Improved Delay-dependent Stability Criteria for Systems with Two Additive Time-varying Delays

Wen-bin Chen*and Fang Gao

Abstract—**This paper revisits the problem of stability for linear systems with two additive time-varying delays and improved delay-dependent stability criteria are acquired. First, taking into account the relationship between the two time-varying delays and their upper bounds, a novel augmented Lyapunov-Krasovskill functional is constructed, which includes some augmented single integral terms and single-/double-/triple-integral terms with interval additive time-delay upper bounds. Second, less conservative stability criteria are established through employing more accurate integral inequalities relatively. Third, a new quadratic inequality is applied to transform nonlinear matrix inequalities with two additive time-varying delays into linear matrix inequalities, which can be easily tested by Matlab LMI toolbox. Finally, two numerical examples are presented to verify the advantages and feasible of our results.**

Index Terms—**Linear system, Additive time-varying delays, Lyapunov-Krasovskii functional, Stability.**

I. Introduction

Time delays widely exist in many practical systems, such as power systems, neural networks systems, manufacturing systems, economic systems, and so on [1]–[9], which can degrade the control performance of systems and destabilize the stability of systems. Therefore, the research of time-varying systems has great values both on theory and practice.

During the past two decades, the stability for systems with a single time-varying delay has been concentrated by many researchers. However, few scholars pay attention on the research of multiple time-delay components. In fact, multiple time-delay components with different properties can often be found in many control systems, for instance, in the networked control systems. Hence, the issue of the stability analysis for the systems with different parts of delays is vary important. Especially, many rich achievements for systems with two additive time-varying delays have been emerged [10]–[18]. In these references, it's worthwhile to mention that bounding estimates of integral terms and eliminating time-varying delays are two essential steps during the estimation of derivative of the Lyapunov-Krasonvskii Functionals (LKFs) in order to achieve the goal of obtaining less conservative stability criterion. To this end, many researches have devoted their efforts on bounding the integral terms taking full advantage of various accurate integral inequalities, such as, Jensen inequality [19], Free-weighting matrix method [20], Free-matrix-based integral inequality [21], Reciprocally convex approach [22] Wirtinger-based inequality [23] and Auxiliary function inequality [24]. After that, in [25], single integral term instead of two-step estimation method was presented, which can acquire less conservative criterion than the ones based on the combination of Wirtinger inequality and Reciprocally convex lemma. Moreover, double integral term arising from the derivative of the triple integral term $\int_{t-d}^{t}\int_{\theta}^{t}\int_{u}^{t}\dot{x}^T(s)W\dot{x}(s)dsdud\theta, W>0$ are also usually using two-step estimation method with interval decomposition. Nevertheless, the accompanying single integral term $-(d-d(t))\int_{t-d(t)}^{t}\dot{x}^T(s)W\dot{x}(s)ds$ can probably bring about nonlinear time-varying delay term $d^2(t)$ or $\frac{d-d(t)}{d(t)}$, which makes the stability criterion infeasible in terms of Matlab LMI tool. Generally, the elimination of the nonlinear time-varying delays can be obtained by combining Auxiliary function inequality with Reciprocally convex lemma (see [24], [26]), which is still room for improvement in some way. In addition, the integral terms on the interval $[t-d_2, t-d_1]$ wherein d_1 and d_2 are two additive time-delay upper bounds respectively should be fully considered to achieve the reduction of the conservatism. However, the integral inequalities mentioned above in [19]–[24] are invalid due to the fact the sizes between the time-delay upper bounds d_1 and d_2 are unknown. To sum up, many theoretical issues need to be further studied and less conservative approaches remain open.

Motivated by the above discussion, we investigate, in this paper, the stability analysis for linear systems with additive time-varying delays. The established stability criteria may give larger stability regions in comparison with some of the latest existing criteria. This claim is due to in the following several aspects. Firstly, some new augmented integral terms with intersection elements related to the time-varying delays and their upper bounds are exploited. Secondly, single- and double-integral terms are accurately estimated based on free-matrix integral inequalities and integral intervals decomposition technology. Thirdly, a novel stability criteria with nonlinear time-varying delays are easy to deal with by utilizing a new quadratic inequalities. Finally, two numerical examples are given to demonstrate the less conservatism and the advantages of the proposed criteria.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ are respectively the $n-$dimensional Euclidean space and the set of $m\times n$ real matrix. P^T and P^{-1} mean the transpose and the inverse of the matrix P. $\mathbb{S}^n$ and $\mathbb{S}^n_+$ represent the sets of symmetric and symmetric positive definite matrices of $\mathbb{R}^{n\times n}$, respectively. $P>0$ $(P\geq 0)$ denotes that the matrix P is a real symmetric and positive definite matrix (non-negative). $I_m, 0_m, I_{m\times n}$ and $0_{m\times n}$ mean, respectively, $m\times m$ identity matrix, $m\times m$ zero matrix, $m\times n$ identity matrix, and $m\times n$ zero matrix. The notation $||\cdot||$ refers to Euclidean vector norm. $diag\{\cdots\}$ denotes a block-diagonal matrix, $sym\{P\}=P+P^T$, and $col\{p_1,p_2,...,p_m\}=[p_1^T,p_2^T,...,p_m^T]^T$.

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The symmetric term in a symmetric matrix is denoted by $\star$.

II. Problem statements

Consider the following delay systems with two additive time-varying delays:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t - d_1(t) - d_2(t)), t > 0, \\ x(t) = \phi(t), t \in [-d_3, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state. The initial conditions $\phi(t)$ is a continuous vector-valued function. A and A_d are constant matrices. The additive time-varying delays, $d_1(t), d_2(t) \in C^1(\mathbb{R}, \mathbb{R})$, satisfy:

$$0 \le d_1(t) \le d_1 < +\infty, 0 \le d_2(t) \le d_2 < +\infty, \tag{2}$$

$$\dot{d}_1(t) \le \mu_1 < +\infty, \dot{d}_2(t) \le \mu_2 < +\infty, \tag{3}$$

where d_1, d_2, μ_1 and μ_2 are constants. Let

$$d_3(t) = d_1(t) + d_2(t), d_3 = d_1 + d_2, \mu_3 = \mu_1 + \mu_2.$$

Lemma 1. *[29] For any constant matrix $W \in \mathbb{S}_+^n$, two scalars $u \ge v > 0$, such that the integrations concerned are well defined, then*

$$\begin{aligned} & -(u-v)\int_{t-u}^{t-v} x^T(s)Wx(s)ds \\ \le & -\left(\int_{t-u}^{t-v} x(s)ds\right)^T W \left(\int_{t-u}^{t-v} x(s)ds\right), \\ & -\frac{u^2 - v^2}{2}\int_{t-u}^{t-v}\int_{\theta}^{t} x^T(s)Wx(s)dsd\theta \\ \le & -\left(\int_{t-u}^{t-v}\int_{\theta}^{t} x(s)dsd\theta\right)^T W \left(\int_{t-u}^{t-v}\int_{\theta}^{t} x(s)dsd\theta\right). \end{aligned}$$

