0$, $N_1(t - \tau) \leq M_1$. By the second equation of the system (1.3), we get:

$$u^{\Delta}(t) \leq -f_1^l u(t) + p_1^u \exp\{M_1\}, \tag{3.3}$$

in view of Lemma 2.2, it follows from (3.3) that

$$\lim_{t\to+\infty} \sup u(t) \leq \frac{p_1^u \exp\{M_1\}}{f_1^l} = L_1. \tag{3.4}$$

Similarly, by the third equation of system (1.3), we have

$$\begin{aligned} N_2^{\Delta}(t) &\leq d(t) - e(t)(1 + N_2(t) - N_1(t)) \\ &\leq d^u - e^l - e^l N_2(t) + e^l M_1. \end{aligned} \tag{3.5}$$

It follows from Lemma 2.2 and condition (H2) that

$$\lim_{t\to+\infty} \sup N_2(t) \leq \frac{d^u + e^l(M_1 - 1)}{e^l} = M_2. \tag{3.6}$$

With (3.6) we know that there exists a $t_3 \in \mathbb{T}$ such that, for any $t \geq t_3 > 0$, $N_2(t) \leq M_2$. Therefore, there exists $t_4 = t_3 + \tau$, for any $t \geq t_4 > 0$, such that $N_2(t - \tau) \leq M_2$. Then, we can get the following results easily:

$$v^{\Delta}(t) \leq -f_2^l v(t) + p_2^u \exp\{M_2\}, \tag{3.7}$$

$$\lim_{t\to+\infty}\sup v(t) \le \frac{p_2^u \exp\{M_2\}}{f_2^l} = L_2. \tag{3.8}$$

(2) In view of (3.4), there exists a $t_5 \in \mathbb{T}$, for any $t \ge t_5 > 0$, such that $u(t) \le L_1$. Let $t_6 = \max\{t_3, t_5\}$, for $t > t_6$, we get $N_2 \le M_2, u(t) \le L_1$ and

$$\begin{aligned} & N_1^{\Delta}(t) \\ \ge\ & a(t) - b(t)\exp\{\theta N_1(t)\} - c(t)\exp\{mN_2(t)\} - \alpha(t)L_1 \\ \ge\ & a^l - b^u \exp\{\theta N_1(t)\} - c^u \exp\{mM_2\} - \alpha^u L_1. \end{aligned} \tag{3.9}$$

Hence, when $t \ge t_5 > 0$, we get

$$a^l - b^u \exp\{\theta N_1(t)\} - c^u \exp\{mM_2\} - \alpha^u L_1 \le 0 \tag{3.10}$$

Otherwise, suppose that exists $t' \ge t_5$ such that

$$a^l - b^u \exp\{\theta N_1(t)\} - c^u \exp\{mM_2\} - \alpha^u L_1 > 0,$$

and for $t \in [t_5, t')_T$, we get

$$a^l - b^u \exp\{\theta N_1(t)\} - c^u \exp\{mM_2\} - \alpha^u L_1 \le 0.$$

So

$$N_1(t') < \frac{1}{\theta}\ln\frac{a^l - c^u\exp\{mM_2\} - \alpha^u L_1}{b^u},$$

for $t \in [t_5, t')_T$, we obtain that

$$N_1(t') \ge \frac{1}{\theta}\ln\frac{a^l - c^u\exp\{mM_2\} - \alpha^u L_1}{b^u}, x^{\Delta}(t') < 0,$$

which conflicts with known conditions.

Therefore, (3.10) is establishment, hence, for $t \ge t_5$,

$$N_1(t) \ge \frac{1}{\theta}\ln\frac{a^l - c^u\exp\{mM_2\} - \alpha^u L_1}{b^u} = m_1,$$

thus

$$\lim_{t\to+\infty}\inf N_1(t) \ge m_1. \tag{3.11}$$

From the second equation of system (1.3), we obtain that: $u^{\Delta}(t) \ge -f_1^u u(t) + p_1^l \exp\{m_1\}$. By Lemma 2.2, we have:

$$\lim_{t\to+\infty}\inf u(t) \ge \frac{p_1^l \exp\{m_1\}}{f_1^u} = l_1. \tag{3.12}$$

Similarly, we can get:

$$\lim_{t\to+\infty}\inf N_2(t) \ge m_1 + \ln\frac{d^l - \beta^u L_2}{e^u} = m_2, \tag{3.13}$$

$$\lim_{t\to+\infty}\inf v(t) \ge \frac{p_2^l m_2}{f_2^u} = l_2. \tag{3.14}$$

With equation (3.2),(3.4),(3.6),(3.8) and (3.11)-(3.14), the system (1.3) is permanence.