Lemma 2. *[26] Let x be a differential function $x : [v, u] \to \mathbb{R}^n$. For a matrix $W \in \mathbb{S}_+^n$, scalars $u > v$ and any matrices $M_{1i} \in \mathbb{R}^{4n \times n}$, $i = 1, 2, 3$, the following integral inequalities hold:*

$$-\int_v^u \dot{x}^T(s)W\dot{x}(s)ds \le \tilde{\xi}^T(t)\Lambda_1\tilde{\xi}(t), \tag{4}$$

$$-\int_v^u\int_{\theta}^u \dot{x}^T(s)W\dot{x}(s)dsd\theta \le \tilde{\xi}^T(t)\Lambda_2\tilde{\xi}(t), \tag{5}$$

where

$$\begin{aligned} \Lambda_1 =& (u-v)\Upsilon_1^T(M_{11}W^{-1}M_{11}^T + \frac{1}{3}M_{12}W^{-1}M_{12}^T \\ & + \frac{1}{5}M_{13}W^{-1}M_{13}^T)\Upsilon_1 + sym\{\Upsilon_1^T M_{11}\Pi_{21} \\ & + \Upsilon_1^T M_{12}\Pi_{22} + \Upsilon_1^T M_{13}\Pi_{23}\}, \\ \Lambda_2 =& -2\Pi_{11}^T W\Pi_{11} - 16\Pi_{12}^T W\Pi_{12} - 54\Pi_{13}^T W\Pi_{13}, \\ \tilde{\xi}(t) =& col\Bigg\{x(u), x(v), \frac{1}{u-v}\int_v^u x(s)ds, \\ & \frac{1}{(u-v)^2}\int_v^u\int_{\theta}^u x(s)dsd\theta, \\ & \frac{1}{(u-v)^3}\int_v^u\int_{\theta}^u\int_r^u x(s)dsdrd\theta\Bigg\}, \\ \Pi_{11} =& e_1 - e_3, \Pi_{12} = \frac{1}{2}e_1 + e_3 - 3e_4, \\ \Pi_{13} =& \frac{1}{3}e_1 - e_3 + 8e_4 - 20e_5, \Pi_{21} = e_1 - e_2, \\ \Pi_{22} =& e_1 + e_2 - 2e_3, \Pi_{23} = e_1 - e_2 + 6e_3 - 12e_4, \\ \Upsilon_1 =& col\{e_1, e_2, e_3, e_4\}, \\ e_i =& [0_{n\times(i-1)n} \quad I_n \quad 0_{n\times(5-i)n}], i = 1, 2, 3, 4, 5. \end{aligned}$$

Lemma 3. *[25] For a block symmetric matrix $W_1 = diag\{W, 3W, 5W\}$ with $W > 0$ and any matrix P_1, then the following single integral inequality holds:*

$$\begin{aligned} & -\int_{t-d}^{t}\dot{x}^T(s)W\dot{x}(s)ds \\ =& -\int_{t-d(t)}^{t}\dot{x}^T(s)W\dot{x}(s)ds - \int_{t-d}^{t-d(t)}\dot{x}^T(s)W\dot{x}(s)ds \\ \le& -\frac{1}{d}\zeta^T(t)\begin{bmatrix}\Pi_1\\ \Pi_2\end{bmatrix}^T\Bigg\{\begin{bmatrix} W_1 & P_1 \\ \star & W_1\end{bmatrix} \\ & + \begin{bmatrix}\frac{d-d(t)}{d}H_1 & 0 \\ 0 & \frac{d(t)}{d}H_2\end{bmatrix}\Bigg\}\begin{bmatrix}\Pi_1\\ \Pi_2\end{bmatrix}\zeta(t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} H_1 =& W_1 - P_1W_1^{-1}P_1^T, H_2 = W_1 - P_1^T W_1^{-1}P_1, \\ \zeta(t) =& col\Bigg\{x(t), x(t-d(t)), x(t-d), \frac{1}{d(t)}\int_{t-d(t)}^{t} x(s)ds, \\ & \frac{1}{d-d(t)}\int_{t-d}^{t-d(t)} x(s)ds, \frac{1}{d^2(t)}\int_{t-d(t)}^{t}\int_{\theta}^{t} x(s)dsd\theta, \\ & \frac{1}{(d-d(t))^2}\int_{t-d}^{t-d(t)}\int_{\theta}^{t-d(t)} x(s)dsd\theta\Bigg\}, \\ \Pi_1 =& col\{e_1 - e_2, e_1 + e_2 - 2e_4, e_1 - e_2 + 6e_4 - 12e_6\}, \\ \Pi_2 =& col\{e_2 - e_3, e_2 + e_3 - 2e_5, e_2 - e_3 + 6e_5 - 12e_7\}, \\ e_i =& [0_{n\times(i-1)n} \quad I_n \quad 0_{n\times(7-i)n}], i = 1, 2, ...7. \end{aligned}$$

Lemma 4. *Let a quadratic function $g(x_1, x_2) = b_6x_1^2 + b_5x_1 + b_4x_2^2 + b_3x_2 + b_2x_1x_2 + b_1$, $b_i \in \mathbb{R}, i = 1, ..., 6$. For $b_6 \le 0, b_4 \le 0$ and any constants d_1, d_2 satisfied $\forall x_1 \in [0, d_1], \forall x_2 \in [0, d_2]$, if*

(i) $g(d_1, d_2) < 0$,
(ii) $g(d_1, 0) - b_4d_2^2 < 0$,
(iii) $g(0, d_2) - b_6d_1^2 < 0$,
(iv) $g(0, 0) - b_6d_1^2 - b_4d_2^2 < 0$,

then $g(x_1, x_2) < 0$.

Proof : By Lemma 2 of [27] and $b_6 \le 0$, it is easy to get $g(d_1, x_2) < 0$, $-b_6d_1^2 + g(0, x_2) < 0$. Similarly, utilizing $b_4 \le 0$, the conditions $(i) - (iv)$ can easily be obtained. This completes the proof.

III. Main results

Theorem 1. *For some given scalars d_1, d_2, μ_1 and μ_2, system (1) is asymptotically stable with two additive time-varying delay $d_1(t), d_2(t)$ satisfying (2)-(3) if there exist matrices $G \in \mathbb{S}_+^{5n}$, $W_i \in \mathbb{S}_+^n, i = 1, 2, ..., 7$, $S_j \in \mathbb{S}_+^{2n}$, $F_j, G_j \in \mathbb{S}_+^n, j = 1, 2, 3, 4$, and any matrices $P_l \in \mathbb{R}^{3n\times 3n}$, $M_{kl} \in \mathbb{R}^{4n\times n}$, $k = 2, 3, 4, l = 1, 2, 3$, such that the following LMIs hold:*

$$\begin{bmatrix}\Sigma_\kappa & \Xi_\kappa \\ \star & \Phi_\kappa\end{bmatrix} < 0, \kappa = 1, 2, 3, 4, \tag{7}$$

where

$$\begin{aligned} \Sigma_1 =& \left[\Gamma_1 + \Gamma_2 + \bar{\Gamma}_3 + \bar{\Gamma}_4\right]_{(d_1(t)=d_1, d_2(t)=d_2)}, \\ \Xi_2 =& \left[\Gamma_1 + \Gamma_2 + \bar{\Gamma}_3 + \bar{\Gamma}_4\right]_{(d_1(t)=d_1, d_2(t)=0)} \\ & + \tfrac{d_2^3}{2}sym\{\gamma_3^T M_{31}I_{21} + \gamma_3^T M_{32}I_{22} + \gamma_3^T M_{33}I_{23}\} \\ & + \tfrac{d_3^2}{2}d_2 sym\{\gamma_4^T M_{41}I_{31} + \gamma_4^T M_{42}I_{32} + \gamma_4^T M_{43}I_{33}\}, \end{aligned}$$