This completes the proof. ∎

IV. Uniform Asymptotical Stability

The main result of this paper concerns the uniformly asymptotically stable of system (1.3).

Theorem 2. *Let $\tau = 1$ in system (1.3). Suppose $(H1)$-$(H5)$ and the following condition hold:*

$(H6)$ *there exists a constant μ such that*

$$\theta b^l e^{\theta m_1} - e^u e^{M_1+M_2-2m_1} - p_1^u e^{M_1} > \mu,$$

$$e^l e^{m_2-M_1} - mc^u e^{mM_2} - p_2^u e^{M_2} > \mu,$$

$$f_1^l - \alpha^u > \mu, \quad f_2^l - \beta^u > \mu,$$

where M_1, M_2, m_1 and m_2 are defined as that in Theorem 1. Then system (1.3) is uniformly asymptotically stable.

Proof: Suppose that $Z(t) = (N_1(t), N_2(t), u(t), v(t))^T$ and $Z^*(t) = (N_1^*(t), N_2^*(t), u^*(t), v^*(t))^T$ are any two solutions of system (1.3).

In view of system (1.3), we have

$$\begin{aligned} & [N_1(t) - N_1^*(t)]^{\Delta} \\ =\ & -b(t)[\exp\{\theta N_1(t)\} - \exp\{\theta N_1^*(t)\}] \\ & -c(t)[\exp\{mN_2(t)\} - \exp\{mN_2^*(t)\}] \\ & -\alpha(t)[u(t) - u^*(t)], \end{aligned}$$

which implies that

$$\begin{aligned} D^+|N_1(t) - N_1^*(t)|^{\Delta} \le\ & -\theta b^l e^{\theta m_1}|N_1(t) - N_1^*(t)| \\ & + mc^u e^{mM_2}|N_2(t) - N_2^*(t)| \\ & + \alpha^u|u(t) - u^*(t)|, \end{aligned}$$

similarly,

$$\begin{aligned} D^+|N_2(t) - N_2^*(t)|^{\Delta} \le\ & -e^l e^{m_2-M_1}|N_2(t) - N_2^*(t)| \\ & + e^u e^{M_1+M_2-2m_1}|N_1(t) - N_1^*(t)| \\ & + \beta^u|v(t) - v^*(t)|, \end{aligned}$$

$$\begin{aligned} D^+|u(t) - u^*(t)|^{\Delta} \le\ & -f_1^l|u(t) - u^*(t)| \\ & + p_1^u e^{M_1}|N_1(t) - N_1^*(t)|, \end{aligned}$$

$$\begin{aligned} D^+|v(t) - v^*(t)|^{\Delta} \le\ & -f_2^l|v(t) - v^*(t)| \\ & + p_2^u e^{M_2}|N_2(t) - N_2^*(t)|. \end{aligned}$$

Set $V(t) = |N_1(t) - N_1^*(t)| + |N_2(t) - N_2^*(t)| + |u(t) - u(t)| + |v(t) - v(t)|$. Then

$$\begin{aligned} & D^+V(t)^{\Delta} \\ \le\ & [-\theta b^l e^{\theta m_1} + e^u e^{M_1+M_2-2m_1} + p_1^u e^{M_1}]|N_1(t) - N_1^*(t)| \\ & + [-e^l e^{m_2-M_1} + mc^u e^{mM_2} + p_2^u e^{M_2}]|N_2(t) - N_2^*(t)| \\ & + [-f_1^l + \alpha^u]|u(t) - u^*(t)| + [-f_2^l + \beta^u]|v(t) - v^*(t)| \\ \le\ & -\mu V(t). \end{aligned} \tag{4.1}$$

Therefore, V is non-increasing. Integrating (4.1) from 0 to t leads to

$$V(t) + \mu\int_0^t V(s)\,\mathrm{d}s \le V(0) < +\infty, \quad \forall t \ge 0,$$

that is,

$$\int_0^{+\infty} V(s)\,\mathrm{d}s < +\infty,$$

which implies that

$$\lim_{s\to+\infty}|N_1(t) - N_1^*(t)| = \lim_{s\to+\infty}|N_2(t) - N_2^*(t)| = 0,$$

$$\lim_{s\to+\infty}|u(t) - u^*(t)| = \lim_{s\to+\infty}|v(t) - v^*(t)| = 0.$$

Thus, system (1.3) is uniformly asymptotically stable. This completes the proof. ∎

V. An example and numerical simulations

Example 1. Let $\mathbb{T} = \mathbb{R}$, considering the coefficients of system (1.2) as follows:

$$a = 3.2 + 0.4\sin 2t, b = 1.1 + 0.2\sin t, c = 0.08,$$

$$d = 1.5 + 0.2\cos 2t, e = 0.9 + 0.3\sin t, m = 0.02, \theta = 0.8,$$

$$f_1 = 1.8 + 0.6\sin 3t, f_2 = 2.5 + 0.3\cos 3t, \alpha = 0.015,$$

$$p_1 = 0.3 + 0.2\sin 2t, p_2 = 0.4 + 0.2\cos 2t, \beta = 0.013.$$

By calculating, it satisfies the conditions (H1)-(H5), in view of Theorem 3.1, we obtain that system (1.2) is permanent. This following figure establishes the dynamic behavior of system (1.2).