$$\Sigma_3 = \left[\Gamma_1+\Gamma_2+\bar{\Gamma}_3+\bar{\Gamma}_4\right]_{(d_1(t)=0,d_2(t)=d_2)}$$
$$+\frac{d_1^3}{2}sym\{\gamma_2^T M_{21}I_{11}+\gamma_2^T M_{22}I_{12}+\gamma_2^T M_{23}I_{13}\}$$
$$+\frac{d_3^3}{2}d_1 sym\{\gamma_4^T M_{41}I_{31}+\gamma_4^T M_{42}I_{32}+\gamma_4^T M_{43}I_{33}\},$$
$$\Sigma_4 = \left[\Gamma_1+\Gamma_2+\bar{\Gamma}_3+\bar{\Gamma}_4\right]_{(d_1(t)=0,d_2(t)=0)}$$
$$+\frac{d_1^3}{2}sym\{\gamma_2^T M_{21}I_{11}+\gamma_2^T M_{22}I_{12}+\gamma_2^T M_{23}I_{13}\}$$
$$+\frac{d_2^3}{2}sym\{\gamma_3^T M_{31}I_{21}+\gamma_3^T M_{32}I_{22}+\gamma_3^T M_{33}I_{23}\}$$
$$+\frac{d_3^3}{2}sym\{\gamma_4^T M_{41}I_{31}+\gamma_4^T M_{42}I_{32}+\gamma_4^T M_{43}I_{33}\},$$
$$\Xi_1 = [P_1^T\Pi_{12}, P_2^T\Pi_{22}, P_3^T\Pi_{32}],$$
$$\Xi_2 = \Big[P_1^T\Pi_{12}, P_2^T\Pi_{21}, \sqrt{\tfrac{d_1}{d_3}}P_3^T\Pi_{32}, d_2^2\gamma_3^T M_{31}, d_2^2\gamma_3^T M_{32}, d_2^2\gamma_3^T M_{33}, \sqrt{d_2^2+d_1d_2d_3}\gamma_4^T M_{41}, \sqrt{d_2^2+d_1d_2d_3}\gamma_4^T M_{42}, \sqrt{d_2^2+d_1d_2d_3}\gamma_4^T M_{43}, \sqrt{\tfrac{d_2}{d_3}}P_3\Pi_{31}\Big],$$
$$\Xi_3 = \Big[P_1\Pi_{11}, P_2^T\Pi_{22}, \sqrt{\tfrac{d_2}{d_3}}P_3^T\Pi_{32}, d_1^2\gamma_2^T M_{21}, d_1^2\gamma_2^T M_{22}, d_1^2\gamma_2^T M_{23}, \sqrt{d_1^2+d_1d_2d_3}\gamma_4^T M_{41}, \sqrt{d_1^2+d_1d_2d_3}\gamma_4^T M_{42}, \sqrt{d_1^2+d_1d_2d_3}\gamma_4^T M_{43}, \sqrt{\tfrac{d_1}{d_3}}P_3\Pi_{31}\Big],$$
$$\Xi_4 = \Big[P_1\Pi_{11}, P_2\Pi_{21}, P_3\Pi_{31}, d_1^2\gamma_2^T M_{21}, d_1^2\gamma_2^T M_{22}, d_1^2\gamma_2^T M_{23}, d_2^2\gamma_3^T M_{31}, d_2^2\gamma_3^T M_{32}, d_2^2\gamma_3^T M_{33}, d_3^2(d_1^2+d_2^2)\gamma_4^T M_{41}, d_3^2(d_1^2+d_2^2)\gamma_4^T M_{42}, d_3^2(d_1^2+d_2^2)\gamma_4^T M_{43}\Big],$$
$$\Gamma_1 = sym\{\Psi_1^T G\Psi_2\},$$
$$\Gamma_2 = \sum_{i=1}^{3}\left(e_1^T W_i e_1-(1-\mu_i)e_{i+4}^T W_i e_{i+4}\right)$$
$$+e_2^T W_4 e_2-(1-\mu_2)e_8^T W_4 e_8+e_3^T W_5 e_3$$
$$-(1-\mu_1)e_9^T W_5 e_9+K_1^T S_1 K_1-K_2^T S_1 K_2+K_3^T S_2 K_3$$
$$-K_4^T S_2 K_4+e_1^T W_6 e_1-e_4^T W_6 e_4+K_1^T S_3 H_1$$
$$-(1-\mu_1)K_5^T S_3 K_5+K_3^T S_4 K_3-(1-\mu_2)K_6^T S_4 K_6$$
$$+(d_2-d_1)(e_2^T W_7 e_2-e_3^T W_7 e_3),$$
$$\bar{\Gamma}_3 = \sum_{i=1}^{3} d_i^2(Ae_1+A_d e_7)^T F_i(Ae_1+A_d e_7)$$
$$+(d_2-d_1)^2(Ae_1+A_d e_7)^T F_4(Ae_1+A_d e_7)$$
$$-\sum_{i=1}^{3}\begin{bmatrix}\Pi_{i1}\\ \Pi_{i2}\end{bmatrix}^T\begin{bmatrix}(2-\frac{d_i(t)}{d_i})\bar{F}_i & P_i\\ \star & (1+\frac{d_i(t)}{d_i})\bar{F}_i\end{bmatrix}$$
$$\times\begin{bmatrix}\Pi_{i1}\\ \Pi_{i2}\end{bmatrix}-(e_2-e_3)^T F_4(e_2-e_3),$$
$$\bar{\Gamma}_4 = \sum_{i=1}^{3}\frac{d_i^4}{4}(Ae_1+A_d e_7)^T G_i(Ae_1+A_d e_7)$$
$$+\frac{(d_2-d_1)^3}{2}(Ae_1+A_d e_7)^T G_4(Ae_1+A_d e_7)$$
$$-\sum_{i=1}^{3}\frac{d_i^2}{2}[2J_{i1}^T G_i J_{i1}+16J_{i2}^T G_i J_{i2}+54J_{i3}^T G_i J_{i3}$$
$$+2J_{i4}^T G_i J_{i4}+16J_{i5}^T G_i J_{i5}+54J_{i6}^T G_i J_{i6}]+\sum_{i=1}^{3}\frac{d_i^2}{2}\Big[(d_i-d_i(t))sym\{\gamma_{i+1}^T M_{(i+1)1}I_{i1}+\gamma_{i+1}^T M_{(i+1)2}I_{i2}+\gamma_{i+1}^T M_{(i+1)3}I_{i3}\}\Big]-(d_2-d_1)^2(e_1-e_{28})^T G_4(e_1-e_{28}),$$
$$\gamma_2 = col\{e_1,e_5,e_{10},e_{11}\},$$
$$\gamma_3 = col\{e_1,e_6,e_{16},e_{17}\}, \gamma_4 = col\{e_1,e_7,e_{22},e_{23}\},$$
$$\Phi_1 = diag\{-\bar{F}_1,-\bar{F}_2,-\bar{F}_3\},$$
$$\Phi_2 = diag\{-\bar{F}_1,-\bar{F}_2,-\bar{F}_3,-2G_2,-6G_2,-10G_2,-2G_3,-6G_3,-10G_3,-\bar{F}_3\},$$
$$\Phi_3 = diag\{-\bar{F}_1,-\bar{F}_2,-\bar{F}_3,-2G_1,-6G_1,-10G_1,-2G_3,-6G_3,-10G_3,-\bar{F}_3\},$$
$$\Phi_4 = diag\{-\bar{F}_1,-\bar{F}_2,-\bar{F}_3,-2G_1,-6G_1,-10G_1,-2G_2,-6G_2,-10G_2,-2G_3,-6G_3,-10G_3,-\bar{F}_3\},$$
$$\bar{F}_1 = diag\{F_1,3F_1,5F_1\},$$
$$\bar{F}_2 = diag\{F_2,3F_2,5F_2\}, \bar{F}_3 = diag\{F_3,3F_3,5F_3\},$$
$$\Psi_1 = col\{Ae_1+A_d e_7, e_1-e_2, e_1-e_3, e_1-e_4, e_2-e_3\},$$
$$\Psi_2 = col\{e_1, d_1(t)e_{10}+(d_1-d_1(t))e_{13}, d_2(t)e_{16}+(d_2-d_2(t))e_{19}, d_3(t)e_{22}+(d_3-d_3(t))e_{25}, (d_2-h_1)e_{28}\},$$
$$I_{11} = e_1-e_5, I_{12} = e_1+e_5-2e_{10},$$
$$I_{13} = e_1-e_5+6e_{10}-12e_{11},$$
$$I_{21} = e_1-e_6, I_{22} = e_1+e_6-2e_{16},$$
$$I_{23} = e_1-e_6+6e_{16}-12e_{17},$$
$$I_{31} = e_1-e_7, I_{32} = e_1+e_7-2e_{22},$$
$$I_{33} = e_1-e_7+6e_{22}-12e_{23},$$
$$\Pi_{11} = col\{I_{11},I_{12},I_{13}\}, \Pi_{12} = col\{e_5-e_2, e_2+e_5-2e_{13}, e_5-e_2+6e_{13}-12e_{14}\},$$
$$\Pi_{21} = col\{I_{21},I_{22},I_{23}\}, \Pi_{22} = col\{e_6-e_3, e_3+e_6-2e_{19}, e_6-e_3+6e_{19}-12e_{20}\},$$
$$\Pi_{31} = col\{I_{31},I_{32},I_{33}\}, \Pi_{32} = col\{e_7-e_4, e_7+e_4-2e_{25}, e_7-e_4+6e_{25}-12e_{26}\},$$
$$J_{11} = e_1-e_{10}, J_{12} = \tfrac{1}{2}e_1+e_{10}-3e_{11},$$
$$J_{13} = \tfrac{1}{3}e_1-e_{10}+8e_{11}-20e_{12},$$
$$J_{14} = e_5-e_{13}, J_{15} = \tfrac{1}{2}e_5+e_{13}-3e_{14},$$
$$J_{16} = \tfrac{1}{3}e_5-e_{13}+8e_{14}-20e_{15},$$
$$J_{21} = e_1-e_{16}, J_{22} = \tfrac{1}{2}e_1+e_{16}-3e_{17},$$
$$J_{23} = \tfrac{1}{3}e_1-e_{16}+8e_{17}-20e_{18},$$
$$J_{24} = e_6-e_{19}, J_{25} = \tfrac{1}{2}e_6+e_{19}-3e_{20},$$
$$J_{26} = \tfrac{1}{3}e_6-e_{19}+8e_{20}-20e_{21},$$
$$J_{31} = e_1-e_{22}, J_{32} = \tfrac{1}{2}e_1+e_{22}-3e_{23},$$
$$J_{33} = \tfrac{1}{3}e_1-e_{22}+8e_{23}-20e_{24},$$
$$J_{34} = e_7-e_{25}, J_{35} = \tfrac{1}{2}e_7+e_{25}-3e_{26},$$
$$J_{36} = \tfrac{1}{3}e_7-e_{25}+8e_{26}-20e_{27},$$
$$K_1 = col\{e_1,e_3\}, K_2 = col\{e_2,e_4\},$$
$$K_3 = col\{e_1,e_2\},$$
$$K_4 = col\{e_3,e_4\}, K_5 = col\{e_5,e_9\},$$
$$K_6 = col\{e_6,e_8\},$$
$$e_i = \begin{bmatrix}0_{n\times(i-1)n} & I_n & 0_{n\times(28-i)n}\end{bmatrix},$$
$$i = 1,2,\ldots,28.$$

Proof. *Construct the following LKFs candidate:*

$$V(t) = \sum_{i=1}^{4}V_i(t),$$

where

$$V_1(t) = \zeta^T(t)G\zeta(t),$$
$$V_2(t) = \sum_{i=1}^{3}\int_{t-d_i(t)}^{t}x^T(s)W_i x(s)ds+\int_{t-d_1-d_2(t)}^{t-d_1}x^T(s)W_4 x(s)ds$$
$$+\int_{t-d_2-d_1(t)}^{t-d_2}x^T(s)W_5 x(s)ds+\int_{t-d_3}^{t}x^T(s)W_6 x(s)ds$$
$$+\sum_{i=1}^{2}\int_{t-d_i}^{t}\zeta_i^T(s)S_i\zeta_i(s)ds+\sum_{i=1}^{2}\int_{t-d_i(t)}^{t}\zeta_i^T(s)S_{i+2}\zeta_i(s)ds$$
$$+(d_2-d_1)\int_{t-d_2}^{t-d_1}x^T(s)W_7 x(s)ds,$$

$$\begin{aligned}
&V_3(t)\\
=&\sum_{i=1}^{3} d_i\int_{t-d_i}^{t}\int_{u}^{t}\dot{x}^T(s)F_i\dot{x}(s)dsdu\\
&+(d_2-d_1)\int_{t-d_2}^{t-d_1}\int_{u}^{t}\dot{x}^T(s)F_4\dot{x}(s)dsdu,\\
&V_4(t)\\
=&\sum_{i=1}^{3}\frac{d_i^2}{2}\int_{t-d_i}^{t}\int_{\theta}^{t}\int_{u}^{t}\dot{x}^T(s)G_i\dot{x}(s)dsdud\theta\\
&+\frac{(d_2^2-d_1^2)}{2}\int_{t-d_2}^{t-d_1}\int_{\theta}^{t}\int_{u}^{t}\dot{x}^T(s)G_4\dot{x}(s)dsdud\theta
\end{aligned}$$

with

$$\begin{aligned}
\zeta(t)=&col\Big\{x(t),\int_{t-d_1}^{t}x(s)ds,\int_{t-d_2}^{t}x(s)ds,\int_{t-d_3}^{t}x(s)ds,\\
&\int_{t-d_2}^{t-d_1}x(s)ds\Big\},\\
\zeta_1(t)=&col\{x(t),x(t-d_2)\},\zeta_2(t)=col\{x(t),x(t-d_1)\}.
\end{aligned}$$

Calculating the time derivative of $V(t)$ along the solutions of systems (1)-(3) leads to

$$\begin{aligned}
\dot{V}_1(t)=&\xi^T(t)sym\{\Psi_1^TG\Psi_2\}\xi(t)\\
=&\xi^T(t)\Gamma_1\xi(t). \qquad (8)
\end{aligned}$$

Calculating the derivative of $V_2(x_t)$, we have

$$\begin{aligned}
&\dot{V}_2(t)\\
\leq&\xi^T(t)\Big\{\sum_{i=1}^{3}\left(e_1^TW_ie_1-(1-\dot{d}_i(t))e_{i+4}^TW_ie_{i+4}\right)+e_2^TW_4e_2\\
&-(1-\dot{d}_2(t))e_8^TW_4e_8+e_3^TW_5e_3-(1-\dot{d}_1(t))e_9^TW_5e_9\\
&+K_1^TS_1K_1-K_2^TS_1K_2+K_3^TS_2K_3-K_4^TS_2K_4\\
&+e_1^TW_6e_1-e_4^TW_6e_4+K_1^TS_3K_1-(1-\dot{d}_1(t))K_5^TS_3K_5\\
&+K_3^TS_4K_3-(1-\dot{d}_2(t))K_6^TS_4K_6\\
&+(d_2-d_1)(e_2^TW_7e_2-e_3^TW_7e_3)\Big\}\xi(t)\\
\leq&\xi^T(t)\Gamma_2\xi(t). \qquad (9)
\end{aligned}$$

The derivative of $V_3(t)$ and $V_4(t)$ can be obtained respectively by lemma 1- 3.

$$\begin{aligned}
&\dot{V}_3(t)\\
=&\sum_{i=1}^{3}\left(d_i^2\dot{x}^T(t)F_i\dot{x}(t)-d_i\int_{t-d_i}^{t}\dot{x}^T(s)F_i\dot{x}(s)ds\right)\\
&+(d_2-d_1)^2\dot{x}^T(s)F_4\dot{x}(s)-(d_2-d_1)\int_{t-d_2}^{t-d_1}\dot{x}^T(s)F_4\dot{x}(s)ds\\
\leq&\xi^T(t)\Big\{\sum_{i=1}^{3}d_i^2(Ae_1+A_de_7)^TF_i(Ae_1+A_de_7)\\
&+(d_2-d_1)^2(Ae_1+A_de_7)^TF_4(Ae_1+A_de_7)\\
&-\sum_{i=1}^{3}\begin{bmatrix}\Pi_{i1}\\ \Pi_{i2}\end{bmatrix}^T\begin{bmatrix}(2-\frac{d_i(t)}{d_i})\bar{F}_i & P_i\\ \star & (1+\frac{d_i(t)}{d_i})\bar{F}_i\end{bmatrix}\begin{bmatrix}\Pi_{i1}\\ \Pi_{i2}\end{bmatrix}\\
&+\sum_{i=1}^{3}\begin{bmatrix}\Pi_{i1}\\ \Pi_{i2}\end{bmatrix}^T\begin{bmatrix}\frac{d_i-d_i(t)}{d_i}P_i\bar{F}_i^{-1}P_i^T & 0\\ \star & \frac{d_i(t)}{d_i}P_i^T\bar{F}_i^{-1}P_i\end{bmatrix}\\
&\times\begin{bmatrix}\Pi_{i1}\\ \Pi_{i2}\end{bmatrix}-(e_2-e_3)^TF_4(e_2-e_3)\Big\}\xi(t)\\
=&\xi^T(t)\Gamma_3\xi(t), \qquad (10)
\end{aligned}$$