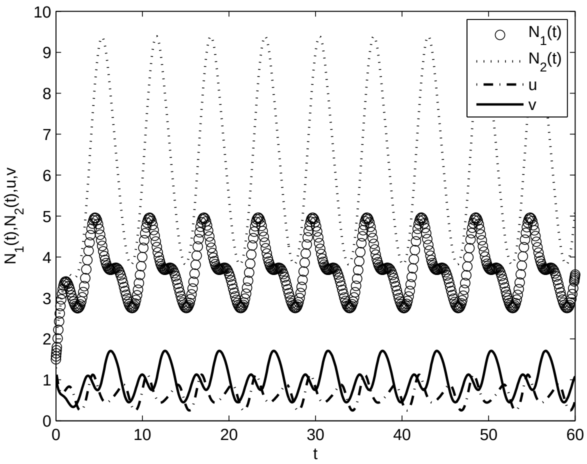

Fig. 1 Dynamic behavior of system (1.2) with the initial condition $(N_1(0), u(0), N_2(0), v(0)) = (1.5, 1.3, 2.1, 1.1)^T$, and $\tau = 0$.

Remark 1. Let $\theta = 1, m = 1, \tau = 0$ in system (1.3) and $\mathbb{T} = \mathbb{R}$, the system (1.3) can be simplified as follows:

$$\begin{cases} N_1'(t) = N_1(t)[a(t) - b(t)N_1(t) - c(t)N_2(t)] \\ \qquad\quad -\alpha(t)u(t), \\ u'(t) = -f_1(t)u(t) + p_1(t)N_1(t), \\ N_2'(t) = N_2(t)[d(t) - e(t)\frac{N_2(t)}{N_1(t)}] - \beta(t)v(t), \\ v'(t) = -f_2(t)v(t) + p_2(t)N_2(t), \end{cases} \quad (5.1)$$

we can get the system (4.1) is permanent by using theorem 3.1, and if we let the coefficients of system (4.1) and (1.2) to be the same, we can use figure 2 to establish the dynamic of system (4.1).

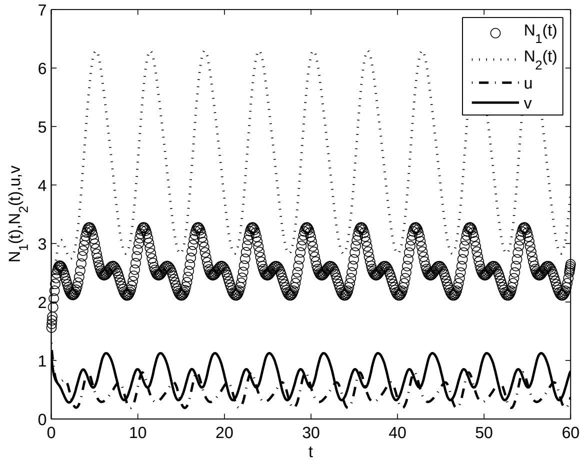

Fig. 2 Dynamic behavior of system (1.2)with the initial condition $(N_1(0), u(0), N_2(0), v(0)) = (1.5, 1.3, 2.1, 1.1)^T$, and $\tau = 0$.

Remark 2. Suppose that system (1.3) without feedback control, and let $\theta = 1, m = 1, \tau = 0$, we can get the following system:

$$\begin{cases} N_1^\Delta(t) = a(t) - b(t)\exp\{N_1(t)\} - c(t)\exp\{N_2(t)\}, \\ N_2^\Delta(t) = d(t) - e(t)\exp\{N_2(t) - N_1(t)\}. \end{cases} \quad (5.2)$$

Suppose condition (H1)-(H3) hold, then the system (3.16) is permanent. Suppose further that $\mathbb{T} = \mathbb{R}$ and all the coefficients of system (3.16) to be the same with example (3.1), we use figure 3 to establish the dynamic.

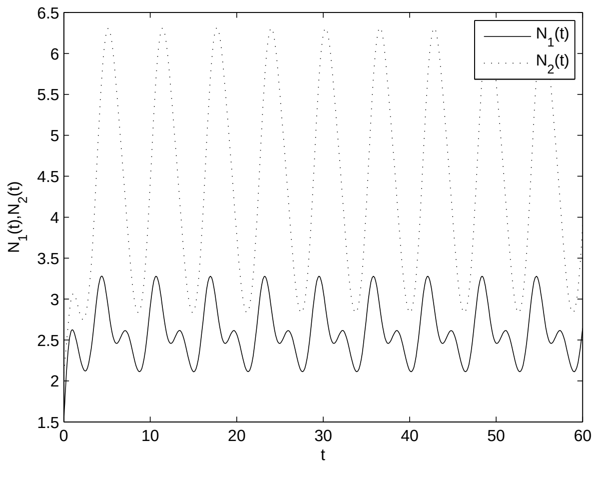

Fig. 3 Dynamic behavior of system (3.16) with the initial condition $(N_1(0), N_2(0)) = (1.5, 2.1)^T$.

By comparing figure 2 and figure 3, we can obtain that the feedback control is harmless to the permanence of system.

Remark 3. Suppose that system (1.3) without feedback control but has mutual interference just as follows:

$$\begin{cases} N_1^\Delta(t) = a(t) - b(t)\exp\{\theta N_1(t)\} - c(t)\exp\{mN_2(t)\}, \\ N_2^\Delta(t) = d(t) - e(t)\exp\{N_2(t) - N_1(t)\}, \end{cases}$$

With the condition (H1)-(H3), we get this system is permanence. Let $\mathbb{T} = \mathbb{R}$, the above system can be simplified to be as follows:

$$\begin{cases} N_1'(t) = N_1(t)[a(t) - b(t)N_1^{\theta}(t)] - c(t)N_1(t)N_2^m(t), \\ N_2'(t) = N_2(t)[d(t) - e(t)\frac{N_2(t)}{N_1(t)}]. \end{cases} \quad (5.3)$$

The coefficients $a(t), b(t), c(t), d(t), e(t)$ and the initial condition $N_1(0), N_2(0)$ are the same to example 1, let $\theta = 0.5, m = 0.6$ and $\theta = 0.3, m = 0.8$ in addition, by calculating, we know they satisfy all the condition of theorem (3.1), the figures ia as follows:

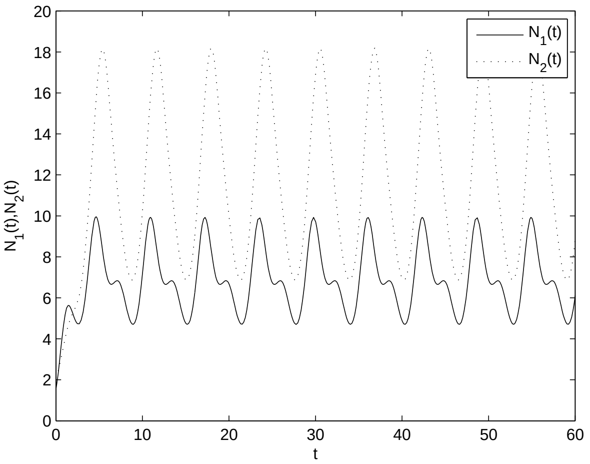

Fig. 4 The dynamic behavior of system 4.3 with $\theta = 0.5, m = 0.6$.

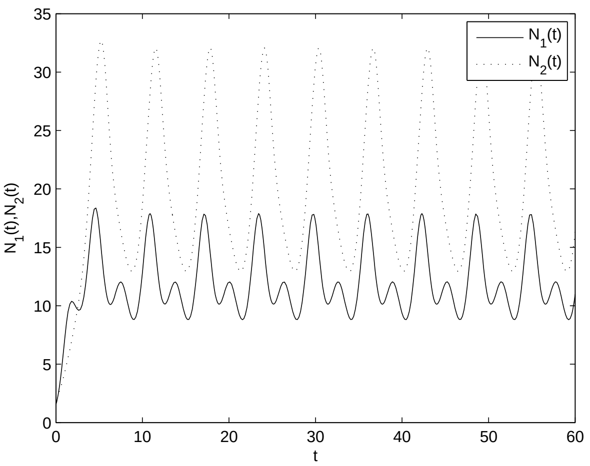

Fig. 5 The dynamic behavior of system 4.3 with $\theta = 0.3, m = 0.8$.

By comparing with the figure 4 and figure 5, we can get the mutual interference is harmless for the permanence of the system but has a great influence on the population density of the resource and consumer.

VI. Conclusion

Make a comparison with our results, from the remark 1-3, we can easy to see that the C-R system (1.3) can maintain persistence with the appropriate condition, feedback control and mutual interference have less influence on the permanence of the C-R system, while the latter has a great influence on the population density of the resource and consumer. In addition system (1.3) can be seen a generalization of paper [11].

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