$$\begin{aligned}
&\dot{V}_4(t)\\
=&\sum_{i=1}^{3}\frac{d_i^2}{2}\left(\frac{d_i^2}{2}\dot{x}^T(t)G_i\dot{x}(t)-\int_{t-d_i}^{t}\int_{\theta}^{t}\dot{x}^T(s)G_i\dot{x}(s)dsd\theta\right)\\
&+\frac{(d_2^2-d_1^2)^2}{4}\dot{x}^T(t)G_4\dot{x}(t)\\
&-\frac{(d_2^2-d_1^2)}{2}\int_{t-d_2}^{t-d_1}\int_{\theta}^{t}\dot{x}^T(s)G_4\dot{x}(s)dsd\theta\\
=&\sum_{i=1}^{3}\frac{d_i^2}{2}\Big(\frac{d_i^2}{2}\dot{x}^T(t)G_i\dot{x}(t)-\int_{t-d_i(t)}^{t}\int_{\theta}^{t}\dot{x}^T(s)G_i\dot{x}(s)dsd\theta\\
&-\int_{t-d_i}^{t-d_i(t)}\int_{\theta}^{t-d_i(t)}\dot{x}^T(s)G_i\dot{x}(s)dsd\theta\\
&-(d_i-d_i(t))\int_{t-d_i(t)}^{t}\dot{x}^T(s)G_i\dot{x}(s)ds\Big)\\
&+\frac{(d_2^2-d_1^2)^2}{4}\dot{x}^T(t)G_4\dot{x}(t)\\
&-\frac{(d_2^2-d_1^2)}{2}\int_{t-d_2}^{t-d_1}\int_{\theta}^{t}\dot{x}^T(s)G_4\dot{x}(s)dsd\theta\\
\leq&\xi^T(t)\Big\{\sum_{i=1}^{3}\frac{d_i^4}{4}(Ae_1+A_de_7)^TG_i(Ae_1+A_de_7)\\
&+\frac{(d_2^2-d_1^2)^2}{4}(Ae_1+A_de_7)^TG_4(Ae_1+A_de_7)\\
&-\sum_{i=1}^{3}\frac{d_i^2}{2}[2J_{i1}^TG_iJ_{i1}+16J_{i2}^TG_iJ_{i2}+54J_{i3}^TG_iJ_{i3}\\
&+2J_{i4}^TG_iF_{i4}+16J_{i5}^TG_iJ_{i5}+54J_{i6}^TG_iJ_{i6}]\\
&+\sum_{i=1}^{3}\frac{d_i^2}{2}\Big[(d_i-d_i(t))d_i(t)\gamma_{i+1}^T(M_{(i+1)1}G_i^{-1}M_{(i+1)1}^T\\
&+\frac{1}{3}M_{(i+1)2}G_i^{-1}M_{(i+1)2}^T+\frac{1}{5}M_{(i+1)3}G_i^{-1}M_{(i+1)3}^T)\gamma_{i+1}\\
&+(d_i-d_i(t))sym\{\gamma_{i+1}^TM_{(i+1)1}I_{i1}\\
&+\gamma_{i+1}^TM_{(i+1)2}I_{i2}+\gamma_{i+1}^TM_{(i+1)3}I_{i3}\}\Big]\\
&-(d_2-d_1)^2(e_1-e_{28})^TG_4(e_1-e_{28})\Big\}\xi(t)\\
=&\xi^T(t)\Gamma_4\xi(t), \qquad (11)
\end{aligned}$$

where

$$\begin{aligned}
&\xi(t)=\\
&col\Big\{x(t),x(t-d_1),x(t-d_2),x(t-d_3),x(t-d_1(t)),\\
&x(t-d_2(t)),x(t-d_3(t)),x(t-d_1-d_2(t)),\\
&x(t-d_2-d_1(t)),\nu_1,\nu_2,\nu_3,\frac{1}{(d_2-d_1)}\int_{t-d_2}^{t-d_1}x(s)ds\Big\},
\end{aligned}$$

$$
\begin{aligned}
\nu_i = & \\
& \left\{\frac{1}{d_i(t)}\int_{t-d_i(t)}^{t}x(s)ds, \frac{1}{d_i^2(t)}\int_{t-d_i(t)}^{t}\int_{\theta}^{t}x(s)dsd\theta,\right. \\
& \frac{1}{d_i^3(t)}\int_{t-d_i(t)}^{t}\int_{\theta}^{t}\int_{u}^{t}x(s)dsdud\theta, \\
& \frac{1}{d_i-d_i(t)}\int_{t-d_i}^{t-d_i(t)}x(s)ds, \\
& \frac{1}{(d_i-d_i(t))^2}\int_{t-d_i}^{t-d_i(t)}\int_{\theta}^{t-d_i(t)}x(s)dsd\theta, \\
& \left.\frac{1}{(d_i-d_i(t))^3}\int_{t-d_i}^{t-d_i(t)}\int_{\theta}^{t-d_i(t)}\int_{u}^{t-d_i(t)}x(s)dsdud\theta\right\}.
\end{aligned}
$$

Combining with (8)-(11) yields

$$
\dot{V}(t) \le \xi^T(t)\sum_{k=1}^{4}\Gamma_k\xi(t).
$$

By Lemma 4, $\sum_{k=1}^{4}\Gamma_k < 0$ with time-varying delay is equivalent to (7). Therefore, if (7) hold, then $\dot{V}(t) \le -\varepsilon||x(t)||^2$ holds for a sufficiently small scalar $\varepsilon > 0$, which means system (1) is asymptotically stable. This completes the proof.

Remark 1. *In fact, an appropriate Lyapunov-Krasovskill functional plays a key role in the field of stability analysis to get the purpose of reducing conservatism. In this paper, some augmented integral terms associated with $d_1 + d_2(t), d_2 + d_1(t), d_k$ and $d_k(t), k = 1,2,3$, are fully developed. For example, augmented integral terms $\sum_{i=1}^{2}\int_{t-d_i}^{t}\zeta_i^T(s)S_i\zeta_i(s)ds$ and $\sum_{i=1}^{2}\int_{t-d_i(t)}^{t}\zeta_i^T(s)S_{i+2}\zeta_i(s)ds$ are constructed such that the stability criterion obtained has less conservativeness. To this end, the following Corollary 1 is given to illustrate the validity of the constructed augmented integral terms. On the other hand, considering the cross relationship between two additive time-varying delays, the integral terms on interval $[t-d_2, t-d_1]$ are effectively created, which are capable of reducing the conservativeness. Meanwhile, the advantages of this approach can be shown in the following Corollary 2.*

Remark 2. *By using integral interval decomposition method, integral terms including $\int_{t-d_i(t)}^{t}\int_{\theta}^{t}\dot{x}^T(s)G_i\dot{x}(s)ds\, d\theta$, $\int_{t-d_i}^{t-d_i(t)}\int_{\theta}^{t-d_i(t)}\dot{x}^T(s)G_i\dot{x}(s)dsd\theta$ and $-(d_i-d_i(t))\int_{t-d_i(t)}^{t}\dot{x}^T(s)G_i\dot{x}(s)ds$ are presented to better reflect the time-varying dependence and decrease conservatism. It is worth noting that the nonlinear time-varying delays terms related to $d_1^2(t), d_2^2(t)$ and $d_1(t)d_2(t)$ may be emerged arising from the existence of the single integral terms $-(d_i - d_i(t))\int_{t-d_i(t)}^{t}\dot{x}^T(s)G_i\dot{x}(s)ds$ in terms of some accurate integral inequalities. Nevertheless, the stability criterion with nonlinear time-delay is accompanied and infeasible by Matlab LMI tool. For this reason, taking advantage of a new quadratic inequality and combining with free-matrix integral inequality, this issue is easy to deal with in this paper.*

Assuming $S_i = 0, i = 1,2,3,4$ in the Theorem 1, the following Corollary 1 is given.

Corollary 1. *For some given scalars d_1, d_2, μ_1 and μ_2, system (1) is asymptotically stable with two additive time-varying delay $d_1(t), d_2(t)$ satisfying (2)-(3) if there exist matrices $G \in \mathbb{S}_+^{5n}$, $W_i \in \mathbb{S}_+^n, i = 1,2,...,7$, $F_j, G_j \in \mathbb{S}_+^n, j = 1,2,3,4$ and any matrices $P_l \in \mathbb{R}^{3n\times 3n}$, $M_{kl} \in \mathbb{R}^{4n\times n}$, $k = 2,3,4, l = 1,2,3$, such that the following conditions hold:*

$$
\Gamma_1 + \bar{\Gamma}_2 + \Gamma_3 + \Gamma_4 < 0, \tag{12}
$$

where

$$
\begin{aligned}
\bar{\Gamma}_2 = & \sum_{i=1}^{3}\left(e_1^TW_ie_1 - (1-\mu_i)e_{i+4}^TW_ie_{i+4}\right) + e_2^TW_4e_2 \\
& - (1-\mu_2)e_8^TW_4e_8 + e_3^TW_5e_3 - (1-\mu_1)e_9^TW_5e_9 \\
& + e_1^TW_6e_1 - e_4^TW_6e_4 + (d_2-d_1)(e_2^TW_7e_2 - e_3^TW_7e_3).
\end{aligned}
$$

Assuming $G_4 = 0, F_4 = 0, W_7 = 0$ and $\zeta(t) = col\left\{x(t), x(t-d_1), x(t-d_2), \int_{t-d_1}^{t}x(s)ds\right\}$, the following Corollary 2 is acquired.

Corollary 2. *For some given scalars d_1, d_2, μ_1 and μ_2, system (1) is asymptotically stable with two additive time-varying delay $d_1(t), d_2(t)$ satisfying (2)-(3) if there exist matrices $\tilde{G} \in \mathbb{S}_+^{4n}$, $W_i \in \mathbb{S}_+^n, i = 1,2,...,6$, $S_j \in \mathbb{S}_+^{2n}, j = 1,2,3,4$, $F_k, G_k \in \mathbb{S}_+^n, k = 1,2,3$ and any matrices $P_l \in \mathbb{R}^{3n\times 3n}$, $M_{kl} \in \mathbb{R}^{4n\times n}$, $k = 2,3,4, l = 1,2,3$, such that the following conditions hold:*

$$
\tilde{\Gamma}_1 + \tilde{\Gamma}_2 + \tilde{\Gamma}_3 + \tilde{\Gamma}_4 < 0, \tag{13}
$$

where

$$
\begin{aligned}
\tilde{\Gamma}_1 = & sym\{\tilde{\Psi}_1^T\tilde{G}\tilde{\Psi}_2\}, \\
\tilde{\Gamma}_2 = & \textstyle\sum_{i=1}^{3}\left(e_1^TW_ie_1 - (1-\mu_i)e_{i+4}^TW_ie_{i+4}\right) + \\
& e_2^TW_4e_2 - (1-\mu_2)e_8^TW_4e_8 + e_3^TW_5e_3 - (1-\mu_1)e_9^TW_5e_9 \\
& + K_1^TS_1K_1 - K_2^TS_1K_2 + K_3^TS_2K_3 - K_4^TS_2K_4 + e_1^TW_6e_1 - \\
& e_4^TW_6e_4 + K_1^TS_3K_1 - (1-\mu_1)K_5^TS_3K_5 + K_3^TS_4K_3 - (1- \\
& \mu_2)K_6^TS_4K_6, \\
\tilde{\Gamma}_3 = & \sum_{i=1}^{3}d_i^2(Ae_1+A_de_7)^TZ_i(Ae_1+A_de_7) \\
& + (d_2-d_1)^2(Ae_1+A_de_7)^TZ_4(Ae_1+A_de_7) \\
& - \sum_{i=1}^{3}\begin{bmatrix}\Pi_{i1}\\ \Pi_{i2}\end{bmatrix}^T\begin{bmatrix}(2-\frac{d_i(t)}{d_i})\bar{F}_i & P_i\\ \star & (1+\frac{d_i(t)}{d_i})\bar{F}_i\end{bmatrix} \\
& \times\begin{bmatrix}\Pi_{i1}\\ \Pi_{i2}\end{bmatrix} + \sum_{i=1}^{3}\begin{bmatrix}\Pi_{i1}\\ \Pi_{i2}\end{bmatrix}^T \\
& \times\begin{bmatrix}\frac{d_i-d_i(t)}{d_i}P_i\bar{F}_i^{-1}P_i^T & 0\\ \star & \frac{d_i(t)}{d_i}P_i^T\bar{F}_i^{-1}P_i\end{bmatrix}\begin{bmatrix}\Pi_{i1}\\ \Pi_{i2}\end{bmatrix}, \\
\tilde{\Gamma}_4 = & \left\{\sum_{i=1}^{3}\frac{d_i^4}{4}(Ae_1+A_de_7)^TG_i(Ae_1+A_de_7)\right. \\
& + \frac{(d_2-d_1)^3}{2}(Ae_1+A_de_7)^TG_4(Ae_1+A_de_7) \\
& - \sum_{i=1}^{3}\frac{d_i^2}{2}[2J_{i1}^TG_iJ_{i1} + 16J_{i2}^TG_iJ_{i2} + 54J_{i3}^TG_iJ_{i3} \\
& + 2J_{i4}^TG_iJ_{i4} + 16J_{i5}^TG_iJ_{i5} + 54J_{i6}^TG_iJ_{i6}] \\
& + \sum_{i=1}^{3}\frac{d_i^2}{2}\Big[(d_i-d_i(t))d_i(t)\gamma_{i+1}(M_{(i+1)1}G_i^{-1}M_{(i+1)1}^T \\
& + \tfrac{1}{3}M_{(i+1)2}G_i^{-1}M_{(i+1)2}^T + \tfrac{1}{5}M_{(i+1)3}G_i^{-1}M_{(i+1)3}^T) \\
& + (d_i-d_i(t))sym\{\gamma_{i+1}M_{(i+1)1}I_{i1} + \gamma_{i+1}M_{(i+1)2}I_{i2} \\
& \left. + \gamma_{i+1}M_{(i+1)3}I_{i3}\}\Big]\right\}, \\
\tilde{\Psi}_1 = & col\{Ae_1+A_de_7, e_1-e_2, e_1-e_3, e_1-e_4\}, \\
\tilde{\Psi}_2 = & col\{e_1, d_1(t)e_{10} + (d_1-d_1(t))e_{13}, d_2e_{16} \\
& + (d_2-d_2(t))e_{19}, d_3(t)e_{22} + (d_3-d_3(t))e_{25}\}.
\end{aligned}
$$

Remark 3. *Combined with Lemma 4 and as the same procedure to Theorem 1, (12) and (13) can be easily translated into LMIs conditions, which can be readily tested by using the Matlab LMI toolbox.*

TABLE I: Upper bounds of $d_2(t)$ for given d_1 and upper bounds of $d_1(t)$ for given d_2

Criteria	Delay bound d_2			Delay bound d_1		
	$d_1=1.0$	$d_1=1.2$	$d_1=1.5$	$d_2=0.3$	$d_2=0.4$	$d_2=0.5$
[11]	0.512	0.406	0.283	1.453	1.214	1.021
[12]	0.596	0.463	0.313	1.532	1.313	1.140
[13]	0.873	0.673	0.373	1.573	1.473	1.373
[14]	0.982	0.782	0.482	1.682	1.582	1.482
[15]	0.999	0.9725	0.6807	1.8804	1.7798	1.6759
Corollary 1 [16]	1.075	0.834	0.416	1.827	1.727	1.626
Theorem 1 [16]	1.163	0.965	0.669	1.875	1.773	1.671
Corollary 1	1.2545	1.1948	1.0910	1.9346	1.8433	1.7516
Corollary 2	1.1535	0.9560	0.6590	1.8525	1.7520	1.6520
Theorem 1	1.6077	1.5521	1.4779	2.0230	1.9091	1.8124

TABLE II: Upper bounds of $d_2(t)$ for given d_1 and upper bounds of $d_1(t)$ for given d_2

Criteria	Delay bound d_2			Delay bound d_1		
	$d_1=1.0$	$d_1=1.2$	$d_1=1.5$	$d_2=2.0$	$d_2=3.0$	$d_2=4.0$
[28]	4.803	4.603	4.303	3.803	2.803	1.803
[16]	5.882	5.682	5.383	4.892	3.886	2.885
Corollary 1	6.6175	6.5046	6.2101	5.5230	4.1143	3.8027
Corollary 2	4.8430	4.7895	4.6514	4.0623	3.7120	3.1052
Theorem 1	7.0319	6.9372	6.6802	6.4432	6.0093	5.4612

IV. Numerical Examples

Example 1. Consider system (1) with

$$A=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\ A_d=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},$$

$$\dot{d}_1(t)\le 0.1, \dot{d}_2(t)\le 0.8.$$

In Table I, the admissible upper bounds d_2 with various d_1, i.e., $d_1\in\{1.0,1.2,1.5\}$ and d_1 with various d_2, i.e., $h_2\in\{0.3,0.4,0.5\}$ are listed for comparison according to the stability criteria in [11]–[16]. There are three observations need to be summarized from Table I. Firstly, it's not hard to find that the stability criteria in Theorem 1 can produce the larger admissible upper bounds d_2 than those given in the above mentioned literatures, which means that our methods are more relaxed than the ones. Secondly, Corollary 1 is directly obtained when $S_i=0, i=1,2,3,4,$ in the Theorem 1 and the admissible upper bounds under d_2 with various d_1 and d_1 with various d_2 are larger than ones in above-mentioned literatures. However, numerical results in TableI show that Corollary 1 is more conservative than Theorem 1, which indicates augmented integral terms $\sum_{i=1}^{2}\int_{t-d_i}^{t}\zeta_i^T(s)S_i\zeta_i(s)ds$, $\sum_{i=1}^{2}\int_{t-d_i(t)}^{t}\zeta_i^T(s)S_{i+2}\zeta_i(s)ds$ in LKFs play a prominent role. Finally, the results of the admissible upper bounds in Corollary 2 are less than ones in Theorem 1, which illustrates single-integral, double-integral and triple-integral terms in the interval $[t-d_2, t-d_1]$ can really contribute to reduce the conservative of results. So our methods reflect its superiority.

Example 2. Consider the following closed-loop Load Frequency Control (LFC) systems:

$$\dot{\bar{x}}(t)=A\bar{x}(t)+A_d\bar{x}(t-d_1(t)-d_2(t)),$$

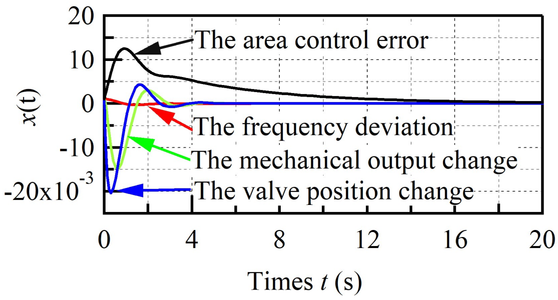

Fig. 1: State responses of the closed-loop system LFC

where

$$\bar{x}(t)=\begin{bmatrix}\Delta f\\ \Delta P_m\\ \Delta P_v\\ \int ACE\end{bmatrix},\ A=\begin{bmatrix}-\frac{D}{M} & \frac{1}{M} & 0 & 0\\ 0 & -\frac{1}{T_{ch}} & \frac{1}{T_{ch}} & 0\\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} & 0\\ \beta & 0 & 0 & 0\end{bmatrix},$$

$$A_d=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -\frac{K_p\beta}{T_g} & 0 & 0 & -\frac{K_I}{T_g}\\ 0 & 0 & 0 & 0\end{bmatrix},$$

$$\dot{d}_1(t)\le 0.1, \dot{d}_2(t)\le 0.8,$$

with $M=10, D=1, T_{ch}=0.3, T_g=0.1, R=0.05, \beta=21, K_I=0.2, K_p=0.1$. Δf, ΔP_m, ΔP_v are the frequency deviation, the mechanical output change, and the valve position change, respectively; M, D, T_g, T_{ch}, R are the moment of inertia of the generator, generator damping coefficient, time constant of governor, time constant of the turbine, and speed drop, respectively; $\int ACE$ is the integration of the area control error ACE. On the one hand, the results in Table II show Theorem 1 possesses the less conservative stability criteria than the ones in [16] and [28]. On the other hand, let $d_1(t)=\frac{1.5}{2}\sin(\frac{2}{15}t)+\frac{1.5}{2}$, $d_2(t)=\frac{6.937}{2}\sin(\frac{1}{6.937}t)+\frac{6.937}{2}$ satisfied $d_1(t)\le 1.5, d_2(t)\le 6.9372, \dot{d}_1(t)\le 0.1$ and $\dot{d}_2(t)\le 0.8$, a simulation verification for LFC systems is given. Finally, The simulation results are shown in Fig. 1, which illustrate the LFC systems are stable and confirm the advantages and the validity of the proposed method.

V. Conclusions

By employing a new Lyapunov-Krasovskill functional, in this paper, the issue of the stability analysis has been investigated. A novel stability criterion with two additive time-varying delays is acquired through utilizing some accurate integral inequalities and integral intervals decomposition method. In addition, the nonlinear matrix inequalities are prone to be processed by a quadratic inequality. Numerical examples show the remarkable accuracy and efficiency of our method compared with other schemes.

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